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1 Insertion sort algorithm and proof

1.1 Algorithm

Algorithm 1 Insertion Sort

Require: array of comparable elements

```
1: function INSERTION_SORT(array) ▷ 0 indexed array assumed
2:   for  $j = 1$  to  $array.length - 1$  do
3:      $key = array[j]$ ;
4:      $i = j - 1$ ;
5:     while  $i \geq 0$  and  $array[i] > key$  do
6:        $array[i + 1] = array[i]$ ;
7:        $i = i - 1$ ;
8:     end while
9:      $array[i + 1] = key$ ;
10:  end for
11: end function
```

1.2 Proof

Invariant: The subarray $array[0..j-1]$, is always in sorted order.

Initialization: Initially we have $j = 1$, so the $array[0]$ (a single element) is always sorted.

Maintenance: For loop works by moving forward from $j = 1$, in each iteration, it searches for the position of current key then places it there while shifting every value down the line by one.

Termination: when the loop terminates, $j = array.length = n$, so $array[0..n-1]$ is in sorted order, which is the entire space value of array. So we conclude that entire array is sorted.

Time	Best Case	Worst Case
Complexity	$\Theta(n)$	$\Theta(n^2)$

2 Linear search algorithm and proof

2.1 Algorithm

Algorithm 2 Linear Search Algorithm

Require: Array to search given element in and the value to search for

```
1: function LINEAR_SEARCH(array, value)
2:   for index = 0 to array.length - 1 do
3:     if array[index] == value then
4:       return index;
5:     end if
6:   end for
7:   return index;
8: end function
```

2.2 Proof

Invariant: The value is not within the range $\text{array}[0..\text{index}-1]$

Initialization: Initially we have $\text{index} = 0$, so the value isn't within the array range $\text{array}[0..-1]$ which is empty array.

Maintenance: Within each for loop if the value pointed by index isn't equal to the value, we increment otherwise we return the index and terminate the function. In either case, the value will not be contained within the range $\text{array}[0..\text{index}-1]$.

Termination: The return value of index could either point to the value itself, or point to the value one after the size limit of array. In former case, since the value was first found at that index, there is no possibility for the same value to exist within the sub-array, $\text{array}[0..\text{index}-1]$. In later case, the value of index is one plus the total size of array. This implies, that the value isn't contained within the array at all. So, we conclude that linear search will always aware us about the presence or absence of the value in the given array.

Time	Best Case	Worst Case
Complexity	$\Theta(1)$	$\Theta(n)$

3 Selection sort

3.1 Algorithm

Algorithm 3 Selection sort algorithm

```
1: function SELECTION_SORT(array)
2:   for  $i = 0$  to  $array.length - 2$  do
3:     index =  $i$ ;
4:     for  $j = i + 1$  to  $array.length - 1$  do
5:       if  $array[index] > array[j]$  then
6:         index =  $j$ ;
7:       end if
8:     end for
9:     SWAP( $array[index]$ ,  $array[i]$ );
10:  end for
11: end function
```

3.2 Proof

Invariant: The sub array $array[0..i-1]$ is always in sorted order.

Initialization: Initially we have $i = 1$, so the $array[0]$ (a single element) is always sorted.

Maintenance: In each iteration, we move in the array by one position. At each position we find the smallest value since that position then swap the value at given position with found smallest value. This ensures that the $array[0..i-1]$ is always sorted in ascending order. We only need to move by $n-1$ position, as the final will be sorted implicitly.

Termination: When the loop ends, we have $i = array.length$, so $array[0..i-1]$ which comprises of the entire array is sorted. Hence we can conclude that the entire array is sorted.

Time	Best Case	Worst Case
Complexity	$\Theta(n^2)$	$\Theta(n^2)$