

Nhóm 05:

Họ tên các thành viên trong nhóm

1. Lê Thị Thu An, MSV: 28001975, K63 TN Toán học
2. Thiên Đình Minh Hùng, MSV: 21000006, K66 TN Toán học

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Bài tập 1:

- Bổ sung thêm biến bù ta có bài toán tương đương:

$$\begin{aligned} \max \quad & 10x_1 - 5x_2 - 9x_3 - 29x_4 \\ \text{s.t.} \quad & \frac{1}{2}x_1 - \frac{11}{2}x_2 - \frac{5}{2}x_3 + 9x_4 + x_5 = 0 \\ & \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 + x_4 + x_6 = 0 \\ & x_1 + x_7 = 1 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0.$$

Bảng đơn vị:

$$\begin{array}{ccccccc|c} \frac{1}{2} & -\frac{1}{2} & -\frac{5}{2} & 9 & 1 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline 10 & -52 & -9 & -29 & 0 & 0 & 0 & 0 \end{array}$$

- Áp dụng quy tắc Bland ~~đối~~ với trục liên dưới

$$\begin{array}{cccccccc|c} \text{Hàng xỏ} \leftarrow & \frac{1}{2} & -\frac{11}{2} & -\frac{5}{2} & 9 & 1 & 0 & 0 & 0 \\ & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 1 & 0 & 0 \\ & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline & 20 & -57 & -9 & -29 & 0 & 0 & 0 & 0 \\ & \downarrow & & & & & & & \\ & \text{cột xỏ} & & & & & & & \end{array}$$

$$\Rightarrow \begin{array}{ccccccc|c} \frac{1}{2} & -\frac{11}{2} & -\frac{5}{2} & 9 & 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & -8 & -1 & 1 & 0 & 0 \\ 0 & 12 & 5 & -18 & -2 & 0 & 1 & 1 \end{array}$$

$$\begin{array}{ccccccc|c} 0 & 53 & 41 & -204 & -20 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & -11 & -5 & 18 & 2 \\ 0 & 12 & 5 & 18 & 0 \end{array}$$

Handwritten notes: $H_1 \times 2$, $H_2 \times 2$, $H_3 \times 2$, $H_4 \times 2$

$$\Rightarrow \begin{array}{ccccccc|c} 1 & -11 & -5 & 18 & 2 & 0 & 0 & 0 \\ 0 & 4 & 2 & -8 & -2 & 1 & 0 & 0 \\ 0 & 12 & 5 & -18 & -2 & 0 & 1 & 1 \end{array}$$

$$\begin{array}{ccccccc|c} 0 & 53 & 41 & -204 & -20 & 0 & 0 & 0 \end{array}$$

Handwritten notes: $H_1 \times 2$, $H_2 \times 2$, $H_3 \times 2$, $H_4 \times 2$, $Ty' le?$

$$\Rightarrow \begin{array}{ccccccc|c} 1 & 0 & \frac{1}{2} & -4 & -\frac{3}{4} & \frac{11}{4} & 0 & 0 \\ 0 & 4 & 2 & -8 & -1 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 4 & \frac{3}{4} & -\frac{11}{4} & 1 & 1 \end{array}$$

$$\begin{array}{ccccccc|c} 0 & 0 & \frac{29}{2} & -98 & -\frac{27}{4} & -\frac{53}{4} & 0 & 0 \end{array}$$

Handwritten notes: $H_1 \times 2$, $H_2 \times 2$, $H_3 \times 2$, $H_4 \times 2$, $Cot' xoay$

$$\Rightarrow \begin{array}{ccccccc|c} 1 & 0 & \frac{1}{2} & -4 & -\frac{3}{4} & \frac{11}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & -2 & -\frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 4 & \frac{3}{4} & -\frac{11}{4} & 1 & 1 \end{array}$$

$$\begin{array}{ccccccc|c} 0 & 0 & \frac{29}{2} & -98 & -\frac{27}{4} & -\frac{53}{4} & 0 & 0 \end{array}$$

Handwritten notes: $H_2/4$, $H_3/4$, $H_4/4$, $Cot' xoay$

\Rightarrow

$$\begin{array}{ccccccc|c} 1 & 0 & \frac{1}{2} & -4 & -\frac{3}{4} & \frac{11}{4} & 0 & 0 \\ \cancel{1} & 1 & 0 & 2 & \frac{1}{2} & -\frac{5}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline -2y & 0 & 0 & 18 & 15 & -93 & 0 & 0 \end{array}$$

$H_2 \times 2$

\Rightarrow Hàng Xảy

$$\begin{array}{ccccccc|c} 2 & 0 & 1 & -8 & -\frac{3}{2} & \frac{11}{2} & 0 & 0 \\ -1 & 1 & 0 & 2 & \frac{1}{2} & -\frac{5}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline -2y & 0 & 0 & 18 & 15 & -93 & 0 & 0 \end{array}$$

\downarrow
Cột Xảy

\Rightarrow

$$\begin{array}{cccc|ccc|c} -2 & 4 & 1 & 0 & \frac{1}{2} & -\frac{9}{2} & 0 & 0 \\ -1 & 1 & 0 & 2 & \frac{1}{2} & -\frac{5}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline \end{array}$$

$H_2/2$ Hàng Xảy

$$\begin{array}{cccc|ccc|c} -20 & -9 & 0 & 0 & \frac{21}{2} & -\frac{141}{2} & 0 & 0 \\ -2 & 4 & 1 & 0 & \frac{1}{2} & -\frac{9}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} & -\frac{5}{4} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline -20 & -9 & 0 & 0 & \frac{21}{2} & -\frac{141}{2} & 0 & 0 \end{array}$$

\downarrow
Cột Xảy

\Rightarrow

$$\begin{array}{ccccccc|c} -2 & 4 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline 22 & -93 & -22 & 0 & 0 & 24 & 0 & 0 \end{array}$$

$H_1 \times 2$

\Rightarrow Hàng
Xoay

$$\begin{array}{ccccccc|c} -4 & 8 & 2 & 0 & 1 & -9 & 0 & 0 & \text{Ty lệ?} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 22 & -93 & -22 & 0 & 0 & 24 & 0 & 0 & 0 \end{array}$$

\Rightarrow

$$\begin{array}{ccccccc|c} 0 & -4 & -2 & 8 & 1 & -9 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & -2 & 0 & -2 & 1 & 1 \\ \hline 0 & -27 & 1 & -22 & 0 & -20 & 0 & 0 \end{array}$$

$H_2 \times 2$

\Rightarrow

Hàng
Xoay

$$\begin{array}{ccccccc|c} 0 & -4 & -2 & 8 & 1 & -9 & 0 & 0 & \text{Ty lệ?} \\ 1 & -3 & -1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & -2 & 0 & -2 & 1 & 1 & 1 \\ \hline 0 & -27 & 1 & -22 & 0 & -20 & 0 & 0 & 0 \end{array}$$

\downarrow

Cột xoay

\Rightarrow

$$\begin{array}{ccccccc|c} 0 & 2 & \approx 0 & 4 & 1 & -13 & 2 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & -2 & 0 & -2 & 1 & 1 \\ \hline 0 & -30 & 0 & -20 & 0 & -18 & -1 & -1 \end{array}$$

Như vậy, giá trị nghiệm cơ sở tối ưu là (x_1, x_3, x_5) và ta có nghiệm tối ưu:

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (1, 0, 1, 0, 2, 0, 0).$$

Giá trị tối ưu là 1.

BT2. $\max C^T x$
s.t. $Ax \leq b$
 $x \geq 0$

thay $C = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 3 & -4 \\ -4 & 3 \end{bmatrix}$

$b = \begin{bmatrix} 9 \\ -1 \\ -1 \end{bmatrix}$

Được b1:

$$(P) \quad \begin{array}{lll} \max & 2x_1 - x_2 & \\ \text{s.t.} & x_1 + x_2 & \leq 9 \\ & 3x_1 - 4x_2 & \leq -1 \\ & -4x_1 + 3x_2 & \leq -1 \\ & x_1, x_2 & \geq 0 \end{array}$$

Ta thấy b và c đều có cả phần > 0 và < 0
Nên ta sử dụng thuật toán đơn hình 2 pha.

Pha 1: Giải bt phụ:

$(Q) \quad \begin{array}{lll} \max & -x_0 & \\ \text{s.t.} & -x_0 + x_1 + x_2 & \leq 9 \\ & -x_0 + 3x_1 - 4x_2 & \leq -1 \\ & -x_0 - 4x_1 + 3x_2 & \leq -1 \\ & x_1, x_2 & \geq 0 \end{array}$

Bảng đơn hình. (có thêm biến bù x_3, x_4, x_5).

	x_0	x_1	x_2	x_3	x_4	x_5	b
x_3	-1	1	1	1	0	0	9
x_6	$\ominus 1$	3	-4	0	1	0	-1
x_5	-1	-4	3	0	0	1	-1
Z	0	2	-1	0	0	0	0
W	-1	0	0	0	0	0	0

$$\begin{aligned} \frac{-1}{-1} &= 1 \\ \frac{-1}{-1} &= 1 \end{aligned} \left. \vphantom{\begin{aligned} \frac{-1}{-1} &= 1 \\ \frac{-1}{-1} &= 1 \end{aligned}} \right\} \max = 1.$$

Cột xoay: x_0

Hàng xoay: x_4

	x_0	x_1	x_2	x_3	x_4	x_5	
x_3	0	-2	5	1	-1	0	10
x_0	1	-3	4	0	-1	0	1
x_5	0	-7	$\oplus 7$	0	-1	1	0
Z	0	2	-1	0	0	0	0
W	0	-3	$\oplus 4$	0	-1	0	1

$$\begin{aligned} \frac{10}{5} &= 2 \\ \frac{1}{4} \\ 0 &= 0 \end{aligned} \left. \vphantom{\begin{aligned} \frac{10}{5} &= 2 \\ \frac{1}{4} \\ 0 &= 0 \end{aligned}} \right\} \min = \frac{1}{4}$$

Hàng xoay: x_5

	x_0	x_1	x_2	x_3	x_4	x_5	
x_3	0	3	0	1	$\frac{2}{7}$	$\frac{-5}{7}$	10
x_0	1	$\ominus 1$	0	0	$\frac{-3}{7}$	$\frac{4}{7}$	1
x_2	0	-1	1	0	$\frac{-1}{7}$	$\frac{1}{7}$	0
Z	0	1	0	0	$\frac{-1}{7}$	$\frac{1}{7}$	0
W	0	1	0	0	$\frac{-3}{7}$	$\frac{-4}{7}$	1

Cột xoay: x_1

$$\begin{aligned} \frac{10}{3} \\ 1 \end{aligned} \left. \vphantom{\begin{aligned} \frac{10}{3} \\ 1 \end{aligned}} \right\} \min = 1$$

Hàng xoay: x_0

	x_0	x_1	x_2	x_3	x_4	x_5	
x_3	-3	0	0	1	1	1	7
x_1	1	1	0	0	$-\frac{3}{7}$	$-\frac{4}{7}$	1
x_2	1	0	1	0	$-\frac{4}{7}$	$-\frac{3}{7}$	1
Z	-1	0	0	0	$\frac{2}{7}$	$\frac{5}{7}$	-1
W	-1	0	0	0	0	0	0

Vậy bt (P) có nghiệm ^{coi} CNĐ là: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Pha 2:

	x_1	x_2	x_3	x_4	x_5	
x_3	0	0	1	①	1	7
x_1	1	0	0	$-\frac{3}{7}$	$-\frac{4}{7}$	1
x_2	1	1	0	$-\frac{6}{7}$	$-\frac{3}{7}$	1
Z	0	0	0	$\frac{2}{7}$	$\frac{5}{7}$	-1

Cột xoay: x_4
 $\frac{7}{1}$

Hàng xoay: x_3

	x_1	x_2	x_3	x_4	x_5	
x_4	0	0	② 1	1	① $\frac{1}{7}$	7
x_1	1	0	$\frac{3}{7}$	0	$-\frac{1}{7}$	4
x_2	0	1	$\frac{4}{7}$	0	$\frac{1}{7}$	5
Z	0	0	$-\frac{2}{7}$	0	$\frac{3}{7}$	-3

Cột xoay x_5

$\frac{7}{1}$ } min = 7

$\frac{5}{\frac{1}{7}} = 35$

Hàng xoay x_4

	x_1	x_2	x_3	x_4	x_5	
x_5	0	0	1	1	1	7
x_1	1	0	10 $\frac{4}{7}$	$\frac{1}{7}$	0	5
x_2	0	1	$\frac{3}{7}$	$-\frac{1}{7}$	0	4
Z	0	0	$-\frac{5}{7}$	$-\frac{3}{7}$	0	$-\frac{24}{7}$ -6

Vậy nghiệm tối ưu của btoán (P) là

$$\begin{pmatrix} x_1 = 5 \\ x_2 = 4 \end{pmatrix}$$

Khi đó $z = \cancel{24} 6$