

312-3302
*Artificial
Intelligence*

Lecture 1
Introduction
to Artificial
Intelligence



Course Outline

Class Time and Location:

Mon 09:00 – 12:00 Room 15309

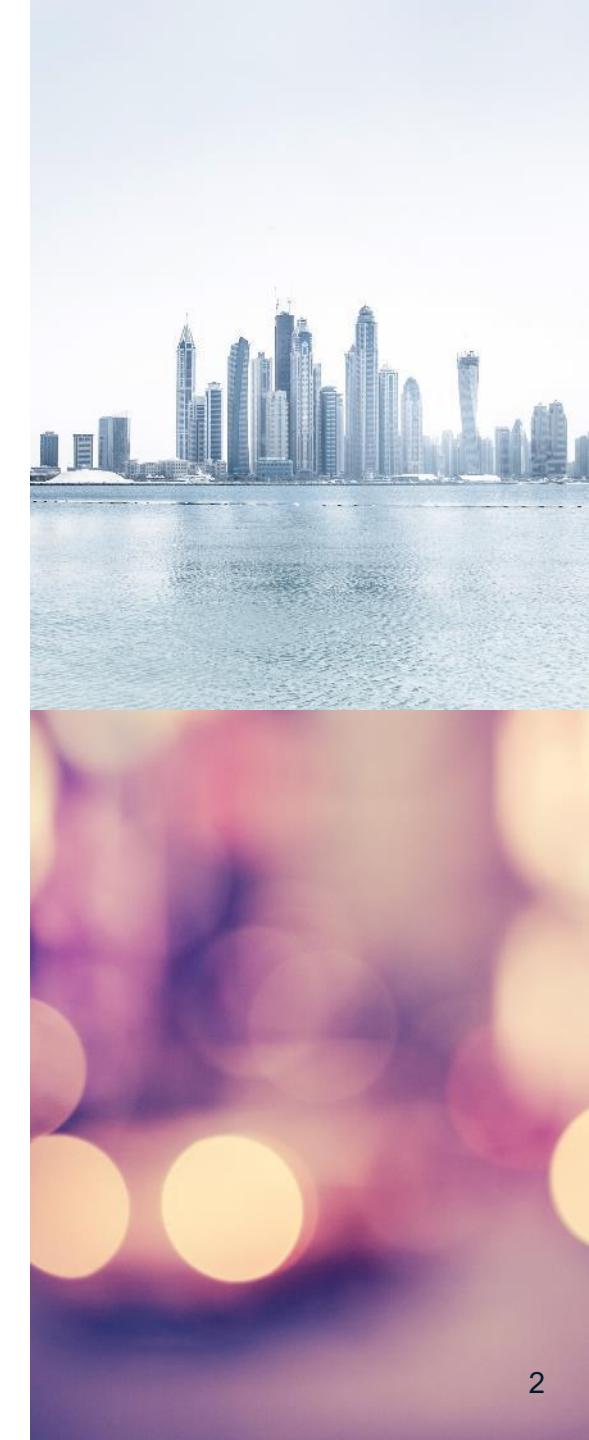
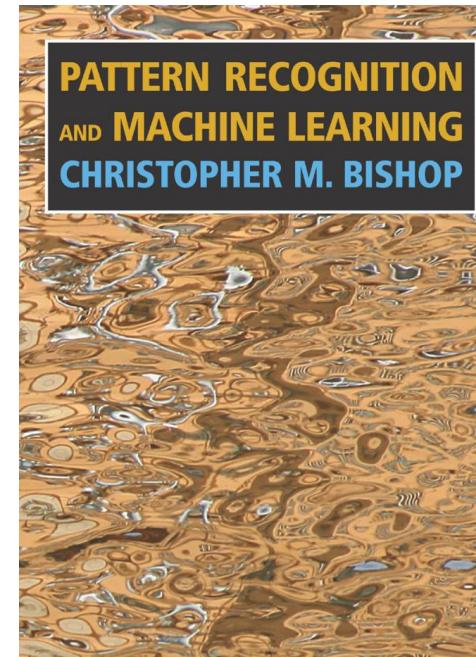
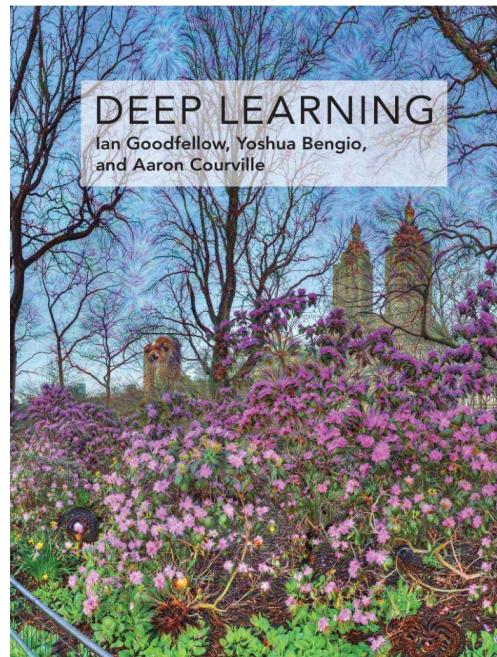
Grading:

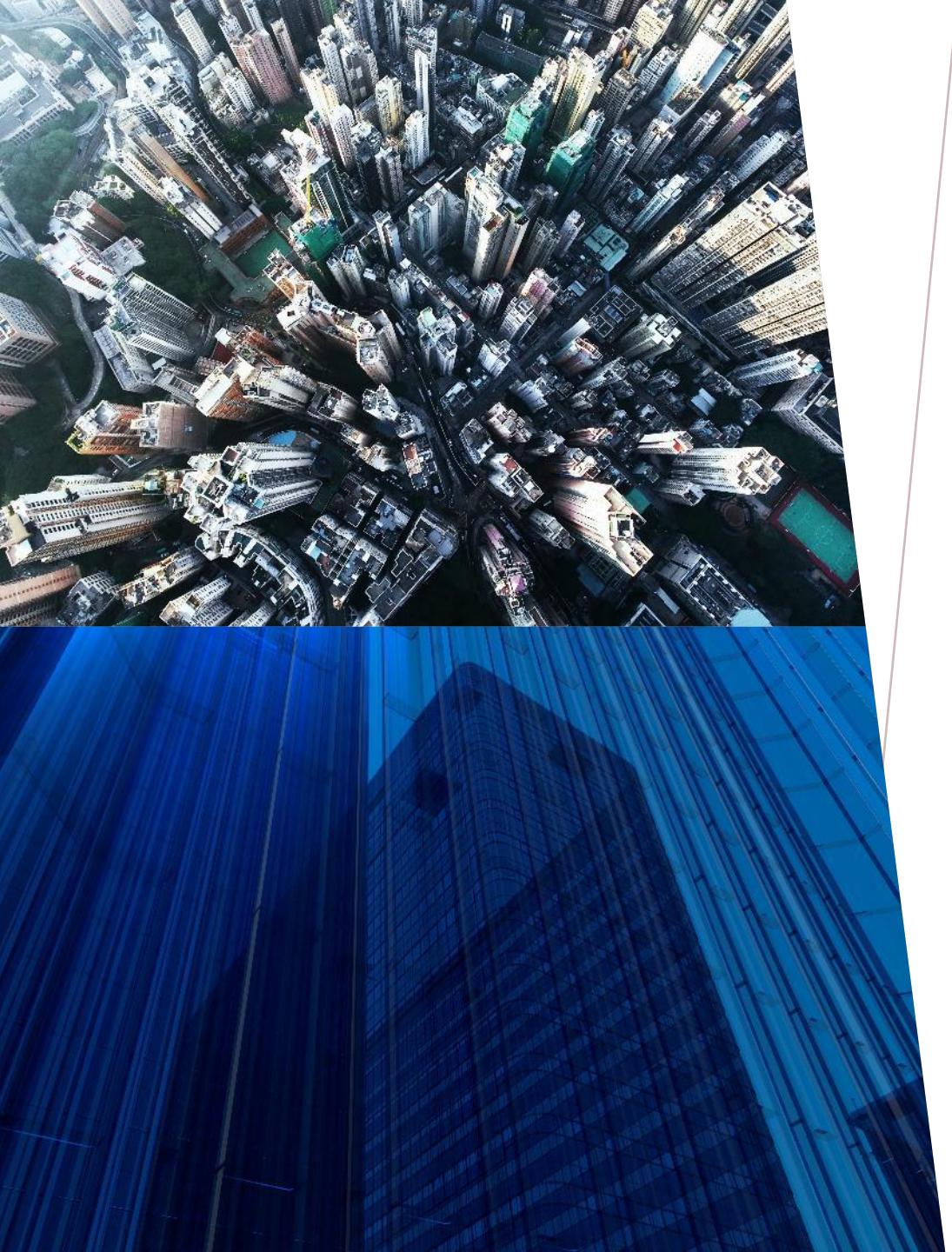
Classwork	20 %
Midterm	30 %
Final	30 %
Project	20 %

FDT Classroom:

<https://persevere.cdti.ac.th>

Textbook:



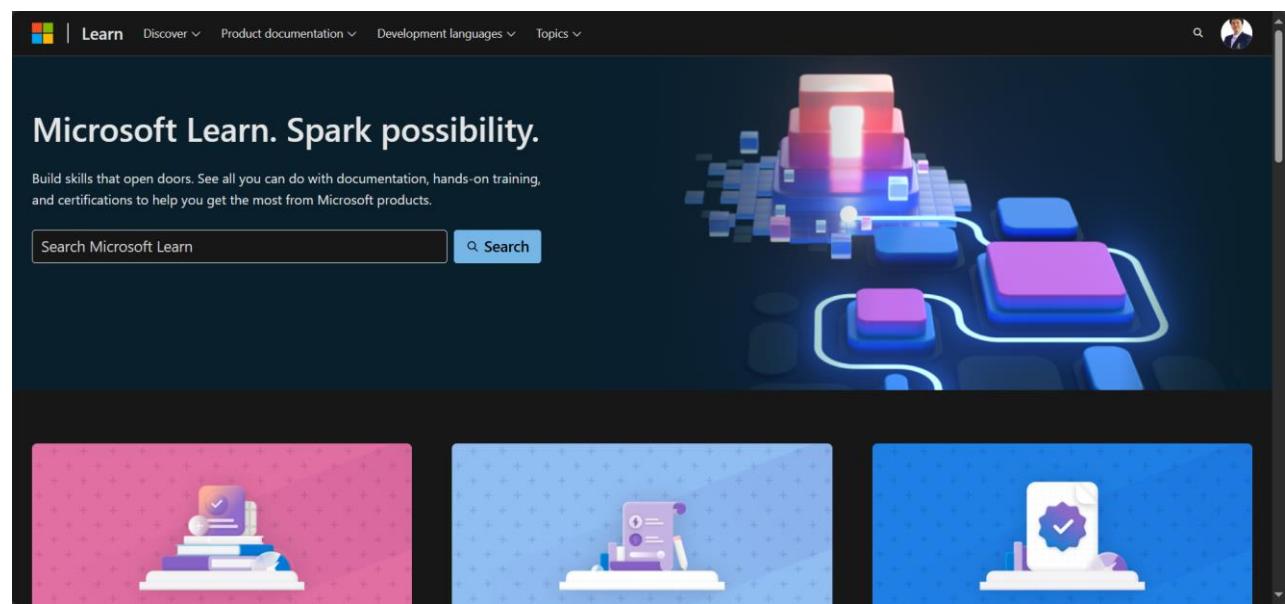


How we learn in this course

1.) การเรียนรู้ด้วยตนเองผ่านระบบ Microsoft Learn

<https://learn.microsoft.com>

เข้าใช้งานด้วย Microsoft account



เข้าไปดูหัวข้อการเรียนรู้ที่ training หรือเข้า URL ต่อไปนี้
<https://learn.microsoft.com/en-us/training/browse>

เลือก Subject เป็น Artificial intelligence และเลือก Levels เป็น Beginner

Levels

Beginner

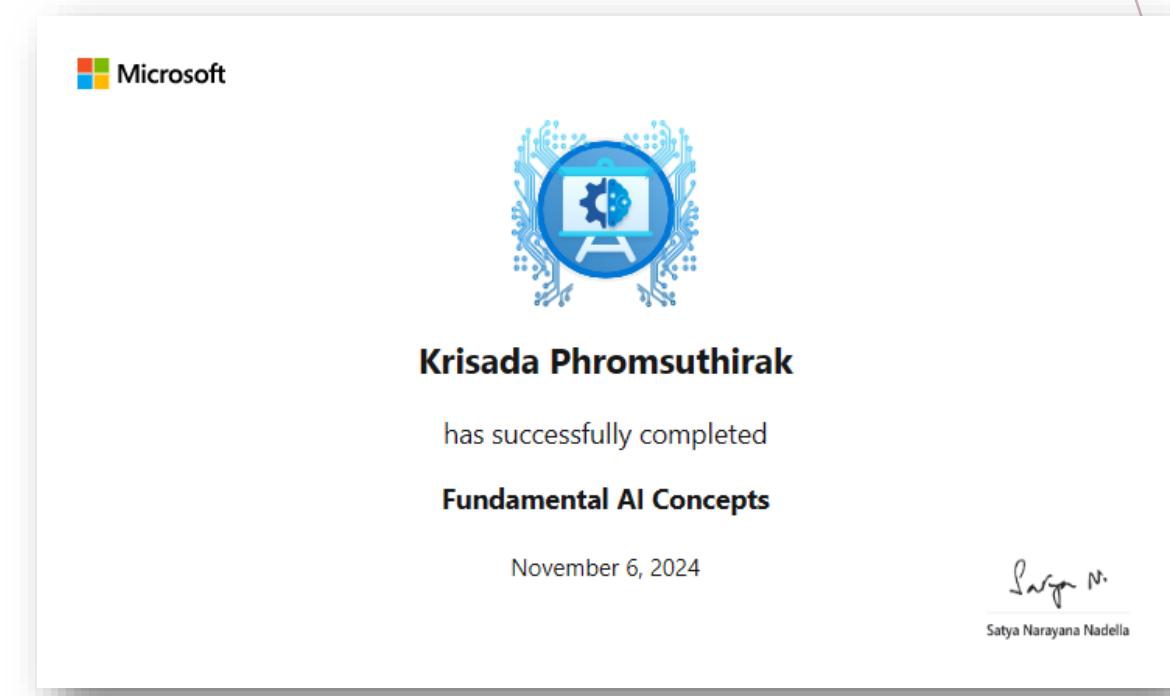
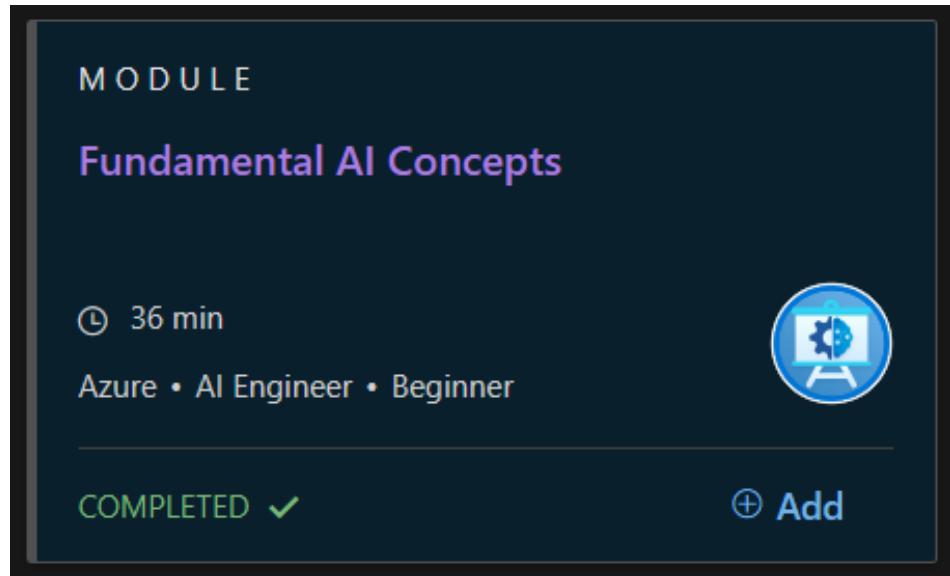
Subjects

Find a subject

- Application development
- Artificial intelligence
- Business applications
- Data management
- Security
- Technical infrastructure

<p>MODULE Fundamental AI Concepts</p> <p>⌚ 36 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>	<p>MODULE Fundamentals of Facial Recognition</p> <p>⌚ 27 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>	<p>MODULE Fundamentals of Azure AI Document Intelligence</p> <p>⌚ 26 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>
<p>MODULE Explore generative AI with Copilot in Bing</p> <p>⌚ 40 min Bing • AI Engineer • Beginner</p> <p><input type="button" value="Add"/></p>	<p>LEARNING PATH Microsoft Azure AI Fundamentals: AI Overview</p> <p>⌚ 3 hr 7 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>	<p>MODULE Fundamentals of optical character recognition</p> <p>⌚ 27 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>
<p>MODULE Fundamentals of question answering with the Language Service</p> <p>⌚ 29 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>	<p>MODULE Fundamentals of Knowledge Mining and Azure AI Search</p> <p>⌚ 53 min Azure • Developer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>	<p>MODULE Fundamentals of Azure AI Speech</p> <p>⌚ 31 min Azure • AI Engineer • Beginner</p> <p>COMPLETED ✓ <input type="button" value="Add"/></p>

แนะนำให้นักศึกษาเรียนตาม Module ที่ครูแนะนำก่อน
(แต่ถ้ามีความสนใจในเรื่องใดเป็นพิเศษจะเรียนเพิ่มก็ได้)



นี่คือ Module แรกที่พากเราจะเรียนกัน

เรียนสำเร็จ 1 Module จะได้รับ Certificate



Fundamental AI Concepts

36 min • Module • 10 Units

1100 XP

Feedback



ใน 1 Module อาจจะมีหลาย Units

Beginner

AI Engineer

Data Scientist

Student

Azure AI Bot Service

Azure Machine Learning

With AI, we can build solutions that seemed like science fiction a short time ago; enabling incredible advances in health care, financial management, environmental protection, and other areas to make a better world for everyone.

Learning objectives

In this module, you'll learn about the kinds of solutions AI can make possible and considerations for responsible AI practices.

⊕ Add

Prerequisites

None

This module is part of these learning paths

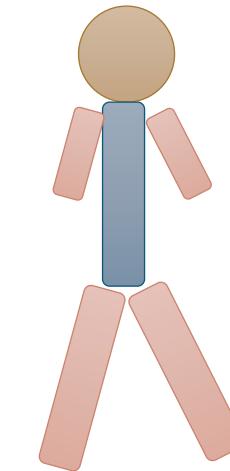
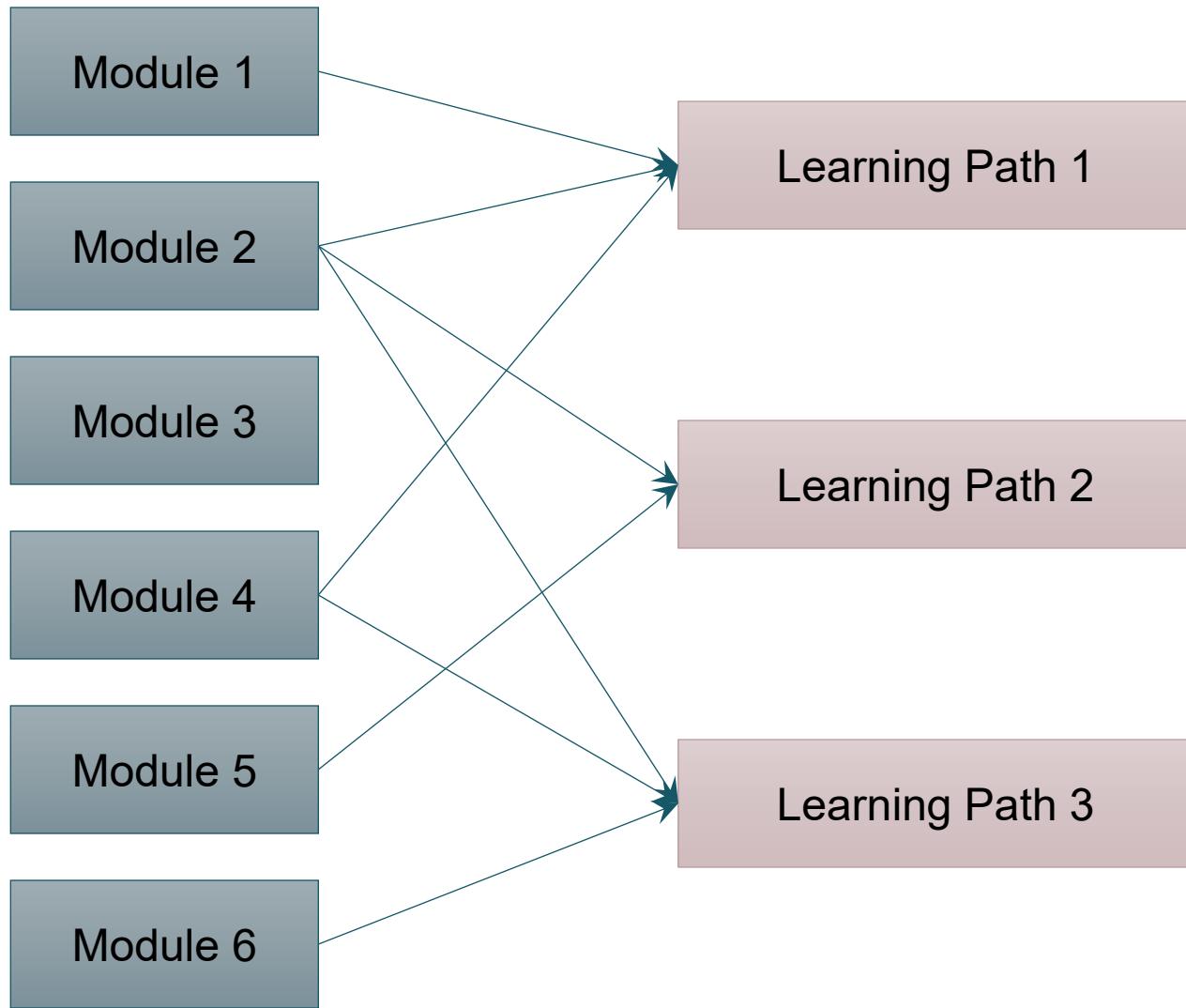
Accessibility fundamentals

Ethical AI

Microsoft Azure AI Fundamentals: AI Overview

หลาย Module รวมกันจะเรียกว่า
Learning paths





Personal Credentials & Skills

สรุปแล้ว
มันดียังไง

???

Krisada Phromsuthirak

Username: krisada
Contact email: Krisada.Phr@ms.cdti.ac.th

Modules completed	Training hours completed
26	20 hr 34 min

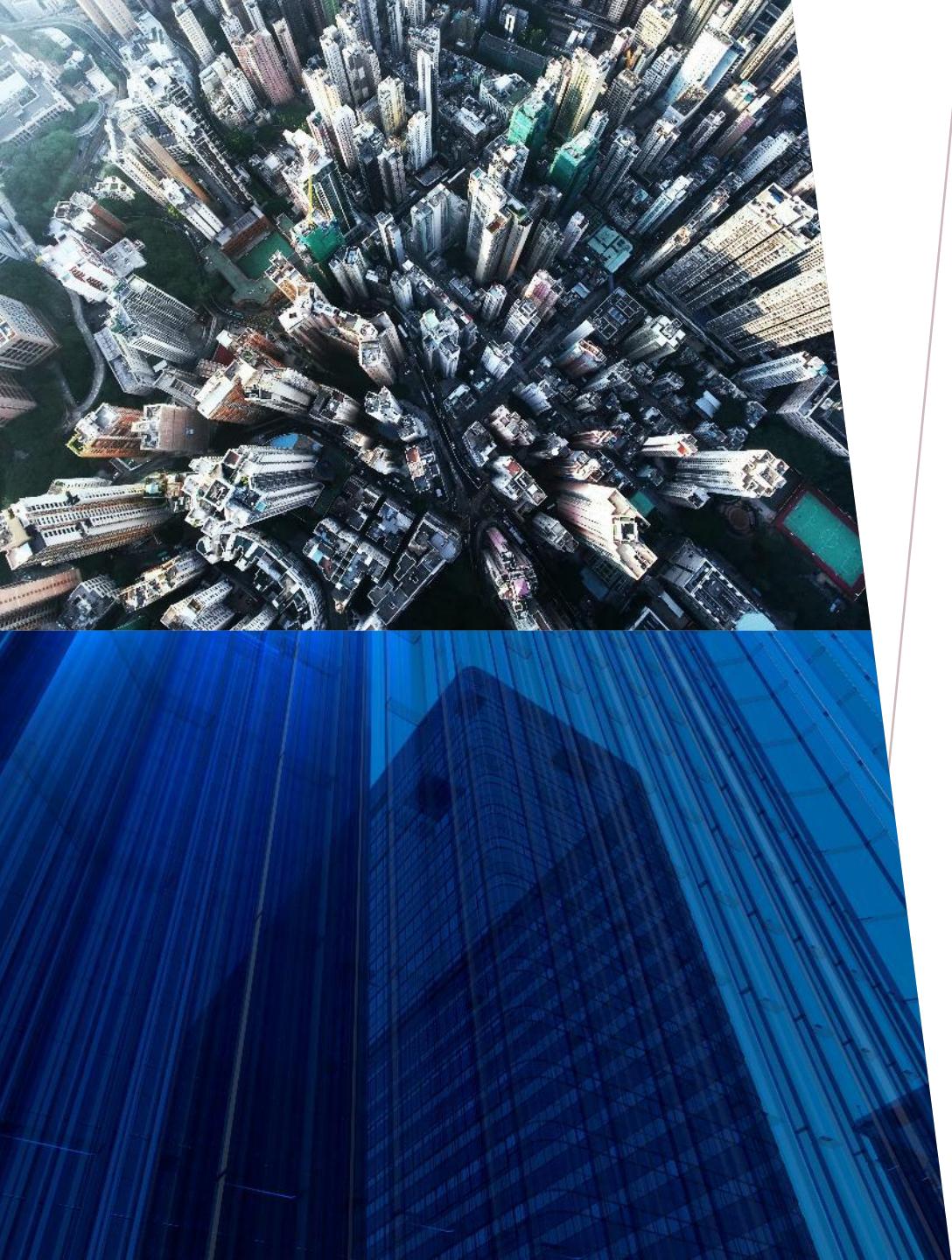
Modules completed

Module title	Description	Completed on	Duration
Build classical machine learning models with supervised learning	Supervised learning is a form of machine learning where an algorithm learns from examples of data. We progressively paint a picture of how supervised learning automatically generates a model that can make predictions about the real world. We also touch on how these models are tested, and difficulties that can arise in training them.	Nov 8, 2024	52 min
Introduction to GitHub	Learn to use key GitHub features, including issues, notifications, branches, commits, and pull requests.	Nov 8, 2024	1 hr 32 min
Introduction to Git	Use Git to track changes and collaborate with other developers.	Nov 8, 2024	31 min
Contribute to an open-source project on GitHub	Learn how to contribute to an open-source project on GitHub.	Nov 8, 2024	27 min

มี Transcript ที่ระบุ Module ที่เราเรียนสำเร็จ
พร้อมบอก Learning Outcome ที่เราทำได้

มีระบบ Simulation ให้เราเขียนโปรแกรมในระหว่างการเรียน
(ต้องเป็น Module ที่มีการปฏิบัติ)

ถ้ามีความพร้อมในความสามารถแล้วสามารถ
สมัครสอบ (Exam) เพื่อขอรับ credential
สำหรับยืนยันความสามารถว่าเรามีทักษะพร้อมทำงาน
โดยสามารถใช้เครื่องมือ AI ของทาง Microsoft ได้



How we learn in this course

2.) การเรียนรู้ผ่านการแก้ปัญหาจากการทำ โครงการ

- เป็นงานกลุ่ม 2 คน ช่วยกันคิด ช่วยกันทำ
- เน้นการสร้างและฝึกสอน Model โดยไม่ให้ใช้ Pre-train model
- จะเริ่มกำหนดรายละเอียดหลังจากที่มีพื้นฐานเรื่อง Model และการ Train แล้ว
- ต้องมีกระบวนการวัดประสิทธิภาพที่ถูกต้อง (ไม่เกี่ยวกับว่าผลการทดลองแม่นยำหรือไม่)

How we learn in this course

3.) การเรียนจากวิธีการ Lecture

- เนื้อหาโดยส่วนใหญ่มาจาก Textbook โดยจะปรับวิธีการอธิบายกับเลือกเนื้อหาโดยมีเนื้อหาอยู่ 2 ลักษณะ
 - หัวข้อที่เน้นทฤษฎี การคำนวณ พิสูจน์
 - หัวข้อที่เกี่ยวข้องกับเครื่องมือที่นิยมในปัจจุบัน



Before we go to AI era let look back to the past...

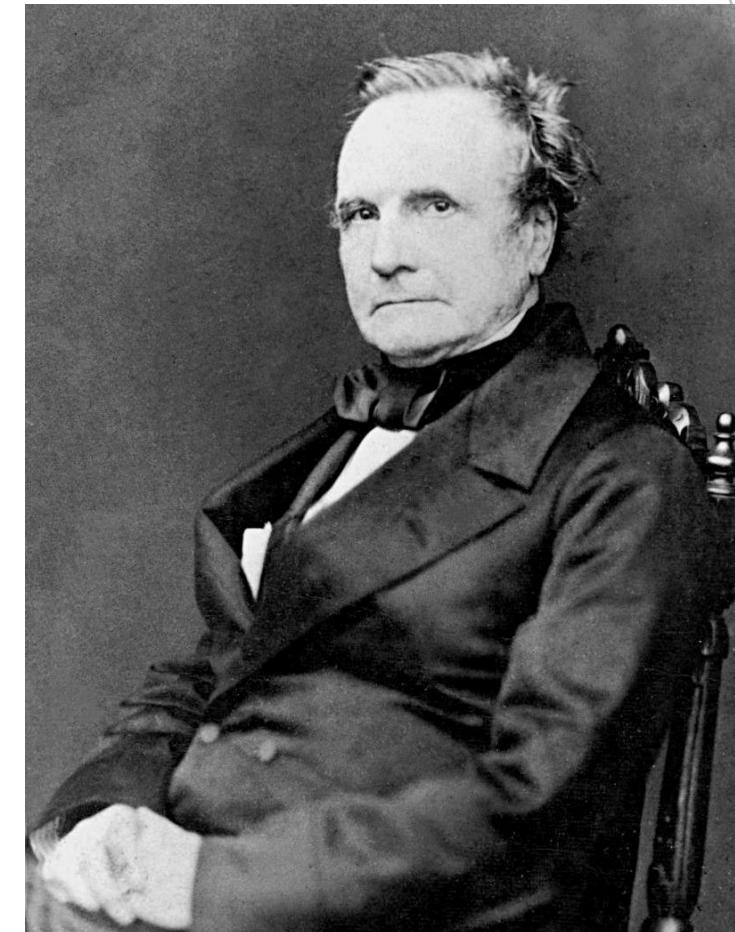
Charles Babbage

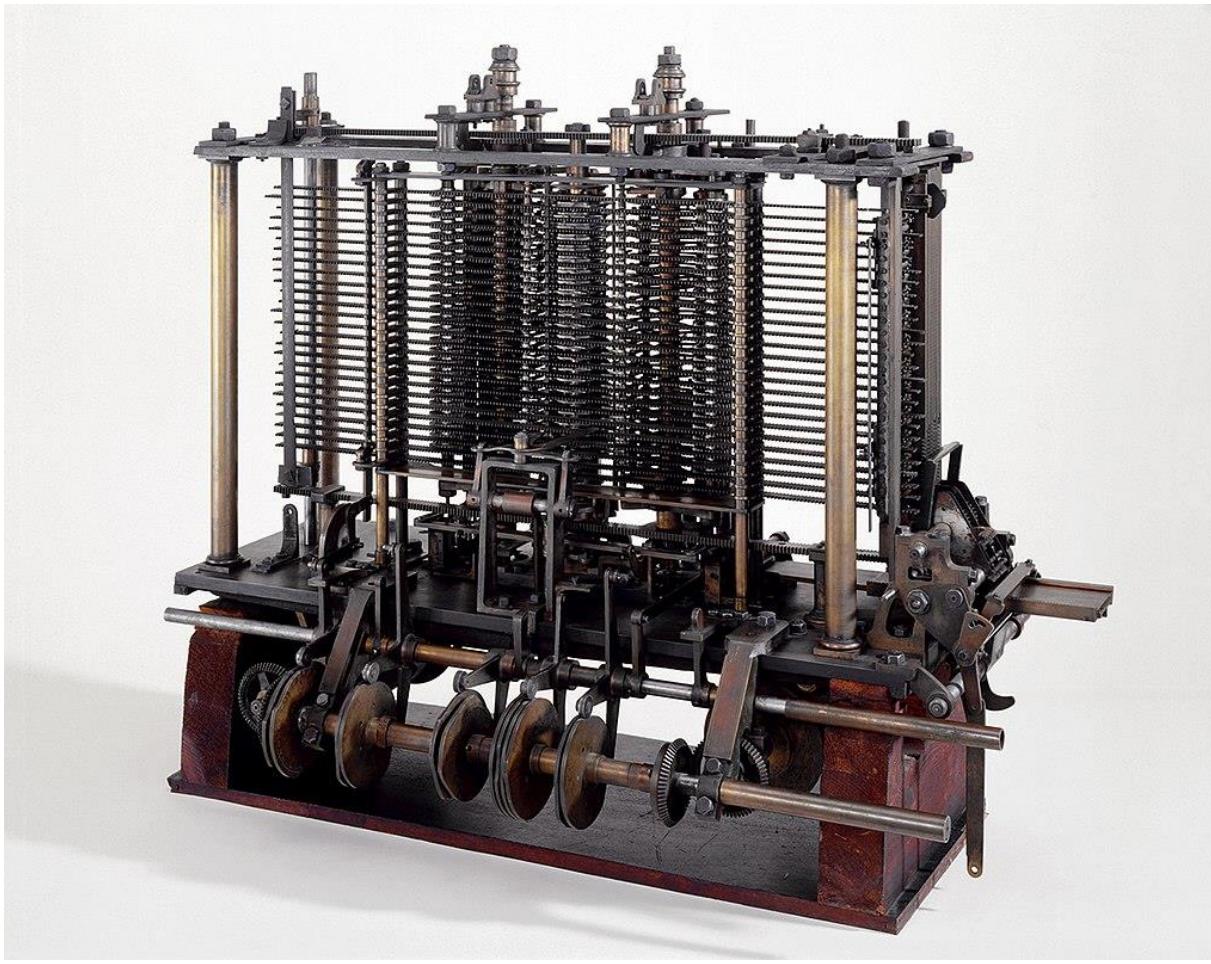
Charles Babbage was a British **mathematician** and **inventor**.

In 1833, he designed the **Analytical Engine**. If it had been built, it would have been the very first **computer**!

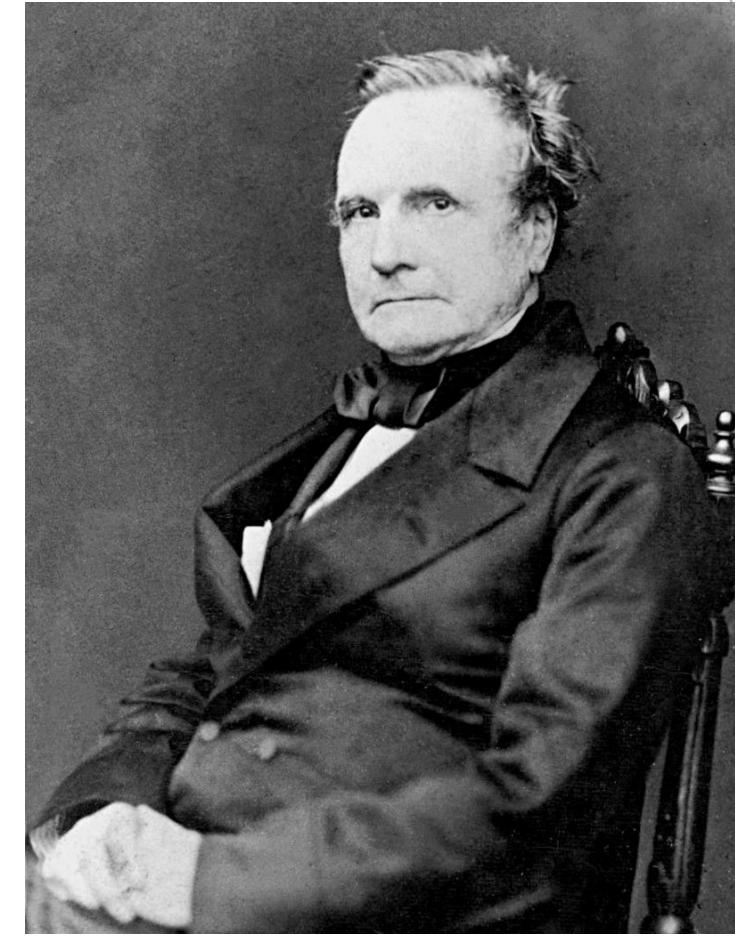
Fascinating Fact: The Analytical Engine was designed to do calculations with different sets of numbers and was **powered by steam**!

เครื่องจักรพลังงานไอน้ำ → คำนวณตัวเลขได้





Design Hardware <Not Built>





Ada Lovelace

Ada Lovelace was another British **mathematician**, who loved solving puzzles.

She studied the plans **Charles Babbage** had made for the Analytical Engine and made notes on how it could be used.

These notes were the **first computer program**.

Fascinating Fact: A programming language was invented in the **1990s** called **Ada**, after Ada Lovelace.

Design Algorithm

- ❖ First published algorithm
- ❖ First computer programmer
- ❖ Compute Bernoulli sequence

ส่วนหนึ่งของ “งานเขียน” เป็นแรงผลักดัน
“Computer สามารถทำงานอื่นๆ ได้
มากกว่าการคำนวณทั่วไป”

Number of Operation.	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.	Data.		Working Variables.								Result Variables.															
						1	2	n	IV ₁	IV ₂	IV ₃	oV ₄	oV ₅	oV ₆	oV ₇	oV ₈	oV ₉	oV ₁₀	oV ₁₁	IV ₁₂	oV ₁₂	IV ₂₁	IV ₂₂	IV ₂₃	oV ₂₁	oV ₂₂	oV ₂₃	B ₁	B ₂	B ₃	B ₄
1	\times	IV ₂ \times^3 V ₃	IV ₄ , IV ₅ , IV ₆	$\left\{ \begin{array}{l} IV_2 = IV_2 \\ IV_3 = IV_3 \\ IV_4 = IV_4 \end{array} \right.$	= 2 n	2	n	2 n	2 n	2 n																				
2	-	IV ₄ - IV ₁	2V ₄	$\left\{ \begin{array}{l} IV_4 = IV_4 \\ IV_1 = IV_1 \end{array} \right.$	= 2 n - 1	1	2 n - 1																						
3	+	IV ₅ + IV ₁	2V ₅	$\left\{ \begin{array}{l} IV_5 = IV_5 \\ IV_1 = IV_1 \end{array} \right.$	= 2 n + 1	1	2 n + 1																			
4	+	2V ₆ + 2V ₄	IV ₁₁	$\left\{ \begin{array}{l} 2V_6 = IV_6 \\ 2V_4 = IV_4 \end{array} \right.$	$\frac{2n-1}{2n+1}$	0	0				
5	+	IV ₁₁ + IV ₂	2V ₁₁	$\left\{ \begin{array}{l} IV_{11} = IV_{11} \\ IV_2 = IV_2 \end{array} \right.$	$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$	2				
6	-	oV ₁₃ - 2V ₁₁	IV ₁₂	$\left\{ \begin{array}{l} oV_{13} = IV_{13} \\ 2V_{11} = IV_{11} \end{array} \right.$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$	0				
7	-	IV ₃ - IV ₁	IV ₁₀	$\left\{ \begin{array}{l} IV_3 = IV_3 \\ IV_1 = IV_1 \end{array} \right.$	= n - 1 (= 3)	1	...	n	n - 1	...									
8	+	IV ₂ + 9V ₇	IV ₇	$\left\{ \begin{array}{l} IV_2 = IV_2 \\ 9V_7 = IV_7 \end{array} \right.$	= 2 + 0 = 2	2	2				
9	+	IV ₆ + IV ₇	2V ₁₁	$\left\{ \begin{array}{l} IV_6 = IV_6 \\ IV_7 = IV_7 \end{array} \right.$	$\frac{2n}{2} = A_1$	2 n	2	$\frac{2n}{2} = A_1$...								
10	\times	IV ₂₁ \times^3 V ₁₁	IV ₁₂	$\left\{ \begin{array}{l} IV_{21} = IV_{21} \\ 3V_{11} = 3V_{11} \end{array} \right.$	$= B_1 \cdot \frac{2n}{2} = B_1 A_1$	$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1 A_1$	B ₁			
11	+	IV ₁₂ + IV ₁₃	2V ₁₂	$\left\{ \begin{array}{l} IV_{12} = IV_{12} \\ IV_{13} = IV_{13} \end{array} \right.$	$= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$	0	$\left\{ -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2} \right\}$...								
12	-	IV ₁₀ - IV ₁	2V ₁₀	$\left\{ \begin{array}{l} IV_{10} = IV_{10} \\ IV_1 = IV_1 \end{array} \right.$	= n - 2 (= 2)	1	n - 2	...									
13	-	IV ₆ - IV ₁	2V ₆	$\left\{ \begin{array}{l} IV_6 = 2V_6 \\ IV_1 = IV_1 \end{array} \right.$	= 2 n - 1	1	2 n - 1																			
14	+	IV ₁ + IV ₇	2V ₇	$\left\{ \begin{array}{l} IV_1 = IV_1 \\ IV_7 = IV_7 \end{array} \right.$	= 2 + 1 = 3	1	3																			
15	+	2V ₆ + 2V ₂	IV ₈	$\left\{ \begin{array}{l} 2V_6 = 2V_6 \\ 2V_2 = 2V_2 \end{array} \right.$	$\frac{2n-1}{3}$	2 n - 1	3	2 n - 1	3	3	2 n - 1	3	2 n - 1	3	2 n - 1	3	2 n - 1	3	2 n - 1	3	2 n - 1	3	2 n - 1	3		
16	\times	IV ₈ \times^3 V ₁₁	IV ₁₁	$\left\{ \begin{array}{l} IV_8 = IV_8 \\ 3V_{11} = 3V_{11} \end{array} \right.$	$= \frac{2n}{3}$	0		
17	-	IV ₆ - IV ₁	IV ₆	$\left\{ \begin{array}{l} IV_6 = IV_6 \\ IV_1 = IV_1 \end{array} \right.$	= 2 n - 2	1	2 n - 2																			
18	+	IV ₁ + 2V ₂	IV ₂	$\left\{ \begin{array}{l} IV_1 = IV_1 \\ 2V_2 = 2V_2 \end{array} \right.$	= 3 + 1 = 4	1	4																			
19	-	3V ₆ + 3V ₇	V ₉	$\left\{ \begin{array}{l} 3V_6 = 3V_6 \\ 3V_7 = 3V_7 \end{array} \right.$	$\frac{2n-2}{4}$	2 n - 2	4	2 n - 2	4	4	2 n - 2	4	2 n - 2	4	2 n - 2	4	$\left\{ \frac{2n}{4}, \frac{2n-1}{4}, \frac{2n-2}{4} \right\}$...								
20	\times	IV ₉ \times^3 V ₁₁	IV ₁₁	$\left\{ \begin{array}{l} IV_9 = IV_9 \\ 3V_{11} = 3V_{11} \end{array} \right.$	$= \frac{2n}{3}$	0		
21	\times	IV ₂₂ \times^3 V ₁₁	IV ₁₂	$\left\{ \begin{array}{l} IV_{22} = IV_{22} \\ 3V_{11} = 3V_{11} \end{array} \right.$	$= B_3 \cdot \frac{2n}{3}$	0	B ₃		
22	+	2V ₁₂ + 2V ₁₃	IV ₁₂	$\left\{ \begin{array}{l} 2V_{12} = 2V_{12} \\ 2V_{13} = 2V_{13} \end{array} \right.$	$= A_0 + B_1 A_1 + B_3 A_2$	0	$\left\{ A_0 + B_1 A_1 + B_3 A_2 \right\}$...								
23	-	2V ₁₀ - IV ₁	IV ₁₀	$\left\{ \begin{array}{l} 2V_{10} = 2V_{10} \\ IV_1 = IV_1 \end{array} \right.$	= n - 3 (= 1)	1	n - 3	...								
24	+	IV ₁₃ + oV ₂₄	IV ₂₄	$\left\{ \begin{array}{l} IV_{13} = oV_{13} \\ oV_{24} = IV_{24} \end{array} \right.$	= B ₇	B ₇		
25	+	IV ₁ + IV ₂	IV ₃	$\left\{ \begin{array}{l} IV_1 = IV_1 \\ IV_2 = IV_2 \end{array} \right.$	$= n + 1 = 4 + 1 = 5$	1	...	n + 1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			

Here follows a repetition of Operations thirteen to twenty-three.

การ “สั่งงาน” (ลองคิดดูว่าถ้าเป็น “ปัญญาประดิษฐ์” ส่วนนี้จะเป็นอย่างไร)



History of Computer

Name	First operational	Numerical system	Computing mechanism	Programming	Turing complete	Memory
Difference engine	Not built until the 1990s (design 1820s)	Decimal	Mechanical	Not programmable; initial numerical constants of polynomial differences set physically	No	Physical state of wheels in axes
Analytical Engine	Not built (design 1830s)	Decimal	Mechanical	Program-controlled by punched cards	Yes	Physical state of wheels in axes
Ludgate's Analytical Engine	Not built (design 1909)	Decimal	Mechanical	Program-controlled by punched cards	Yes	Physical state of rods
Torres' Analytical Machine	1920	Decimal	Electro-mechanical	Not programmable; input and output settings specified by patch cables	No	Mechanical relays

ສັງເກດຸດູຕຽນ “Turing Complete” ສ່ວນໃຫຍ່ມີແຕ່ “No” (ສືແດງ)

Zuse Z1 (Germany)	1939	Binary floating point	Mechanical	Not programmable; cipher input settings specified by patch cables	No	Physical state of rods
Bombe (Poland, UK, US)	1939 (Polish), March 1940 (British), May 1943 (US)	Character computations	Electro-mechanical	Not programmable; cipher input settings specified by patch cables	No	Physical state of rotors
Zuse Z2 (Germany)	1940	Binary floating point	Electro-mechanical (Mechanical memory)	Program-controlled by punched 35 mm film stock	No	Physical state of rods
Zuse Z3 (Germany)	May 1941	Binary floating point	Electro-mechanical	Program-controlled by punched 35 mm film stock	In principle	Mechanical relays
Atanasoff–Berry Computer (US)	1942	Binary	Electronic	Not programmable; linear system coefficients input using punched cards	No	Regenerative capacitor memory
Colossus Mark 1 (UK)	December 1943	Binary	Electronic	Program-controlled by patch cables and switches	No	Thermionic valves (vacuum tubes) and thyratrons
Harvard Mark I – IBM ASCC (US)	May 1944	Decimal	Electro-mechanical	Program-controlled by 24-channel punched paper tape (but no conditional branch)	No	Mechanical relays ^[54]
Zuse Z4 (Germany)	March 1945 (or 1948) ^[55]	Binary floating point	Electro-mechanical	Program-controlled by punched 35 mm film stock	In 1950	Mechanical relays

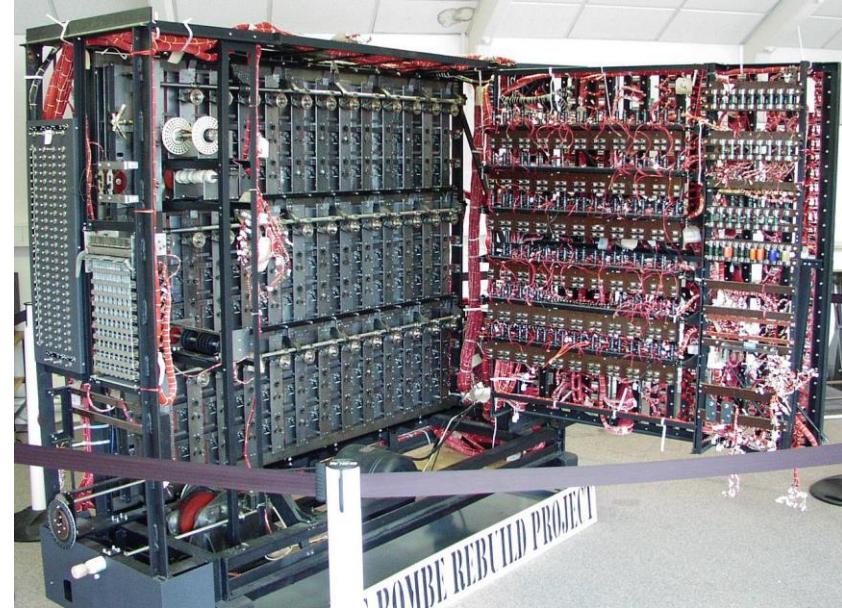
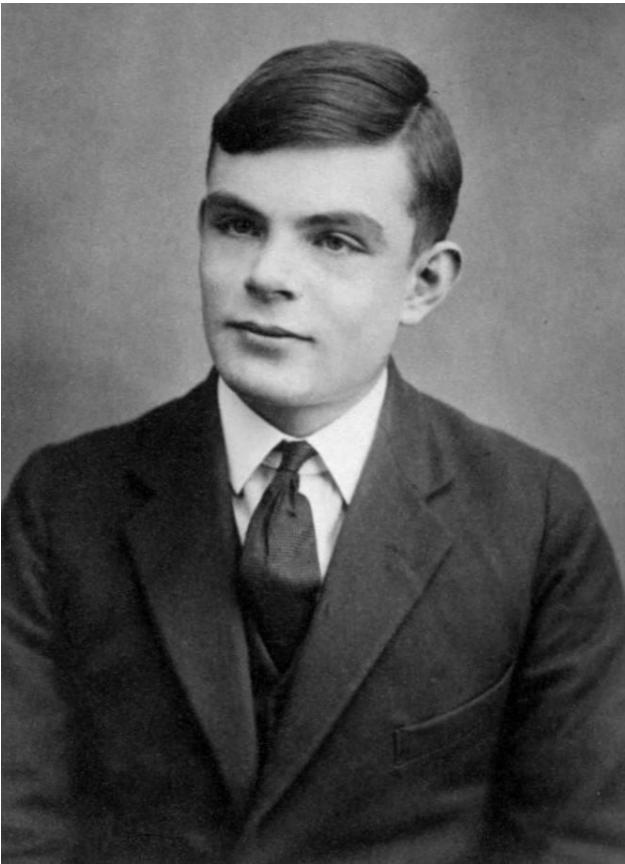
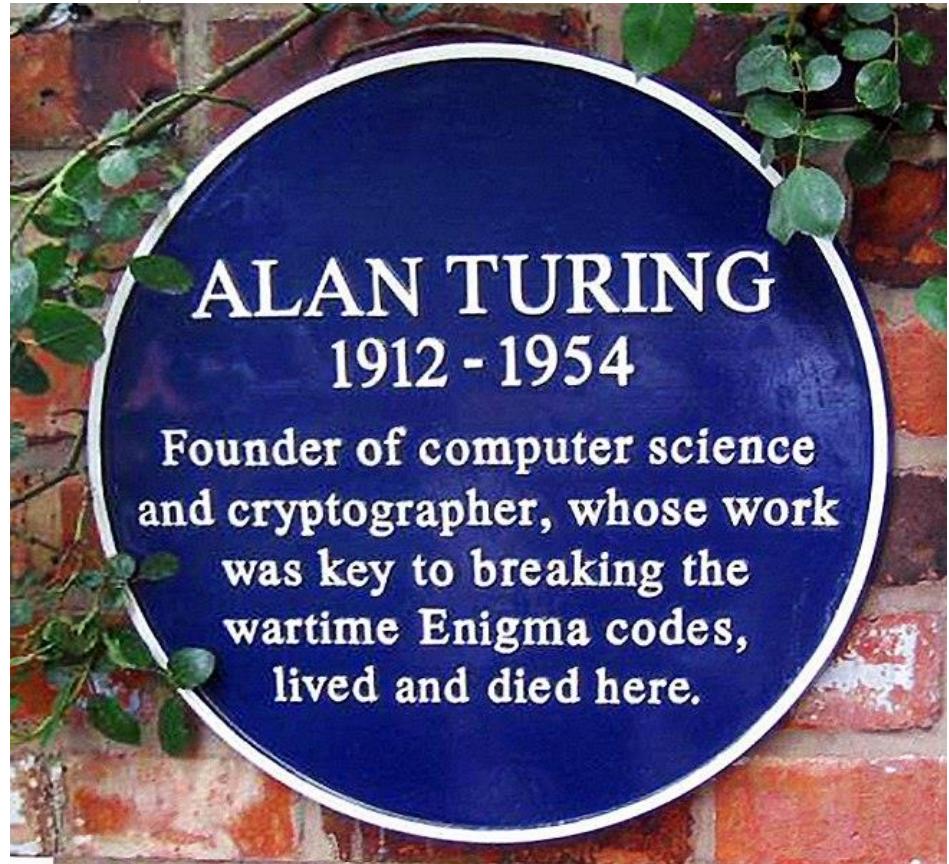
History of Computer

ENIAC (US)	July 1946	Decimal	Electronic	Program-controlled by patch cables and switches	Yes	Vacuum tube triode flip-flops
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ในช่วงหลังสงครามโลกครั้งที่ 2 (1945) จบลง มีการถือกำเนิดของเครื่องจักรชนิดหนึ่งที่ทำงานด้วยไฟฟ้า และเปลี่ยนการทำงานจากบัตรเจ้าสูญ (punch card) เป็น cables และ switches รวมทั้งผ่านเงื่อนไข “Turing Complete” เป็นครั้งแรก

ENIAC

<https://youtu.be/9PYWXBkgIkl>



ALAN TURING

<https://youtu.be/7thziYzrPs4>

ประวัติศาสตร์
คอมพิวเตอร์



Alan Turing

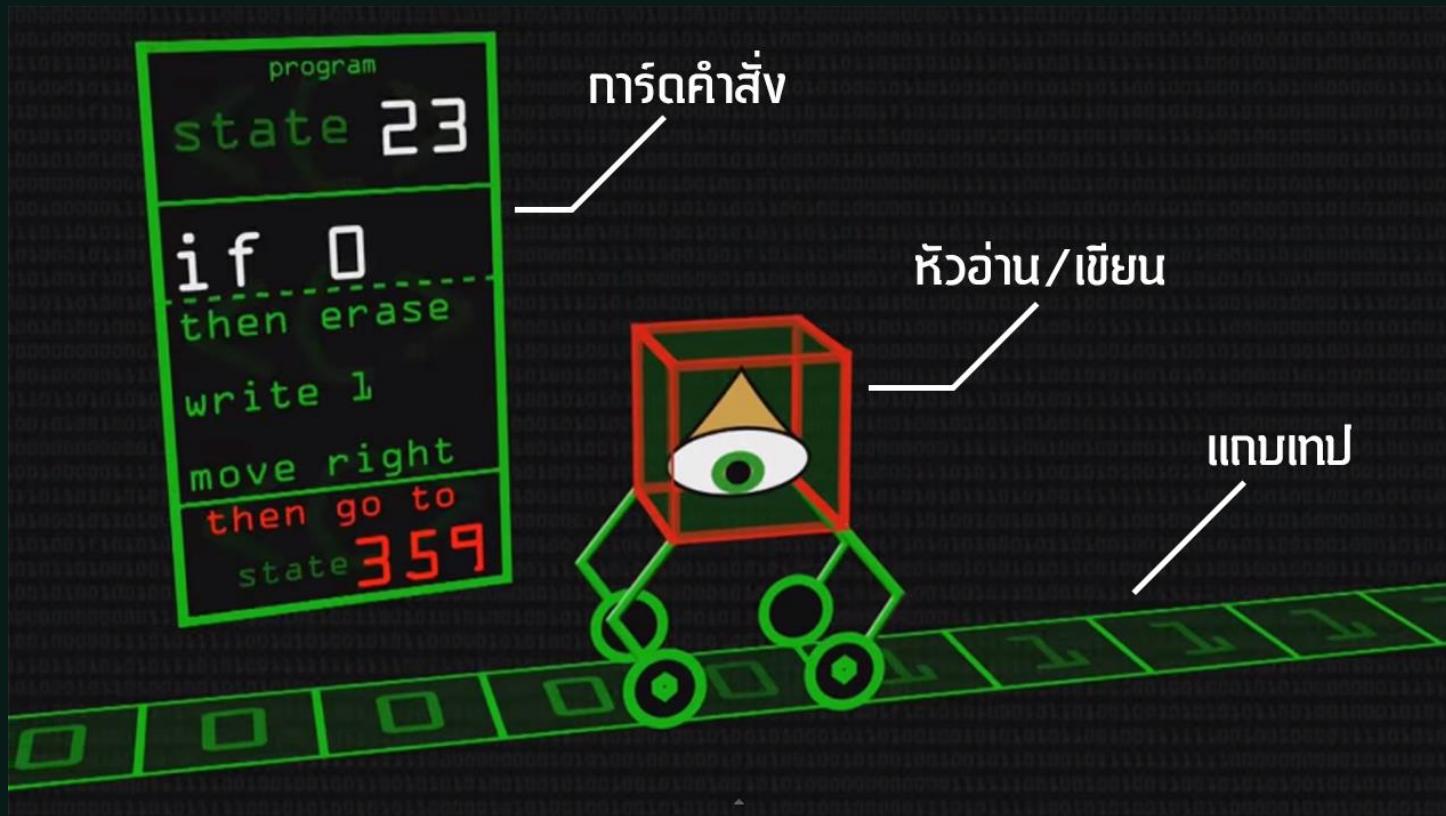
Alan Turing was a British **mathematician** who worked with a team of people during **World War Two**.

He created a machine that could **read secret coded messages** and helped save many lives!

Fascinating fact: Alan Turing's face is on the English £50 note.



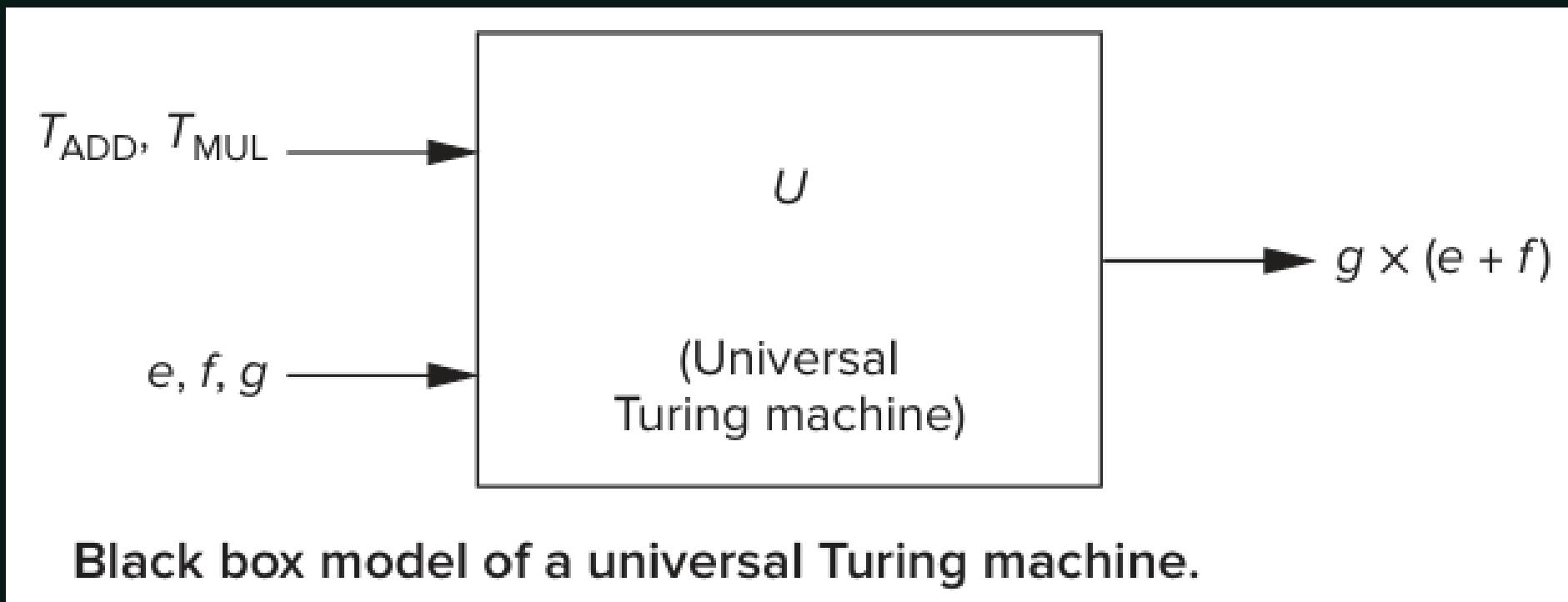
Turing Machine



- ❖ ระบบในจินตนาการ
- ❖ เครื่องอ่านคำสั่งได้
- ❖ เครื่องแก้ไขข้อมูลบนเทปได้
- ❖ เลื่อนตำแหน่งไปที่เทปซ่องไหนก็ได้
- ❖ ข้อมูลที่เขียนมีแค่ 0 หรือ 1 เท่านั้น
- ❖ เมื่อเครื่องเริ่มต้นแล้วจะทำงานไปเรื่อยๆ จนกว่าจะเจอการ์ด “หยุด (Halt)”

ถ้าทำได้ครบ Turing Complete
➔ Core ของ Computer Theory

Turing Machine



Computer Creator

เครื่องคอมพิวเตอร์แบบอิเล็กทรอนิกส์เครื่องแรกของโลกคือ ENIAC
(the Electronic Numerical Integrator and Calculator)

- เป็น General purpose electronic device ที่สามารถ Reprogrammed ได้
- สร้างขึ้นในช่วงปี ค.ศ. 1943 – 1945
- ขนาดเครื่องกินพื้นที่ประมาณ 300 ตารางฟุต (ห้องใหญ่ๆ 1 ห้อง)
- เขียนโปรแกรมด้วยการเปลี่ยนตำแหน่ง สายไฟ / หลอดสูญญากาศ (Vacuum Tube)
- Turing Complete



LANL 15736

Two Very Important Ideas

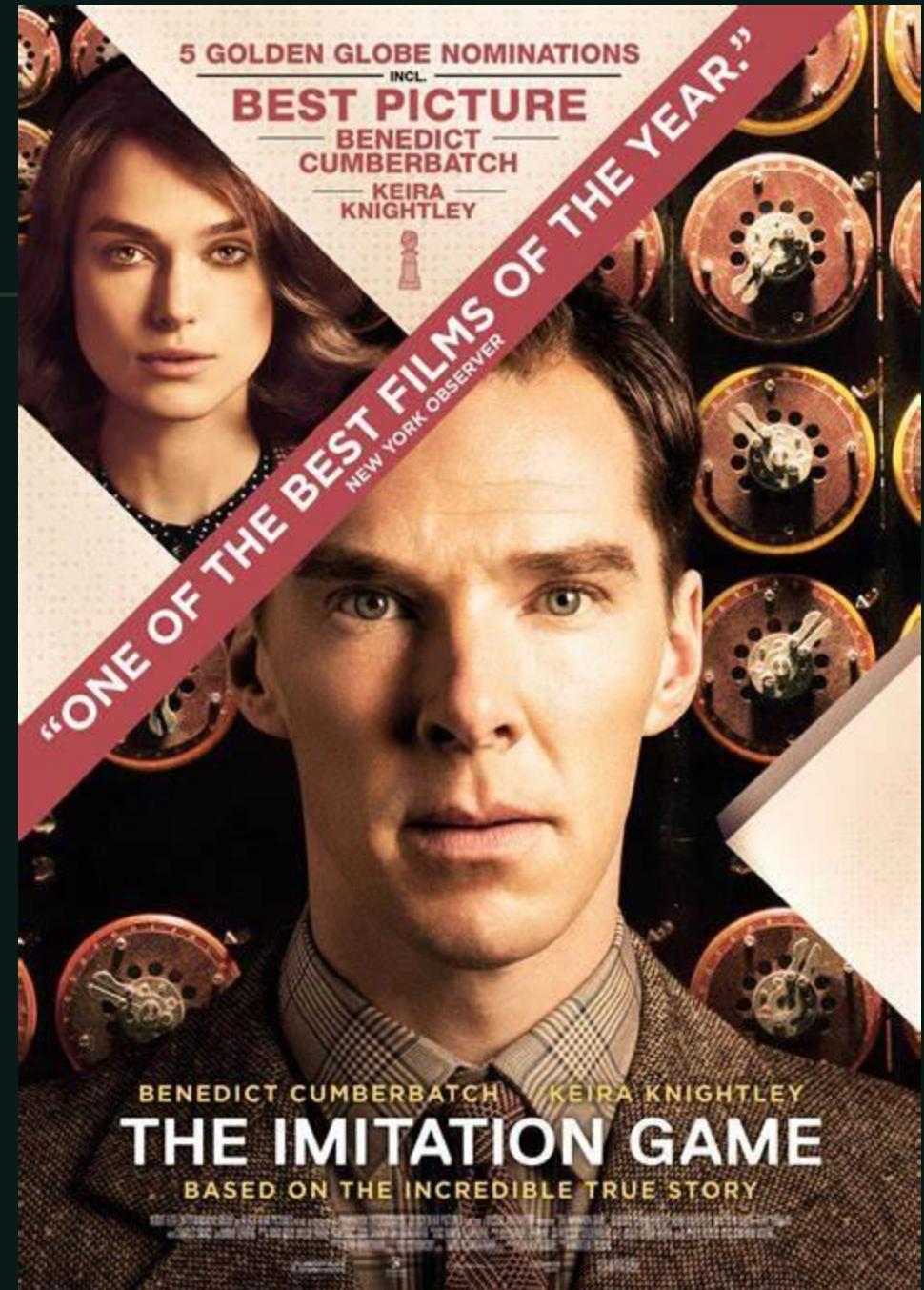
- ❑ Idea 1: All computers are capable of computing exactly the same things if they are given enough time and enough memory.
- ❑ Idea 2: We describe our problems in some language spoken by people.
Yet the problems are solved by electrons running around inside the computer.
It is necessary to transform our problem from the language of humans to the voltages that influence the flow of electrons.

Turing Test

AI หรือ Artificial Intelligence (ปัญญาประดิษฐ์) สาขานี้
อยากรู้ว่าทำให้เครื่องจักรสามารถ “คิด” ได้เหมือนมนุษย์

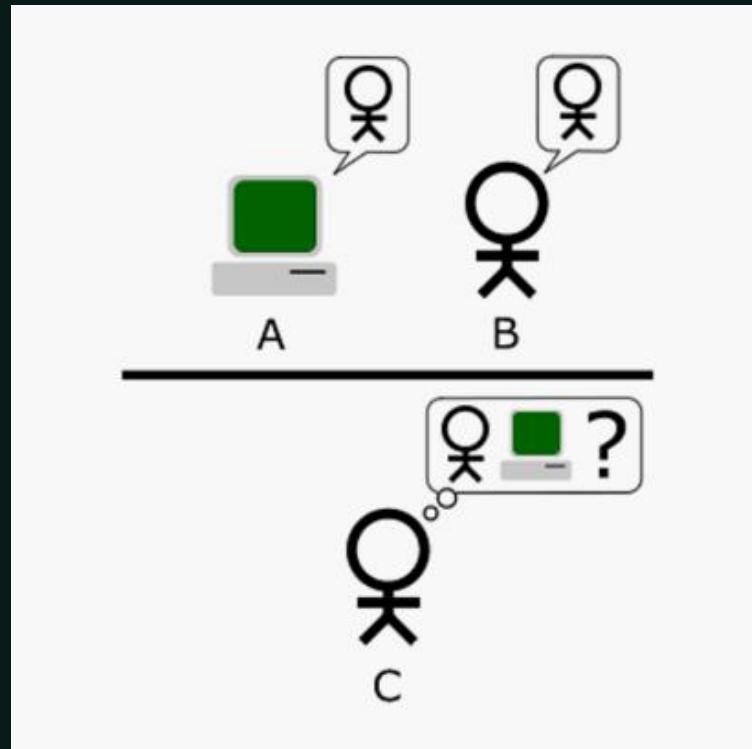
อะไรคือการ “คิด”?

จะวัดผลยังไงว่าเครื่องจักรคิดได้เหมือน
มนุษย์?



เดิมที่ Imitation Game เป็นเกมง่ายๆ ก็คือให้เราเขียนโน้ตคุยกับคนปริศนา 2 คน ชาย 1 หญิง 1 ชายคนนั้นจะพยายามตีเนียนเป็นผู้หญิง (หรือกลับกัน) และเราต้องพยายามว่าคนไหนเป็นผู้หญิง? (หรือผู้ชาย)

เมื่อไอเดียนี้เปลี่ยนมาใช้กับ AI ก็จะแทนที่สองคนนั้นด้วย คอมพิวเตอร์ กับ คน ถ้าคุยกัน 5 นาทีแล้วเครื่องหลอกคนทายว่าเป็นคนได้สำเร็จ ก็ถือว่าผ่าน ซึ่ง Imitation Game เวร์ชั่น “คนหรือเครื่อง?” นี้แหละครับที่เรียกว่า Turing Test



taepras

Computer Engineering Student • Front-end Web Developer • UI/UX Design Enthusiast • Design Geek

Activity

1. Turing Test มีจุดอ่อนหรือไม่ ในปัจจุบัน AI อย่างเช่น ChatGPT, CoPilot มีความสามารถในการผ่าน Turing Test หรือไม่ (เตรียมพูดคุยกันสับดาห์หน้า)
2. ให้เลือกทำ 1 อย่าง
 - ❖ Assignment “เจาะ Use Case AI ที่เด็ก MIT เค้าคุยกัน”
 - ❖ รวมกลุ่มหัดใช้เครื่อง Enigma และนำมาสอนการใช้งานให้เพื่อนที่ไปแข่ง TESA พังให้เข้าใจ <https://enigma.virtualcolossus.co.uk>

MATHEMATICS REVIEW

- Function**
- Linear Equation**
- Matrix**
- Probability**



ความสัมพันธ์และฟังก์ชัน

คู่ลำดับ (Ordered Pairs)

ในทางคณิตศาสตร์เมื่อเราพิจารณาถึงคู่ของสิ่งที่สัมพันธ์กัน เรามักจะเขียนเป็นคู่ลำดับ เมื่อจะเขียนถึงคู่ลำดับ a,b ก็จะใช้สัญลักษณ์ (a,b)

คู่ลำดับ สลับตำแหน่งหน้า-หลังไม่ได้ ยกเว้นกรณี $a = b$

ตัวหน้า	ตัวหลัง
1	10
2	20
3	30
4	40
5	50



ตัวหน้า	ตัวหลัง
10	1
20	2
30	3
40	4
50	5

ความสัมพันธ์และฟังก์ชัน

ความสัมพันธ์ (Relating)

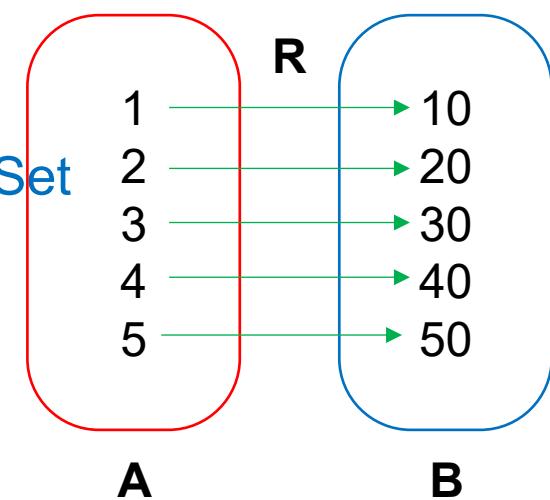
ความสัมพันธ์ คือ Set ที่สมาชิกทุกตัวเป็น คู่ลำดับ (แสดงว่าทุกตัวต้องมีคู่เสมอ)

กำหนดให้เซต $A = \{ 1, 2, 3, 4, 5 \}$

$B = \{ 10, 20, 30, 40, 50 \}$

ความสัมพันธ์ R คือ ความสัมพันธ์จาก $A \rightarrow B$ โดยเราจะเขียนในรูปของ Set

$R = \{ (1,10), (2,20), (3,30), (4,40), (5,50) \}$



ความสัมพันธ์และฟังก์ชัน

ความสัมพันธ์ (Relating)

ความสัมพันธ์ R

โดเมน A	เรนจ์ B
1	10
2	20
3	30
4	40
5	50

Domain

Range

ฟังก์ชัน (Function)

คืออะไร?



คือ ความสัมพันธ์ (Relationship)

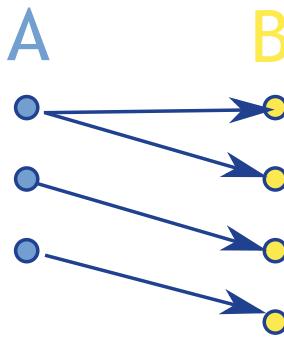
ระหว่าง Input กับ Output + (เงื่อนไข...)



$$f(x) = x^2$$

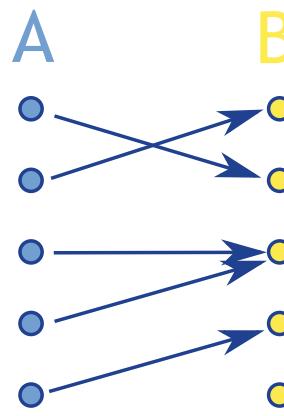
function name input what to output

ความสัมพันธ์และฟังก์ชัน



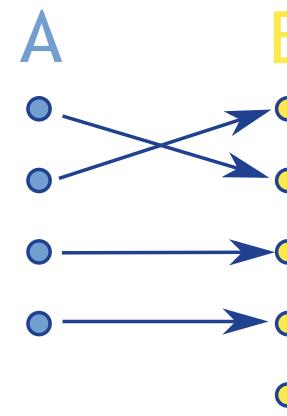
NOT a
Function

A has many B



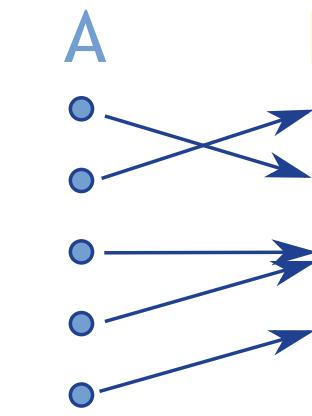
General
Function

B can have many A



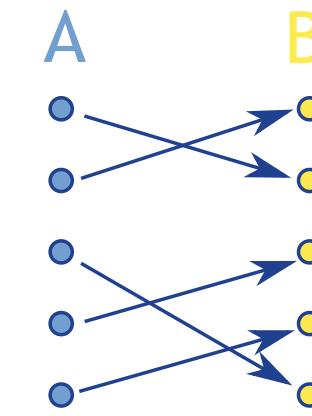
Injective
(not surjective)

B can't have many A



Surjective
(not injective)

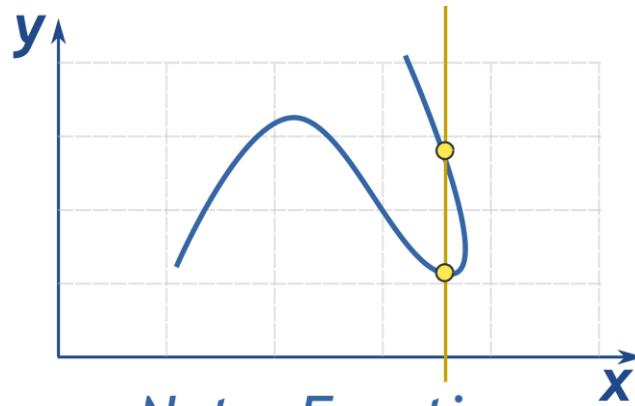
Every B has some A



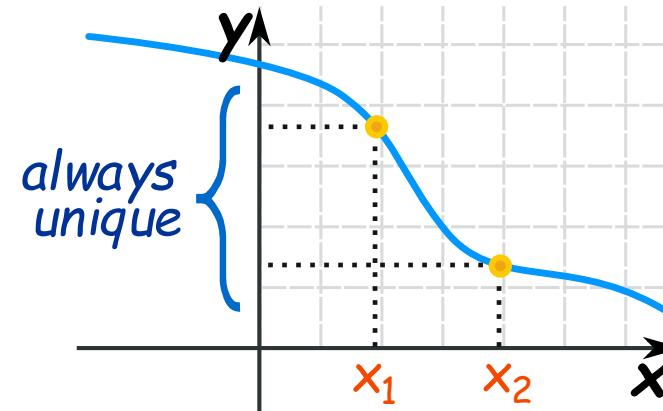
Bijective
(injective, surjective)

A to B, perfectly

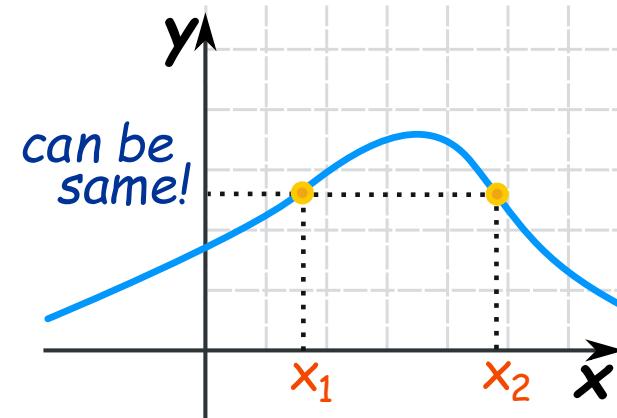
ความสัมพันธ์และฟังก์ชัน



Not a Function
(a vertical line crosses 2 values)



Injective



General Function

B มีค่าทุกตัว
Surjective

ความสัมพันธ์และฟังก์ชัน

Example

This tree grows 20 cm every year, so the height of the tree is **related** to its age using the function ***h***:



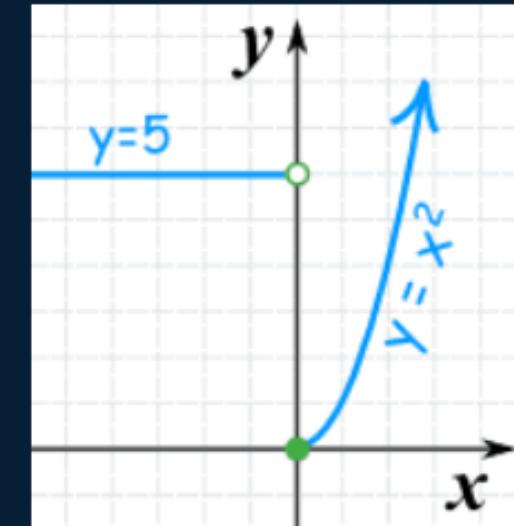
$$h(\text{age}) = \text{age} \times 20$$

age	$h(\text{age})$
0	0
1	20
3.2	64
15	300
...	...

ความสัมพันธ์และฟังก์ชัน

Example: A function with two pieces:

- when x is less than 0, it gives 5,
- when x is 0 or more it gives x^2



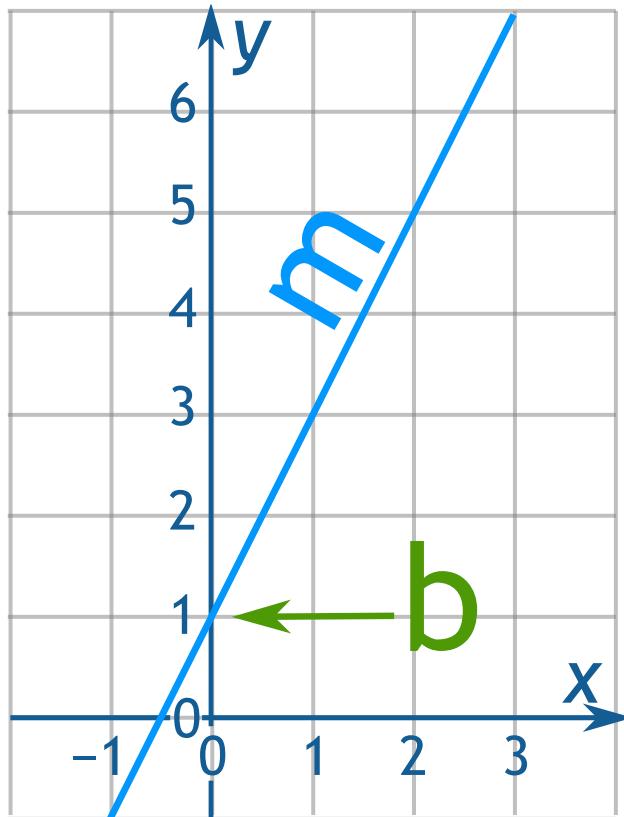
Here are some example values:

x	y
-3	5
-1	5
0	0
2	4
4	16
...	...

ในทางคณิตศาสตร์เรียก Input
ที่เข้าไปใน Function ว่า
Argument และเรียก Output
ที่ออกจาก Function ว่า **Value**

สมการเชิงเส้น (Linear Equation)

ส่วนประกอบ



$$y = mx + b$$

Slope or Gradient y value when $x=0$

How do you find "m" and "b"?

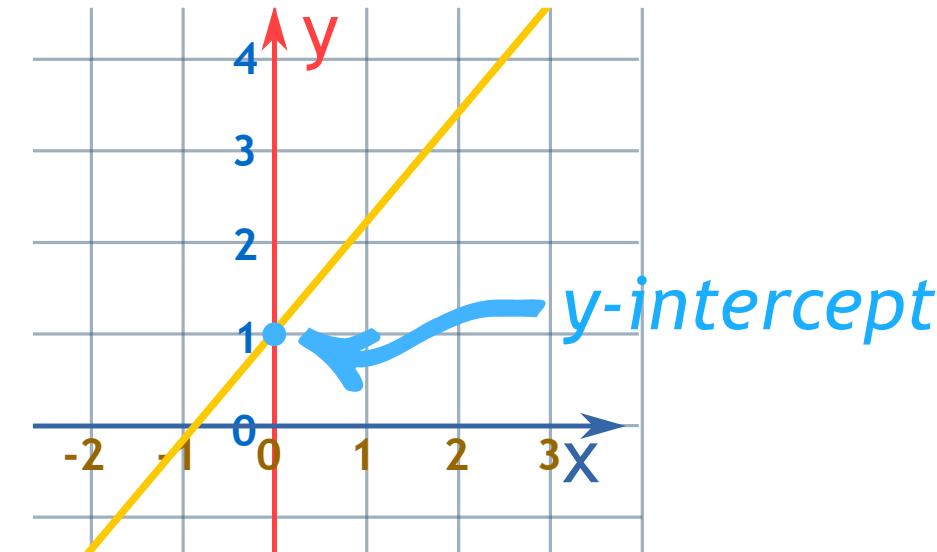
สมการเชิงเส้น (Linear Equation)

ส่วนประกอบ

$$y = mx + b$$

Slope or Gradient y value when $x=0$

find "b"



Just find the value of y when x equals 0

สมการเชิงเส้น (Linear Equation)

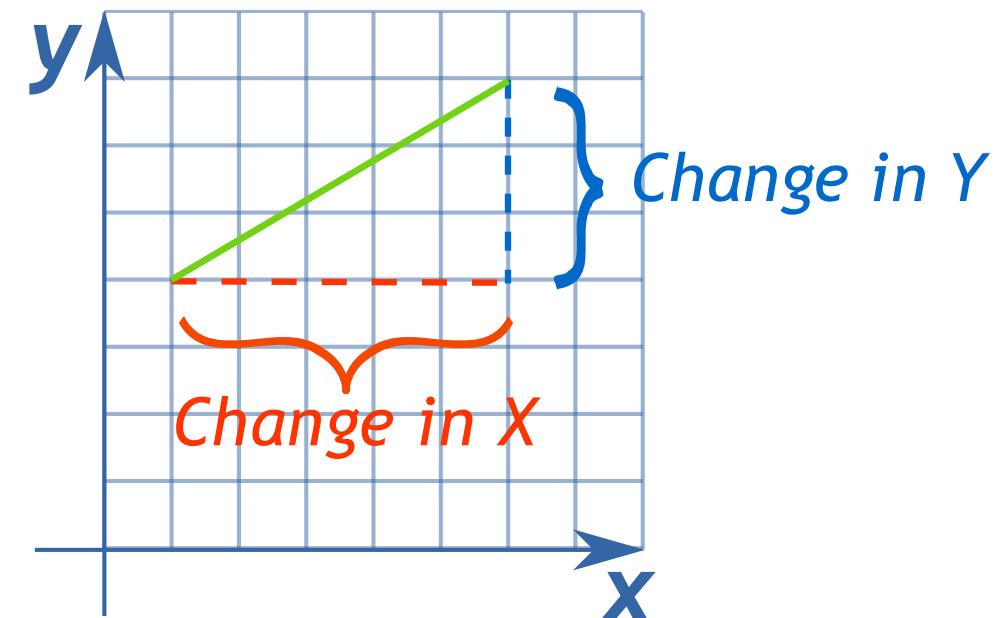
ส่วนประกอบ

$$y = mx + b$$

Slope or Gradient y value when $x=0$



find "m"



$$m = \frac{\text{Change in Y}}{\text{Change in X}}$$

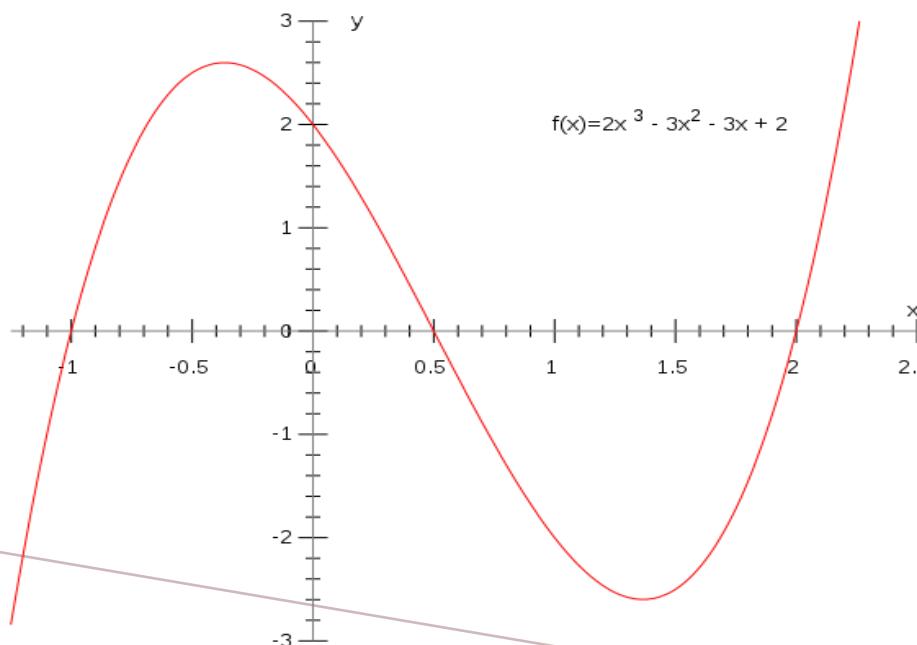
Polynomials

ส่วนประกอบ

Constant: ตัวเลขที่เป็นค่าคงที่

Variable: ตัวแปร เช่น x, y, a, b, \dots

Exponent: เลขชี้กำลัง ($0, 1, 2, 3, \dots$)



$$f(x) = 2x^3 - 3x^2 - 3x + 2$$

$$4xy^2 + 3x - 5$$

terms

exponents: 0, 1, 2, ...

$$5xy^2 - 3x + 5y^3 - 3$$

terms

A Polynomial

~~$3xy^{-2}$~~

~~$\frac{2}{x+2}$~~

Not Polynomials

Logarithms

How many of one number multiply together to make another number?

Example: How many 2s multiply together to make 8?

Answer: $2 \times 2 \times 2 = 8$, so we had to multiply 3 of the 2s to get 8
So, the logarithm is 3

$$\underbrace{2 \times 2 \times 2}_{3} = 8 \leftrightarrow \log_2(8) = 3$$

Exponent

Exponents and Logarithms are related...

A diagram illustrating the components of an exponent. It shows the number 2^3 with arrows pointing from the base '2' to the exponent '3'. The word 'exponent' is written above the '^' symbol, and the word 'base' is written below the '2'.

The **exponent** says **how many times** to use the number in a multiplication.

In this example: $2^3 = 2 \times 2 \times 2 = 8$

(2 is used 3 times in a multiplication to get 8)

A diagram showing the inverse relationship between exponentiation and logarithms. It compares $2^3 = 8$ with $\log_2(8) = 3$. Two sets of arrows connect the two equations: one set of arrows points from $2^3 = 8$ up to $\log_2(8) = 3$, and another set of arrows points from $\log_2(8) = 3$ down to $2^3 = 8$.

A diagram showing the inverse relationship between exponentiation and logarithms. It compares $a^x = y$ with $\log_a(y) = x$. Two sets of arrows connect the two equations: one set of arrows points from $a^x = y$ up to $\log_a(y) = x$, and another set of arrows points from $\log_a(y) = x$ down to $a^x = y$.

A diagram illustrating the equivalence between exponential and logarithmic forms. It shows the equation $2^3 = 8$ on the left and $\log_2(8) = 3$ on the right, connected by double-headed arrows. Above the first equation, the word 'exponent' is written with an arrow pointing to the '3'. Below the first equation, the word 'base' is written with an arrow pointing to the '2'. Above the second equation, the word 'exponent' is written with an arrow pointing to the '3'. Below the second equation, the word 'base' is written with an arrow pointing to the '2'.

Logarithms & Exponent

Number	How Many 10s	Base-10 Logarithm	
.. etc..			
1000	$1 \times 10 \times 10 \times 10$	$\log_{10}(1000)$	= 3
100	$1 \times 10 \times 10$	$\log_{10}(100)$	= 2
10	1×10	$\log_{10}(10)$	= 1
1	1	$\log_{10}(1)$	= 0
0.1	$1 \div 10$	$\log_{10}(0.1)$	= -1
0.01	$1 \div 10 \div 10$	$\log_{10}(0.01)$	= -2
0.001	$1 \div 10 \div 10 \div 10$	$\log_{10}(0.001)$	= -3
.. etc..			

10x Larger
10x Smaller

Example: Powers of 5			
	.. etc..		
5^2	5×5		25
5^1	5		5
5^0	1		1
5^{-1}	$\frac{1}{5}$		0.2
5^{-2}	$\frac{1}{5} \times \frac{1}{5}$		0.04
	.. etc..		

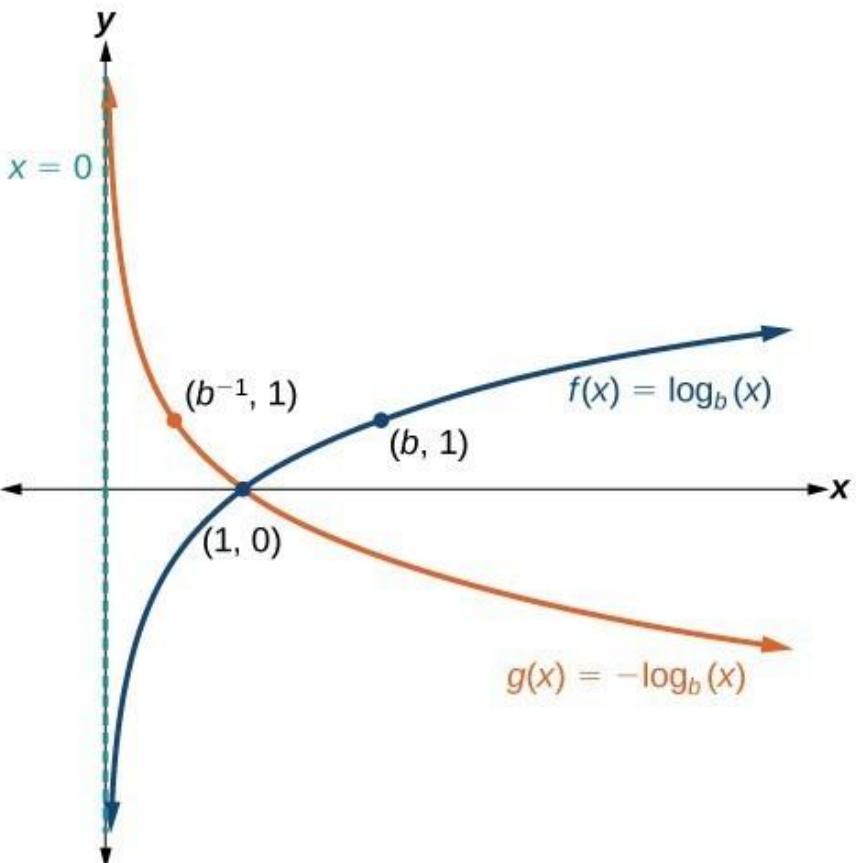
5x Larger
5x Smaller

Don't forget to write "base" of logarithms

Example	Engineer Thinks	Mathematician Thinks	
$\log(50)$	$\log_{10}(50)$	$\log_e(50)$	confusion
$\ln(50)$		$\log_e(50)$	no confusion
$\log_{10}(50)$		$\log_{10}(50)$	no confusion

Reflection about the x-axis

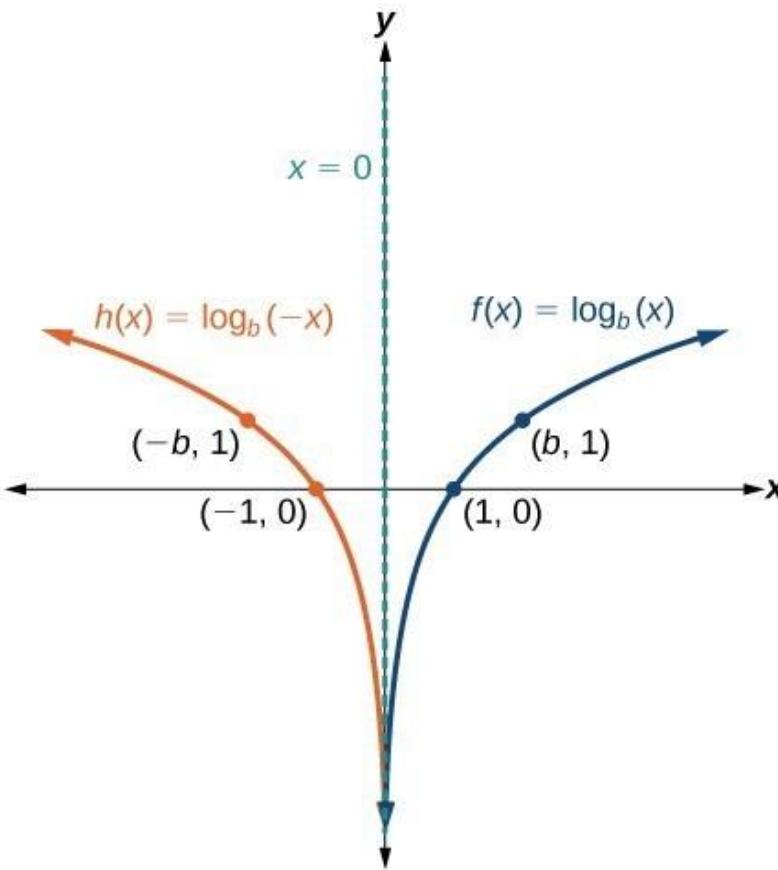
$$g(x) = \log_b(x), b > 1$$



- The reflected function is decreasing as x moves from zero to infinity.
- The asymptote remains $x = 0$.
- The x -intercept remains $(1, 0)$.
- The key point changes to $(b^{-1}, 1)$
- The domain remains $(0, \infty)$.
- The range remains $(-\infty, \infty)$.

Reflection about the y-axis

$$h(x) = \log_b(-x), b > 1$$



- The reflected function is decreasing as x moves from negative infinity to zero.
- The asymptote remains $x = 0$.
- The x -intercept changes to $(-1, 0)$.
- The key point changes to $(-b, 1)$
- The domain changes to $(-\infty, 0)$.
- The range remains $(-\infty, \infty)$.

Matrix

A Matrix is an array of numbers:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix
(This one has 2 Rows and 3 Columns)

$A_{m \times n}$ หมายถึง matrix ขนาด m และ n หลัก

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

$$\begin{matrix} & 1 & 2 & \cdots & n \\ 1 & a_{11} & a_{12} & \cdots & a_{1n} \\ 2 & a_{21} & a_{22} & \cdots & a_{2n} \\ 3 & a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m & a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix}$$

Matrix

Operation

Adding

To add two matrices: add the numbers in the matching positions:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

3+4=7 8+0=8
4+1=5 6-9=-3

The diagram shows two 2x2 matrices being added. Yellow circles highlight the numbers 3, 4, 8, and 0 from the first matrix, and 4, 1, 0, and -9 from the second matrix. Yellow arrows point from each highlighted number to its corresponding sum in the result matrix: 3+4=7, 8+0=8, 4+1=5, and 6-9=-3.

These are the calculations:

3+4=7	8+0=8
4+1=5	6-9=-3

Matrix

Operation

Subtracting

To subtract two matrices: subtract the numbers in the matching positions:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

3-4 = -1

These are the calculations:

3-4=-1	8-0=8
4-1=3	6-(-9)=15

Note: subtracting is actually defined as the **addition** of a negative matrix: $A + (-B)$

Matrix

Operation

Multiply by a Constant

We can multiply a matrix by a **constant** (*the value 2 in this case*):

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

The diagram shows the scalar 2 being multiplied by each element of the 2x2 matrix. A yellow circle contains the scalar 2, which is multiplied by the first element 4 to produce 8. A yellow arrow labeled "2x4=8" points from the scalar to the result. This process is repeated for all four elements of the matrix.

These are the calculations:

$2 \times 4 = 8$	$2 \times 0 = 0$
$2 \times 1 = 2$	$2 \times -9 = -18$

We call the constant a **scalar**, so officially this is called "scalar multiplication".

Multiplying a Matrix by Another Matrix

But to multiply a matrix **by another matrix** we need to do the "dot product" of rows and columns ... what does that mean? Let us see with an example:

To work out the answer for the **1st row** and **1st column**:

The diagram shows the multiplication of two matrices. On the left is a blue-bordered matrix with 2 rows and 3 columns, containing the numbers 1, 2, 3 in the top row and 4, 5, 6 in the bottom row. To its right is a blue-bordered matrix with 3 rows and 2 columns, containing the numbers 7, 8 in the top row, 9, 10 in the middle row, and 11, 12 in the bottom row. Between them is a blue multiplication sign (\times). To the right of the second matrix is an equals sign (=). To the right of the equals sign is a blue-bordered matrix with 2 rows and 1 column, containing the number 58. Above the first matrix, a yellow curved arrow labeled "Dot Product" points from the first row of the first matrix to the first column of the second matrix. A yellow arrow points from the number 58 to the right side of the equals sign.

To multiply an **$m \times n$** matrix by an **$n \times p$** matrix, the **n s** must be the same, and the result is an **$m \times p$** matrix.

$$m \times n \times n \times p \rightarrow m \times p$$

Why Do It This Way?

This may seem an odd and complicated way of multiplying, but it is necessary!

I can give you a real-life example to illustrate why we multiply matrices in this way.

Example: The local shop sells 3 types of pies.

- Apple pies cost **\$3** each
- Cherry pies cost **\$4** each
- Blueberry pies cost **\$2** each

And this is how many they sold in 4 days:

	Mon	Tue	Wed	Thu
Apple	13	9	7	15
Cherry	8	7	4	6
Blueberry	6	4	0	3

Now think about this ... the **value of sales** for Monday is calculated this way:

$$\begin{aligned} &\rightarrow \text{Apple pie value} + \text{Cherry pie value} + \text{Blueberry pie value} \\ &\rightarrow \$3 \times 13 + \$4 \times 8 + \$2 \times 6 = \$83 \end{aligned}$$

So it is, in fact, the "dot product" of prices and how many were sold:

$$(\$3, \$4, \$2) \bullet (13, 8, 6) = \$3 \times 13 + \$4 \times 8 + \$2 \times 6 = \$83$$

We **match** the price to how many sold, **multiply** each, then **sum** the result.

Matrix

Operation

$$AB \neq BA$$

Example:

See how changing the order affects this multiplication:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 2 \times 1 & 1 \times 0 + 2 \times 2 \\ 3 \times 2 + 4 \times 1 & 3 \times 0 + 4 \times 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 0 \times 3 & 2 \times 2 + 0 \times 4 \\ 1 \times 1 + 2 \times 3 & 1 \times 2 + 2 \times 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

The answers are different!

Matrix

Operation

การคูณแบบ Hadamard product (element-wise product)

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 8 & -2 \end{bmatrix} \circ \begin{bmatrix} 3 & 1 & 4 \\ 7 & 9 & 5 \end{bmatrix} = \begin{bmatrix} 2 \times 3 & 3 \times 1 & 1 \times 4 \\ 0 \times 7 & 8 \times 9 & -2 \times 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 4 \\ 0 & 72 & -10 \end{bmatrix}$$

$$A \circ B = C$$

Matrix

Operation

Transposing

To "transpose" a matrix, swap the rows and columns.

We put a "T" in the top right-hand corner to mean transpose:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

Matrix

Identity Matrix

An **Identity Matrix** has **1s** on the main diagonal and **0s** everywhere else:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A 3×3 Identity Matrix

- It is square (same number of rows as columns)
- It can be large or small (2×2 , 100×100 , ... whatever)
- Its symbol is the capital letter **I**

It is the matrix equivalent of the number "1", when we multiply with it the original is unchanged:

$$\mathbf{A} \times \mathbf{I} = \mathbf{A}$$

$$\mathbf{I} \times \mathbf{A} = \mathbf{A}$$

Matrix

Diagonal Matrix

A diagonal matrix has zero anywhere not on the main diagonal:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A diagonal matrix

Scalar Matrix

A scalar matrix has all main diagonal entries the same, with zero everywhere else:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

A scalar matrix

Matrix

Triangular Matrix

Lower triangular is when all entries above the main diagonal are zero:

$$\begin{bmatrix} 5 & 0 & 0 \\ 2 & 1 & 0 \\ 7 & 6 & -3 \end{bmatrix}$$

A lower triangular matrix

Upper triangular is when all entries below the main diagonal are zero:

$$\begin{bmatrix} 2 & -2 & 7 \\ 0 & 4 & 11 \\ 0 & 0 & 5 \end{bmatrix}$$

An upper triangular matrix

Zero Matrix (Null Matrix)

Zeros just everywhere:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Zero matrix

Symmetric

In a Symmetric matrix matching entries either side of the main diagonal are **equal**, like this:

$$\begin{bmatrix} 3 & 2 & 11 & 5 \\ 2 & 9 & -1 & 6 \\ 11 & -1 & 0 & 7 \\ 5 & 6 & 7 & 9 \end{bmatrix}$$

Symmetric matrix

It must be square, and is equal to its own transpose

$$A = A^T$$

What is the Inverse of a Matrix?

Just like a **number** has a reciprocal ...

$$\frac{1}{8}$$

Reciprocal of a Number (note: $\frac{1}{8}$ can also be written 8^{-1})

... a **matrix** has an **inverse** :

$$A^{-1}$$

We write A^{-1} instead of $\frac{1}{A}$ because we don't divide by a matrix!

The inverse of A is A^{-1} only when:

$$AA^{-1} = A^{-1}A = I$$

Sometimes there is no inverse at all.

What is the Inverse of a Matrix?

Just like a **number** has a reciprocal ...

$$8 \xrightarrow{\text{Reciprocal}} \frac{1}{8}$$

Reciprocal of a Number (note: $\frac{1}{8}$ can also be written 8^{-1})

... a **matrix** has an **inverse** :

$$A \xrightarrow{\text{Inverse}} A^{-1}$$

Inverse of a Matrix

We write A^{-1} instead of $\frac{1}{A}$ because we don't divide by a matrix!

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}^{-1} = \frac{1}{3 \times 8 - 4 \times 6} \begin{bmatrix} 8 & -4 \\ -6 & 3 \end{bmatrix}$$
$$= \frac{1}{24-24} \begin{bmatrix} 8 & -4 \\ -6 & 3 \end{bmatrix}$$

24-24? That equals 0, and **1/0 is undefined**.
We cannot go any further! This matrix has no Inverse.

Inverse Matrix

$$\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}^{-1} = \frac{1}{3 \times 8 - 4 \times 6} \begin{bmatrix} 8 & -4 \\ -6 & 3 \end{bmatrix}$$
$$= \frac{1}{24 - 24} \begin{bmatrix} 8 & -4 \\ -6 & 3 \end{bmatrix}$$

24–24? That equals 0, and **1/0 is undefined.**
We cannot go any further! This matrix has no Inverse.

Such a matrix is called "Singular",
which only happens when the determinant is zero.



A Real Life Example: Bus and Train



A group took a trip on a **bus**, at \$3 per child and \$3.20 per adult for a total of \$118.40.

They took the **train** back at \$3.50 per child and \$3.60 per adult for a total of \$135.20.

How many children, and how many adults?

First, let us set up the matrices (be careful to get the rows and columns correct!):

$$\begin{matrix} \text{Child} & \text{Adult} \\ \left[\begin{matrix} x_1 & x_2 \end{matrix} \right] \end{matrix} \begin{matrix} \text{Bus} & \text{Train} \\ \left[\begin{matrix} 3 & 3.5 \\ 3.2 & 3.6 \end{matrix} \right] \end{matrix} = \begin{matrix} \text{Bus} & \text{Train} \\ \left[\begin{matrix} 118.4 & 135.2 \end{matrix} \right] \end{matrix}$$

$$XA = B$$

So to solve it we need the inverse of "A":

$$\begin{bmatrix} 3 & 3.5 \\ 3.2 & 3.6 \end{bmatrix}^{-1} = \frac{1}{3 \times 3.6 - 3.5 \times 3.2} \begin{bmatrix} 3.6 & -3.5 \\ -3.2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -9 & 8.75 \\ 8 & -7.5 \end{bmatrix}$$

Now we have the inverse we can solve using:

$$X = BA^{-1}$$
$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} 118.4 & 135.2 \end{bmatrix} \begin{bmatrix} -9 & 8.75 \\ 8 & -7.5 \end{bmatrix}$$
$$= \begin{bmatrix} 118.4 \times -9 + 135.2 \times 8 & 118.4 \times 8.75 + 135.2 \times -7.5 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & 22 \end{bmatrix}$$

There were 16 children and 22 adults!

Probability



Probability

How **likely** something is to happen

Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using the idea of probability.

Tossing a Coin



When a coin is tossed, there are two possible outcomes:

Heads (H) or
Tails (T)

Also:

- the probability of the coin landing **H** is $\frac{1}{2}$
- the probability of the coin landing **T** is $\frac{1}{2}$

Probability

How **likely** something is to happen

Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using the idea of probability.

Throwing Dice



When a single **die** is thrown, there are six possible outcomes: **1, 2, 3, 4, 5, 6**.

The probability of any one of them is $\frac{1}{6}$

Probability

How **likely** something is to happen

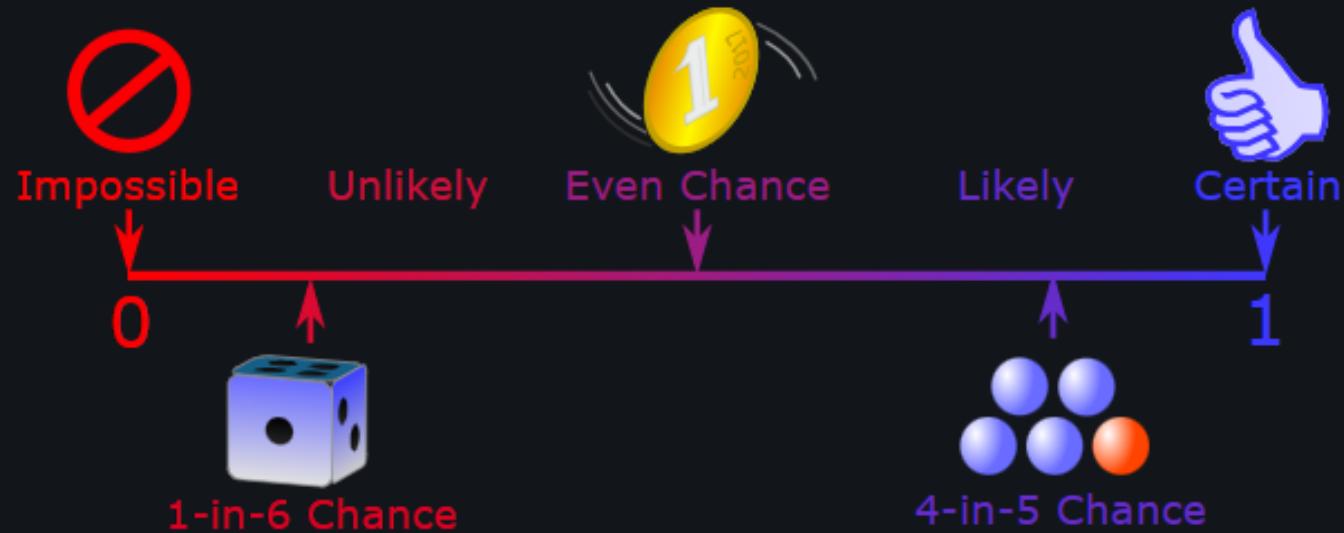
Many events can't be predicted with total certainty. The best we can say is how likely they are to happen, using the idea of probability.

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Probability

Probability Line

We can show probability on a Probability Line:



Probability is always between 0 and 1

Probability



Experiment: a repeatable procedure with a set of possible results.

Example: Throwing dice

We can throw the dice again and again, so it is repeatable.



The set of possible results from any single throw is $\{1, 2, 3, 4, 5, 6\}$



Outcome: A possible result.

Example: "6" is one of the outcomes of a throw of a die.



Probability



Trial: A single performance of an experiment.

Example: I conducted a coin toss experiment. After 4 trials I got these results:



Outcome	Trial	Trial	Trial	Trial
Head	✓	✓		✓
Tail			✓	

Three trials had the outcome "Head", and one trial had the outcome "Tail"

Probability



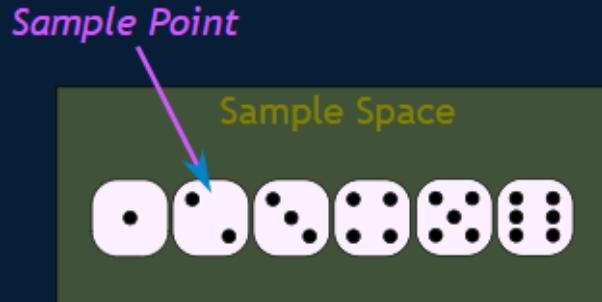
Sample Space: all the possible outcomes of an experiment.



Sample Point: just one of the possible outcomes

Example: Throwing dice

There are 6 different sample points in that sample space.



Probability



Event: one or more outcomes of an experiment

Example Events:

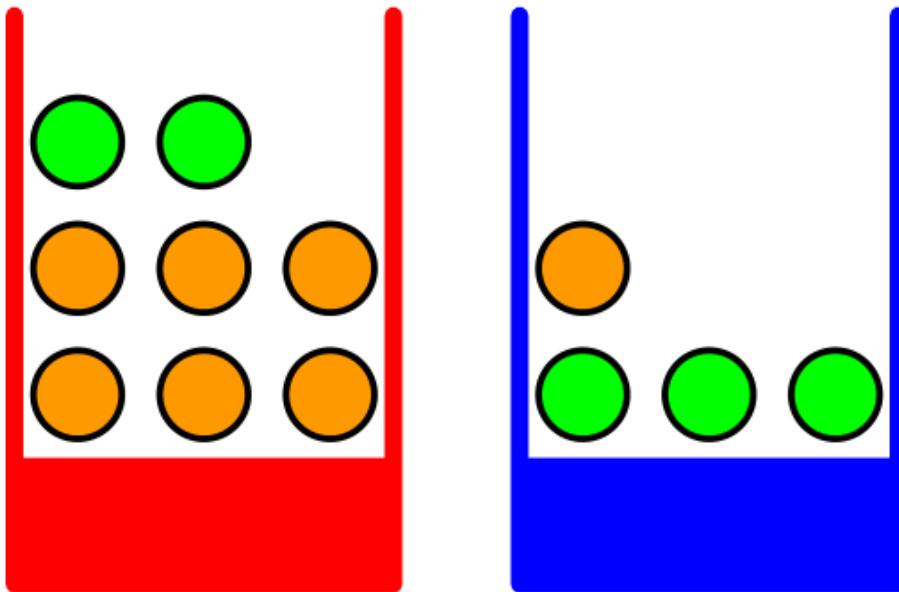
An event can be just one outcome:

- Getting a Tail when tossing a coin
- Rolling a "5"

An event can include more than one outcome:

- Choosing a "King" from a deck of cards (any of the 4 Kings)
- Rolling an "even number" (2, 4 or 6)

Probability

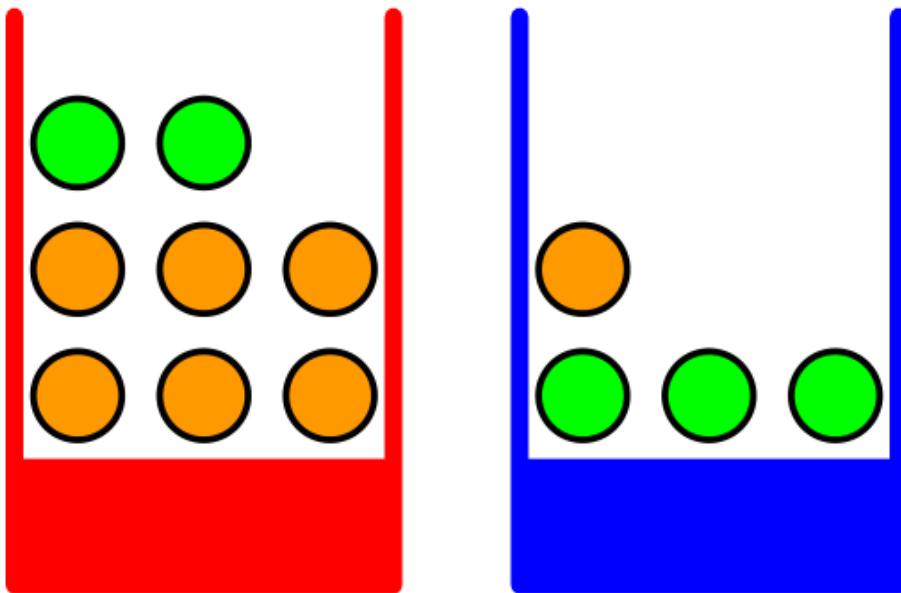


Box of Orange & Apple

Suppose we randomly pick one of the boxes and from that box we randomly select an item of fruit and having observed which sort of fruit it is we replace it in the box from which it came.

We could imagine repeating this process many times. Let us suppose that in so doing **we pick the red box 40% of the time and we pick the blue box 60% of the time.**

Probability



Box of Orange & Apple

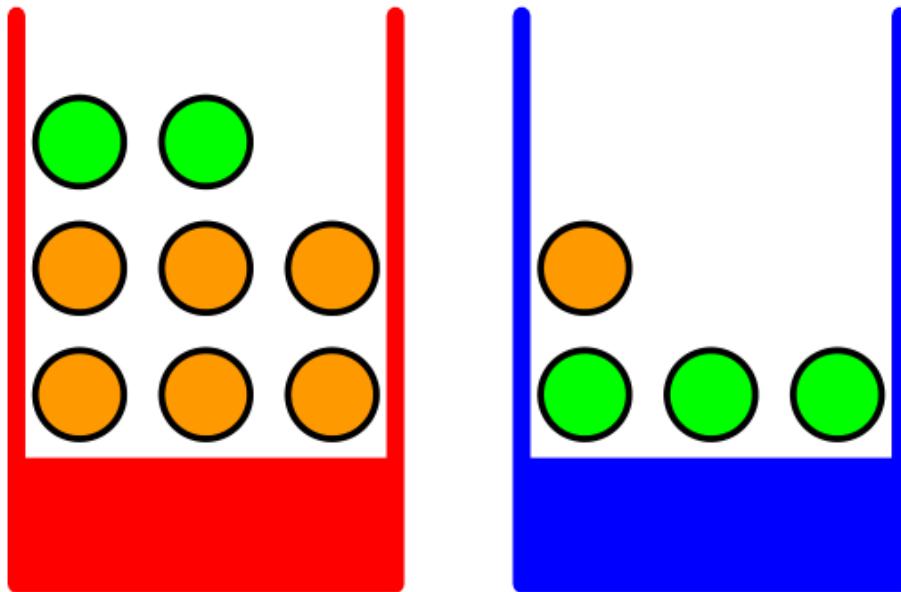
In this example, the **identity of the box** that will be chosen is a **random variable**, which we shall denote by **B**

Two Possible values
r (red box) and b (blue box)

Similarly, the **identity of the fruit** is also a **random variable** and will be denoted by **F**

Two Possible values
a (apple) and o (orange)

Probability



Box of Orange & Apple

ความน่าจะเป็นในการเลือกกล่องแต่ละลิ

$$p(B = r) = \frac{4}{10}$$
$$p(B = b) = \frac{6}{10}$$

*** ค่าของความน่าจะเป็นต้องอยู่ในช่วง $[0, 1]$

ตัวอย่างความที่เกิดขึ้น

- ความน่าจะเป็นที่จะหยิบได้ Apple คือเท่าใด
- ถ้าเราหยิบได้ Orange ความน่าจะเป็นที่หยิบจากกล่องสีน้ำเงิน (Blue) คือ เท่าใด

KEEP IT UP

