New York City Interscholastic Mathematics League

Senior A Division

Spring 2025

PART 1 **Spring 2025** **Time: 10 Minutes**

S25SA01 Find the smallest positive integer x for which

$$(3x)^{x^2} > (x+28)^{13x+30}$$

holds.

S25SA02 Regular polygons $ZERO$ and ONE are oriented in the plane so that R is inside ONE and $NZ = 2$. Find the area of $\triangle ORZ$.

PART 2 **Spring 2025** **Time: 10 Minutes**

S25SA03 Gi-hun has a box containing nine balls, three of which are red, three of which are yellow, and three of which are green. He removes one ball at a time from the box, without replacement. What is the probability that the first and last balls he removes are the same color?

S25SA04 Find the smallest positive integer n for which the leftmost digits of n, n^2, and n^3 are all equal to the same digit $d \neq 1$.

PART 3 **Spring 2025** **Time: 10 Minutes**

S25SA05 Find the smallest odd, composite, positive integer that is relatively prime to 2025. (Two integers are relatively prime to each other if they do not share any factors other than 1.)

S25SA06 On each edge of a square, five points are drawn to partition the edge into six congruent segments. Let S be the set containing these twenty points and the vertices of the square. How many ways are there to choose four distinct points from S such that they form a parallelogram with positive area?

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PART 1 **Spring 2025** **Time: 10 Minutes**

S25SA07 Aditya is flipping a fair coin. After each flip, if the number of heads flipped so far is equal to the number of tails flipped so far, Aditya earns one dollar. Find the probability that Aditya earns exactly one dollar after four flips.

S25SA08 Noam has four cubes, which have side lengths 1, 2, 5, and 10. He wishes to glue them together to form a single solid. What is the minimum possible surface area of the resulting solid?

PART 2 **Spring 2025** **Time: 10 Minutes**

S25SA09 Find the area of the set of all points (x, y) satisfying both of the following equations:

- $x^2 + y^2 \leq 1$
- $(x-1)^2 + (y-1)^2 \geq 1$

S25SA10 Find the sum of all primes dividing $12^5 + 13$.

PART 3 **Spring 2025** **Time: 10 Minutes**

S25SA11 Seven students took the AMC 12, and the average of their scores was 103.5. If the average of the lowest four scores was 81 and the average of the highest four scores was 123, find the median score.

S25SA12 Let $\triangle ABC$ be a triangle with $AB = AC$. Point E lies on $\overline{BC}$ such that $AE = BE$, and segment $\overline{BE}$ is rotated about A such that B coincides with C and E gets sent to point G. Given that the areas of $\triangle ABG$ and $\triangle ABC$ are 5 and 7 respectively, compute $\cos(\angle BAC)$.

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S25SA13 Aditya sees a bowl with 2025 candies and decides to take x of them. Mark, who has a sweet tooth, takes x^2 candies from the bowl. Aditya is horrified, and steals half of Mark's candies. Finally, Mark takes the rest of the candies in the bowl and says to Aditya, "Nice try, but I have one more candy than you." Find the value of x.

S25SA14 Thomas the tank engine is traveling at 60 mph down a straight track from point A to point B, starting at A. The distance between A and B is 5 miles, but there are four junctions in between them at 1, 2, 3, and 4 miles along the way. Each junction contains an "in" portal facing towards A, and an "out" portal facing towards B. Each time Thomas enters an "in" portal, he teleports randomly and uniformly to one of the "out" portals. What is the probability that Thomas is able to reach B within five minutes?

PART 2 **Spring 2025** **Time: 10 Minutes**

S25SA15 A sequence a_n is defined by $a_1 = \frac{37}{71}$ and $a_n = \frac{1}{1-a_{n-1}}$ for all $n \geq 2$. Find the value of a_{2025}.

S25SA16 Jun-ho and 455 other people are playing one round of the Octopus Game. In this game, a positive integer factor of 456 is chosen uniformly at random, and that many people are eliminated uniformly at random. Find the probability that Jun-ho remains.

PART 3 **Spring 2025** **Time: 10 Minutes**

S25SA17 Equilateral hexagon $SQUARE$ with side length 4 has all interior angles measuring less than $180°$. If four of its vertices are the vertices of a square, find its area.

S25SA18 Compute

$$\frac{23}{24} + \frac{26}{60} + \frac{29}{120} + \frac{32}{210} + \frac{35}{336} + \frac{38}{504} + \frac{41}{720} + \frac{44}{990}$$

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PART 1 — Spring 2025 — Time: 10 Minutes

S25SA19 In the following addition problem, all letters are digits from 0 to 9, inclusive, and $S \neq 0$.

$$\begin{array}{r} STAN \\ STA \\ ST \\ +S \\ \hline 3238 \end{array}$$

Find the value of the four-digit number $STAN$.

S25SA20 Find the smallest positive integer that ends in 9999 and is divisible by 97.

PART 2 — Spring 2025 — Time: 10 Minutes

S25SA21 Gary the grasshopper is at 0 on the number line. Gary can travel in either direction along the number line in two ways: short jumps of length 5, and long jumps of length 17. Find the smallest number of jumps Gary must make in order to land on 2025.

S25SA22 Triangle $\triangle ABC$ has circumcenter O. The bisector of $\angle BAC$ intersects $\overline{BC}$ at D, and the perpendicular bisector of $\overline{AD}$ intersects $\overline{BC}$ at E and $\overline{AO}$ at F. If $AD = 20$ and $DE = 25$, compute the length of $\overline{AF}$.

PART 3 — Spring 2025 — Time: 10 Minutes

S25SA23 In acute $\triangle ABC$, D is the foot of the altitude from A to $\overline{BC}$. If $AB = 9$, $BD = 1$, and $CD = 8$, find AC.

S25SA24 Find the sum of all positive integers $x < 10^4$ that have a digit sum of 7.

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PART 1 **Spring 2025** **Time: 10 Minutes**

S25SA25 A square with side length 8 and a circle have the same center. The circle intersects the sides of the square at 8 distinct points, which are the vertices of an octagon with area 46. What is the radius of the circle?

S25SA26 The outer surface of a white $3 \times 3 \times 3$ cube is painted red. Then, the cube is split into 27 unit cubes, one of which is chosen uniformly at random and rolled. Given that the chosen unit cube's top face is white, what is the probability that it has exactly one red face?

PART 2 **Spring 2025** **Time: 10 Minutes**

S25SA27 Let r and s be the zeroes of $x^2 + 5x + 1$. Find $\left(r + \frac{1}{r}\right)^3 + \left(r - \frac{1}{r}\right)^3 + \left(s + \frac{1}{s}\right)^3 + \left(s - \frac{1}{s}\right)^3$.

S25SA28 Find the largest integer that is equal to twice the sum of the squares of its digits.

PART 3 **Spring 2025** **Time: 10 Minutes**

S25SA29 A square pool table with side length 9 has a pool ball at one corner. Noam strikes the ball, which bounces off the rails twice before reaching the diagonally opposite corner. Find the total length of the ball's path.

S25SA30 Compute the number of triples of integers (a, b, c) such that $0 \leq a, b, c < 288$ and $a^n + bn + c$ is divisible by 288 for all positive integers n.