New York City Interscholastic Mathematics League

Senior A Division

Fall 2024

PART 1 **Fall 2024** **Time: 10 Minutes**

F24SA01 While reading the last 120 pages of a book, Aditya counted 417 total digits over all of the page numbers of the pages he read. How many pages does the book have? (Assume that the first page of the book is numbered 1, and that pages are numbered sequentially.)

F24SA02 Let $ABCD$ be a unit square and let M be the midpoint of side $\overline{CD}$. Point P is positioned on segment $\overline{AM}$ such that $PM = MC$. What is the area of quadrilateral $PBCM$?

PART 2 **Fall 2024** **Time: 10 Minutes**

F24SA03 Positive integers a and b satisfy $a^b + b^a = 2530$. Find the sum of all possible values of a.

F24SA04 How many ordered pairs of positive integers (x, y) satisfy $x < y \leq 2024$ and $\frac{\text{lcm}(x,y)}{\text{gcd}(x,y)} = 2024$?

PART 3 **Fall 2024** **Time: 10 Minutes**

F24SA05 Three rooks are randomly placed on an 8×8 chessboard such that no two rooks occupy the same square. Compute the probability that no two rooks attack each other. (Two rooks are said to attack each other if they are in the same row or the same column.)

F24SA06 Let $f(x) = x^2 - 20x + 24$. Find all real numbers x satisfying $f(f(x)) = x$.

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PART 1 — Fall 2024 — Time: 10 Minutes

F24SA07 Let $\tau(k)$ be the number of positive integers dividing k. For example, $\tau(1) = 1$ and $\tau(2025) = 15$. Compute $\tau(1) + \tau(2) + \tau(3) + \cdots + \tau(10)$.

F24SA08 How many ways are there to select three letters from the string NYCNYCNYCNYCNYCNYC (NYC repeated six times) so that they spell out "NYC", in that order?

PART 2 — Fall 2024 — Time: 10 Minutes

F24SA09 Compute the value of $\sqrt{3^2 + 4^2 + 12^2 + 84^2 - 77^2 + 48^2}$.

F24SA10 Let $\triangle ABC$ be equilateral. A semicircle ω with diameter $\overline{BC}$ is drawn such that the interiors of ω and $\triangle ABC$ do not overlap. Point P is chosen on segment $\overline{BC}$, and line $\overline{AP}$ intersects ω at Q. Given that $\frac{AP}{PQ} = 2$, compute the sum of all possible values of $\frac{BP}{CP}$.

PART 3 — Fall 2024 — Time: 10 Minutes

F24SA11 A word bank consists of the four words YOU, BELONG, WITH, and ME. Brandon selects one of the words uniformly at random, and then selects one of the letters in that word uniformly at random. What is the probability that Brandon selects a vowel? (Y is **not** considered a vowel.)

F24SA12 Let $\tau(k)$ be the number of positive integers dividing k. For example, $\tau(1) = 1$ and $\tau(2025) = 15$. Compute $\tau(1) + \tau(2) + \tau(3) + \cdots + \tau(100)$.

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F24SA13 A pentagon's angle measures form an arithmetic sequence. Its smallest angle measures $5x - 58$ degrees, and its second-largest angle measures $155 - x$ degrees. Find the measure of its largest angle, in degrees.

F24SA14 Find the sum of all prime factors of 8040201.

PART 2 **Fall 2024** **Time: 10 Minutes**

F24SA15 The $5! = 120$ positive integers whose digits are a permutation of 1, 2, 3, 4, and 5 are written down in increasing order. (So the first three terms are 12345, 12354, and 12435.) What is the 55^{th} number in this list?

F24SA16 Let P be a point on the graph of $x^2 + y^2 = 1$ and let Q be a point on the graph of $y = \sqrt{x^4 + 4x^3 + 5x^2 + 4x + 50}$. Find the smallest possible distance between P and Q.

PART 3 **Fall 2024** **Time: 10 Minutes**

F24SA17 A triangle with positive area has two sides whose lengths differ by 19. Find the minimum possible integer value of the perimeter of this triangle.

F24SA18 Let $\triangle ABC$ have angles $\angle B = 18^\circ$ and $\angle C = 72^\circ$. The altitude from A to $\overline{BC}$ intersects $\overline{BC}$ at D. What is the degree measure of the acute angle formed by line $\overline{BC}$ and the line through the incenters of $\triangle ABD$ and $\triangle ACD$?

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F24SA19 Circles ω_1 and ω_2, both with radius 1, are externally tangent to each other and are both internally tangent to another circle ω_3. If the centers of ω_1, ω_2, and ω_3 make an equilateral triangle, what is the area of ω_3?

F24SA20 A hunter and an invisible rabbit are playing a series of ping pong games, none of which end in a tie. The first player to win 3 games wins the series. They are equally skilled, so normally they both have a 50% chance of winning each game. However, due to self confidence issues, the rabbit's chances of winning a game drop to 25% if and only if the hunter has won strictly more games than the rabbit so far. Compute the probability that the rabbit wins the series.

PART 2 **Fall 2024** **Time: 10 Minutes**

F24SA21 Let $f(x) = x^2 - 10x + 28$ and $g(x) = f(f(f(x)))$. Compute $g(1) + g(4) - g(9)$.

F24SA22 Let $\mathcal{S}_1$ be a semicircle with diameter $\overline{AB}$ and center O. Point C lies on segment $\overline{AB}$ such that $AC = 20$ and $BC = 25$. Semicircle $\mathcal{S}_2$ is drawn with diameter $\overline{AC}$ in the region bounded by arc $\mathcal{S}_1$ and line $\overline{AB}$. Point P on $\mathcal{S}_2$ is equidistant from A and C, and circle ω is tangent to $\overline{OP}$ at P and internally tangent to $\mathcal{S}_1$. What is the radius of ω?

PART 3 **Fall 2024** **Time: 10 Minutes**

F24SA23 A right triangle has area 7 and hypotenuse 6. Find its perimeter.

F24SA24 For a positive integer n, define

$$f_n(x) = \sum_{k=1}^{n} \lfloor kx \rfloor.$$

What is the smallest integer n such that $f_n(x)$ achieves at least 50 values as x varies across the interval $(0, 1)$?

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F24SA25 Aditya randomly and uniformly chooses two distinct integers from the set $\{1, 2, 3, \ldots, 2025\}$. Find the probability that the sum of these two integers is even.

F24SA26 Andrew's K-pop playlist has an average song length of 2 minutes and 59 seconds. His friend recommends the song "Fatal Trouble" by ENHYPEN, which is 2 minutes and 50 seconds long. Andrew adds it to his playlist, and the average song length becomes 2 minutes and 58 seconds. Andrew likes the song so much that he also adds five similar songs, which total 16 minutes and 56 seconds in length. Now, what is his playlist's average song length, in seconds?

PART 2 **Fall 2024** **Time: 10 Minutes**

F24SA27 Let $ABCD$ be a trapezoid with area 588 and parallel bases $AB = 7$ and $CD = 17$. The line through A parallel to $\overline{BC}$ intersects line $\overline{CD}$ at E, and the line through B parallel to $\overline{AD}$ intersects $\overline{CD}$ at F. What is the area of the quadrilateral formed by A, B, E, and F?

F24SA28 How many positive integers $n \leq 2024$ are there such that $n^{22} - 1$ is divisible by 2024?

PART 3 **Fall 2024** **Time: 10 Minutes**

F24SA29 Real numbers x and y satisfy $x + y = xy = \frac{x}{y}$. Find the value of $x - y$.

F24SA30 Let S be the set of lattice points with $1 \leq x \leq 4$ and $1 \leq y \leq 6$. How many ways are there to color each of the points in S either red or blue such that, for every four distinct points in S that form a rectangle with sides parallel to the axes, not all of them are the same color?