TECHNICAL NOTE

MOMENT ARMS AND LENGTHS OF HUMAN UPPER LIMB MUSCLES AS FUNCTIONS OF JOINT ANGLES

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Abstract—Modeling of musculoskeletal structures requires accurate data on anatomical parameters such as muscle lengths (MLs), moment arms (MAs) and those describing the upper limb position. Using a geometrical model of planar arm movements with three degrees of freedom, we present, in an analytical form, the available information on the relationship between MAs and MLs and joint angles for thirteen human upper limb muscles. The degrees of freedom included are shoulder flexion/extension, elbow flexion/extension, and either wrist flexion/extension (the forearm in supination) or radial/ulnar deviation (the forearm in mid-pronation). Previously published MA/angle curves were approximated by polynomials. ML/angle curves were obtained by combining the constant values of MLs (defined by the distance between the origin and insertion points for a specific upper limb position) with a variable part obtained by multiplying the MA (joint radius) and the joint angle. The MAs of the prime wrist movers in radial/ulnar deviation were linear functions of the joint angle ($R^2 \geqslant 0.9954$), while quadratic polynomials accurately described their MAs during wrist flexion/extensions. The relationship between MAs and the elbow angle was described by 2nd, 3rd or 5th-order polynomials ($R^2 \geqslant 0.9904$), with a lesser quality of fit for the anconeus ($R^2 = 0.9349$). In the full range of angular displacements, the length of wrist, elbow and shoulder muscles can change by 8.5, 55 and 200%, respectively. Copyright © 1996 Elsevier Science Ltd.

Keywords: Biomechanic models; Human upper limb; Moment arms; Muscle lengths; Joint angles.

INTRODUCTION

In some biomechanical models, motor control is essentially associated with the anatomical arrangement of muscles (Feldman and Levin, 1993; Flanagan *et al.*, 1993; Flash and Mussa-Ivaldi, 1990). These and other models require precise data relating muscle lengths (MLs) and moment arms (MAs) with joint angles. Such data have not been presented in a systematic form for the human upper limb, and, in addition, were simplified by assuming that MAs are constant (Happee, 1994; Zajac *et al.*, 1984) and MLs are linear functions of joint angles (Flanagan *et al.*, 1993). Trigonometric models of the arm were also used to estimate MAs (Stern, 1971; Van Zuylen *et al.*, 1988; Yeo, 1976). In the present report, we derive the equations for the relationship between the MAs, MLs and joint angles for 13 upper limb muscles subserving three degrees of freedom (DFs) for horizontal planar arm movements: shoulder flexion/extension, elbow flexion/extension, and either wrist flexion/extension or wrist radial/ulnar deviation. The approach involves curve fitting previously published MA/angle data and formulation of equations for ML/angle curves.

METHODS

The equations relating MAs and joint angles were obtained using anatomical or model data (Amis *et al.*, 1979; Horii *et al.*, 1993; Winters and Kleweno, 1993; Wood *et al.*, 1989). The

MatLab Polyfit function (The MathWorks, Inc., 1992) was used for the fitting. Based on these equations, we then derived the equations for MLs. A ML was a function of the joint angles, MAs and also included a constant representing a portion of the length ($L_1 + L_3$ or $L_1 + L_3 + L_5$ in Fig. 1(A), similar to Fig. 2 in Winters and Stark, 1988). This constant was taken for each muscle from the measurements made by Seireg and Arvikar (1989) who used Braus' scaled diagrams (Braus, 1954). Seireg and Arvikar (1989) modeled muscles as strings between the points of origin and insertion in the bone-fixed coordinate system. They measured MLs for an erect body posture with the upper limb hanging vertically and forearm supinated. To obtain a ML for any limb position, we multiplied the MA (radius of the joint) by the joint angles and added it to the constant portion of the length defined above. Thus, if a MA for a DF was described by a second-order polynomial, the order of the polynomial for the ML increased by a unit. For a laterally extended limb (when the wrist radial/ulnar deviation as well as the wrist, elbow, and shoulder flexion/extensions are zero), the ML coincided with its constant portion [$q_j = 0°$, Fig. 1(A)]. Ulnar deviations, extensions of the wrist, and elbow and shoulder flexions were considered positive and the sign of the MAs remained constant in the full range of movements since no muscle changed the direction of its mechanical action [i.e. from flexion to extension, Fig. 1(B)].

Eight mono- and five bi-articular muscles contributing to torques about the three DFs with the arm held horizontally at shoulder level were considered: the flexor carpi ulnaris (FCU), extensor carpi radials brevis (ECRB), anconeus (AN), brachialis (BS), brachioradialis (BR), the clavicular portion of pectoralis (PC) and the anterior (DA) and posterior (DP) portions of the deltoid muscle; the flexor carpi radialis (FCR), extensor carpi radialis longus (ECRL), extensor carpi ulnaris (ECU), biceps brachii (BB) and triceps brachii (TB). The MA equations were obtained based on the available data for the wrist flexion/

extension and radial/ulnar deviation (Horii et al., 1993) and for the elbow flexion/extension (Amis et al., 1979; Winters and Kleweno, 1993). Data for the AN, FCR, ECU and TB were taken from Amis et al. (1979) and those for the BS, BR, ECRL and BB from Winters and Kleweno (1993). Two MA/angle BB curves (Winters and Kleweno, 1993) for elbow flexion/extension, one for pronation and the other for supination, were averaged to get a curve for a mid-pronation. MAs of the DA, DP, PC, BB and TB (Wood et al., 1989) measured on a cadaver for one specific position were used for shoulder flexion/extension. MAs

and MLs were described by the following equations:

$$MA_m^{DF} = x_n q_j^n + x_{n-1} q_j^{n-1} + \cdots + x_1 q_j + x_0, \qquad (1)$$

$$ML_m = cst + e_m^{wRU}(\text{or } e_m^{wFE}) + e_m^{eFE} + e_m^{sFE}$$

$$= cst + \sum_{j=1}^{3} (y_n q_j^n + y_{n-1} q_j^{n-1} + \cdots + y_1 q_j), \qquad (2)$$

where MA_m^{DF} is the moment arm, in mm, of muscle m for a particular DF; x_i stands for coefficients a_i, b_i, c_i or d_0 for wrist radial/ulnar deviation (wRU), wrist flexion/extension (wFE), elbow flexion/extension (eFE), or shoulder flexion/extension (sFE), respectively; i varies from 0 to n; n is the order of the polynomial fitting the data; q_j is the joint angle, in degrees (q_1, q_2, q_3 for wrist radial/ulnar deviation or wrist flexion/extension, elbow flexion/extension and shoulder flexion/extension, respectively), ML is the length of muscle m in mm; cst is the constant ML portion; e_m is the excursion of muscle m due to rotation in a specific DF; y_i stands for coefficients r_i, s_i, t_i or u_i for wrist radial/ulnar deviation, wrist flexion/extension, elbow flexion/extension, and shoulder flexion/extension; j is the DF causing the excursion.

MAs were functions of a single joint angle while MLs were functions of one or two joint angles for mono- and bi-articular muscles, respectively. Since we considered only horizontal planar movements of the upper limb held at shoulder level, the wrist DF in equation (2) depended on the forearm orientation (for a mid-pronated forearm, wrist radial/ulnar deviation was the DF producing horizontal planar movements of the hand; wrist flexion/extension did the same for a supinated forearm). The goodness of fit was estimated by the coefficient of determination, R^2 (correlation coefficient squared). The order of the interpolation polynomial was found when R^2 was not less than 0.99.

RESULTS

MAs of the FCU and ECRB were interpolated by linear functions for wrist radial/ulnar deviations, and quadratic polynomials for wrist flexion/extensions (Table 1). The slope (a_1) of MA/angle curves for ECRB and ECRL for wrist radial/ulnar deviations was small (Table 2) compared to other muscles.

The MLs of single-joint wrist muscles were calculated using equation (2). For example, the ML of the FCU (Table 3) for wrist radial/ulnar deviations is given by:

$$ML_{FCU} = cst + e_{FCU}^{wRU}$$

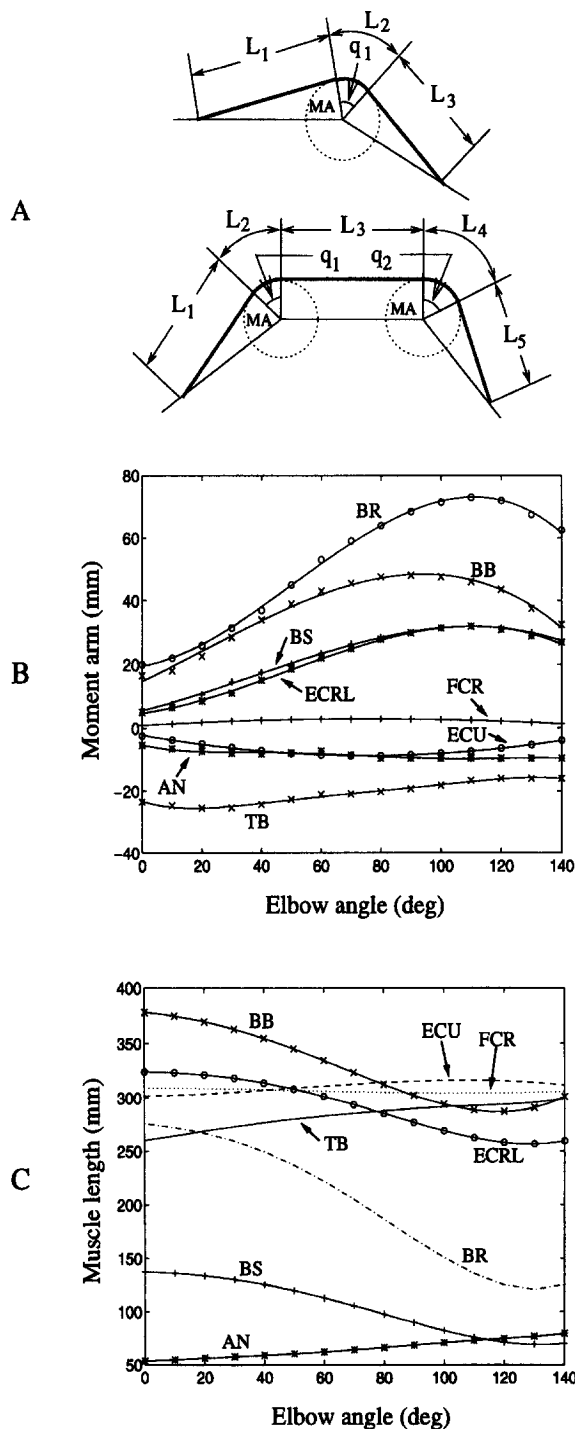$$= 289.40 + 5.1871 \times 10^{-3} q_1^2 - 0.2853 q_1. \qquad (3)$$

A

B

C

Fig. 1. (A) Schematic representation of the relationship between joint angles and MLs showing the constant portion of the length of mono- ($L_1 + L_3$) and bi-articular ($L_1 + L_3 + L_5$) muscles as well as the changes in the length (L_2 and L_4) due to joint rotation (q_j). (B) MA/angle curves of elbow muscles. Except for the FCR, the MAs of flexors are described by third-order polynomials. Fifth-order interpolations were used for the MAs of AN and TB while a quadratic one was used for the MA of the ECU. The MA of the BB is sensitive to forearm rotation and is shown here for a mid-pronated forearm. The symbols on the curve represent the previously published MA data (Amis et al., 1979; Winters and Kleweno, 1993) used in the interpolations. (C) ML/angle curves of elbow muscles. Some flexors shorten by only 1% (FCR) whereas others by 55% (BR). Some extensor MLs increase with elbow flexion by only 3% (ECU) and others by 49% (AN). The values at 0° (fully extended elbow) correspond to the constant portion of MLs (Table 3). Unlike in (B), the symbols on the curve are used to distinguish between different curves.

Table 1. Coefficients of determination (R^2) of the polynomials for MAs for wrist radial/ulnar deviation (wRU), wrist flexion/extension and elbow flexion/extension

						Coefficients of determination, R^2					
DF	wRU		Wrist flexion/extension				Elbow flexion/extension				
n	1	2	1	2	3	4	1	2	3	4	5
FCU	0.9998*	0.9998	0.6374	0.9913*	0.9979	0.9992					
ECRB	0.9978*	0.9978	0.9708	0.9969*	0.9996	0.9997					
AN							0.8016	0.8790	0.8792	0.9056	0.9349*
BS							0.8427	0.9783	0.9988*	0.9989	0.9991
BR							0.8528	0.9533	0.9989*	0.9994	0.9994
FCR	0.9989*	0.9989	0.7675	0.9957*	0.9961	0.9980	0.0438	0.9975*	0.9979	0.9981	0.9995
ECRL	0.9954*	0.9955	0.3823	0.9203	0.9944*	0.9944	0.8470	0.9596	0.9986*	0.9995	0.9996
ECU	0.9997*	0.9998	0.7476	0.9983*	0.9987	0.9998	0.0795	0.9966*	0.9979	0.9995	0.9999
BB							0.4669	0.9612	0.9918*	0.9972	0.9988
TB							0.9154	0.9270	0.9797	0.9837	0.9904*

Note: R^2 increased or remained constant with increasing order of the polynomial, n (the chosen order is indicated by an asterisk and, except for the AN, is defined as the minimal n at which $R^2 \geqslant 0.99$). Linear and quadratic interpolations were used for the wrist muscles (except for the ECRL) in radial/ulnar deviations and wrist flexion/extensions, respectively; second, third and fifth order polynomials were used for elbow MAs.

Table 2. Coefficients a_i, b_i, c_i and d_i of equation (1) for the MA/angle curves

							Moment arm coefficients							
DF	wRU		Wrist flexion/extension				Elbow flexion/extension						sFE
Muscle	a_1	a_0	$b_3 \times 10^6$	$b_2 \times 10^4$	$b_1 \times 10^2$	b_0	$c_5 \times 10^9$	$c_4 \times 10^7$	$c_3 \times 10^5$	$c_2 \times 10^3$	$c_1 \times 10^1$	c_0	d_0
FCU	-0.2972	16.349		8.3020	-2.6875	-16.247							
ECRB	-0.0842	-10.124		-1.8656	2.5315	10.780							
AN							-2.7306	10.448	-14.329	8.4297	-2.2841	-5.3450	
BS									-2.0530	2.3425	2.3080	5.5492	
BR									-6.5171	10.084	1.6681	19.490	
DA													33.02
DP													-78.74
PC													50.80
FCR	-0.1445	-6.6164		10.111	-3.6152	-15.079			-3.0229	-0.3627	0.5375	0.9351	
ECRL	0.0411	-17.475	4.7731	-3.2685	0.0892	7.9605				4.4347	1.2590	4.7304	
ECU	-0.2205	19.726		-9.2926	3.6494	6.0834				1.1491	-1.7386	-2.1826	
BB									-2.9883	1.8047	4.5322	14.660	29.21
TB							-3.5171	13.277	-19.092	12.886	-3.0284	-23.287	-25.40

Note: The first eight muscles are mono-articular and the last five are bi-articular. Depending on forearm orientation, the MAs for either wrist radial/ulnar deviations (forearm mid-pronated) or wrist flexion/extensions (forearm supinated) were used. Constant values in the last column reflect the lack of experimental data on the variations of shoulder MAs during planar shoulder flexion/extensions. For abbreviation, some coefficients of MAs were multiplied by powers of 10 (e.g for the FCU, $b_2 \times 10^4 = 8.3020$, thus $b_2 = 8.3020 \times 10^{-4}$).

Table 3. Coefficients r_i, s_i, t_i and u_i of equation (2) for ML/angle curves computed based on the MA/angle data

		Muscle length coefficients												
	DF	wRU			Wrist flexion/extension			Elbow flexion/extension						sFE
Muscle	cst(mm)	$r_2 \times 10^3$	r_1	$s_4 \times 10^8$	$s_3 \times 10^6$	$s_2 \times 10^4$	s_1	$t_6 \times 10^{11}$	$t_5 \times 10^8$	$t_4 \times 10^7$	$t_3 \times 10^5$	$t_2 \times 10^3$	$t_1 \times 10^2$	$u_1 \times 10^1$
FCU	289.40	5.1871	−0.2853		−14.490	4.6906	0.2836							
ECRB	316.03	1.4696	0.1767		3.2561	−4.4183	−0.1881							
AN	53.57							4.7658	−1.8235	25.008	−14.713	3.9865	9.3288	
BS	137.48									3.5832	−4.0884	−4.0282	−9.6852	
BR	276.13									11.374	−17.600	−2.9114	−34.017	
DA	172.84													−5.7631
DP	157.64													13.743
PC	155.19													−8.8663
FCR	309.19	2.5220	0.1155		−17.647	6.3097	−0.2632				0.6330	−0.9381	−1.6321	
ECRL	324.06	−0.7173	0.3050	−8.3306	5.7046	−0.1557	−0.1389			5.2706	−7.7400	2.1974	−8.2561	
ECU	301.90	3.8485	−0.3443		16.219	−6.3694	−0.1062				−2.0056	3.0344	3.8094	
BB	378.06									5.2156	−3.1498	7.9101	−25.587	−5.0981
TB	260.05							6.1385	−2.3174	33.321	−22.491	5.2856	40.644	4.4331

Note: Values in the second column (cst) give constant portion of MLs for a laterally extended limb (the wrist radial/ulnar deviation angle as well as the wrist, elbow, and shoulder flexion/extension angles are 0°). As in Table 2, for abbreviation, some coefficients of MLs were multiplied by powers of 10.

In particular, when $q_1 = 5°$, equation (3) yields $ML_{FCU} = 288.10$ mm. FCU shortens as the wrist moves into ulnar deviation. The lengths of the FCU and ECRB change, respectively, by 3 and 1.5% for the full range of wrist radial/ulnar deviations or by 8.5 and 5.5% for full wrist flexion/extensions.

The MAs of single-joint elbow muscles (BS and BR) were described by third-order polynomials (Table 1). The R^2 value for the MA of AN reached the threshold 0.99 limit at $n = 11$. The R^2 values were, however, very close to the threshold at $n = 5$, and we chose this order for the AN interpolation. The same order was used for the interpolation of the MA of TB for elbow flexion/extensions. For other muscles, the R^2 values were greater than 0.99 when the order of the polynomials was 3 or less. For full elbow flexions, the ML of the AN increased by 49% while MLs of the BS and BR shortened by 49% and 55%, respectively [Fig. 1(C)].

The MAs for DA, DP and PC for shoulder flexion/extensions were constant (Table 2) resulting in linear ML functions. The ML of DP at 100° of shoulder flexion was twice as much as the ML at −45° while the opposite was true for the PC.

The MAs of double-joint muscles (FCR, ECRL and ECU) for wrist radial/ulnar deviations and wrist flexion/extensions were approximated by linear and quadratic polynomials, respectively, except for the MA of the ECRL for wrist flexion/extensions which required a third order interpolation. The MLs changed by 1–7% during the wrist full rotation. For the same muscles acting at the elbow joint, the second and third orders were used for interpolations (Table 1). The MA of the FCR appeared nearly constant for elbow flexion/extensions but a good fit was only reached when a second order interpolation was used. The MLs of the FCR and ECU changed less than 5% with elbow flexion/extensions, while ECRL by 20% [Fig. 1(C)].

The MAs of the BB and TB for elbow flexion/extensions were described by a third and a fifth order curve, respectively. The MA of the BB changed substantially more than that of the TB [Fig. 1(B)]. The BB shortened by 20% and 18% during elbow and shoulder flexions, respectively, while the TB lengthened by 15% and 27% [Fig. 1(C)].

The MA and ML equations (Tables 2 and 3) were valid in the ranges: − 17° to 10°, for wrist radial/ulnar deviations; − 45° to 45°, for wrist flexion/extensions; 0° (complete extension) to 140°, for elbow flexion/extensions; and − 45° to 100° for shoulder flexion/extensions.

DISCUSSION

The models of motor control tend to become more complex by integrating the mechanical, control and reflex mechanisms (Feldman and Levin, 1993; Flanagan et al., 1990; Flash and Mussa-Ivaldi, 1990), requiring, in particular, an improvement of the mathematical description of anatomical structures (Winters and Stark, 1988). In the present study, based on previously published data we derived the equations describing MA/angle and ML/angle relationships for 13 muscles and planar arm movements. Our results support the notion that the MAs of upper limb muscles depend considerably on position and that the assumption of constant MAs is inaccurate for all muscles except for the ECRB and ECRL for wrist radial/ulnar deviations. Furthermore, the assumption that the MAs of elbow flexors vary sinusoidally with elbow angle suggested in trigonometric models (Van Zuylen et al., 1988; Yeo, 1976) results in an error by predicting negligible MAs near full extension. The error in these models occurs since they ignore the fact that muscle tendons arc the bony prominences and surrounding soft tissues (Winters and Stark, 1988; Kleweno and Winters, 1988).

The equations presented here have limitations partly related to those of the data used in the present study. For example, the data on the MAs of shoulder muscles measured on an embalmed cadaver (Wood et al., 1989) do not account for variations in the MAs during planar shoulder flexion/extensions. In contrast, during abduction, the MAs of the anterior and posterior portions of the deltoid are reported to vary between 19.9 and

15.8 mm, and 2.3 and 25.6 mm, respectively (Pio et al., 1993), thus suggesting that the MA of this muscle may also change during horizontal planar motion at the shoulder. In situ, the constant portion of MLs (Seireg and Arvikar, 1989) depends on the initial limb position. For example, when the arm is raised, the humeral ball translates laterally increasing the initial ML of the BB. Full pronation stretches the ML of BB up to 2–3 cm (Winters and Kleweno, 1993); thus, during wrist radial/ulnar deviations when the forearm is mid-pronated, the ML of BB increases. The MA of the BB during elbow flexion/extensions depends somewhat on the initial forearm position (Winters and Kleweno, 1993). Neglecting this dependency, one may underestimate the ML by about 5 mm when the elbow angle is 140° and the forearm is supinated. Finally, muscles with multiple heads (the BB and TB) are considered to have unique MAs and MLs. Actually, only the long head of the TB spans the shoulder joint, and the MAs of the two heads of the BB have disparity of 2.5 mm during shoulder flexion/extensions (Wood et al., 1989).

The MAs of muscles during wrist flexion/extensions described by Jacobson et al. (1993) resemble those obtained by Horii et al. (1993). The MAs measured for elbow flexion/extensions (An et al., 1984; Jorgensen and Bankov, 1971) and wrist radial/ulnar deviations and flexion/extensions (Ohnishi et al., 1991; Tolbert et al., 1985) are within the range given by our equations. The maximal and minimal values of the MA of the TB are reached when the elbow joint angle is about 30° and 115°, respectively (Gerbeaux et al., 1993; Murray et al., 1995). The peak value of the MAs of elbow flexors (the BR, BB and BS) is reached when the angle is greater than 75° (Murray et al., 1995), which resembles to our interpolations [Fig. 1(B)]. The experimental shoulder MLs measured at one position (Van der Helm and Veenbaas, 1991) are consistent with our values. Also, the large changes in the MLs of the DP and PC for shoulder flexion/extensions in our equations are consistent with the published data for humeral abduction (Van der Helm, 1994).

Our analysis shows that the more a MA varies with joint angle, a higher order of polynomial is required to accurately describe the ML/angle relationship. A good example is the MA of the BB at the elbow [Fig. 1(B)] for which a linear interpolation results in a low R^2 value (0.4669, Table 1). An acceptable accuracy is reached when the order of the MA curve is 3 so that the order of the ML/angle curve is 4 [Fig. 1(C)]. Generally, elbow MAs require a higher order interpolation than wrist MAs. MAs are maximal for the elbow flexors. Wrist MAs vary linearly with radial/ulnar deviation, and for all wrist muscles except ECRL, MA/angle relationships for wrist flexion/extensions are quadratic. The MA/angle curves of shoulder muscles for horizontal flexion/extensions at the shoulder are likely linear as is the case for adduction/abductions (Kuechle et al., 1993; Pio et al., 1993). Changes in MLs during full joint excursions may be small (e.g. 1% for the FCR during elbow flexion/extensions) and large (200% for the DP during shoulder flexion/extensions).

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