Data-Driven Modeling of Landau Damping by Physics-Informed Neural Networks

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Abstract

In this report, we successfully simulate the Landau damping using finite difference algorithm. The contruction of physics-informed neural network (PINN) is also constructed.

1 Introduction

Why are we doing this?

2 Landau Damping

Consider a collisionless plasma in the absence of magnetic field, the dynamics of the plasma can be described by the particle distribution $f_s(\mathbf{x}, \mathbf{v}, t)$, where $s = \{electron, ion\}$ stands for species. The evolution of distribution function is governed by the Vlasov equation,

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \boldsymbol{\nabla}_\mathbf{r} f_s + \frac{q_s}{m_s}\mathbf{E} \cdot \boldsymbol{\nabla}_\mathbf{v} f_s = 0 \tag{1}$$

where $\boldsymbol{\nabla}_\mathbf{r} = (\partial_x, \partial_y, \partial_z)$ and $\boldsymbol{\nabla}_\mathbf{v} = (\partial_{v_x}, \partial_{v_y}, \partial_{v_z})$. In this report, we consider one-dimensional model in $x - v_x$ space for convenience.

To complete the system, we introduce the Poisson equation for electric potential,

$$\nabla^2 \phi = -\rho/\epsilon_0 \tag{2}$$

For convenience, we set the vacuum permittivity $epsilon_0 = 1$, and the charge density ρ is defined by,

$$\rho = \sum_s q_s n_s \tag{3}$$

where the number density $n_s(x, t)$ is defined as

$$n_s(x, t) = \int f_s(x, v_s, t) dv_s \tag{4}$$

By Gauss's law, we can get electric field,

$$E(x,t) = -\nabla\phi \tag{5}$$

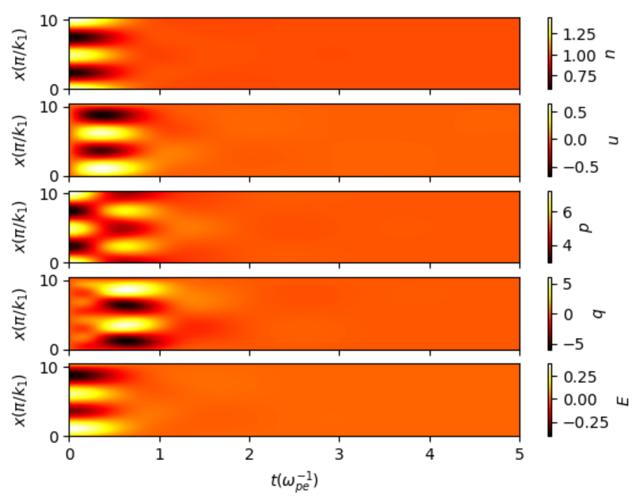
Equations (1)-(5) forms a complete PDE system. We can investigate the evolution of distribution $f_s(x, v_s, t)$.

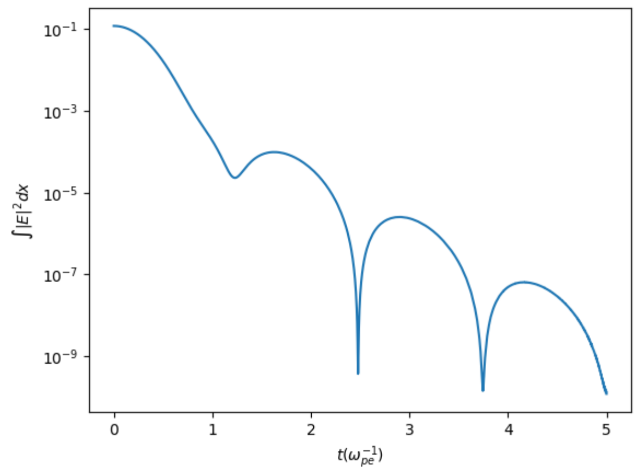
Starting from a plasma at equilibrium. We assume the ions are too heavy to move in a short period of time. Then two perturbation modes is given to the number density of electron. Then the initial number densities of ions and electrons are

$$n_e(x, t=0) = n_0(1 + A_1\cos(k_1x) + A_2\cos(k_2x + \varphi)) \tag{6}$$
$$n_i(x, t=0) = n_0 \tag{7}$$

where n_0 is the number density of each species at equilibrium. k_1 and k_2 are the wavenumbers of the two perturbation modes, A_1 and A_2 are their amplitudes, and φ is a random phase.

The result of simulation is shown below.





3 Physics-Informed Nueral network (PINN)

4 Methodology

4.1 Data Preparation

4.2 Construction of PINN

4.3 Training the PINN

5 Result

References

[1] Yilan Qin, Jiayu Ma, Mingle Jiang, Chuanfei Dong, Haiyang Fu, Liang Wang, Wenjie Cheng, and Yaqiu Jin. Data-Driven Modeling of Landau Damping by Physics-Informed Neural Networks, November 2022. arXiv:2211.01021 [astro-ph, physics:physics].