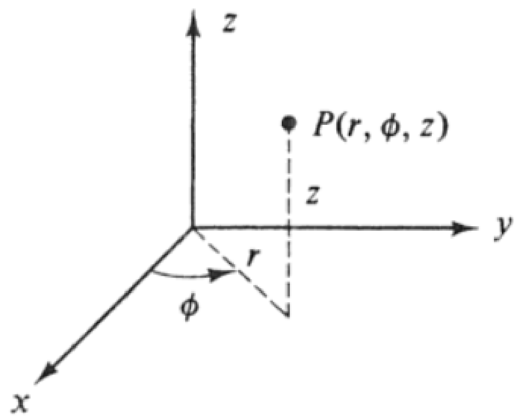
1 Rectangular coordinate system

This is trivial, there are three coordinates, they are x, y and z, respectively. A vector in this coordinate with a tip at position (x,y,z) and tail at the origin is usually denoted by (x, y, z) or $\langle x, y, z\rangle$ or 3 by 1 matrix $[x, y, z]^T$.

the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) can be calculated by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

2 Cylindrical coordinate system



In cylindrical coordinate, instead of using x, y and z as coordinates we express the vector in terms of the length of the vector r, azimuth angle ϕ, and height z, i.e. (r, ϕ, z). The transformation is shown below:

$$\begin{aligned}
x &= r\cos\phi \\
y &= r\sin\phi \\
z &= z
\end{aligned}$$

or

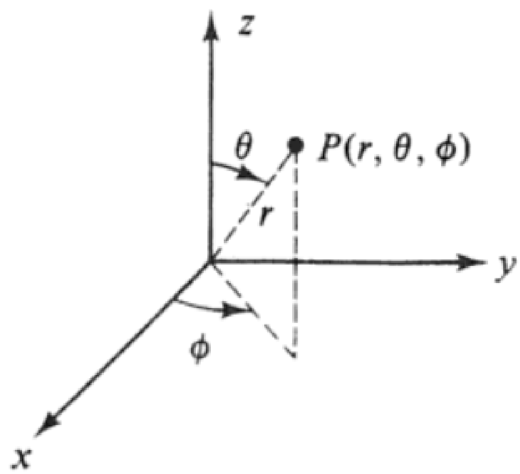
$$\begin{aligned}
r &= \sqrt{x^2 + y^2},\ r \in [0, \infty) \\
\phi &= \arctan(y/x),\ \phi \in [0, 2\pi) \\
z &= z
\end{aligned}$$

Example 1. *A vector in rectangular has an expression* $(1, 1, 1)$*, then if we use* *cylindrical coordinate to express it, then*

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\phi = \arctan(1/1) = \frac{\pi}{4}$$
$$z = 1$$

Thus the representation of this vector in cylindrical system is $(\sqrt{3}, \frac{\pi}{4}, 1)$.

3 Spherical coordinate system



The representation of a vector in spherical coordinate system is (ρ, θ, ϕ). Where ρ is the length of the vector, θ the polar angle, and ϕ the azimuth angle. The transformation between rectangular coordinate and spherical coordinate is given by

$$x = \rho \sin\theta \cos\phi$$
$$y = \rho \sin\theta \sin\phi$$
$$z = \rho \cos\theta$$

or

$$\rho = \sqrt{x^2 + y^2 + z^2}, \ \rho \in [0, \infty)$$
$$\theta = \arccos(z/\rho)$$
$$\phi = \arctan(y/x)$$

Example 2. *A vector in spherical coordinate has a representation of* $(4, \pi/3, 0)$*. Then its representation in rectangular coordinate will be given by*

$$\begin{aligned} x &= 4\sin(\pi/3)\cos(0) = 2\sqrt{3} \\ y &= 4\sin(\pi/3)\sin(0) = 0 \\ z &= 4\cos(\pi/3) = 2 \end{aligned}$$

Thus its representation in rectangular coordinate is $(2\sqrt{3}, 0, 2)$*.*