

Definition 1. Dot product can be defined between two vectors. Suppose two vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$. Then

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$

or

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

where θ is the acute angle between two vectors.

Property.

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \quad (1)$$

$$\vec{u} \cdot \vec{v} = 0 \text{ (if they are othorgonal)} \quad (2)$$

$$(\vec{u} \cdot \vec{v}) \cdot \vec{w} = \vec{u} \cdot (\vec{v} \cdot \vec{w}) \quad (3)$$

$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w} \quad (4)$$

Note: The result of dot product is a scalar.

Definition 2. Cross product can be defined between two vectors. Suppose two vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$. Then

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

or

$$|\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin \theta$$

where θ is the acute angle between two vectors.

Property.

$$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u} \quad (5)$$

$$\vec{u} \times \vec{v} = 0 \text{ (if they are parallel)} \quad (6)$$

$$(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w}) \quad (7)$$

$$\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w} \quad (8)$$

Note: The result of a cross product is a vector, its direction can be determined by right-hand rule.