Definition 1. *If a discrete random variable X is distributed according Poisson distribution, then it has pmf*

$$p_X(x) = \left\{ \begin{array}{cc} \lambda^x \frac{e^{-\lambda}}{x!} & , x = 1, 2, ... \\ 0 & , otherwise \end{array} \right.$$

where $\lambda \in \mathbb{R}^+$ is the parameter of Poisson distribution, and we denote $X \sim Poisson(\lambda)$. Again Poisson distribution is not just one distribution, but a family of a set of distributions with parameter λ.

Poisson distribution is especially good for modeling rare events.
Poisson distribution also has that two properties.

Property 1. $p(x) \geq 0, \ \forall x$

Property 2. $\sum_x p(x) = 1$

Proof. First one is obvious, I am going prove the second one.

$$\sum_{x=0}^{\infty} \lambda^x \frac{e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} e^{\lambda} = 1$$

$\square$