Definition 1. ***Predicate*** *is a statement that its truth value depends on one or more variables. The set of values that can assign to the variables are called the* ***domain****(or* ***universe****) of the variable.*

Example 1.

$$p(x) \equiv x \textit{ is odd. } x \in \mathbb{N} \tag{1}$$
$$q(x,y) \equiv x < y \quad x,y \in \mathbb{R} \tag{2}$$
$$r(x,y,z) \equiv x + y = z \quad x,y,z \in \mathbb{Z} \tag{3}$$

These are all predicates since their truth values are all depend on the variables.

Definition 2. *The universal quantifier,* $\forall$*, means: for all, every, any, etc...*

Example 2.

$$\forall x \in \mathbb{N}, x \textit{ is an integer.} \tag{4}$$
$$\forall x \in \mathbb{R}, x^2 + 1 > 0 \tag{5}$$

Definition 3. *The existential quantifier,* $\exists$*, means: for some, there is, exists, at least one, etc...*

Example 3.

$$\exists x \in \mathbb{Z} \textit{ such that } x \textit{ is even} \tag{6}$$
$$\exists x \in \mathbb{N} \textit{ such that } x > 5 \tag{7}$$

Property 1.

$$\neg(\forall x, p(x)) \Leftrightarrow \exists x \textit{ s.t. } \neg p(x)$$
$$\neg(\exists x \textit{ s.t. } p(x)) \Leftrightarrow \forall x, \neg p(x)$$