1 Average Rate of Change

Definition 1. *The average rate of change of a continuous function $f(x)$ during the time interval $[a, b]$ is defined by*

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example 1. *Suppose a car is moving with constant velocity, and its position can be described by function $r(t) = 5t$ (t in second, r in meter). Then its average rate of change(in this case, average velocity) of the car during the time interval $[0, 2]$ can be calculated by*

$$\frac{\Delta r}{\Delta t} = \frac{r(2) - r(0)}{2 - 0} = \frac{10 - 0}{2 - 0} = 5$$

In other words, the average velocity of the car is 5m/s.

Example 2. *Suppose a chemical reaction occurs between some materials A and B, and the mass of the material A can be described by a function $m_A(t) = \frac{5}{t+1}$ (t in second, mass in gram). Then the average rate of change(in this case, average reaction rate) during the time interval $[1, 2]$ can be computed by*

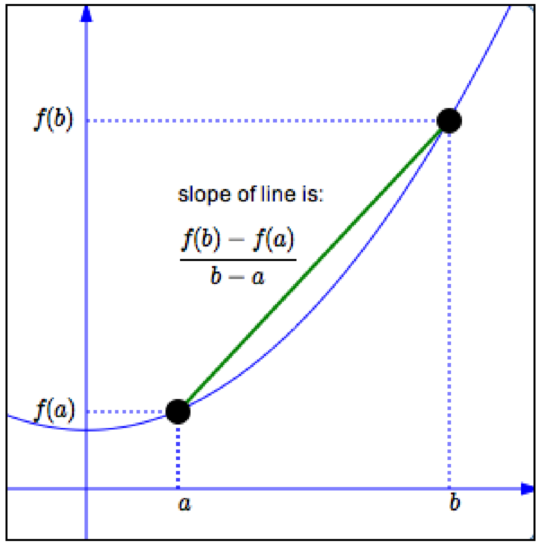
$$\frac{\Delta m_A}{\Delta t} = \frac{m_A(2) - m_A(1)}{2 - 1} = \frac{5/3 - 5/2}{1} = -\frac{5}{6}$$

The average reaction rate of this reaction is $-\frac{5}{6}$gram/s.

2 Geometrical Interpretation

Definition 2. *The slope of a secant line that intersecting the function $f(x)$ at two points $(a, f(a))$ and $(b, f(b))$ can be calculated by*

$$slope = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$



Example 3. *Draw a secant line that intersects the function $f(x) = x^2$ at $x = 1$ and $x = 2$, then its slope is*

$$\frac{\Delta y}{\Delta x} = \frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = 3$$

The average rate of change of a function $f(x)$ during the time interval $[a, b]$ is the same as the slope of the secant line intersecting the function $f(x)$ at points $x = a$ and $x = b$.