The general rule of computing expectation (or easier word, mean) of some real value $g(x)$, is defined by

$$E(g(X)) = \sum_x g(x)p_X(x)$$

Expectation has a very nice property, it is a linear operator.

**Property 1.**

$$E\left(\sum_{i=1}^{n} a_i g_i(X)\right) = \sum_{i=1}^{n} a_i E(g_i(x))$$

*Where* $a_i, g_i(x) \in \mathbb{R}, \forall i = 1, 2, ..., n.$

*Proof.*

$$\begin{aligned}
E\left(\sum_{i=1}^{n} a_i g_i(X)\right) &= \sum_x \left(\sum_{i=1}^{n} a_i g_i(X)\right) p_X(x) \\
&= \sum_x (a_1 g_1(x) + a_2 g_2(x) + \cdots + a_n g_n(x))p_X(x) \\
&= a_1 \sum_x g_1(x)p_X(x) + a_2 \sum_x g_2(x)p_X(x) + \cdots + a_n \sum_x g_n(x)p_X(x) \\
&= \sum_{i=1}^{n} a_i E(g(X))
\end{aligned}$$

□

Thus the expectation value of $X$ is defined by

$$E(X) = \sum_x x p_X(x)$$

Since the expectation value is linear, we easily see that

**Property 2.**

$$E(aX + b) = aE(X) + b, \quad a, b \in \mathbb{R}$$

This property states that if a random variable $X$ is transformed linearly to $aX + b$, then the expectation value of this new random variable is just the transformed version of the original one, namely, $aE(X) + b$.

Geometrically, if all data on the histogram is transformed to some where else, then the measure of location(in this case, mean) will be also transformed in the same manner.

The variance of the $X$ is defined by

$$\begin{aligned}
Var(X) &= E\left((X - E(X))^2\right) \\
&= E(X^2) - (E(X))^2 \\
&= \sum_x x^2 p_X(x) - \left(\sum_x x p_X(x)\right)^2
\end{aligned}$$

Variance also has a meaningful property due to the linearity of expectation.

**Property 3.**

$$Var(aX + b) = a^2 Var(X), \quad a, b \in \mathbb{R}$$

*Proof.*

$$\begin{aligned} Var(aX + b) &= E((aX + b)^2) - (E(aX + b))^2 \\ &= E((aX)^2) + E(2abX) + E(b^2) - (aE(X) + b)^2 \\ &= a^2 E(X^2) + 2abE(X) + b^2 - a^2(E(X))^2 - 2abE(X) - b^2 \\ &= a^2(E(X^2) - (E(X))^2) \\ &= a^2 Var(X) \end{aligned}$$

□

This property states that if data is being shifted left or right, this spread of the data would not changed, thus the value $b$ does not have effect on variance; however, if the scale of the data is shrink or stretched, then the spread of the data would also be changed, thus the value $a$ has effect on variance.