

Theorem 1. ***u-substitution** is the reverse application of the chain. Suppose $F'(x) = f(x)$ and then the derivative of a composite function can be calculated by chain rules*

$$\frac{d}{dx}F(u(x)) = f(u(x))u'(x)$$

Thus if $f(u(x))u'(x)$ is going to be integrated, then

$$\int f(u(x))u'(x)dx = \int \frac{d}{dx}F(u(x))dx = F(u(x)) + C$$

Example 1. *Find the indefinite integral of $f(x) = \cos 2x$.*

$$\int \cos 2x dx$$

Let $u = 2x$, then $du = 2dx$. Thus

$$\int \cos 2x dx = \int \cos u \frac{du}{2} = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2x + C$$

Example 2. *Find the integral of $f(x) = x^2(x^3 - 2)^3$.*

$$\int x^2(x^3 - 2)^3$$

Let $u = x^3 - 2$, then $du = 3x^2 dx$. Hence

$$\int x^2(x^3 - 2)^3 = \int x^2(u)^3 \frac{du}{3x^2} = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}(x^3 - 2)^4 + C$$