Definition 1. *A mapping, φ, defined on set D, is a rule that assign each element in D a value. The set D is called the domain of the mapping, and the set of values we get (let's give it a name, R) after applying the mapping is called the range of the mapping. We denote this by*

$$\varphi : D \to R$$

Definition 2. *A function $f : D \to R$ is a special mapping in which $\forall x \in D$, there is only one $y \in R$ such that $y = f(x)$.*

Example 1.

$$\begin{aligned}
&y = 3x - 1,\ D = \mathbb{R} && \textit{is a function}\\
&y = x^2 + 2x - 1,\ D = \mathbb{R} && \textit{is a function}\\
&x^2 + y^2 = 1,\ D = [-1, 1] && \textit{is not a function}\\
&y^2 = x\ D = \mathbb{R}^+ && \textit{is not a function}
\end{aligned}$$

Definition 3. *A function $f : D \to R$ is said to be 1-1 if $\forall y \in R$, there is only one $x \in D$ such that $y = f(x)$.*

Example 2.

$$\begin{aligned}
&y = 3x - 1,\ D = \mathbb{R} && \textit{is a 1-1 function}\\
&y = x^2 + 2x - 1,\ D = \mathbb{R} && \textit{is not a 1-1 function}\\
&y = x^2 + 2x - 1,\ D = \mathbb{R}^+ && \textit{is a 1-1 function}
\end{aligned}$$

Definition 4. *An inverse function of a function $f : D \to R$ is a mapping from R to D, and we denote it as $f^{-1} : R \to D$. Its action is defined by*

$$f^{-1}(f(x)) = x$$

Example 3. *Let $f(x) = 3x + 1$, then its inverse function f^{-1} can be calculated as follow:*

$$\begin{aligned}
&y = 3x + 1 && \textit{change the } f(x) \textit{ symbol to } y\\
&x = \frac{y - 1}{3} && \textit{express } x \textit{ in terms of } y\\
&y = \frac{x - 1}{3} && \textit{switch the symbol of } x \textit{ and } y\\
&f^{-1}(x) = \frac{x - 1}{3} && \textit{change the symbol } y \textit{ to } f^{-1}(x)
\end{aligned}$$

What inverse function does is the inverse action of the original function. For example:

$$\begin{aligned}
&f(1) = 4\ \ (f \textit{ maps 1 to 4}) && f^{-1}(4) = 1\ \ (f^{-1} \textit{ maps 4 back to 1})\\
&f(2) = 7\ \ (f \textit{ maps 2 to 7}) && f^{-1}(7) = 2\ \ (f^{-1} \textit{ maps 7 back to 2})
\end{aligned}$$

Proposition 1. *The graph of the function and its inverse are symmetric about the $y = x$ line.*

Proof. It is not hard to see that if the graph of the function consists of points of the form (x, y), where $x \in D$ and $y \in R$, then the inverse function takes the values in R and maps them back to D, thus the graph of the inverse function is consisted of points of the form (y, x). □

Example 4. *The following is the graph of $f(x) = 3x + 1$ and its inverse.*