There are three types of discontinuities.

1 Removable Discontinuity

Definition 1. *A function $f(x)$ has a removable discontinuity at $x = x_0$ if*

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) \neq f(x_0)$$

Example 1. *the rational function*

$$f(x) = \frac{x^2 + 2x + 1}{x + 1}$$

has a removable discontinuity at $x = -1$.

Since at $x = -1$ this is a hole. So the left right limits exist and equal, but the function value is missing.

!!!!!!!!!!!!!!!!!!!!!! desmos graph !!!!!!!!!!!!!!!!!!!!!!!!!!

2 Jump Discontinuity

Definition 2. *A function is said to have a jump discontinuity at $x = x_0$ if*

$$\lim_{x \to x_0^-} f(x) \neq \lim_{x \to x_0^+} f(x)$$

Example 2. *The piecewise function*

$$f(x) = \begin{cases} x, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$$

has a jump discontinuity at $x = 0$. Since $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$, yet $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = 1$.

!!!!!!!!!!!!!!!!!!!!!! desmos graph !!!!!!!!!!!!!!!!!!!!!!!!!!

3 Essential Discontinuity

Definition 3. *A function has essential discontinuity at $x = x_0$ if one (both) of one-sided limits does(do) not exist or be infinity.*

Example 3.

$$f(x) = \sin\left(\frac{1}{x}\right)$$

has an essential discontinuity at $x = 0$, since both left and right limits at $x = 0$ do not exist.

!!!!!!!!!!!!!!!!!!!!!! desmos graph !!!!!!!!!!!!!!!!!!!!!!!!!!