Definition 1. *We say that a function's limit exits at $x = x_0$,i.e*

$$\lim_{x \to x_0} f(x) = L$$

if this statement is true: $\forall \epsilon > 0, \exists \delta > 0$ *such that if* $0 < |x - x_0| < \delta$*, then*

$$|f(x) - L| < \epsilon$$

Example 1. *Let* $f(x) = \frac{(x+1)(x-1)}{(x+1)x}$.

!!!!!!!!!!!!!!!! demos graph !!!!!!!!!!!!!!!!!!!!!!

We see that from the graph

$$\lim_{x \to 1} f(x) = 0$$

$$\lim_{x \to 0} f(x) = DNE$$

Notes: The limit of a function at some points x_0 has nothing to do with the function value at that point.

Theorem 1.

$$\lim(f(x) \pm g(x) = \lim f(x) \pm \lim g(x)$$
$$\lim(f(x) \times g(x) = \lim f(x) \times \lim g(x)$$
$$\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$$

Example 2.

$$\lim_{x \to 0}(\cos x + \sin x) = \lim_{x \to 0} \cos x + \lim_{x \to 0} \sin x = 1 + 0 = 1$$

Example 3.

$$\lim_{x \to 1} \frac{x^2}{x+1} = \frac{\lim_{x \to 1} x^2}{\lim_{x \to 1}(x+1)} = \frac{1}{2}$$