There are few important transformations we have to know. We first start by talking about linear transformation of a function.

Linear Transformations

By apply the linear transformations on the graph, shifts, stretch and compression:

$$f(x) \to af(b(x+c)) + d$$

the point (x, y) on the original graph will transformed to $(\frac{x}{b} - c, ay + d)$.

There are four linear transformations on a function $f(x)$, controlled by four parameters a,b,c,d.

We first talk about the actions of c and d.

c controls the horizontal shift and d controls the vertical shift.

Example 1. *!!!!!!!!!!!!!!!!!!!!!!!!!!!!! add demos graph here !!!!!!!!!!!!!!!!!!!!!!!!*

a and b controls horizontal and vertical stretch and compression. However, their behaviors are a little different. For a, it stretches(if $|a| > 1$) or compresses(if $0 < |a| < 1$) the graph vertically by a factor factor of $|a|$, and if $a < 0$, then the graph will be flipped about the x-axis. Meanwhile, b stretches(if $0 < |b| < 1$) or compresses(if $|b| > 1$) the graph by a factor of $1/|b|$. If $b < 0$, the graph is flipped about the y-axis.

Example 2. *!!!!!!!!!!!!!!!!!!!!!!!!!!!!! add demos graph here !!!!!!!!!!!!!!!!!!!!!!!!*

Absolute Value

Let's see what can absolute value function do on the graphs.

Typically, there are two most common-used such transformations in Calculus.

$$f(x) \to |f(x)| \tag{1}$$
$$f(x) \to f(|x|) \tag{2}$$

The first one is easy, the absolute value function flips the part where the function values $f(x) < 0$ about the x-axis.

Example 3. *!!!!!!!!!!!!!!!!!!!!!!!!!!!!! add demos graph here !!!!!!!!!!!!!!!!!!!!!!!!*

The second one is a little bit tricky. The absolute value function only acts on the variable x, we can see that $f(|x|)$ actually becomes an even function since

$$f(|-x|) = f(|x|)$$

and we by the definition of absolute value function,

$$f(|x|) = \begin{cases} f(x) \ , x \geq 0 \\ f(-x) \ , x < 0 \end{cases}$$

we see that after the transformation, the left part($x < 0$) of the graph disappeared and became the the mirror image of the right part($x > 0$) of the graph.

Example 4. *!!!!!!!!!!!!!!!!!!!!!!!!!!!!! add demos graph here !!!!!!!!!!!!!!!!!!!!!!!!*