

Definition 1. A general form of a plane is

$$Ax + By + Cz = D$$

It is a set of points that satisfy this equation.

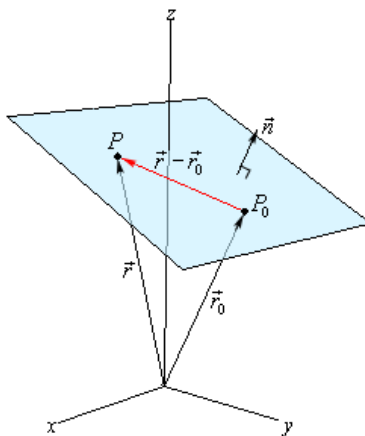
However, practically the general form is not as useful as the normal-point form.

Definition 2. The **normal-point** form of a plane is defined by the normal vector $\vec{n} = (n_x, n_y, n_z)$ of the plane and a point $\vec{r}_0 = (x_0, y_0, z_0)$ the plane contains.

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

or

$$n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$$



Problem 1. Determine the equation of the plane which contains the three points $\vec{P} = (1, -2, 0)$, $\vec{Q} = (3, 1, 4)$, $\vec{R} = (0, -1, 2)$.

Solution 1. First we need to find out the normal of this plane.

$$\begin{aligned} \vec{n} &= \overrightarrow{PQ} \times \overrightarrow{PR} \\ &= (\vec{Q} - \vec{P}) \times (\vec{R} - \vec{P}) \\ &= (2, 3, 4) \times (-1, 1, 2) \\ &= (2, -8, 5) \end{aligned}$$

Since the plane contains all these points, we choose one, say P , then the normal-point form equation of this plane is

$$2(x - 1) - 8(y + 2) + 5(z - 0) = 0$$

or written in general form

$$2x - 8y + 5z = 18$$