Definition 1. *If* $\forall \epsilon > 0$, $\exists N > 0$ *such that when* $x > N$, *then*

$$|f(x) - L| < \epsilon$$

where $L \in \mathbb{R}$. *We say that the limit of the function when* $x \to \infty$ *is* L. *In other words,*

$$\lim_{x\to\infty} f(x) = L$$

Example 1. *One very important example that worths to remember is this*

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \ \ (n \in \mathbb{N})$$

We can easily see this from the graph.

!!!!!!!!!!!!!!!!!!!! demos graph !!!!!!!!!!!!!!!!!!!!!!!!!

Example 2.

$$lim_{x\to\infty} = \frac{3x^2+2x+1}{2x^2-3}$$

Graphically, we know that this just asking us to find the horizontal asymptote of this rational function. And the answer is 3/2.

Algebraically, for the numerator and the denominator, we both divide the leading term of the polynomial with smaller degree. In this case, we divide x^2.

$$\begin{aligned} lim_{x\to\infty}\frac{3x^2+2x+1}{2x^2-3} &= \lim_{x\to\infty}\frac{(3x^2+2x+1)/x^2}{(2x^2-3)/x^2} \\ &= \lim_{x\to\infty}\frac{3+2\frac{1}{x}+\frac{1}{x}}{2-3\frac{1}{x}} \\ &= \frac{3}{2} \end{aligned}$$

Example 3.

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{x}$$

When x *is very large, we see that* $x^2+2 \approx 2$. *Then*

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{x} = \lim_{x\to\infty}\frac{\sqrt{x^2}}{x}$$

One important thing to remember is that $\sqrt{x^2} = |x|$. *So*

$$\lim_{x\to\infty}\frac{\sqrt{x^2}}{x} = \lim_{x\to\infty}\frac{|x|}{x} = 1$$

Example 4.

$$\lim_{x\to\infty}\sqrt{x^2-x}+x$$

Be careful when dealing with square root.

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2-x}+x &= \lim_{x\to\infty} \frac{\sqrt{x^2-x}+x}{1} \\
&= \frac{(x^2-x)-x^2}{\sqrt{x^2-x}-x} \\
&= \frac{-x}{\sqrt{x^2-x}-x} \\
&= \frac{-1}{\frac{\sqrt{x^2-x}}{x}-1} \\
&= \frac{-1}{\frac{|x|}{x}-1} \\
&= \frac{-1}{-1-1} \\
&= 2
\end{aligned}$$