Definition 1. *The derivative of a function $f(x)$ is noted as $f'(x)$. It is the same as computing the instantaneous rate of change at point x.*

Theorem 1. *We have a table of rules of taking derivatives.*

$f(x)=k\in\mathbb{R}\Rightarrow f'(x)=0$	$f(x)=e^x\Rightarrow f'(x)=e^x$
$f(x)=x\Rightarrow f'(x)=1$	$f(x)=a^x\Rightarrow f'(x)=a^x\ln a$
$f(x)=x^k\Rightarrow f'(x)=kx^{k-1}$	$f(x)=\sin x\Rightarrow f'(x)=\cos x$
$f(x)=\frac{1}{x}\Rightarrow f'(x)=-\frac{1}{x^2}$	$f(x)=\cos x\Rightarrow f'(x)=-\sin x$
$f(x)=\sqrt{x}\Rightarrow f'(x)=\frac{1}{2\sqrt{x}}$	$f(x)=\tan x\Rightarrow f'(x)=\sec^2 x=1+\tan^2 x$
$f(x)=\ln x\Rightarrow f'(x)=\frac{1}{x}$	$f(x)=\arcsin x\Rightarrow f'(x)=\frac{1}{\sqrt{1-x^2}}$
$f(x)=\log_a x\Rightarrow f'(x)=\frac{1}{x\ln a}$	$f(x)=\arctan x\Rightarrow f'(x)=\frac{1}{1+x^2}$

Example 1. *Find the derivative of the function $f(x)=\sqrt{x^3}$ at point $x=4$.*

Proof. From the table, we see that

$$f'(x)=(x^{\frac{3}{2}})'=\frac{3}{2}x^{\frac{1}{2}}$$

Thus, $f'(4)=\frac{3}{2}4^{\frac{1}{2}}=3$ □

In order to compute the derivative of more complicated functions, we need the following properties.

Property 1.

$$(f\pm g)'=f'\pm g' \tag{1}$$
$$(cf)'=cf' \tag{2}$$
$$(fg)'=f'g+fg' \tag{3}$$
$$(\frac{f}{g})'=\frac{f'g-fg'}{g^2} \tag{4}$$
$$f(g)'=f'(g)g' \tag{5}$$

Example 2. *Find the derivative of the function $f(x)=x^2+\cos x$.*

Proof.

$$f'(x)=(x^2)'+(2\cos x)'=2x-2\sin x$$

□

Example 3. *Find the derivative of the function $f(x) = x \sin x$.*

Proof.
$$f'(x) = (x)' \sin x + x(\sin x)' = \sin x + x \cos x$$

□

Example 4. *Find the slope of the tangent line of the function*

$$f(x) = \frac{x^2}{x+1}$$

at point $x = 1$.

Proof.

$$\begin{aligned} f'(x) &= \left(\frac{x^2}{x+1}\right)' \\ &= \frac{(x^2)'(x+1) - (x^2)(x+1)'}{(x+1)^2} \\ &= \frac{2x(x+1) - x^2}{(x+1)^2} \\ &= \frac{x^2 - 2x}{(x+1)^2} \end{aligned}$$

Thus the slope of $f(x)$ at $x = 1$

$$f'(1) = \frac{1^2 - 2 \times 1}{(1+1)^2} = -\frac{1}{4}$$

□

Example 5. *Find the derivative of the composite function $f(x) = (x^2 + 2x)^4$.*

Proof. Let $g(x) = x^4$ and $h(x) = x^2 + 2x$. Then $f(x) = g(h(x))$. Thus

$$f'(x) = g'(h(x))h'(x) = 4(x^2 + 2x)^3(2x + 2)$$

□