Definition 1. *A random variable X is a function from a sample space S to a set of numbers E (i.e. $X : S \to E$). If $E = \mathbb{Z}$, then X is said to be a discrete random variable; If $E = \mathbb{R}$, then X is said to be a continuous random variable.*

It might seem so abstract in this way, but what it says is just: X maps the outcomes to some numbers. We will see that in the following examples.

Example 1. *Let there be a fair coin, when I toss it, the sample space $S = \{H, T\}$. Let X be a function defined on S such that*

$$X(A) = \begin{cases} 0 & ,if\ A = T \\ 1 & ,if\ A = H \end{cases}$$

Then X is a discrete random variable.

Example 2. *Suppose I rolled a dice 5 times. Let $X \equiv$ sum of 5 upward-faced numbers I got. This X is a discrete random variable*

Example 3. *Let $Y \equiv$ waiting time at bus stop. This Y is a continuous random variable.*

Since we have defined the random variable, we can now introduce the so-called **probability mass function**.

Definition 2. *The probability mass function(pmf) of a discrete random variable X is*

$$p_X(x) = P(\{A \in S | X(A) = x\})$$

The pmf has the following two properties:

Property 1. $p_X(x) \geq 0, \forall x \in X(S)$

Property 2. $\sum_x p_X(x) = 1$

Finally, we can define **cumulative distribution function** now.

Definition 3. *The cumulative distribution function(cdf) is defined as*

$$F_X(x) = P(X \leq x) = \sum_{y<x} p_X(y)$$