

Definition 1. *Antiderivative or Indefinite integral* is the reverse operation of derivative. Suppose a function $F(x)$ has derivative $f(x)$, then the antiderivative of $f(x)$ is

$$\int f(x)dx = F(x) + C$$

Where $C \in \mathbb{R}$ is a constant.

Property 1.

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx \quad (1)$$

$$\int kf(x)dx = k \int f(x)dx \quad (k \in \mathbb{R}) \quad (2)$$

$$(3)$$

Since the antiderivative is just the reverse operation of derivative, thus we have a set of formulas

Differentiation Formulas:

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(ax) = a$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(\cos x) = -\sin x$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$
7. $\frac{d}{dx}(\cot x) = -\csc^2 x$
8. $\frac{d}{dx}(\sec x) = \sec x \tan x$
9. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = (\ln a)a^x$
13. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

Integration Formulas:

1. $\int 1 dx = x + C$
2. $\int a dx = ax + C$
3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
4. $\int \sin x dx = -\cos x + C$
5. $\int \cos x dx = \sin x + C$
6. $\int \sec^2 x dx = \tan x + C$
7. $\int \csc^2 x dx = -\cot x + C$
8. $\int \sec x (\tan x) dx = \sec x + C$
9. $\int \csc x (\cot x) dx = -\csc x + C$
10. $\int \frac{1}{x} dx = \ln |x| + C$
11. $\int e^x dx = e^x + C$
12. $\int a^x dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$
13. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
14. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
15. $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C$

Example 1. Find the antiderivative of $f(x) = x^3 + \frac{\cos x}{2}$

$$\begin{aligned} \int [x^3 + \frac{\cos x}{2}] dx &= \int x^3 dx + \frac{1}{2} \int \cos x dx \\ &= \frac{x^4}{4} + \sin x + C \end{aligned}$$