Stability eigen-value problem (1)

General eqs.

$$\frac{\partial}{\partial t} n + B \frac{\partial}{\partial z}\left(\frac{nV}{B}\right) = 0$$

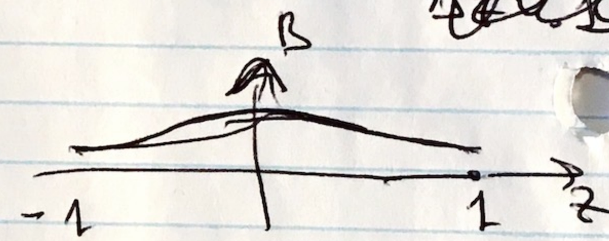
$$\frac{\partial}{\partial t} V + V \frac{\partial V}{\partial z} = -C_s^2 \frac{1}{n}\frac{\partial n}{\partial z} \qquad B = B(z)$$

$$B(z) = B_0\left[1 + R \exp\left(-\frac{z^2}{\delta^2}\right)\right]$$

$$B(z) = B_0\left[1 + R \exp\left(-(z/L)^2 \frac{L^2}{\delta^2}\right)\right]$$

$$B(z) = B_0\left[1 + R \exp\left(-\frac{z'^2}{\Delta^2}\right)\right] \qquad \Delta \equiv \frac{\delta}{L}$$

$R = 1 \div 2 \qquad \Delta = 0.1 \div 0.3$



Equilibrium
$$\frac{\partial}{\partial z}\left(\frac{n_0 V_0}{B}\right) = 0$$

$$V_0 \frac{\partial V_0}{\partial z} = -C_s^2 \frac{1}{n_0}\frac{\partial n_0}{\partial z}.$$

Solution in terms of Lambert function

$W \exp(W) = y \quad \Rightarrow \quad W = W(y)$

↑ Lambert function

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General eqs.

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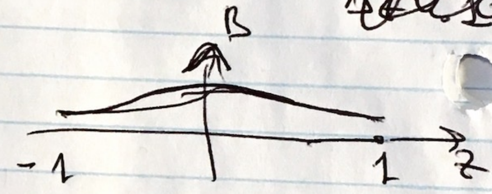

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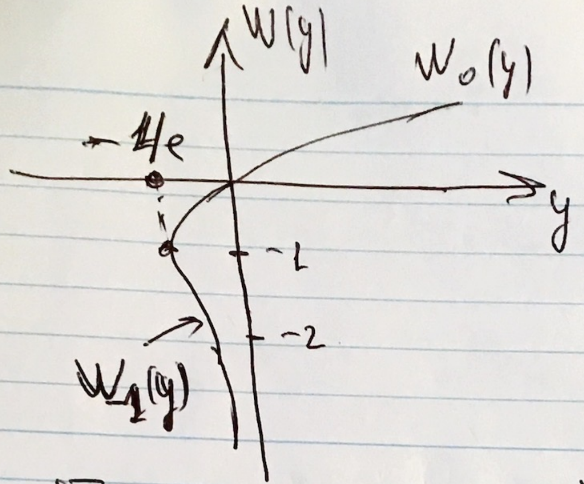
↑ Lambert function.

$$\frac{V_0}{c_s} \equiv M$$

$$-M^2 e^{-M^2} = -M_m^2 \left(\frac{B(z)}{B_m}\right)^2 e^{-M_m^2}.$$

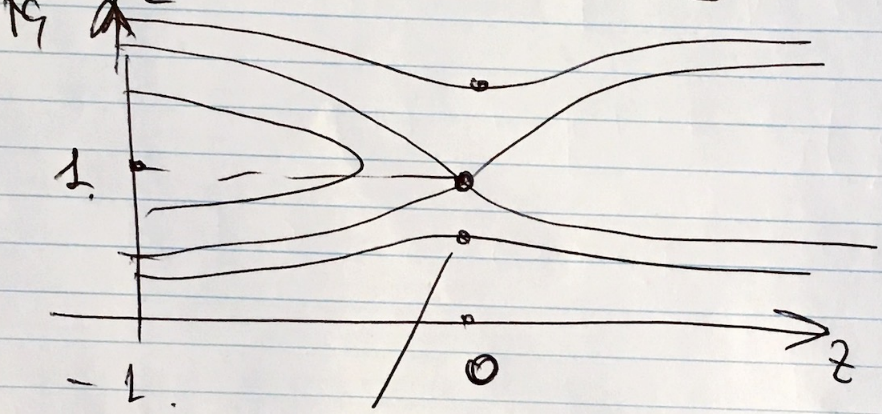
$$-M^2 = W(y) \qquad y \equiv -M_m^2 \left(\frac{B(z)}{B_m}\right)^2 e^{-M_m^2}.$$



$W_0(y)$

$W_{-1}(y)$

$$M = \left[-W\left(-M_m^2 \frac{B(z)^2}{B_m^2} e^{-M_m^2}\right)\right]^{1/2}$$

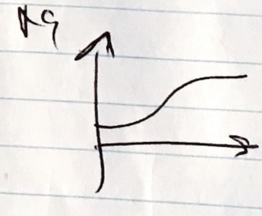


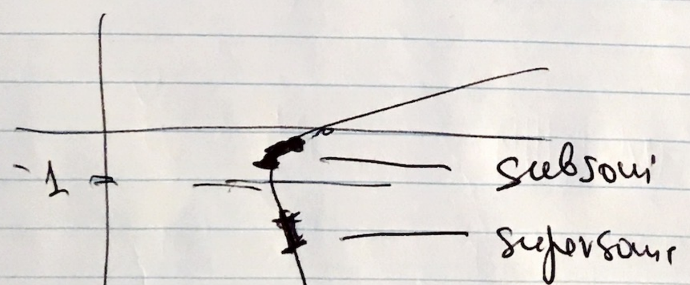
M_m

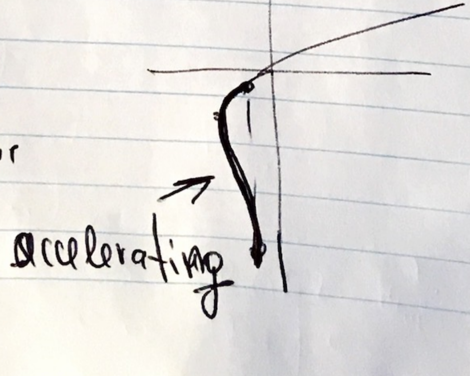
Accelerating solution $M_m = 1$

supersonic $M_m > 1$

Subsonic $M_m < 1$







Subsonic solution

③

$$M = \left[-W_0 \left(-M_m^2 \frac{B^2(z)}{B_m^2} e^{-M_m^2} \right) \right]^{1/2}$$

$$B_m = 1 + R \qquad M_m = 0.5 \div 0.9$$

$$n_0 \frac{(z)\, V_0(z)}{B(z)} = \frac{n_m V_m}{B_m}$$

$$\frac{\partial}{\partial Z} \ln \tilde{u} = -\frac{\partial}{\partial \tau}\tilde{v} - v_0 \frac{\partial \tilde{v}}{\partial Z} - \tilde{v}\frac{\partial}{\partial Z}$$

$$\frac{\partial}{\partial \tau} \ln \hat{u} + v_0 \frac{\partial \ln \hat{u}}{\partial Z} + \tilde{v}\left(\frac{\partial}{\partial Z}\ln\frac{u}{B}\right)_0 + \frac{\partial}{\partial Z}$$

$$\frac{\partial}{\partial Z}\left(\ln \frac{u}{B}\right) = + \cancel{\frac{\partial}{\partial Z}\ln B = \frac{B'}{B}}.$$

$$= -\frac{\partial}{\partial Z} v_0 = -\frac{v_0'}{v_0}$$

$Z = z/L$

$\tau = \frac{t}{L} c_s$

$$v_0 = \left[-W\left(-M_w^2 \frac{B(z)^2}{B_w^2} e^{-M_w^2}\right)\right]^{1/2}$$

$$W'(y) = \frac{1}{[1+W(y)]\exp(W(y))} =$$

$$= \frac{W(y)}{y(1+W(y))}$$