On the Instability of Magnetic Nozzle

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Abstract

Linear instability of plasma flow in paraxial magnetic nozzle is studied under the framework of polynomial eigenvalue problem. The linear instability for subsonic and supersonic case are found using spectral method. Spectral pollution is filtered using convergence test. The linear stability for transonic velocity profile is investigaed by employing shooting method. The regular solution to the polynomial eigenvalue problem near the singularity is picked up through Frobenius method. Solutions that are crossing the sonic point smoothly were obtained for transonic case.

1 Introduction

- Mangetic nozzle.
- Magnetic mirror configuration.
- Relavent research.

2 Polynomial Eigenvalue Problem

- Fluid description.
- Equilibrium velocity profile.
- Results for subsonic and supersonic case?

3 Singular Perturbation

- Expansion of $\tilde{v}$ near singularity.
- Results for transonic cases.

4 Conclusion

- Breifly talk about the result.
- Obtained smooth slutions crossing sonic point.

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