

Magnetic Nozzle - Filtering Spurious Modes Ny Convergen Test

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Problem

The polynomial eigenvalue problem is given by

$$\begin{aligned} \omega^2 v + 2i\omega \left(v_0 \frac{\partial}{\partial z} + \frac{\partial v_0}{\partial z} \right) v \\ + \left[(1 - v_0^2) \frac{\partial^2}{\partial z^2} - \left(3v_0 + \frac{1}{v_0} \right) \frac{\partial v_0}{\partial z} \frac{\partial}{\partial z} - \left(1 - \frac{1}{v_0^2} \right) \left(\frac{\partial v_0}{\partial z} \right)^2 - \left(v_0 + \frac{1}{v_0} \right) \frac{\partial^2 v_0}{\partial z^2} \right] v = 0 \end{aligned} \quad (1)$$

This problem can be transformed to an eigenvalue problem

$$\begin{bmatrix} O & I \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v \\ \omega v \end{bmatrix} = \omega \begin{bmatrix} v \\ \omega v \end{bmatrix} \quad (2)$$

where

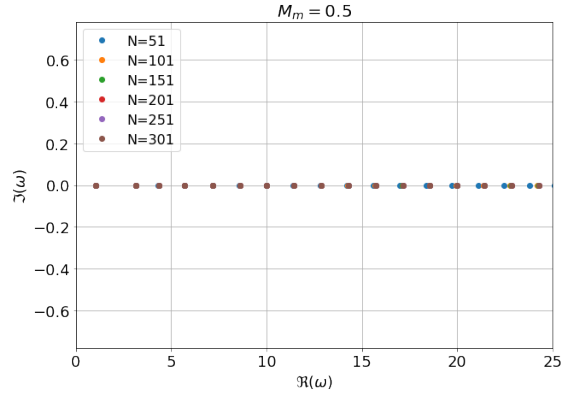
$$\begin{aligned} A_{21} &= - \left[(1 - v_0^2) \frac{\partial^2}{\partial z^2} - \left(3v_0 + \frac{1}{v_0} \right) \frac{\partial v_0}{\partial z} \frac{\partial}{\partial z} - \left(1 - \frac{1}{v_0^2} \right) \left(\frac{\partial v_0}{\partial z} \right)^2 - \left(v_0 + \frac{1}{v_0} \right) \frac{\partial^2 v_0}{\partial z^2} \right] \\ A_{22} &= -2i \left(v_0 \frac{\partial}{\partial z} + \frac{\partial v_0}{\partial z} \right) \end{aligned}$$

If we try to investigate the eigenvalues of the problem, the spectral pollution will generate spurious modes.

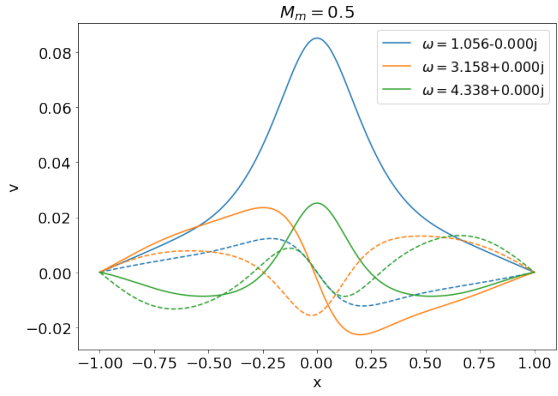
Convergence Test

To filter the eigenvalues, we compute the eigenvalues in different resolutions, and only trust those that are converging.

Subsonic case

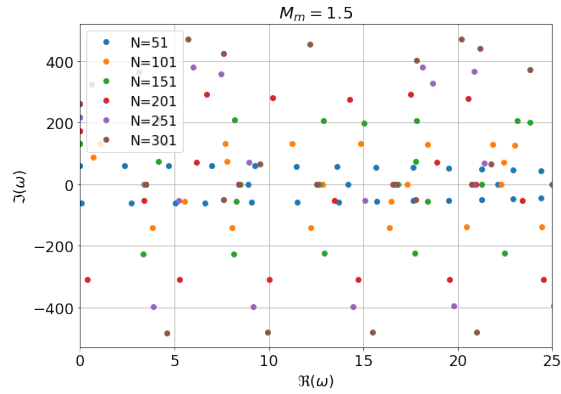


(a) No spurious modes in subsonic case.

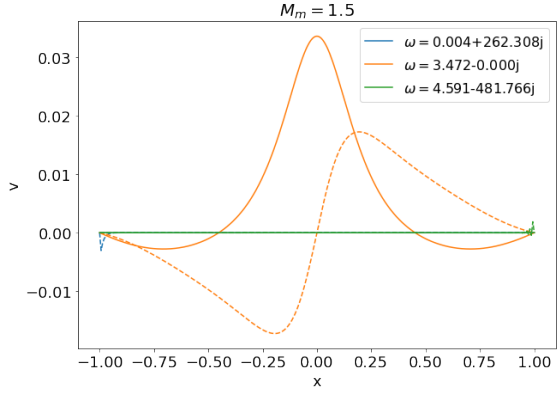


(b) First three eigenfunctions.

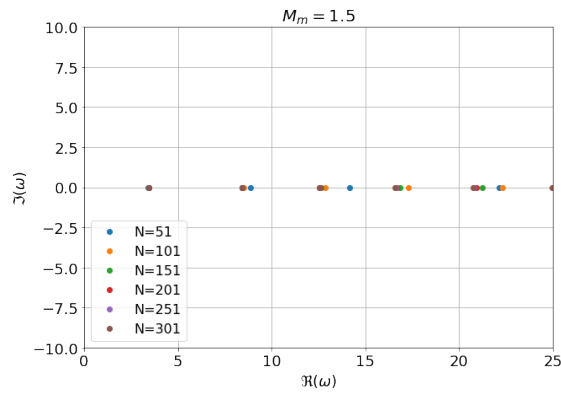
Supersonic case



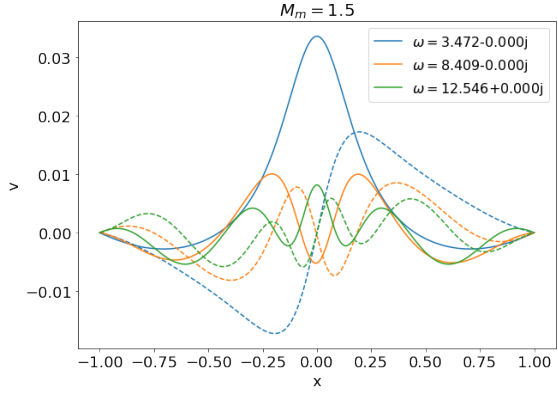
(a) Eigenvalues before filtering.



(b) Spurious eigenfunctions has weird shape.

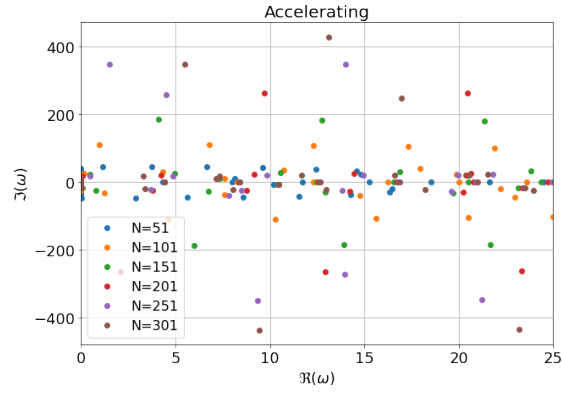


(c) Only these eigenvalues are converging.

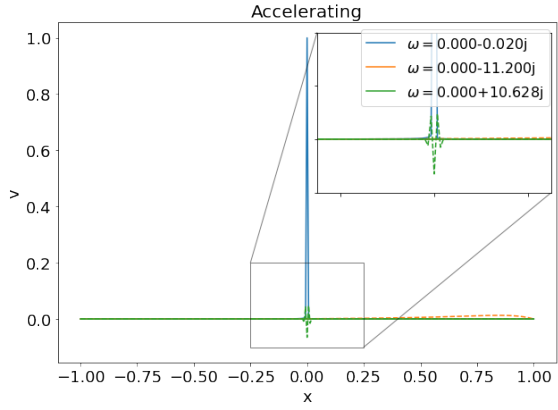


(d) Filtered eigenfunctions.

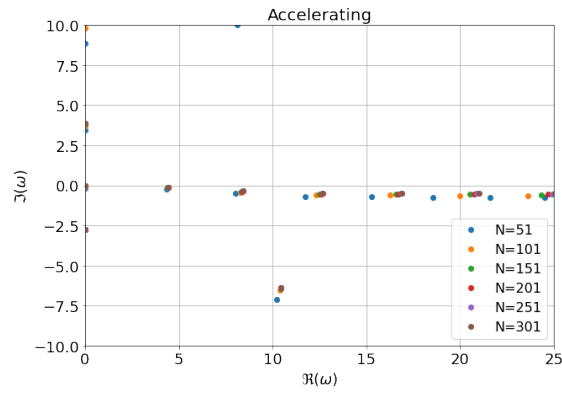
Accelerating case



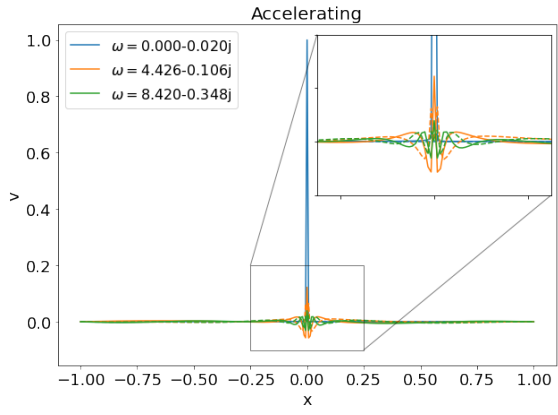
(a) Eigenvalues before filtering.



(b) Spurious eigenfunctions has weird shape.

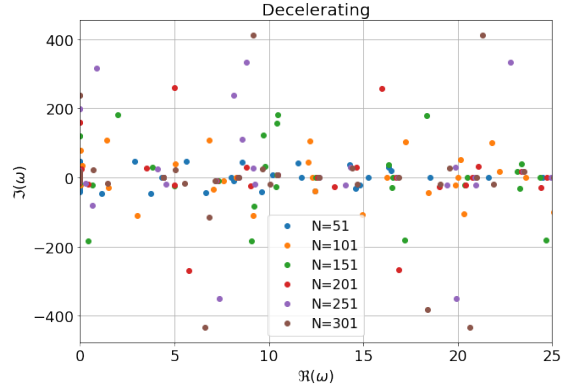


(c) Only these eigenvalues are converging.

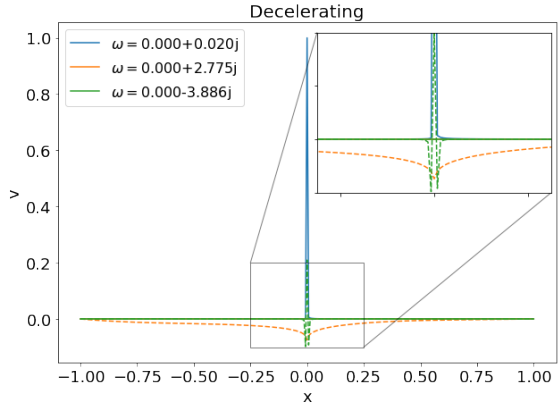


(d) Filtered eigenfunctions.

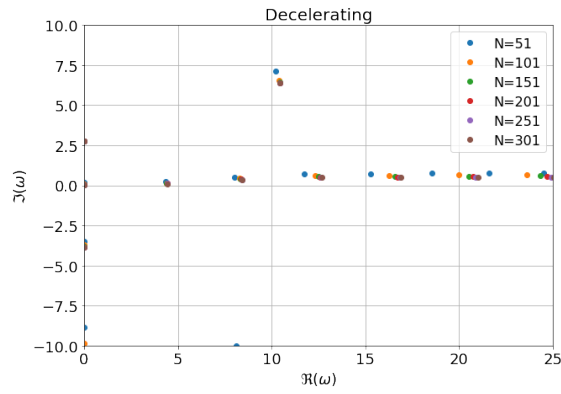
Decelerating case



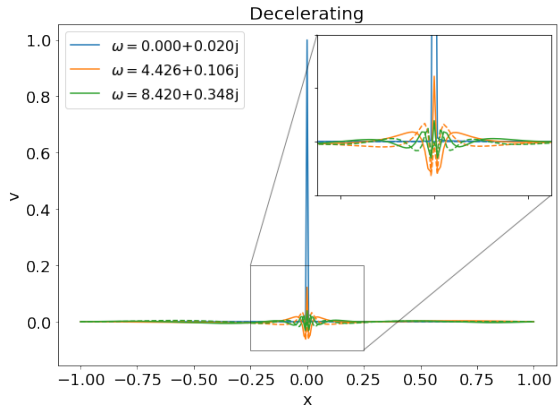
(a) Eigenvalues before filtering.



(b) Spurious eigenfunctions has weird shape.



(c) Only these eigenvalues are converging.



(d) Filtered eigenfunctions.