Non modal instabilities

We will use the recent paper by Squire and Bhattacharjee [1] as a starting poont of the discussion of the various properties of non-modal instabilities. Below I will try to formulate some statements and results of this (and some other works) and questions which arises from these. Some of these questions (and answers) are trivial, some are well known in the literature (but not to me) and some may be unknown. You may also find that some results/answers in the literature may be incomplete and/or contradictory. As we go along with reading of the literature and discussions, you may identify/formulate other questions and issues.

In the end, we would like to get as complete picture as possible on this important topics and (possibly) formulate topics for the original research relevant to instabilities and turbulence in Hall and fusion plasmas (and other e.g. space plasmas).

Brief Synopsis: Typically analysis of the linear instabilities is based on the investigation of the unstable spectrum of the related linearized system of PDEs. That is the solution for the system of the linear PDEs is sought in the form $\mathbf{X}(\mathbf{x},t) = \exp(-i\omega t)\,\widehat{\mathbf{X}}(\mathbf{x})$. Then, the PDEs are converted into the system of ODE in $\mathbf{x}$, which together with appropriate boundary conditions form the eigen-value problem for ω. The eigen-value problem is solved and most unstable ω (with largest positive imaginary part, $\omega = \omega_r + i\gamma$) are considered as the most dynamically relevant modes. Presumably these modes will define the behavior of the system asymptotically, at large times $\mathbf{X} \sim \exp(i\gamma t)$. This paradigm of the linear stability seems work well (and indisputed) if the ODE eigen-value problem is self-adjoint. Note that the properties of the eigen-value problem (e.g. self-adjointness) depend on the form of the ODEs and the form of the boundary conditions. However, when the eigen-value problem is non self-adjoint there exist other solutions of the linearized PDE system that **might be?** growing in time and be dynamically more important? These solutions are not eigen-modes, or non-modal (in Russian "nesobstvennye mody"), most easily these modes can be charactetized in that they are not exponential in time solutions, not separable in time and space?. The paper by Squire and Bhattacharjee [1] investigates the existence and relative importance of such non-modal solutions for a specific problem of magnetorotational instability.

MagnetoRotational Instability (MRI) is a MHD instability of a rotating weakly magnetized plasmas ($\beta \equiv 8\pi p_0/B^2 \gg 1$) described by Velikhov in 1959 (also some work by Chandrasekar in his book). It was rediscovered by Balbus and Hawley in 1991 and suggested as a mechanism of the anomalous transport of the angular momentum in accretion disks. It became hugely polular since then as a basic model explaining accretion, eg. around black holes and other massive objects (rumors are that it was suggested for the Nobel prize, it is not clear if Velikhov would be among the recipients)

Q1. Discuss meaning, range applicability of basic MHD equations (1). Derivation of fluid equations in rotating coordinate system, Coriolis and centripetal forces. Compressibility, fast and slow modes, Alfven vs magnetosonic, etc

Q2. Orr-Sommerfelf variables

Q3. Equations 2-3. I do not understand these. What are the 2-norm, Cholesky decomposition? Derive these equations and explain this approach on a simple example. Is Eq 3 has right dimensionality? Presumably $G^+_{\max}$ has the dimensions of the growth rate, but Λ has also the same dimensionality?

Q4. Shearing and static modes in Eq 4? Derive equations 4.

Q5. Derive the global equations (7)

Q6. As far as I understand, the global equations are solved numerically as an initial value problem and compared with shearing and static (and eigen modes) solutions. What are the equations that were used to obtain the eigen modes solutions? How were they (eigen-modes) obtained and what is the eigen-functions (eluded in Fig 1d) and frequency (eigen) spectrum?

Q7. How are the nonmodal calculations performed giving Fig 1 a-c? How this is done: "..As outlined in Ref 18, the nonmodal calculations solves for the initial conditions that maximize the energy amplification by some chosen time t_M" , end of the second from the bottom paragraph, page 4, left column.

Q8. The Fig 2 represents the main result of the paper, or two most important main results: a) The local shearing mode growth and most unstable global non-modal growth are very similar; b) Both are much faster than the most unstable eigen-mode growth. Therefore the non-modal solutions are most important dynamically!

Q9. The note in the top paragraph, page 3, right column, rises some questions. It suggests that the non-modal solution are purely hydrodynamic or purely magnetic. This would suggest that the results and the conclusions may be dependent on the norm? $E = \int \left(\mathbf{u}^2 + \mathbf{B}^2\right) d\mathbf{x}$, ie. $\mathbf{u} = \mathbf{0}$ for magnetic mode, and $\mathbf{B} = \mathbf{0}$ for hydrodynamic? For Alfven state $\mathbf{u}^2 = \mathbf{B}^2$, what is the energy partition for generic eigen mode of MRI? and how it is different (might be different) for non-modal solutions? Investigate the freedom in non-modal solutions that maximizes magnetic and kinetic energies, as per Ref 18? What is the real energy integral for this system?

Q10. The equations studied in Squire, Bhattacharjee paper are more general than those in NF, Ilgisonis et al., 49 (2009) 035008, due to presence of the azimuthal magnetic field B_θ. However some resuIts of Ilgisonis et all still might be relevant. Is there anything that can be learned from that paper in application to SB paper?

Q11. More conceptual question: Non-modal growth effects appear due to non self-adjoint nature of the eigen-value problem. Most typical reason for this is the presence of the linear shear flow term in the equilibrium flow. The shear flow may be generated self-consistently in nonlinear equations which do not have such terms (i.e. initially self-adjoint) , eg zonal flow generated in HM equation. Physically, does it matter that shear flow is intially imposed or self-consistently generated?

If the non-modal growth effects are real, may such effects appear on nonlinear stages when the shear flow is generated from small scales? How to reconcile shear flow stabilization vs non-modal growth?

References

[1] PRL 113, 025006 (2014) PHYSICAL REVIEW LETTERS .