Summary

Full equations

$$\frac{\partial}{\partial t}\ln n + v\frac{\partial \ln n}{\partial z} + \frac{\partial v}{\partial z} - v\frac{B'}{B} = 0$$

$$\frac{\partial}{\partial t} v + v\frac{\partial v}{\partial z} = -c_s^2 \frac{\partial}{\partial z}\ln n$$

Linearized and dimensionless

$$\frac{\partial}{\partial t}\tilde{v} + v_0 \frac{\partial \tilde{v}}{\partial z} + \tilde{v} \frac{\partial v_0}{\partial z} = -\frac{\partial}{\partial z}\widetilde{\ln n}$$

$$\frac{\partial}{\partial t}\widetilde{\ln n} + \tilde{v} \frac{\partial}{\partial z}\ln\frac{n}{B}\Big|_0 + v_0\frac{\partial}{\partial z}\widetilde{\ln n} + \frac{\partial}{\partial z}\tilde{v} = 0$$

Polynomial $\quad \frac{\partial}{\partial t} \to -i\omega$

$$\boxed{\begin{aligned}&\omega^2\tilde{v} + i\omega\left(2\tilde{v}\frac{\partial v_0}{\partial z} + 2v_0\frac{\partial \tilde{v}}{\partial z}\right)\\&+ \frac{\partial^2\tilde{v}}{\partial z^2}\left(1-v_0^2\right) - 3v_0v_0'\frac{\partial}{\partial z}\tilde{v} + \frac{\partial \tilde{v}}{\partial z}\left(-\frac{v_0'}{v_0}\right)\\&+\tilde{v}\left[-v_0''\left(v_0+\frac{1}{v_0}\right) + v_0'^2\left(\frac{1}{v_0^2}-1\right)\right] = 0\\[1em]&v_0 = \left[-W_0\left(-\frac{B(z)^2}{B_m^2}M_m^2 e^{-M_m^2}\right]^{1/2}\end{aligned}}$$

W_0, W_{-1}