$$\omega^2 + 2i'\omega v_0 \frac{\partial \hat{v}}{\partial z} + 2i\omega \hat{v} \frac{\partial v_0}{\partial z}$$

$$+ \frac{\partial^2 \hat{v}}{\partial z^2} - v_0' \frac{\partial^2 \hat{v}}{\partial z^2}.$$

$$+ \frac{\partial \hat{v}}{\partial z} \left[\frac{\partial}{\partial z} \left(\ln \frac{y}{v_0} \right) - 3 v_0 \frac{\partial v_0}{\partial z} \right]$$

$$+ \hat{v} \left[-\left(\frac{v_0}{v_0}\right)' - v_0'^2 - v_0 v_0'' \right] = Q$$

$$\omega^2 + 2i\omega v_0 \frac{\partial \hat{v}}{\partial z} + 2i\omega \hat{v} \frac{\partial v_0}{\partial z}$$

$$+ \frac{\partial^2 \hat{v}}{\partial z^2} \left(1 - v_0' \right)$$

$$+ \frac{\partial \hat{v}}{\partial z} \left[- \frac{v_0'}{v_0} - 3 v_0 v_0' \right]$$

$$+ \hat{v} \left[- \frac{v_0''}{v_0} + \underbrace{\frac{v_0'^2}{v_0^2} - v_0'^2 - v_0 v_0''}_{} \right] = 0$$

$$Q \equiv v_0'^2 \left(\frac{1}{v_0^2} - 1 \right) - v_0'' \left(\frac{1}{v_0} + v_0 \right)$$