Full equations

(1) $$\frac{\partial}{\partial t} n + n \frac{\partial v}{\partial z} + v \frac{\partial n}{\partial z} - n v \frac{B'}{B} = 0$$

(2) $$\frac{\partial}{\partial t} v + v \frac{\partial v}{\partial z} = -\frac{1}{n} \frac{\partial n}{\partial z}$$

$$\frac{1}{n} \frac{\partial n}{\partial t} + \frac{\partial v}{\partial z} + v \frac{1}{n} \frac{\partial n}{\partial z} - v \frac{B'}{B} = 0$$

Linearized equations

(3) $$\frac{1}{n_0} \frac{\partial \hat{n}}{\partial t} + \frac{\partial \hat{v}}{\partial z} + V_0 \hat{Y} + \hat{v} \frac{n_0'}{n_0} - \hat{v} \frac{B'}{B} = 0$$

(4) $$\frac{\partial}{\partial t} \hat{v} + V_0 \frac{\partial \hat{v}}{\partial z} + \hat{v} \frac{\partial V_0}{\partial z} = -\hat{Y}$$

$$\hat{Y} = -\frac{1}{n_0} \frac{\partial \hat{n}}{\partial z} + \frac{n_0'}{n_0} \frac{\hat{n}}{n_0}$$

Reducing to a single equation

$$-i\omega \frac{\hat{n}}{n_0} + \frac{\partial \hat{v}}{\partial z} + \hat{v} \frac{n_0'}{n_0} - \hat{v} \frac{B'}{B_0}$$

$$+ V_0 \left[i\omega \hat{v} - V_0 \frac{\partial \hat{v}}{\partial z} - \hat{v} \frac{\partial V_0}{\partial z} \right] = 0$$

The latter gives

(5) $$-i\omega \frac{\hat{n}}{n_0} + i\omega V_0 \hat{v} + \frac{\partial \hat{v}}{\partial z} (1 - V_0^2) - \hat{v} V_0' \left(\frac{1}{V_0} + V_0 \right) = 0$$

Using (5) in (4) one gets

$$
\begin{aligned}
&\omega^2 \tilde{v} + 2i\omega \left(V_0 \frac{\partial \tilde{v}}{\partial z} + V_0' \tilde{v} \right) + \\
&+ \frac{\partial^2 \tilde{v}}{\partial z^2} \left(1 - V_0^2\right) - \frac{\partial \tilde{v}}{\partial z} V_0' \left(\frac{1}{V_0} + 3V_0 \right) \\
&- \tilde{v} \left[V_0'' \left(\frac{1}{V_0} + V_0 \right) + V_0'^2 \left(1 - \frac{1}{V_0^2} \right) \right] = 0
\end{aligned}
\tag{6}
$$

$$\omega^2 \tilde{v} + 2i\omega \left(v_0 \frac{\partial \tilde{v}}{\partial z} + v_0' \tilde{v} \right) +$$

$$(6) \qquad + \frac{\partial^2 \tilde{v}}{\partial z^2} (1 - v_0^2) - \frac{\partial \tilde{v}}{\partial z} v_0' \left(\frac{1}{v_0} + 3 v_0 \right)$$

$$- \tilde{v} \left[v_0'' \left(\frac{1}{v_0} + v_0 \right) + v_0'^2 \left(1 - \frac{1}{v_0^2} \right) \right] = 0$$