

# Variational forms

(1)

Original equation

$$\textcircled{1} \quad \omega^2 \hat{V} + 2i\omega \left( V_0 \frac{\partial \hat{V}}{\partial z} + V_0' \hat{V} \right) + \frac{\partial^2 \hat{V}}{\partial z^2} (1 + V_0') \\ - V_0' \left( \frac{1}{V_0} + 3V_0 \right) \frac{\partial \hat{V}}{\partial z} + \\ + \hat{V} \left( V_0'^2 \left( \frac{1}{V_0^2} - 1 \right) - V_0'' \left( \frac{1}{V_0} + \frac{1}{V_0} \right) \right) = 0$$

$$\omega = \omega_r + i\gamma$$

$$\left( \omega_r^2 + 2i\omega_r\gamma - \gamma^2 \right) \hat{V} + 2i\omega_r \left( V_0 \frac{\partial \hat{V}}{\partial z} + V_0' \hat{V} \right) \\ - 2\gamma \left( V_0 \frac{\partial \hat{V}}{\partial z} + V_0' \hat{V} \right) + \frac{\partial^2 \hat{V}}{\partial z^2} (1 + V_0') \\ - V_0' \left( \frac{1}{V_0} + 3V_0 \right) \frac{\partial \hat{V}}{\partial z} + \hat{V} Q = 0$$

$$P = - V_0' \left( \frac{1}{V_0} + 3V_0 \right)$$

$$\left( \omega_r^2 + 2i\omega_r\gamma - \gamma^2 \right) |\hat{V}|^2 + 2i\omega_r \left( V_0 \hat{V}^* \frac{\partial \hat{V}}{\partial z} + V_0' |\hat{V}|^2 \right) \\ - 2\gamma \left( V_0 \hat{V}^* \frac{\partial \hat{V}}{\partial z} + V_0' |\hat{V}|^2 \right) + \hat{V}^* \frac{\partial^2 \hat{V}}{\partial z^2} (1 + V_0') \\ + P \hat{V}^* \frac{\partial \hat{V}}{\partial z} + Q |\hat{V}|^2 = 0$$