Instability of Flow In Magnetic Nozzle

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Outline of Presentation

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Plasma

- Plasma is an ionized gas that consists of highly energized particles, including positively charged ions and negatively charged electrons.

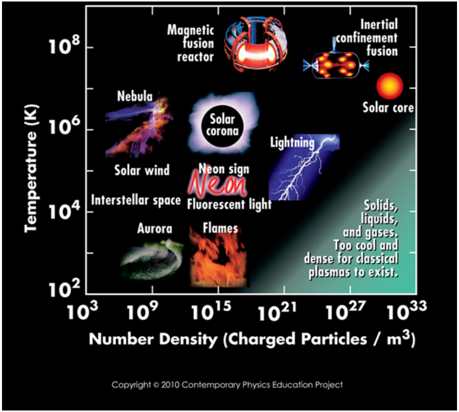

Figure 1: Characteristics of typical plasmas.

Magnetic Nozzle

- A magnetic nozzle is a device that uses a magnetic field to shape and control the flow of charged particles in a plasma propulsion system.

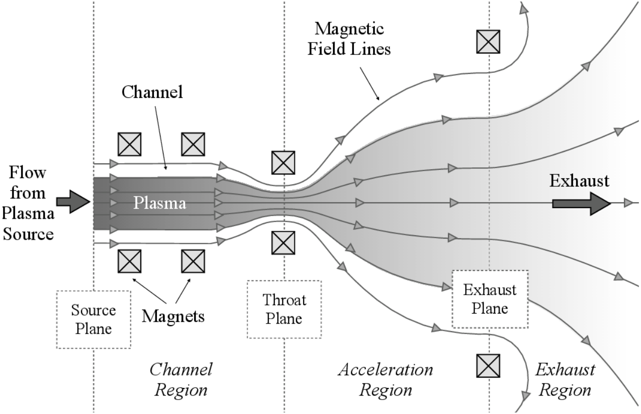

Figure 2: Figure taken from [1]. Example of a magnetic nozzle configuration.

Instability of Plasma Flow

- The instability of plasma flow refers to the tendency of a plasma system to deviate from a stable, equilibrium state and exhibit perturbations or fluctuations in its behavior.
- Plasma instabilities plays a significant role in the transport of energy.

Goals of this thesis

- Understand the standard technique to tackle instability problems.
- Understand and apply spectral method to our problem.
- Investigate the instability of the plasma flow with different velocity profile in magnetic nozzle under different boundary conditions.
- Investigate the effects of the presence of singularity in transonic velocity profile.

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Governing Equations

For convenience, we nondimensionalize the governing equations by normalizing the velocity to c_s, $v \mapsto v/c_s$, z to system length L, coordinate along the nozzle $z \mapsto z/L$ and time $t \mapsto c_s t/L$. The governing equations become

$$\text{Cons. of Den.} \qquad \frac{\partial n}{\partial t} + n\frac{\partial v}{\partial z} + v\frac{\partial n}{\partial z} - nv\frac{\partial_z B}{B} = 0 \tag{1}$$

$$\text{Cons. of Mom.} \qquad n\frac{\partial v}{\partial t} + nv\frac{\partial v}{\partial z} = -\frac{\partial n}{\partial z} \tag{2}$$

where n is nondimensionalized density.

The nondimensionalized equilibrium condition is

$$\frac{\partial}{\partial z}\left(\frac{n_0 v_0}{B}\right) = 0 \tag{3}$$

$$v_0\frac{\partial v_0}{\partial z} = -\frac{1}{n_0}\frac{\partial n_0}{\partial z} \tag{4}$$

where n_0 and v_0 are equilibrium density and velocity, respectively.

Linearized Equations

To linearized the governing equations, we perturbed the density and velocity profiles. Let $n = n_0(z) + \tilde{n}(z,t)$ and $v = v_0(z) + \tilde{v}(z,t)$, where $\tilde{n}$ and $\tilde{v}$ are small perturbed quantities. Then Eq.(3) and Eq.(4) becomes

$$\frac{1}{n_0}\frac{\partial \tilde{n}}{\partial t} + \frac{\partial \tilde{v}}{\partial z} + v_0 \tilde{Y} + \tilde{v}\frac{\partial_z n_0}{n_0} - \tilde{v}\frac{\partial_z B}{B} = 0 \quad (5)$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial (v_0 \tilde{v})}{\partial z} = -\tilde{Y} \quad (6)$$

where

$$\tilde{Y} \equiv \frac{1}{n_0}\frac{\partial \tilde{n}}{\partial z} - \frac{\partial_z n_0}{n_0^2}\tilde{n} = \frac{\partial}{\partial z}\left(\frac{\tilde{n}}{n_0}\right)$$

Polynomial Eigenvalue Problem

Suppose the perturbed quantities are oscillating, $\tilde{n} \sim \exp(-i\omega t)$ and $\tilde{v} \sim \exp(-i\omega t)$. The perturbed quantities will blow up if $\text{Im}(\omega) > 0$, so-called unstable flow.

Substituting them into Eq.(5) and Eq.(6), and combine the two equation, we get the so-called polynomial eigenvalue problem

$$\begin{aligned} &\omega^2 \tilde{v} \\ &+2i\omega \left(v_0 \frac{\partial}{\partial z} + \frac{\partial v_0}{\partial z} \right) \tilde{v} \\ &+ \left[(1 - v_0^2) \frac{\partial^2}{\partial z^2} - \left(3v_0 + \frac{1}{v_0} \right) \frac{\partial v_0}{\partial z} \frac{\partial}{\partial z} \right. \\ &\quad \left. - \left(1 - \frac{1}{v_0^2} \right) \left(\frac{\partial v_0}{\partial z} \right)^2 - \left(v_0 + \frac{1}{v_0} \right) \frac{\partial^2 v_0}{\partial z^2} \right] \tilde{v} = 0 \end{aligned} \tag{7}$$

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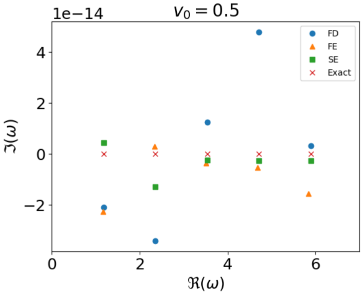
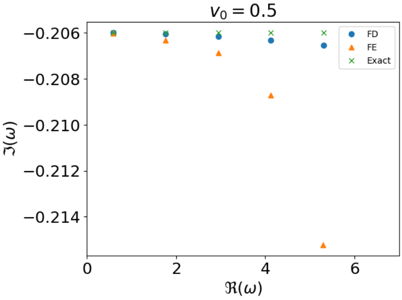
Constant Velocity Case - Subsonic



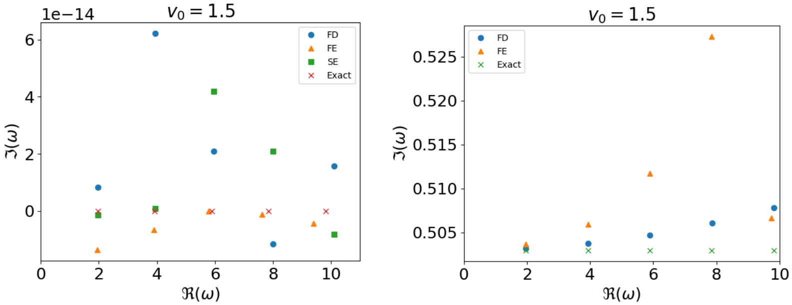
(a) Dirichlet boundary, all modes are stable.



(b) Fixed-open boundary, all modes are stable.

Figure 3: Showing the first 5 eigenvalues. In the Dirichlet boundary case, all methods are close to the exact eigenvalues. Meanwhile, finite-difference method has higher accuracy than finite-element method in fixed-open case.

Constant Velocity Case - Supersonic

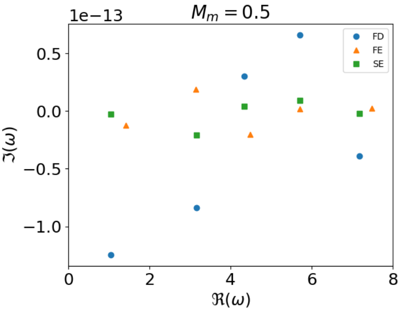


(a) Dirichlet boundary, filtered modes are stable.

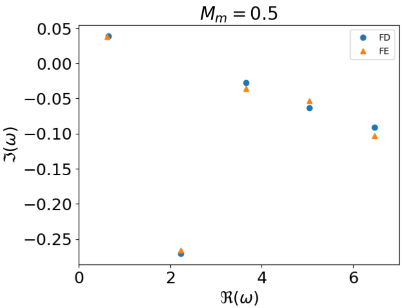
(b) Fixed-open boundary, all modes are unstable.

Figure 4: Showing the first 5 eigenvalues. In the Dirichlet boundary case, all methods are close to the exact eigenvalues. Meanwhile, finite-difference method has higher accuracy than finite-element method in fixed-open case.

Subsonic Case



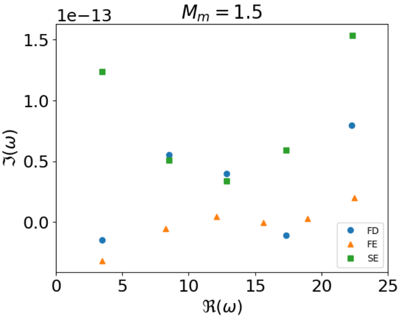
(a) Dirichlet boundary, all modes are stable.



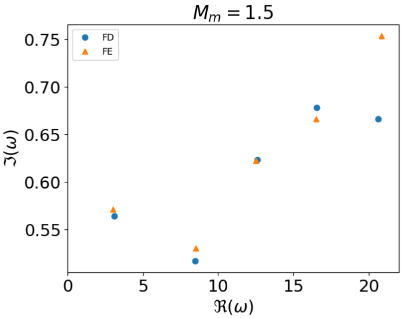
(b) The ground mode is unstable, other modes are stable.

Figure 5: Showing the first 5 modes. It suggests that the subsonic flow in magnetic nozzle is stable.

Supersonic Case



(a) Dirichlet boundary, filtered modes are stable.



(b) Fixed-open boundary, all modes are unstable.

Figure 6: This suggests that the supersonic flow is stable if the boundary is Dirichlet and unstable if the boundary is left-fixed-right-open.

Accelerating Case

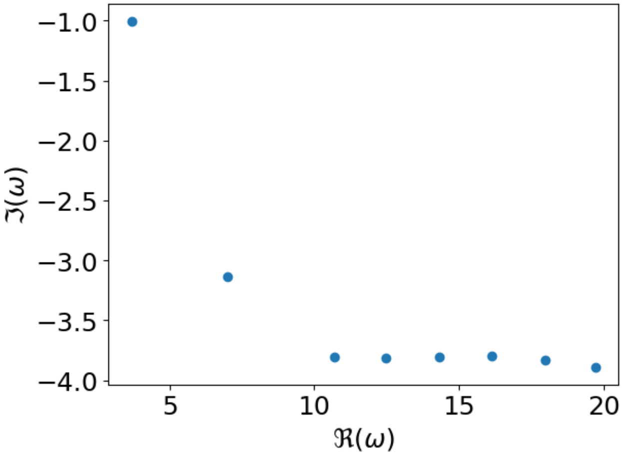


Figure 7: All modes are stable.

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Future Work

To improve the credibility of the results, different numerical calculation methods will be employed.

- Try to match the eigenfunctions to a non-zero left boundary and investigate the corresponding instabilities. [figure]
- Setup a analytically solvable problem with similar configuration. Compare the analytical results to the the experimental computations to better understand the physics. [Some results]

Justin M Little.
Performance scaling of magnetic nozzles for electric propulsion, 2015.
ISBN: 9781321565317.