Stability of the Transonic Plasma Flow in the Magnetic Nozzle

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Goals and Significance of the Research

Goals:

- Investigate the stability of transonic plasma flow in magnetic nozzle.
- Understand the plasma behavior near the nozzle throat.
- Dealing with singularity at nozzle throat both analytically and numerically.

Significance:

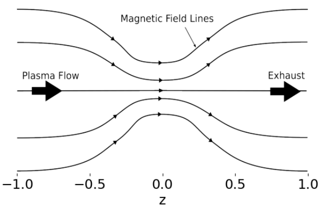
- Understand better the transonic plasma flow in magnetic nozzle.
- Addresses the complexity introduced by the singularity in the governing equations.
- Offer insights into plasma flow in magnetic mirror configuration.

Linear Instability of Plasma Flow

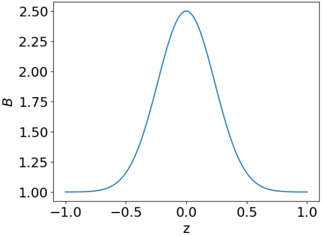
- The instability of plasma flow refers to the tendency of a plasma system to deviate from a stable, equilibrium state and exhibit perturbations or fluctuations in its behavior. [4]
- To investigate linear instability, we assume oscillating perturbed quantities, $\tilde{n}, \tilde{v} \sim \exp(-i\omega t)$.
    1. If $\text{Im}(\omega) > 0$, then it is unstable flow since the perturbations grow exponential in time, $\exp(\text{Im}(\omega)t)$.
    2. If $\text{Im}(\omega) <= 0$, then it is stable flow since the perturbations decay/unchanged in time.
- In this research we will focus on the linear instability only.

Magnetic Nozzle

- A magnetic nozzle is a device that uses a magnetic field to shape and control the flow of charged particles in a plasma propulsion system.
- Instabilities may affect magnetic nozzle operation and the resulting thrust. [7]



(a) Simplified representation of magnetic nozzle.



(b) A simplified magnetic field of magnetic nozzle.

Figure 1: Simplified representation of magnetic nozzle. Length is normalized.

Governing Equations

The nondimensionalized governing equations for the plasma flow in magnetic nozzle are [14]

$$\text{Cons. of Den.} \qquad \frac{\partial n}{\partial t} + n\frac{\partial v}{\partial z} + v\frac{\partial n}{\partial z} - nv\frac{\partial_z B}{B} = 0 \tag{1}$$

$$\text{Cons. of Mom.} \qquad n\frac{\partial v}{\partial t} + nv\frac{\partial v}{\partial z} = -\frac{\partial n}{\partial z} \tag{2}$$

where n, v are density and velocity, respectively.
The equilibrium quantities n_0, v_0 must satify the condition,

$$\frac{\partial}{\partial z}\left(\frac{n_0 v_0}{B}\right) = 0 \tag{3}$$

$$v_0\frac{\partial v_0}{\partial z} = -\frac{1}{n_0}\frac{\partial n_0}{\partial z} \tag{4}$$

Equilibrium Velocity Profiles

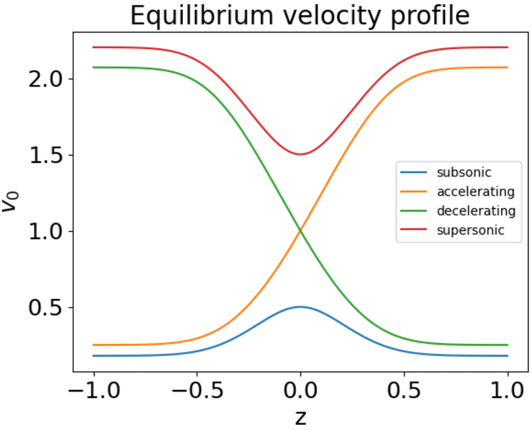


Figure 2: Velocity if normalized to sound speed. There are 4 different cases for velocity profile: subsonic, supersonic, accelerating and decelerating case. In this research we are focusing on the accelerating velocity profile.

Polynomial Eigenvalue Problem

By linearizing the governing equations, and assume oscillating perturbed quantities, $\tilde{n}, \tilde{v} \sim \exp(-i\omega t)$. We can derive the following polynomial eigenvalue problem,

$$
\begin{aligned}
&\omega^2 \tilde{v} \\
&+2i\omega \left(v_0 \frac{\partial}{\partial z} + \frac{\partial v_0}{\partial z} \right) \tilde{v} \\
&+ \left[(1 - v_0^2) \frac{\partial^2}{\partial z^2} - \left(3v_0 + \frac{1}{v_0} \right) \frac{\partial v_0}{\partial z} \frac{\partial}{\partial z} \right. \\
&\quad \left. - \left(1 - \frac{1}{v_0^2} \right) \left(\frac{\partial v_0}{\partial z} \right)^2 - \left(v_0 + \frac{1}{v_0} \right) \frac{\partial^2 v_0}{\partial z^2} \right] \tilde{v} = 0
\end{aligned}
\tag{5}
$$

Notice that the highest derivative term (labeled in red) vanishes at the throat of the nozzle, $z = 0$. Spectral method fails to resolve eigenmodes because of the existence of this singularity.

Existence of Singularity

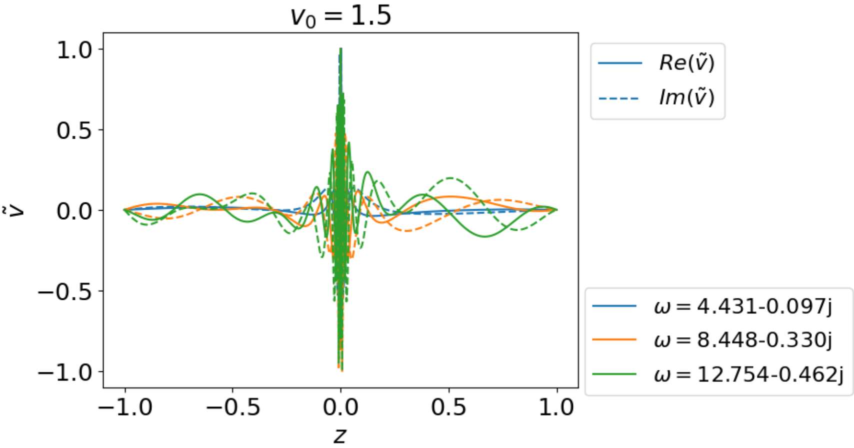


Figure 3: Spectral method failed to resolve meaningful eigenfunctions (and eigenvalues) due to the existence of the singularity at $z = 0$.

Shooting Method

- Expand $\tilde{v}$ near the singularity using Frobenius method.
- Pick up regular solutions and shoot them to the left.
- Eigenvalues are found by matching the Dirichlet BC.

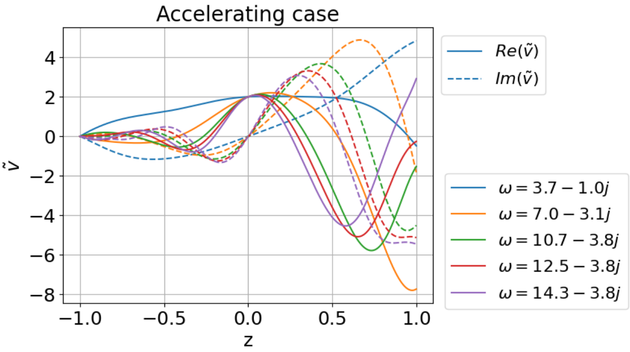


Figure 4: The solutions crosses the singular point smoothly. All modes are stable.

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