Chapter 1 - Fusion

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Outline of Presentation

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Fusion Reactions

There are a few fusion reactions,

- D-T reaction: large cross-section at low temperature, but hard to find Tritium.

$$ {}^2_1D +^2_1 T \rightarrow^4_2 He +^1_0 n + 17.6\text{MeV} $$

- D-D reaction: easy to find the fuel, but small cross-section at low temperature.

$$ {}^2D +^2 D \rightarrow^3 He +^1 n + 3.27\text{MeV} $$

$$ {}^2D +^2 D \rightarrow^3 T + 4.03\text{MeV} $$

- D-He reaction: easy to find the fuel, but small cross-section at low temperature.

$$ {}^2D +^3 He \rightarrow^4 He +^1 H + 18.3\text{MeV} $$

Cross-section

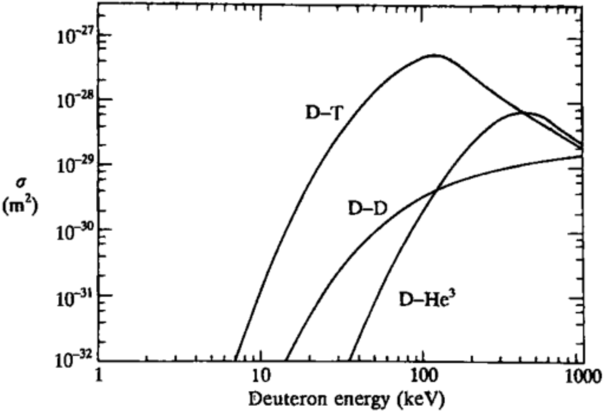

Figure 1: Adapted from [4] Cross-sections for the reactions D-T, D-D and D-^{3}He. The two D-D reactions have similar cross-sections, the graph gives their sum. At 100keV, D-T reaction has the largest cross-section, meaning that more fusion reactions happen in D-T reaction compare to the other two reactions.

Thermonuclear Fusion

The reaction rate is given by

$$R = \left(\frac{8}{\pi}\right)^{1/2} n_1 n_2 \left(\frac{\mu}{T}\right)^{3/2} \frac{1}{m_1^2} \int \sigma(\epsilon)\epsilon \exp\left(-\frac{\mu\epsilon}{m_1 T}\right) d\epsilon \tag{1}$$

where the subscript 1 and 2 are D and T, respectively. And n is the number density, $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and $\epsilon = \frac{1}{2} m_1 (v_1 - v - 2)^2$ is the kinetic energy of D.

- The rate is maximized when $n_1 = n_2$.
- The cross-section σ is given by Fig.1.

$\langle \sigma v \rangle$

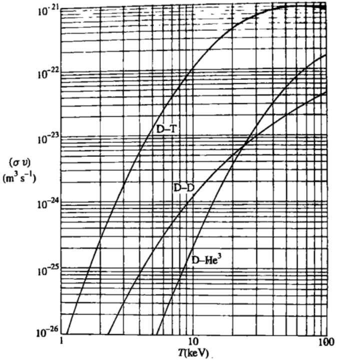

Figure 2: Adapted from [4], $\langle \sigma v \rangle$ for D-T, D-D(total) and D-^{3}He reactions as a function of plasma temperature. $\langle \sigma v \rangle$ for D-D and D-^{3}He are much smaller than that of D-T.

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Power Balance - Thermonuclear Power

For D-T reaction (assuming $n_d = n_t$), the thermonuclear power density is given by

$$p_{Tn} = \frac{1}{4} n^2 \langle \sigma v \rangle \varepsilon \tag{2}$$

where $n = n_d + n_T$ is the total number of density, and $\langle \sigma v \rangle$ is drawn in Fig.2, ε is the energy released per reaction.

- $4/5$ of the reaction energy, ε, is carried away by neutrons, the rest is carried by α-particles, ε_α.
- Neutrons will leave plasma without any interaction.
- The α-particles will be confined by magnetic field, hence self heating the plasma.

Power Balance - α-particle Heating

Since the α-particles are trapped by the magnetic field, so they will transfer their 3.5MeV energy to the plasma through collisions. Thus, the α-particle heating power

$$P_\alpha = \int \frac{1}{4} n^2 \langle \sigma v \rangle \varepsilon_\alpha \, \mathrm{d}^3 x = \frac{1}{4} \overline{n^2 \langle \sigma v \rangle} \varepsilon_\alpha V \tag{3}$$

where the bar means average in the plasma, and V is the volume of the plasma.

Power Balance - Energy Loss

Since each plasma particle has energy $3T/2$ ($T/2$ in each degree of freedom), and there are equal number of electrons and ions, so the total energy of the plasma is

$$W = \int 3nT \, \mathrm{d}^3x = 3\overline{nT}V \tag{4}$$

where V is the volume of the plasma.
If the energy confinement time is τ_E, then the energy loss power is

$$P_L = W/\tau_E \tag{5}$$

- To experimentally determine τ_E, we can maintain a steady state plasma by external heating. In this case the power of energy loss can be estimated by the power of heating, $P_L = P_H$, so

$$\tau_E = W/P_H$$

Ignition - Condition

The requirement for the plasma burn to be self-sustaining is

$$P_\alpha > P_L \tag{6}$$

Take constant density and temperature for simplicity, we have

$$n\tau_E > \frac{12T}{\langle \sigma v \rangle \varepsilon_\alpha} \tag{7}$$

The right-hand-side of the inequality is drawn in Fig.3.
Since τ_E itself is also a function of temperature, so there is a more convenient form,

$$nT\tau_E > 3 \times 10^{21} \text{keV}\cdot\text{s} \tag{8}$$

Ignition - Condition

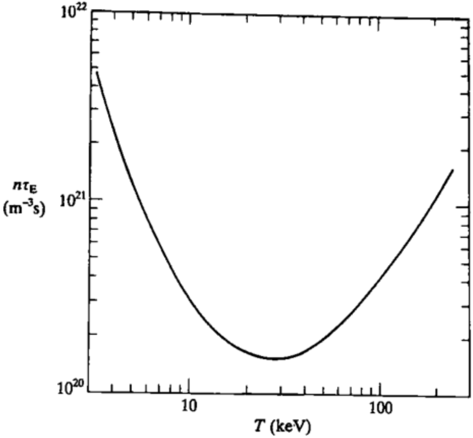

Figure 3: Adapted from [4]. The value of $n\tau_E$ required to obtain ignition, as a function of temperature.

Ignition - Approach

- L-mode: Low confinement mode. Poor confinement in this regime.
- H-mode: High confinement mode. τ_E of plasma is long in this regime.
- With high enough applied power, plasma transition from L to H-mode.
- Once the mode transition happens, the plasma burn is self-sustaining.

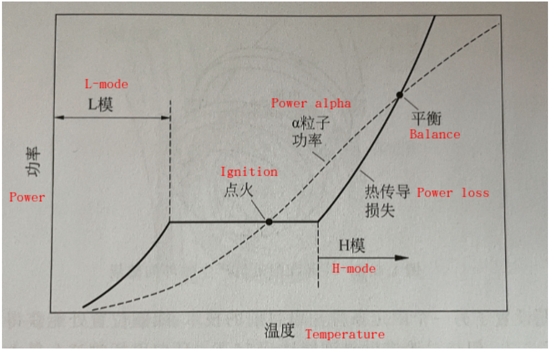

Figure 4: Adapted from [4]. P_L and P_α as function of temperature.

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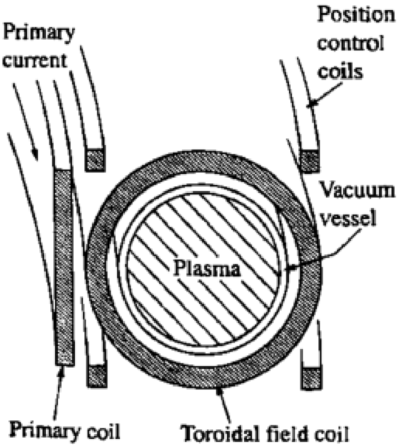
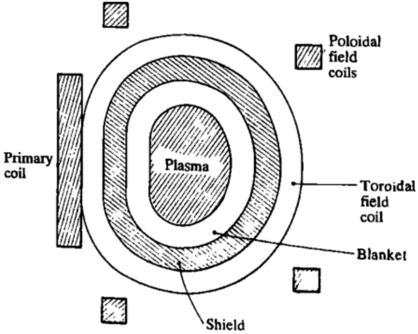
Tokamaks

The Tokamak uses coils to control the plasma in the torus-shape chamber.



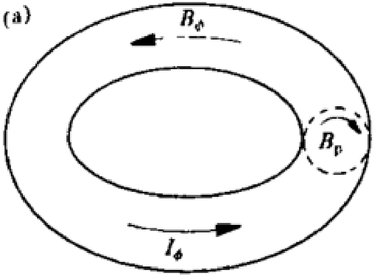
(a) Arrangement of coils in a tokamak.



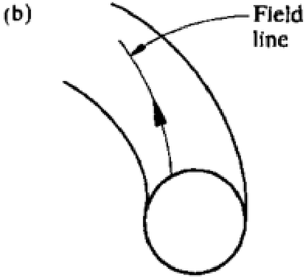
(b) Blanket (compound containing Li) is used to absorb the thermonuclear energy and also for tritium breeding.

Magnetic Field

The poloidal and toroidal magnetic field are essential to stabilize the plasma



(a) Toroidal magnetic field B_ϕ, and poloidal magnetic field B_p due to toroidal current I_ϕ.



(b) Combination of B_ϕ and B_p causes field lines to twist around plasma.

Tokamak Reactor - Structure

In the classical design of tokamak reactor, we only replace the energy source by a tokamak, for the rest of the structure we have mature industrial solutions already.

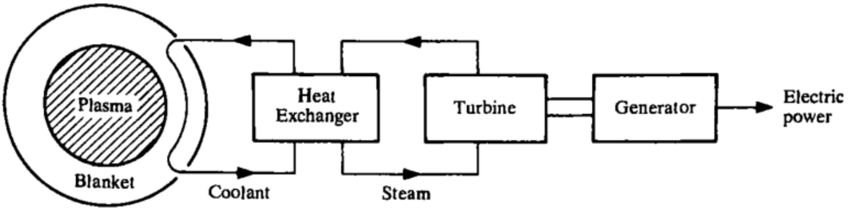


Figure 7: Thermonuclear power absorbed in blanket would be converted into electric power by conventional means.

The power density of the D-T reaction is given by Eq.(2), so

$$P = \frac{\pi}{2}\varepsilon \int n^2 \langle \sigma v \rangle RdS \tag{9}$$

where S is an area element of the poloidal cross-section. We can simplify it by taking R as constant and $\bar{a} = (ab)^{1/2}$. Moreover, $\langle \sigma v \rangle$ can be approximated by $1.1 \times 10^{-24} T^2$, and the pressure profile can be taken as $nT = \hat{n}\hat{T}(1 - r^2/\bar{a}^2)^{\nu}$, so the total power

$$P = \frac{0.15}{2\nu + 1} Rab \left(\frac{\hat{n}}{10^20} \right)^2 \hat{T}^2 \tag{10}$$

where the unit of $\hat{T}$ is keV.

Tokamak Reactor - Impurities

There are two types of impurities:

- Ions coming from solid surfaces (walls). Need to avoid this since it causes plasma energy loss through radiation.
- α-particles, ^{4}He. The α-particles are the byproduct of fusion reaction. It is believed that a magnetic divertor is required to guide the "helium ash" to a "target" surface well separated from the plasma, and to restrict the impurity back-flow.

Outline of Presentation

Commercial Fusion

- I will talk about General Fusion's fusion reactor.
- I think it is useful to include a list of companies working on fusion energy. `https://en.wikipedia.org/wiki/Commercial_fusion`
- General Fusion uses a structure called Magnetized Target Fusion.

General Fusion - Magnetic Target Fusion

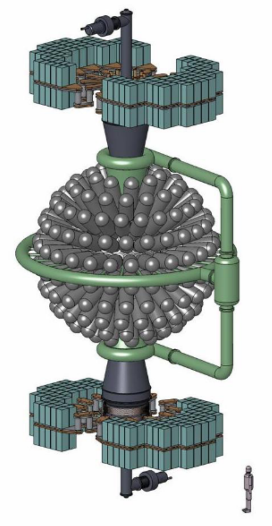


Figure 8: [2] General Fusion's Acoustic Magnetized Target Fusion Reactor Concept.

General Fusion - Plasma in Field-Reverse Configuration

- Plasma injectors inject spheromaks with opposite helicity into the center.
- Spheromaks meet and form a plasma that is in field reverse configuration (FRC).
- Plama in FRC is stable, so no need for the coils. [3]

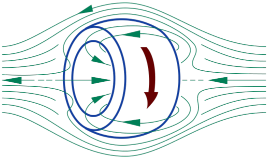


Figure 9: Field-reversed configuration: a toroidal electric current is induced inside a cylindrical plasma, making a poloidal magnetic field, reversed in respect to the direction of an externally applied magnetic field. The resultant high-beta axisymmetric compact toroid is self-confined. Taken from `https://commons.wikimedia.org/wiki/File:Field-Reversed_Configuration.svg`

General Fusion - Liquid Pb-Li as Blanket

In order to absorb the neutrons emitted from the fusion reaction, a liquid metal, Pb-Li, is used as the blanket. Moreover, the Li element can help to breed the tritium through the reaction, $^7Li + n \rightarrow ^4He + ^3H + n$, for further fusion reaction. [3]

- Pb-Li liner is spun up in the device to wrap the plasma.
- Steam piston compresses all the things to create fusion.
- Liquid metal liner is extracted for heat exchange purpose.

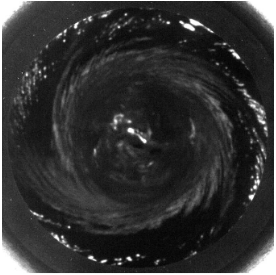


Figure 10: Image of Liquid Pb.

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