# Mathe 2 Prof. Streicher SoSe 22

**A1.)** 1. wahr 2. wahr 3. falsch

4. $f(-x) = \cos(-x)\sin(-x) = -\cos(x)\sin(x) = -f(x)$

5. falsch, z. B. $f(x) = g(x) = |x| \leadsto f \cdot g(x) = x^2$

6. falsch 7. wahr 8. falsch

8. falsch 10. wahr

**A2.)** (a) $\sum\limits_{k=1} \frac{(-1)^k}{k}$ (b) $(0,1]$

(c) $f: \mathbb{R} \to \mathbb{R},\ x \mapsto f(x) = x^2$

(d) $2022 e^{2022\pi i} = 2022 e^{2\pi i} = 2022$

$e^{\frac{3}{2}\pi i} = 0 + i(-1) = -i$

(e) $f: \mathbb{R} \to \mathbb{R},\ x \mapsto f(x) = |x|$

(f) $\ln(\sin(x))' = \frac{\cos(x)}{\sin(x)}$

(g) $f(3) = e^6,\ f'(3) = 2e^6,\ f''(3) = 4e^6$

$\Rightarrow T_{2,f}(x; x_0 = 3) = e^6 + 2e^6(x-x_0) + \frac{4}{2}e^6(x-x_0)^2$

Z: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

$= e^6(1 + 2(x-x_0) + 2(x-x_0)^2)$

(h) $f(x) = \cos(x)$ (jede gerade Funktion)

3) (i) $\boxed{\frac{(n+1)^3}{2^{n+1}} \frac{2^n}{n^3}} = \frac{1}{2}\left(\frac{n+1}{n}\right)^3 \xrightarrow{n\to\infty} \frac{1}{2} < 1$

$\Rightarrow$ konvergent absolut

(ii) $\frac{n^2}{2022n^2+2022} = \frac{\cancel{n^2}}{\cancel{n^2}} \frac{1}{2022 + \frac{2022}{n^2}} \xrightarrow{n\to\infty} \frac{1}{2022} \neq 0$

[4] $f(x,y) = x^2y^2 + \sin(y)$

(a) $\nabla f(x,y) = (2xy^2, 2x^2y + \cos(y))$

(b) $H_f(x,y) = \begin{pmatrix} 2y^2 & 4xy \\ 4xy & 2x^2 - \sin(y) \end{pmatrix}$

(c) $\rightsquigarrow\ 2xy^2 = 0 \rightsquigarrow x = 0$

$2x^2y + \cos(y) = 0 \rightsquigarrow \cos(y) \overset{!}{=} 0$

$\Rightarrow y = \frac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$

(d) $\left| H_f(0, \underbrace{\tfrac{\pi}{2} + k\pi}_{\alpha(k)}) \right|$

$= \left| \begin{pmatrix} 2\alpha(k)^2 & 0 \\ 0 & -\sin(\alpha(k)) \end{pmatrix} \right| = -2\alpha(k)^2 \sin(\alpha(k))$

$\boxed{6}$. 1(a) $\displaystyle\int_{\pi^2}^{4\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ $\boxed{\varphi(x)=\sqrt{x}}$

$$= \int_{\pi^2}^{4\pi^2} 2\,\boxed{\sin(\varphi(x))\,\varphi'(x)}\,dx$$

$= \sin(\sqrt{x})\,\frac{1}{2\sqrt{x}}$

$$= 2\int_{\pi}^{2\pi} \sin(x)\,dx = 2\Big[-\cos(x)\Big]_{\pi}^{2\pi} = -2(1-(-1))$$

$$= -4$$

(b) $\displaystyle\int_{1/2}^{5/2} x e^{2x}\,dx = \left[\frac{xe^{2x}}{2}\right]_{1/2}^{5/2} - \int_{1/2}^{5/2} \frac{e^{2x}}{2}\,dx$

$f\ g'$

$$= \frac{5}{4}e^5 - \frac{1}{4}e - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]_{1/2}^{5/2}$$

$$= \frac{5}{4}e^5 - \frac{1}{4}e - \frac{1}{4}(e^5 - e) = e^5$$

$\boxed{6.}$ $y^{(4)}(t) + y^{(2)}(t) = 0, \; t \in \mathbb{R}$

(a) $\xrightarrow[\text{Pol.}]{\text{char}}$ $\lambda^4 + \lambda^2 \stackrel{!}{=} 0$

$\Leftrightarrow \lambda^2(\lambda^2+1) = 0$

$\Rightarrow \lambda_{1/2} = 0, \; \lambda_3 = i, \; \lambda_4 = -i$

FS $\{1, t\} \cup \{e^{it}, e^{-it}\}$

reelle NS $\qquad$ komplexe NS

(b) $y(0) = [a + bt + ce^{it} + de^{-it}]_{t=0}$

$= a + c + d \stackrel{!}{=} 2$

$y(\pi) = a + b\pi - c - d \stackrel{!}{=} 0$

$c = d = 0$

$a = 2$

$b = -\frac{2}{\pi}$

[7.7(a)] z.z.: $f(J)$ ist Intervall

__Bew:__ Sei $x, z \in J$ und $f(x) \leq f(z)$.

Denn gibt es nach ZWS

$\forall y : f(x) \leq y \leq f(z)$ ein $y_0 \in J$

mit $f(x) \leq f(y_0) = y \leq f(z)$, i.e.

$y \in f(J)$ $\square$

(b) $f(x) = x$, $g(x) = 1$, $x_0 = 0$

$$\rightsquigarrow \lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$$

$$\frac{f'(x)}{g'(x)} = \frac{1}{0}$$ nicht wohldef.

(c) $$|f(x) - f(y)| \overset{MWS}{=} |f'(\xi)(x-y)|$$

mit $\xi$ zwischen $x$ und $y$

$$|f(x) - f(y)| \leq \underbrace{\sup_{\xi \in [0,1]} |f'(\xi)|}_{=:L} \, |x - y| \quad \forall x, y \in [0,1]$$