AMATH 567 COMPLEX ANALYSIS FINAL STUDY GUIDE

The final is scheduled for Monday December 9th at 2:30pm in Johnson Hall 075.

Be able to use the important theorem's and know their respective assumptions and limitations. Be able to state examples and counter examples for each. Be comfortable with all important computation applications of the main principals.

Below I've listed some key topics from memory (trying to force myself to recall). Each of these should be supplemented by looking in the textbook, Bernard's notes, and homework assignments for more information related to the listed topics.

1: Basic complex arithmetic
- $z = x + \mathrm{i}y$
- $z = \rho\, \mathrm{e}^{\mathrm{i}\theta} = \rho(\cos\theta + \mathrm{i}\sin\theta)$
- $\bar{z} = x - \mathrm{i}y$
- $|z| = \sqrt{x^2 + y^2} = \rho$
- $z\bar{z} = |z|^2$

2: Roots of unity, logs, and the square of complex number
- $z^n = a = |a|\, \mathrm{e}^{\mathrm{i}(\theta+2\pi k)} \implies z = |a|^{1/n}\, \mathrm{e}^{\mathrm{i}(\theta+2\pi k)/n}$ with $k \in \mathbb{Z}$
- $\log z = \log r + \mathrm{i}\theta + 2\pi \mathrm{i}k$ with $k \in \mathbb{Z}$

3: Branch Cuts and branch points
- Figure out what the heck they *really* are and how they apply

4: Analyticity, Cauchy Riemann Equations, Cauchy's Theorem, and Cauchy Integral Formula
- Remember the **Cauchy-Riemann equations** ? If $f(z) = f(x,y) = u(x,y) + \mathrm{i}v(x,y)$ then $f(z)$ is analytic if and only if

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}.$$

- **Cauchy's Theorem:** *If $f(z)$ is analytic in a simply connected region Ω which contains the closed contour C then*

$$\oint_C f(z)\mathrm{d}z = 0.$$

- **Cauchy's Integral Formula:** *If $f(z)$ is analytic on and interior to a simple closed contour C then for any point z interior to C*

$$f(z) = \frac{1}{2\pi \mathrm{i}} \oint_C \frac{f(\zeta)\mathrm{d}\zeta}{\zeta - z}.$$

- Derivative's using **Cauchy's Integral Formula.** *If $f(z)$ is analytic on and interior to a simple closed contour C then all derivatives $f^{(k)}(z)$, $k = 1, 2, \ldots$ exist in* *the domain D interior to C, and*

$$f^{(k)}(z) = \frac{k!}{2\pi \mathrm{i}} \oint_C \frac{f(\zeta)\mathrm{d}\zeta}{(\zeta - z)^{k+1}}.$$

5: Contour Integration
Just be really good at it that's it.
TODO: Maybe add a few methods of solving these problems in general cases.

6: Analytic Continuation, Entire Functions, Maximum Modulus Principal, and Liouville's Theorem

7: Taylor and Laurent Series representations of complex valued functions

8: Singularities and their types

9: Residue Theorem and applications

10: Miscellaneous things to have memorized (Taylor/Laurent series representations and others)

(a) Know the series representations of e^z, $\sin z$ and $\cos z$
(b) Know the exponential representation of $\sin z$ and $\cos z$
(c) Know helpful representations for $\tan z$, $\sec z$, $\csc z$, $\cot z$, $\sinh z$, $\cosh z$, $\tanh z$, $\coth z$, $\operatorname{sech} z$, and $\operatorname{csch} z$ in terms of exponentials, sines, and cosines.
(d) Geometric series
(e)