AMATH 561 Autumn 2024
Problem Set 1

Due: Mon 10/7 at 10am

Note: Submit electronically to Canvas.

1. Describe the probability space for the following experiments: a) a biased coin is tossed three times; b) two balls are drawn without replacement from an urn which originally contained two blue and two red balls.

2. (No translation-invariant random integer). Show that there is no probability measure P on the integers $\mathbb{Z}$ with the discrete σ-algebra $2^{\mathbb{Z}}$ with the translation-invariance property $P(E+n) = P(E)$ for every event $E \in 2^{\mathbb{Z}}$ and every integer n. $E+n$ is obtained by adding n to every element of E.

3. (No translation-invariant random real). Show that there is no probability measure P on the reals $\mathbb{R}$ with the Borel σ-algebra $\mathcal{B}(\mathbb{R})$ with the translation-invariance property $P(E+x) = P(E)$ for every event $E \in \mathcal{B}(\mathbb{R})$ and every real x. Borel σ-algebra $\mathcal{B}(\mathbb{R})$ is the σ-algebra generated by intervals $(a, b] \subset \mathbb{R}$.

4. Let $\Omega = \mathbb{R}$, $\mathcal{F}$ = all subsets of $\mathbb{R}$ so that A or A^c is countable. Let $P(A) = 0$ in the first case and $P(A) = 1$ in the second. Show that $(\Omega, \mathcal{F}, P)$ is a probability space.