**AMATH 567 FALL 2024**
**HOMEWORK 1 — DUE SEPT. 30 ON GRADESCOPE BY 1:30PM**
**THE 48 HOUR LATE PENALTY IS WAIVED FOR THIS ASSIGNMENT**

All solutions must include significant justification to receive full credit. If you handwrite your assignment you must either do so digitally or if it is written on paper you must *scan* your work. A standard photo is not sufficient.

If you work with others on the homework, you must name your collaborators.

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**1:** From A&F: 1.1.1: b, e

**2:** From A&F: 1.1.2: b, c, d

**3:** From A&F: 1.1.3: d

**4:** From A&F: 1.1.4: d,f

**5:** For $a, b \in \mathbb{C}$, define

$$a^b = e^{b \log a},$$

where $a = re^{i\theta}$, $-\pi < \theta \leq \pi$ and

$$\log a = \log r + i\theta,$$

is the principal branch of the logarithm. Find the real and imaginary parts of

$$i^i \quad \text{and} \quad (1+i)^i.$$

**6:** Consider the function $e(z) := \sum_{n=0}^{\infty} \frac{z^n}{n!}$, which is defined for all $z \in \mathbb{C}$ (you need not show this). Using only the power series, show that $e(z_1 + z_2) = e(z_1)\, e(z_2)$. Can you find other power series with the same property?

**7:** Consider the complex-valued expression

$$f(z) = z^{1/2}$$

where $z = x + iy$, with $x, y \in \mathbb{R}$. Derive explicit expressions for the real and imaginary part(s) of $f(z)$ in terms of $x$ and $y$. If you make any choices (e.g. for branch cuts), show how they impact your answer. Your answer should not contain any trig functions.

**8:** (Solution of the cubic) Consider the cubic equation

$$x^3 + ax^2 + bx + c = 0,$$

where $a, b$ and $c$ are given numbers.

- Use the change of variables $x = y - a/3$ to reduce the equation to the form

$$y^3 + py + q = 0$$

  Find expressions for $p$ and $q$.
- Let $y = u + v$. We're replacing one unknown with two, so we get to impose another constraint later. Check that

$$u^3 + v^3 + (3uv + p)(u + v) + q = 0.$$

- Now we impose $3uv + p = 0$, so that

$$u^3 v^3 = -p^3/27$$

  Also, from above, we have

$$u^3 + v^3 = -q.$$

  Find a quadratic equation satisfied by both $u^3$ and $v^3$.
- Solve this quadratic equation, finding expressions for $u$ and $v$.
- Finally, obtain an expression for $x$. How many different solutions does your expression give rise to?
- Use your result to solve the cubic $x^3 + 3x^2 + 6x + 8 = 0$.
- (Bombelli's equation) Use your result to solve the cubic $x^3 - 15x - 4 = 0$, writing your result explicitly in terms of real and imaginary parts.