**Machine Learning** **(Due:** Mar. 22**)**

# Assignment #1 (Linear Algebra)

*Instructor:* *Name:* , *ID:*

**Problem 1: Basic Vector Operations** (12 points, 4 points/each)

Let two vectors $\mathbf{a} = (1 \;\; 2 \;\; 3)^T$ and $\mathbf{b} = (-8 \;\; 1 \;\; 2)^T$, answer the following equations:

**(1)** Calculate the $\ell_2$ norm of $\mathbf{a}$ and $\mathbf{b}$.

**(2)** Calculate the Euclidean distance between $\mathbf{a}$ and $\mathbf{b}$ (i.e. $\ell_2$ norm of $\mathbf{a} - \mathbf{b}$).

**(3)** Are $\mathbf{a}$ and $\mathbf{b}$ orthogonal? State you reason.

**Problem 2: Basic Matrix Operations** (40 points, 4 points/each)

Suppose $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$, answer the following questions:

**(1)** Calculate $A^{-1}$ and $\det(A)$.

**(2)** The Rank of $A$ is?

**(3)** The trace of $A$ is?

**(4)** Calculate $A + A^T$.

**(5)** Is $A$ an orthogonal matrix? State your reason.

**(6)** Calculate all the eigenvalue $\lambda$ and corresponding eigenvectors of $A$.

**(7)** Diagonalize the matrix $A$.

**(8)** Calculate the $\ell_{2,1}$ norm $\|A\|_{2,1}$ and the Frobenius norm (i.e. $\ell_2$ norm) $\|A\|_F$. (6 points)

**(9)** Calculate the nuclear norm $\|A\|_*$ and the spectral norm $\|A\|_2$. (6 points)

**Problem 3: Linear Equations** (48 points, 4 points/each)

Please give some proper steps to show how you get the answer.

Let $x = (x_1, x_2, x_3)^T$ and

$$\begin{cases} 2x_1 + 2x_2 + 3x_3 = 1 \\ x_1 - x_2 = -1 \\ -x_1 + 2x_2 + x_3 = 2 \end{cases}$$

Answer the following questions:

**(1)** Solve the linear equations

**(2)** Write it into matrix form(i.e. $Ax = b$) and we will use the same $A$ and $b$ in the following questions.

**(3)** The Rank of $A$ is?

**(4)** Calculate $A^{-1}$ and $\det(A)$

**(5)** Use (4) to solve the linear equations

**(6)** Calculate the inner product and outer product of $x$ and $b$.(i.e. $\langle x, b\rangle$ and $x \otimes b$)

**(7)** Calculate the $\ell_1$, $\ell_2$ and $\ell_\infty$ norm of $b$ (6 points)

**(8)** Suppose $y = (y_1, y_2, y_3)^T$, calculate $y^T Ay$, $\bigtriangledown_y y^T Ay$. (6 points)

**(9)** We add one linear equation $-x_1 + 2x_2 + x_3 = 2$ into linear equations above. Write it into matrix form(i.e. $A_1 x = b_1$)

**(10)** The rank of $A_1$ is?

**(11)** Could these linear equations $A_1 x = b_1$ be solved? State reasons.