Calculating the Optimal Parameters for a Skyhook to Take Single Stage Rockets into Orbit, and Estimating Launch Windows

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*Abstract – The purpose of this paper is to explore the ideal characteristics of a skyhook to launch spacecraft from the atmosphere into orbit. The purpose of this is to reduce the amount of fuel, and subsequently cost, of taking payloads into space. Depending on the desired outcome, different sizes and speeds of a skyhook would be ideal. This paper will look at taking payloads into ‘Low Earth Orbit’.*

*Keywords* - About four, alphabetically ordered, keywords or phrases, separated by commas.

1. Introduction

Ever since the first launch of a satellite into space, Sputnik 1 on 4th October 1957, people have been looking for more efficient, cheaper, and more reusable ways to put objects into orbit, manned or unmanned. Currently, the cheapest way to take payloads into orbit is the SpaceX Falcon 9 rockets. The Falcon 9 can take 22,800kg to LEO if the rocket is expended, costing around $62 million [1]. This makes the cost per kilo in LEO around $2,720.

Presently, the primary method of placing an object in orbit is by using a 2 or 3-stage rocket. In the 1970’s [2], an idea was suggested to use a large rotating structure, already in orbit, to catch a spacecraft at a low altitude and velocity and, through exchange of momentum, carry the spacecraft to a higher altitude and velocity, enough to leave the craft in orbit. Having infrastructure built in orbit to launch payloads would greatly reduce the cost of placing payloads in orbit. This system is known as a skyhook. It has often been suggested to use this approach paired with a hypersonic spaceplane, however in this project I will look at it’s possible use with rockets.

The purpose of this research project is to estimate the ideal characteristics of a skyhook and its capacity to use it to launch payloads from a single stage rocket. The Earth-skyhook system will initially be modelled in an ideal environment, not accounting for forces that may pull or push the satellite out of orbit. Only gravitational forces will be considered in this circumstance. It is useful to look at the ideal conditions as not all celestial bodies have the same properties as the Earth. Natural satellites such as the Moon have much thinner atmospheres, and thus will not face the same drag and gravitational forces, so it is useful to keep them in consideration. A skyhook orbiting the moon may be able to help reduce fuel costs for interplanetary travel.

Once the ideal orbit is evaluated, it is important to evaluate the same system in more realistic Earth conditions. In these more realistic conditions, conditions such as atmospheric drag, the effects of gravity, magnetic fields, and to account for areas of high energy particles such as the Van Allen radiation belts, must be considered. The ideal conditions will look at picking up a spacecraft from the Kármán line, where NASA views as the edge of space [3], while the real conditions will investigate whether this is a sensible altitude to launch craft from.

From there the usability of the skyhook is important to estimate. As the skyhook travels around the world, it will not always be in a position appropriate to catch a spacecraft from an already existing space port, meaning that it may not be able to be used every orbit. Also, the system would not be very efficient if the rocket had to be caught only at the very bottom of the rotation. Considering positioning and the window in which the rocket can be caught by the skyhook, launch windows can be estimated.

1. Mechanics of a Skyhook

Skyhooks work through the principle of exchange of momentum. The tether orbits, gradually rotating as it does so. The tips of the tether would be designed to catch payloads from within the Earth’s atmosphere, and then carry them up through the rotation, and release them at the top of their rotation, or an appropriate point along the rotation. As the spacecraft moves higher up into the orbit, the skyhook will gradually lose momentum, which if ignored could cause the skyhook to fall out of orbit. As a result, it is necessary to place boosters or thrusters to allow the skyhook to rebuild momentum.

The length of the skyhook and its angular velocity will affect the speed difference between the tip of the hook and the payload being picked up from the atmosphere. The longer a skyhook is, the greater its tip speed will be for an equal angular velocity. The ideal result is a combination of both with a low angular velocity, as to allow for delicate payloads, but with a minimal speed difference between the skyhook at the payload it is meeting up with.

The length of the skyhook and its angular velocity will affect the speed difference between the tip of the hook and the payload that is being collected.

1. A Skyhook in Ideal Conditions

While the purpose of this paper is to explore the use of skyhooks in orbit around Earth, skyhooks still have their uses in orbit around other bodies. For this reason, the skyhook is first looked at around an Earth sized planet in ideal conditions. This means that only the effect of gravity is considered, as this is the force to keep the skyhook in orbit.

For the ideal conditions, the skyhook orbits a body the same size as Earth, rotating at the same velocity, one rotation per sidereal day. The two tips of skyhook are treated as two masses rotating around their centre of mass (CoM). The skyhook is assumed to be one rigid body of unvarying mass, consequently the CoM always remains in the centre of the two tips and shows the orbital path that the skyhook is following. The CoM is treated as a satellite in a circular, equatorial orbit, and the minimum altitude for either tip is treated as the Kármán line, 100km, as this is where NASA treats as the beginning of space [3].

1. Orbital Mechanics

From these characteristics, the orbital mechanics of the

CoM can be calculated, and from there the velocities of the

tips can be calculated.

As the orbit is circular, the semi-major and semi-minor axes are equal, which is equal to the radius. The radius of the skyhook, , is half of the complete length of the skyhook, the radius of the orbit can be found.

Using the same gravitational field constant (as Earth, time period for the CoM can be found using (2)

The unit vectors, and used to calculate the position (3) and velocity vectors (4) remain constant for all bodies.

Finally, the satellite position can be found using the satellite orbit equation (5).

Using (1-5), the orbital path of the CoM can be described in a MATLAB function. This can be used as the point around which the skyhook rotates. The results of this show the speed difference between the tips of the skyhook and a payload to be picked up from the central body.

1. Rotational Mechanics

Using the CoM as the basis around which the tip masses

rotate. The angular velocity of the skyhook is one of the

parameters that are intended to be optimised.

The angular velocity of the skyhook is related to the number of rotations it completes per orbit, , and to the time period, T, as calculated using (2).

1. Tip Characteristic Equations

From the orbital rotational mechanics, it is possible to calculate the velocities and positions of the tip masses at any point during the orbit. The tip masses are known as P1, the tip that starts closest to the body, and P2, the tip that starts furthest from the body. The orbit begins at a position where ; the position of the tips, CoM, and centre of the body all line up, such that ; and the initial velocities of the tip masses are .

From the initial conditions, the next step is to calculate the changes in position and velocity. These can be found using the change of and time.

The change in with respect to time can be used alongside the unit vector from (3) to find the change in position of the tip masses.

The simplest way to calculate the velocity of the tip masses is to use the change in position compared to the previous velocity and the difference will be your velocity change.

It is best to use as tiny increment as possible, to achieve the most accurate position and velocity.

Using the angular velocity, it is possible to calculate the g-force experienced by the payload at the tips. As the skyhook rotates around a single point, for the purposes of g-forces, it can be treated as a centrifuge. The Relative Centrifugal Force (RCF) is the g-force experienced at the tip and can be estimated using (11).

In (11) r is the radius of the skyhook in mm. The RPM of a skyhook will simply be the number of rotations per orbit divided by the T in minutes.

1. Evaluation of Ideal Conditions

Using the (1-10), it is possible to create a MATLAB function to calculate the orbit of the skyhook in ideal conditions, called Skyhook\_Ideal\_Conditions (SIC). In order to perform the differentiations to an adequate accuracy the Ordinary Differential Equation (ODE) solver function, ode23, is used. There are five ODE solver function in MATLAB that are suitable for his set of equations. While ode23 is the least accurate, it is also the fastest. ode23 is designed to be used over larger time periods, and as orbits take several hours, the lower tolerances are acceptable. Also, when compared to the other ODE functions, ode23 was between 10% and 60% faster than the more accurate functions, whilst producing the same answer to at least 5 decimal places.

SIC requires two inputs: the number of rotations per orbit; and the length of the skyhook. The function will then output the minimum speed difference between the Earth’s surface and the g-force experienced at the tips.

1. Limitations of the Real Conditions

In an ideal world, a 35,786km long tether could orbit the Earth, pick a payload off the surface of the planet, and place it in a geosynchronous orbit. However, there are a multitude of real factors that limit the size and characteristics of a skyhook.

1. G-force limitations

Both the human body and payload have an upper limit as to the g-force they can withstand. The human body can withstand a sustained g-force of 3g [4], however with training and equipment, such as G-suits, the human body can withstand up to 5g for a sustained period. The g-force that is experienced at the tip of the skyhook varies with the distance between the tips and the number of rotations per orbit. The average g-force at the tip of a 5000km skyhook rotating 10 times per orbit is 28.143g, which is enough to kill a human being. Whereas the average g-force experienced at 5 and 1 rotation per orbit is 8.153g and 0.281g respectively. If the length is shortened, the average g-force reduces as well. A cable 2000km long will experience an average g-force of 5.896g and 0.195g at 5 and 1 rotations per orbit respectively.

The g-force at the tips will change as the skyhook rotates. At all times, the Earth will exert 1g of force on the tip of the skyhook, in the direction of the Earth’s centre. At the bottom of the rotation, where the tip is closest to the Earth, the g-force will be highest. The Earth will exert an extra 1g on the tip due to the gravitation field strength of Earth. Counter to this, at the peak of the rotation, when the tip is furthest from Earth, the tip will experience 1g less than the average as the Earth continues to attract the tip towards itself.

Payloads are less of an issue. As the skyhook is intended to complement rockets, as opposed to completely replacing them, should a payload not be able to withstand the g-forces experienced on the skyhook, they can be sent up by traditional methods. At the same time, non-human payloads are often less affected by g-force and as such, humans will be the limiting factor of the skyhook.

With all these factors taken into consider, the maximum g-force that the tips will be allowed to experience, for use with humans, is 5g, meaning that an average of 4g is the maximum for each rotation.

1. Atmospheric Conditions
2. Momentum

Skyhooks work on the principle of exchange of momentum. If no external forces, except gravity, acted on the skyhook and no craft were attached to the tips at any point, the skyhook would remain rotating, in orbit, indefinitely. However, this skyhook would have no use, and is in unrealistic conditions.

With every spacecraft that attaches to and is carried by the skyhook, the skyhook will lose momentum.

If momentum is not recuperated, then the skyhook will fall out of orbit. There are many methods in which this could be achieved.

1. Minimum Altitude

For the ideal conditions, the minimum altitude of the skyhook is at the Kármán Line, 100km altitude. This is used as it is treated as the edge of space. Under real conditions, this may not be the optimal minimum altitude. There are many factors that could influence the optimal minimum altitude.

Atmospheric drag will slow the skyhook considerably, reducing the momentum, which could cause the skyhook to fall out of orbit. While atmospheric drag could be the end of the skyhook, it could be used to advantage the skyhook. Whether the skyhook penetrates deep into the atmosphere, or just scrapes the surface, momentum will be lost due to atmospheric drag. This momentum needs to be replaced. One way to use (sc)ramjets. Ramjets are an airbreathing reaction engine designed to be used at supersonic speeds. While in the atmosphere, the skyhook will remain supersonic, meaning that it will operate in the most efficient range of speed for the ramjet. However, ramjets require fuel, which will expend over time. If the skyhook is not used frequently enough, the ramjets will run out of fuel, and all advantage of entering the atmosphere will be lost.

1. G-force limitations
2. Figures, Tables and Equations

All figures and tables must fit either one or two-column width, 8.34 cm, or 17.78 cm wide respectively. It is suggested that you use one-column whenever possible. If your table or figure will not fit into one-column, then insert a continuous section break before and after the table or figure, as described above and define it as one-column. To make the paper read easier you may want to position any table or figure that requires one-column either at the bottom of the page or the top of a new page.

Do not abbreviate ‘Table’; use Roman numerals to number tables. Use the following format guidelines for figures and tables:

* **Figure and table headings**: 8 point, Times New Roman UPPERCASE, centred. Place below the figure and above the Table, (this style is defined under the style menu of this document as ‘Figure Heading’)
* Leave one blank line above and below each Table or Figure.
* **Figure and table captions**: 8 point, Times New Roman, Small Caps, centred. Place below the figure or table headings (this style is defined under the style menu of this document as ‘Figure Caption’)
* **Table text**: 8 point, Times New Roman, (this style is defined under the style menu of this document as ‘Table text’)

Table I and Figure 1 below illustrates proper Table and Figure formatting. Avoid placing figures and tables before their first mention in the text.

If you use the ‘caption’ function to label your table or figure, make sure to select which type of figure it is in the options, this will allow you to correctly position table captions above, and image captions below.

Be sure to correctly refer to your tables and figures in text by its label e.g., 'the data (Table 4) was plot against its location from the start and shows a clear exponential trend (Figure 3)”, instead of “the data (see below) was plot against its location from the start and shows a clear exponential trend (see right)”.

TABLE I

Point Sizes and Type Styles

|  |  |  |
| --- | --- | --- |
| Points | Place of Text | Type Styles |
| 8  8  8  8  8  8  10  10  10  10  10  10  11  24 | Table number  Table text  Figure and Table Headings  Figure and Table Captions  Footnote  Reference list  Footer  Abstract  Index Terms  Section Titles  Main Text and Equations  Subheadings  Authors’ names  Title | Roman numerals  UPPERCASE  Small Caps  **Bold**  **Bold**  **Small Caps, Bold**  *Italic*, Left justified  Title Case |



Figure 1

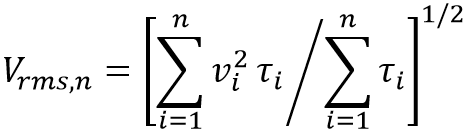
Logo of THE INSTITUTE for Electrical and Electronics Engineers

Number equations consecutively with equation numbers in parenthesis flush with the right margin, as in (1)

*2jk ∂u/∂z = ∂2u/∂x2 + k2 (n2* - β*2) u.* (1)

Refer to ‘(1),’ not ‘Eq. (1)’ or ‘Equation (1),’ except at the beginning of a sentence: ‘Equation (1) is….’

Use the insert ‘symbol’ function to access Greek letters or other mathematical symbols to write your equations. If your equations are complex, use the ‘equation’ tool in a separate word document and insert the equation as a picture but format it within the text as an equation, like this (including an indent):

 (2)

(Use microsoft’s ‘snipping tool’ to copy the equation and paste it as a picture). Be consistant with how you present your equations. Use one style and stick to it.

1. Headers and Footers

Select Page Layout tab, and then select Margins – Custom Margins. This set of actions will open the Page Setup Window; there, select the Layout tab. Set the Header and Footer to 1.27 cm. You should not insert Header text; you may insert footer text if required.

* **Footer text**: should be 10-point Times New Roman, bold (this style is defined under the style menu of this document as ‘Footer’). The text of the footer should say the same as shown on the bottom of this document. Please copy and paste this information into your document exactly as shown on this page.

1. Acknowledgment

Use the singular heading even if you have many acknowledgments. Please put any sponsor acknowledgments in this section; do not use a footnote on the first page for your acknowledgements.

1. References

All source material must be fully referenced in accordance with Swansea University Vancouver Style (please refer to guide for full details). Failure to properly reference all resource material used in a paper leaves the paper’s author open to charges of academic misconduct.

* Vancouver is a specific style of numeric referencing.
* Each reference is given a number as it is cited in the text. The number given becomes the unique identifier fir that reference, and so if it is cited again later in the text, it will still have the same number. The first reference cited with always be number 1 and numbers are allocated sequentially.
* The number should be given in square brackets [].
* Refer simply to the reference number, as [3]. Do not use ‘Ref. [3]’ or ‘reference [3]’ except at the beginning of sentence: “Reference [3] shows…...”.
* In the Reference list at the end of your paper, you should list items in numerical order based on the numbers allocated in the text,
* **Reference text**: 8 point, Times New Roman, full justified, no space between the references (this style is defined under the style menu of this document as ‘References’)

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