Abhyudaya (23010007)

01

Assignment - 01

(a) Test for consistency and solve :—

i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$

$\Rightarrow AX = B$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$\Rightarrow A:B \Rightarrow$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

Applying row operation :—

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\to \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 1 & 4 & -10 & 8 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$R_3 \to \frac{1}{2} R_3$, $R_2 \to 2R_2 - R_1$

$$\Rightarrow \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 0 & 11 & -27 & 11 \\ 0 & 11 & -27 & 13.5 \end{array}\right] \longrightarrow R_3 \to R_3 - R_2$$

$R_3 \to R_3 - R_2$ — Abhyudaya (007) (02)

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 11 & -27 & : & 11 \\ 0 & 0 & 0 & : & 13.5 \end{bmatrix} \to \begin{array}{l} \rho(A) = 2 \\ \rho(A:B) = 3 \end{array}$$

$\Rightarrow \rho(A) \neq \rho(A:B)$

So, the eigen system of equation is inconsistent

(ii) $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$

$$\to A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow A:B$

$$\begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 - R_1 + R_2, \\ R_2 \to 2R_2 + R_1 \end{array}$$

$$\to \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 3 & 5 & : & 16 \\ 0 & 2 & -6 & : & 4 \end{bmatrix} \quad R_2 \to R_2 + 1.5R_3$$

$$\to \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & 0 & -4 & : & 22 \\ 0 & -2 & -6 & : & 4 \end{bmatrix} \quad R_2 \longleftrightarrow R_3$$

$$\begin{bmatrix} 2 & -1 & 3 & : & 8 \\ 0 & -2 & -6 & : & 4 \\ 0 & 0 & -4 & : & 22 \end{bmatrix} \quad \rho(A) = \rho(A:B) = 3$$

$\downarrow$

Consistent & unique solution.

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(iii) $4x - y = 12, \quad -x + 5y - 2z = 0, \quad -2x + 4z = -8$

$\rightarrow AX = B$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$A : B$

$$\Rightarrow \begin{bmatrix} 4 & -1 & 0 & : & 12 \\ -1 & 5 & -2 & : & 0 \\ -2 & 0 & 4 & : & -8 \end{bmatrix}$$

$R_3 \rightarrow 2R_3 + R_1 \qquad R_2 \rightarrow 4R_2 + R_1$

$$\begin{bmatrix} 4 & -1 & 0 & : & 12 \\ 0 & 19 & -8 & : & 12 \\ 0 & -1 & 8 & : & -4 \end{bmatrix}$$

$R_3 \rightarrow R_3 + \frac{1}{19} R_2$

$\frac{-1}{2} + 8$

$$\begin{bmatrix} 4 & -1 & 0 & : & 12 \\ 0 & 19 & -8 & : & 12 \\ 0 & 0 & \frac{-17}{2} & & -64/19 \end{bmatrix}$$

$\rho(A) = \rho(A:B) = 3$

It is consistent and has unique solⁿ

(b) for what value of λ and μ the given system of eqⁿ $x+y+z=6$, $x+2y+3z=0$, $x+2y+\lambda z=\mu$ has (04)

(i) a unique solⁿ

(ii) no solution (iii) infinite no. of solⁿ.

→ [A : B]

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$$

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$R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

(i) for no solution

$\lambda = 3$ & $\mu \neq 10$

(ii) for unique solⁿ

$\lambda \neq 3$ & $\mu \neq 10$

(iii) for Infinite many solⁿ.

$\lambda = 3$, $\mu = 10$

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(05)

(c) find for what values of λ of the given eq^n $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a sol^n and solve them completely in each case

→ $(A:B)$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & \lambda \\ 1 & 4 & 10 & : & \lambda^2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \lambda-1 \\ 0 & 3 & 9 & : & \lambda^2-1 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & \lambda-1 \\ 0 & 0 & 0 & : & (\lambda^2-1-3(\lambda-1)) \end{bmatrix}$$

for system having sol^n:—

$$\lambda^2-1-3\lambda+3=0$$

$$\lambda^2-3\lambda+2=0$$

$$\lambda(\lambda-2)-1(\lambda-2)=0$$

$$(\lambda-1)(\lambda-2)=0$$

$$\boxed{\lambda=2,\ \lambda=1}$$

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(e) find for what values of λ the given eq^n $3x+y-\lambda=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$ may passes non-trivial sol^n & solve them completely in each case.

$$\Rightarrow A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$$

$$|A| = 3(-2\lambda+12) - 1(4\lambda+6\lambda) - \lambda(16+4\lambda) = 0$$

$$\Rightarrow -6\lambda + 36 \quad -10\lambda - 16\lambda - 4\lambda^2 = 0$$

$$\Rightarrow -32\lambda + 36 - 4\lambda^2 = 0$$

$$\Rightarrow 4\lambda^2 + 32\lambda - 36 = 0$$

$$\Rightarrow \lambda^2 + 8\lambda - 9 = 0$$

$$\Rightarrow \lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\Rightarrow \lambda(\lambda+9) - 1(\lambda+9) = 0$$

$$\Rightarrow (\lambda+9)(\lambda-1) = 0$$

$$\Rightarrow \boxed{\lambda = -9, \lambda = 1}$$

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Assignment - 02

Are the following set of vectors linearly dependent or independent.

(i) $[1\ 0\ 0]$, $[1\ 1\ 0]$, $[1\ 1\ 1]$

$\Rightarrow c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1[1 0 0] + c_2 [1 1 0] + c_3[1 1 1] = 0$

$c_1 + c_2 + c_3 = 0$

$c_2 + c_3 = 0$

$c_3 = 0$

$V = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow \text{rank} = 3 = n \rightarrow \boxed{\text{trivial sol}^n}$

→ Linearly dependent

(ii) $[7\ -3\ 11\ -6]$ $[-56\ 24\ -88\ 48]$

$c_1 v_1 + c_2 v_2 = 0$

$V = c_1[7\ -3\ 11\ -6] + c_2[-56\ 24\ -88\ 48]$

$7c_1 - 56c_2 = 0$

$-3c_1 + 24c_2 = 0$

$11c_2 - 82c_2 = 0$

$-6c_1 + 48c_2 = 0$

$A = \begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -82 \\ -6 & 42 \end{bmatrix} \Rightarrow \begin{matrix} R_4 \to R_1 + R_4 \\ R_2 \to R_2 + 3R_4 \\ R_3 \to R_3 - 11R_4 \end{matrix} \begin{bmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{bmatrix}$

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$R_1 \to R_1 - 7R_4$, $R_4 \leftrightarrow R_1$ $\begin{bmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\rho(A) < 2 \to$ infinite sol^n

→ The vector is linearly dependent.

(iii) $[-1\ 5\ 0]$, $[16\ 8\ -3]$, $[-64\ 59\ 9]$

$V = c_1[-1\ 5\ 0] + c_2[16\ 8\ -3] + c_3[-64\ 59\ 9]$

$-c_1 + 16c_2 - 64c_3 = 0$

$5c_1 + 8c_2 + 56c_3 = 0$

$-3c_2 + 9c_3 = 0$

$V = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix} \Rightarrow |V| = -1[72+168] - 5(144+1$

$-24+240 = 0$

$|V| = 0$, $\rho(V) < 3$

System has infinite sol^n.

→ linearly dependent

(iv) $[1\ -1\ 1]$, $[1\ 1\ -1]$, $[-1\ 1\ 1]$, $[0\ 1\ 0]$

$V = c_1[1\ -1,\ 1] + c_2[1\ 1\ -1] + c_3[-1\ 1\ 1] + c_4[0\ 1$

$c_1 + c_2 - c_3 = 0$

$-c_1 + c_2 + c_3 + c_4 = 0$

$c_1 - c_2 + c_3 = 0$

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$$\rightarrow \begin{vmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{vmatrix} \Rightarrow \begin{matrix} R_3 \rightarrow R_3 + R_2 \\ R_2 \rightarrow R_2 + R_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Rank $= 3 \neq n$ → infinite sol^n

→ linearly dependent.

(v) $[2\ -4]$, $[1\ \ 9]$, $[3\ \ 5]$

$$V_1 = c_1[2, -4] + c_2[1\ \ 9] + c_3[3\ \ 5]$$

$$2c_1 + c_2 + 3c_3 = 0$$

$$-4c_1 + 9c_2 + 5c_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix} \Rightarrow R_2 \rightarrow R_2 + 2R_1 \rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 11 & 11 \end{bmatrix}$$

No. of unknown $= 3$ Rank → 2 Rank $\neq n$

∴ → infinite sol^n

→ linearly dependent.

(vi) $[3\ -2\ \ 0\ \ 4]$, $[5\ \ 0\ \ 0\ \ 1]$, $[-6\ \ 1\ \ 0\ \ 1]$, $[2\ \ 0\ \ 0\ \ 3]$

$$V = c_1[3\ -2\ 0\ 4] + c_2[5\ 0\ 0\ 1] + c_3[-6\ 1\ 0\ 1] + c_4[2\ 0\ 0\ 3]$$

$$\rightarrow 3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$$

$$-2c_1 + c_3 = 0$$

$$4c_1 + c_2 + c_3 + c_4 = 0$$

$$V = \begin{bmatrix} 3 & -2 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix} \rightarrow |V| = 0$$

Rank $< 4 \neq n$

→ infinite sol^n or non-trivial sol^n

→ linearly dependent.

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Assignment (03)

find the Eigen values and eigen vectors of the following matrices.

(i) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = A$

$\to A - \lambda I$

$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix}$ $|A - \lambda I| = 0$

$\to (-2-\lambda)[(-\lambda+\lambda^2)-12] - 2[-2\lambda-6] - 3[-4+1-\lambda] = 0$

$\to 2\lambda - 2\lambda^2 + \lambda^2 - \lambda^3 + 24 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$

$\to \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

$(\lambda-5)(\lambda^2+6\lambda+9) = 0$

$(\lambda-5)(\lambda^2+3\lambda+3\lambda+9) = 0$

$(\lambda-5)(\lambda+3)(\lambda+3) = 0$

$\boxed{\lambda = 5 \text{ or } \lambda = -3}$

for $\lambda = 5 \Rightarrow$

$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$ $R_1 \leftrightarrow R_3 \to \begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}$

$\to R_2 \to R_2 + 2R_1$ $R_3 \to R_3 - 7R_1$

$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -6 & -16 \\ 0 & 16 & 32 \end{bmatrix} \to R_3 \to R_3 + 2R_2 \to \begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = K$

$-8x_2 - 16x_3 = 0 \rightarrow -8x_2 = 16x_3$

$x_2 = -2x_3 = -2K$

$x_1 + 2x_2 + 5x_3 = 0$

$x_1 + 2(-2K) + 5(K) = 0$

$x_1 = -K$

$$\begin{bmatrix} -K \\ -2K \\ K \end{bmatrix} \Rightarrow K \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

for $\lambda = -3$

$$\begin{bmatrix} -5 & 2 & -3 \\ 2 & -2 & -6 \\ -1 & -2 & -3 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \rightarrow \begin{bmatrix} -1 & -2 & -3 \\ 2 & -2 & -6 \\ -5 & 2 & -3 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 5R_1 \quad , R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -12 \\ 0 & 12 & 12 \end{bmatrix} \rightarrow R_3 \rightarrow R_3 + 2R_2 \rightarrow \begin{bmatrix} -1 & -2 & -3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} - 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & -12 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 + 3x_3 = 0$

$-6x_2 - 12x_3 = 0$

$-12x_3 = 0$

$x_3 = K$

$\rightarrow x_2 = -2x_3 \rightarrow x_2 = -2K$

$\rightarrow x_1 - 4K + 3K = 0 \rightarrow x_1 = K$

$$\rightarrow \begin{bmatrix} K \\ -2K \\ K \end{bmatrix} \rightarrow K \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

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Eigen value 5, −3 and corresponding Eigen vectors

$$\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

(2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = 0$$

$\rightarrow (4-\lambda)(1-\lambda)^2 + 1(2(1-\lambda)) = 0$

$(4-\lambda)(1+\lambda^2-2\lambda) + (2-2\lambda) = 0$

$4+4\lambda^2-8\lambda-\lambda-\lambda^3+2\lambda^2+2-2\lambda$

$\lambda^3+6\lambda^2-11\lambda+6=0$

$-\lambda^2(\lambda-1)+5\lambda(\lambda-1)-6(\lambda-1)=0$

$(\lambda-1)(\lambda^2-5\lambda+6)=0$

$(\lambda-1)(\lambda-2)(\lambda-3)=0$

for $\lambda=1$

$$\begin{bmatrix} 3 & 0 & 1 \\ 2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + x_3 = 0$

$-2x_2 = 0 \rightarrow x_2 = 0$

$-2x_3 = 0 \rightarrow x_3 = 0$

$\rightarrow \begin{bmatrix} -1/3 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda=2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_1,\ R_2 \rightarrow R_2 + R_1 \rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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$2x_1 + x_3 = 0 \rightarrow x = -K/2$

$-x_2 + x_3 = 0 \rightarrow K \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = x_3 = K$

For $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1 \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$x_1 + x_3 = 0 \quad , -2x_2 + 2x_3 = 0 \quad , x_3 = K$

$x_1 = -x_3$

$K \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\Rightarrow A - \lambda I \rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} \rightarrow (5-\lambda)((-\lambda)(3-\lambda)) = 0$

$\rightarrow (5-\lambda)(-3\lambda + \lambda^2) = 0$

$\rightarrow -15\lambda + 5\lambda^2 + 3\lambda^2 - \lambda^3 = 0$

$\lambda^3 - 8\lambda^2 + 15\lambda = 0$

$(\lambda)(\lambda^2 - 8\lambda + 15) = 0$

$(\lambda)(\lambda^2 - 5\lambda - 3\lambda + 15) = 0$

$\lambda((\lambda-5)(\lambda-3)) = 0$

$\lambda = 0, 3, 5$

For $\lambda = 0$

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for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \quad \begin{matrix} R_1 \leftrightarrow R_3 \\ R_2 \leftrightarrow R_3 \end{matrix} \quad \begin{bmatrix} -1 & 0 & 3 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \to R_2 + 5R_1 \quad \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + 5x_3 = 0 \quad , \quad x_1 = 5x_3$

$15x_2 = 0 \quad , \quad x_2 = 0$

$$K \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_3 \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ +2 & 0 & 0 \end{bmatrix}$$

$$R_3 \to R_3 + 2R_1$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-1x_1 = 0 \to x_1 = +k_1 \quad \to \begin{bmatrix} k_1 \\ k_2 \\ 0 \end{bmatrix}$

$-3x_2 = 0 \to x_2 = +k_2$

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(4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow (A-\lambda I) = 0 \rightarrow \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$\rightarrow (0-\lambda)(-6-3\lambda+2\lambda+\lambda^2) = 0$

$6\lambda - 3\lambda^2 + 2\lambda^2 - \lambda^3 = 0$

$\lambda^3 - \lambda^2 - 6\lambda = 0$

$\lambda(\lambda^2 - \lambda - 6) = 0$

$\lambda(\lambda(\lambda-3) + 2(\lambda-3)) = 0$

$\lambda(\lambda-3)(\lambda+2) = 0$

$\lambda = 0, \lambda = 3, \lambda = -2$

for $\lambda = 0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

$R_1 \leftrightarrow R_3$; $R_3 \leftrightarrow R_2$

$\begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$3x_2 + 4x_3 = 0$

$x_2 = -\frac{4}{3} x_3$

$x_2 \rightarrow K$, $x_3 = -\frac{4}{3} K$, $x_1 = 0$

$K \begin{bmatrix} 0 \\ 1 \\ -4/3 \end{bmatrix}$

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for $\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \quad R_1 \rightarrow -\frac{1}{3} R_1$$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rightarrow -3x_1 = 0 \quad , 4x_2 = 0 \quad , -5x_3 = 0$

$x_1 = K_1 \quad , x_2 = K_2 \quad x_3 = K_3$

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 \rightarrow -\frac{1}{2} R_1$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0 \quad , \quad x_2 + 4x_3 = 0$

$x_2 - 4x_3$

$x_3 \rightarrow K \quad , x_2 = -4K$

$$K \begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}$$

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(5) For the following matrix find one eigen value without calculation and justify your answer.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$\Rightarrow$ $|A| = 0$

as $R_1 = R_2 = R_3$

As we know $\rightarrow$ ~~for~~

Product of eigen value = det. of vector.

Since, det is 0 so one of the eigen value will be 0

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Assignment - 04

(1) find the rank of the matrix A by reducing in Row reduced Echlon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 2R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$R_3 \to R_4 + R_3$

$R_4 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & -1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 4 & 5 & -1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $(A) = 3$

(2) Let W be the vector space of all symmetric 2×2 matrices $T: W \to P_2$ be linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c) + (c-a)x^2$.

find the rank ~~class~~ and nullity of T.

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③ Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. find Eigen value & eigen vectors of A^{-1} and $A + 4I$.

$\rightarrow \quad A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A : I$

$\left[\begin{array}{cc:cc} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right]$

$R_2 \rightarrow R_2 + R_1$

$\left[\begin{array}{cc:cc} 2 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$

$R_2 \leftrightarrow R_1$

$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 0 \end{array}\right]$

$R_2 \leftrightarrow R_2 - R_1$

$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & -1 \end{array}\right]$

$R_2 \rightarrow R_2 \times -1$

$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 1 \end{array}\right]$

$R_2 \rightarrow R_2 + R_1$

$\left[\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 2 \end{array}\right]$

$R_1 \rightarrow 3R_1 - R_2$

$\left[\begin{array}{cc:cc} 3 & 0 & 2 & 1 \\ 0 & 3 & 1 & 2 \end{array}\right]$

$R_1 \rightarrow 1/3 R_1 \quad , R_2 \rightarrow 1/3 R_2$

$\left[\begin{array}{cc:cc} 1 & 0 & 2/3 & 1/2 \\ 0 & 1 & 1/3 & 2/3 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 2/3 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}$

$\rightarrow |A^{-1} - \lambda I| = 0$

$\frac{1}{3}\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right) = \frac{1}{3}\left[(2-\lambda)^2 - 1\right] = 0$

$\rightarrow \lambda^2 + 4 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 1, \lambda = 3}$

<u>for $\lambda = 1$</u>

$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1$

$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ let $x_2 = k$

$x_1 + x_2 = 0 \Rightarrow x_1 - x_2 = -k$

$\rightarrow \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad k = 1 \Rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

<u>for $\lambda = 3$</u>

$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 \rightarrow R_2 + R_1$

$\frac{1}{3}\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 + x_2 = 0, \; x_2 = k \Rightarrow x_1 = x_2 = k$

$\begin{bmatrix} k \\ k \end{bmatrix} \rightarrow k\begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow k = 1, \; \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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$\rightarrow A + 4I \rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

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(4) Solve by Gauss-seided method (Take three iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1 + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial value $x(0)=0,\ y(0)=0,\ z(0)=0$

$\rightarrow x_1 = \frac{7.85x + 0.1y + 0.2z}{3} \xrightarrow{y=0,\, z=0} x_1 = \frac{7.25}{3} = 2.616$

$y_1 = \frac{-19.3 - 0.1x + 0.3z}{7} \xrightarrow[z=0]{x=2.616} \frac{-19.3 - 0.1(2.616)}{7} = -2.7$

$x_2 = \frac{7.85 + 0.1 y_1 + 0.2 \times z_1}{3} = \frac{7.85 + (0.1)(-2.4415) + 0.2(-2.194)}{3}$

$= 2.995$

$y_2 = \frac{-19.3 + 0.3(z) + 0.32}{7} = \frac{-19.3 + 0.3(7.00563) + 0.1(2.9995)}{7}$

$= -2.41418$

$z_2 = \frac{71.4 - 0.3x_2 + 0.2y_2}{10} = \frac{71.4 - 0.3(2.9905) + 0.2(-2.41410)}{7}$

$= 7.0020014$

$x_3 = \frac{7.85 + 0.1y_2 + 0.222}{3} = \frac{7.85 + 0.1(-2.41418) + 0.2(7.0020}{3}$

$\rightarrow 3.0029$

$y_3 = \frac{-19.3 + 0.3(z_2) + 0.1(x_3)}{7} = \frac{-19.3 + 0.3 \times (7.002) + 0.1 \times 3.0029}{7}$

$z_3 = \frac{71.4 - 0.3(x_3) + 0.2(y_3)}{10} = \frac{71.4 - 0.3 \times (3.0029) + 0.2 \times (-2.99}{10}$

$= 7.00125$

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(5) Define consistent and inconsistent system of equations here solve the following system of equation if consistent
$x+3y+2z=0$, $2x-y+3z=0$, $3x-5y+4z=0$, $x+17y+4z=0$

→ $AX=0$ → Homogenous eq^n → Always consistent.

$$A=\begin{bmatrix}1&3&2\\2&-1&3\\3&-5&4\\1&17&4\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$R_2 \to R_2-2R_1$, $R_3 \to R_3-3R_1$, $R_4 \to R_4-R_1$

$$\begin{bmatrix}1&3&2\\0&-7&-1\\0&-14&-2\\0&14&2\end{bmatrix}$$

$R_3 \to R_3-2R_2$, $R_4 \to R_4+R_3$

$$\begin{bmatrix}1&3&2\\0&-7&-1\\0&0&0\\0&0&0\end{bmatrix} \to$$

$\rho(A)=2$

No. of variable $=3$

$\rho(A) < 3n$ → Infinite sol^n

For sol^n →

$$\begin{bmatrix}1&3&2\\0&-7&-1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \to \begin{array}{l}x+3y+2z=0\\-7y-z=0\\z=-7y\end{array}$$

$x+3y-7y=0$

$x+4y$ let $y=k$

$x=4k$, $z=7k$, $y=k$ $\begin{bmatrix}4k\\k\\7k\end{bmatrix}=k\begin{bmatrix}4\\1\\7\end{bmatrix}$

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(7) Determine whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the sub space spanned by S.

$$c_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$c_1, c_2, c_3 \rightarrow$ scalar & $(v_1, v_2, v_3) \in V$.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$AX = B$

$[A : B]$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_2 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 9/5 \, R_2$.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$P = 2 < n$

$\rightarrow$ infinite sol^n

$\rightarrow$ Linear dependent.

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(8) Using Jacobi's method (perfor 3 iterations), solve $3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$, with initial values $x_0 = 1$, $y_0 = 1$, $z_0 = 1$.

→ $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 16$

$x_1 = \frac{23 + 6y_0 - 2z_0}{3} = \frac{0.23 + 6 \times 1 - 2 \times 1}{3} = \frac{27}{3} - 9$

$y_1 = -15 + z_0 + 4x_0 = -15 + 1 + 4 \times 1 = -10$

$z_1 = \frac{16 - x_0 + 3y_0}{7} = \frac{16 - 1 + 3(1)}{7} = \frac{18}{7} = 2.57$

$x_2 = \frac{23 + 6y_1 - 2z_1}{3} = \frac{27 + 6 \times -10 - 2 \times 2.57}{3} = -12.714$

$y_2 = -15 + z_1 + 4x_1 = -15 + 2.57 + 4 \times 9 = 23.5714$

$z_2 = \frac{16 - x_1 + 3y_1}{7} = \frac{16 - 9 + 3 \times -10}{7} = -3.25$

$x_3 = \frac{23 + 6y_2 - 2z_2}{3} = \frac{23 + 6(23.5714) - 2(-3.25)}{3}$

$= 56.99$

$y_3 = -15 + z_2 + 4x_2 = -15 + (3.25) + 4(-12.714) = -69.14$

$z_3 = \frac{16 - x_2 + 3y_2}{7} = \frac{16 - (-12.714) + 3(23.5714)}{7}$

$\Rightarrow 14.20404$