

CISS450: Artificial Intelligence

Lecture 21: Sets

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Agenda

- ♦ Study sets.

Sets

- Sets are extremely important and appears in CS and math.
- A set is just a collection of objects or values.
- $\{1, 3, 6, '2'\}$ is a set. $\{\}$ is the empty set.
- 1 is a member of the set $\{1, 3, 6, '2'\}$
- 6 is a member of the set $\{1, 3, 6, '2'\}$
- 5 is not a member of the set $\{1, 3, 6, '2'\}$
- Things in a set are called elements of the set.
- $1 \in \{1, 3, 6, '2'\}$ means 1 is a member of $\{1, 3, 6, '2'\}$

Subset and Superset

- $\{1, 3, 6, '2'\}$ is a subset of $\{0, 1, 3, 6, '2', 7, 9\}$.
 We also say that $\{0, 1, 3, 6, '2', 7, 9\}$ is a superset of $\{1, 3, 6, '2'\}$.
- In general set X is a subset of set Y if every element of X is an element of Y .
- The set $\{1, 3, 6, '2'\}$ is the same as the set $\{3, '2', 6, 1\}$: Order does not matter.
- The set $\{1, 3, 6, '2'\}$ is the same as the set $\{1, 1, 3, 3, 6, 6, 6, '2'\}$: Repetitions are not important.
- In general two sets X and Y are equal if X is a subset of Y and Y is a subset of X .

Union and Intersection

- The union of sets X and Y is the set containing all the elements either in X or in Y .
- $\{1,2,3,'2',7\}$ is the union of $\{1,3\}$ and $\{2,'2',3,7\}$.
- We write: $\{1,3\} \cup \{2,'2',3,7\} = \{1,2,3,'2',7\}$
- The intersection of sets X and Y is the set containing all the elements in both X and Y .
- $\{3\}$ is the intersection of $\{1,3\}$ and $\{2,'2',3,7\}$.
- We write: $\{1,3\} \cap \{2,'2',3,7\} = \{3\}$

Difference

- If X and Y are sets, the difference $X - Y$ is the set of elements in X but not in Y .
- $\{1, 3, 6, '2'\} - \{0, 3, 7\} = \{1, 6, '2'\}$

Python set

```
x = set([1,3,6, '2'])
print(x, len(x))
print(1 in x, 5 in x, 5 not in x)
print(x == set([3, '2', 6, 1]))
y = set([0, 1, 3, 6, '2', 7, 9])
print(x.issubset(y), y.issuperset(x))
x = set([1,3]); y = set([2, '2', 3, 7])
print(x.union(y))
print(x.intersection(y))
x.clear()
print(x)
x.add(1); x.add(3); x.add(6); x.add('2')
print(x)
x.remove(3)
print(x)
for e in x:
    print(e)
```

Python frozenset

- A set is mutable: you can change it by adding elements to it or removing elements from it.
- A frozenset is the same as a set except it is immutable: cannot be changed
- Sets cannot be used as dictionary keys.
- Frozensets can be used as dictionary keys.

```
d = {}
x = frozenset([1, 3, 6, '2'])
d[x] = 'hello world'
```


Graph

- ♦ Undirected graph:
 - ♦ $G = (V, E)$ where V is set of nodes
 - ♦ E is a set containing (u,v) where u,v are nodes.
 Since G is undirected, if E contains (u,v) , then it also contains (v,u) . This means that in G if you can go from u to v , then you can go from v to u .
 - ♦ Example: $G = (\text{set}([1, 2, 3]), \text{set}([(1,2),(2,1)]))$
 - ♦ OR ...
 - ♦ E is a set containing $\{u,v\}$.
 - ♦ Example: $G = (\text{set}([1, 2, 3]), \text{set}([\text{set}([1,2])]))$

Graph

- ♦ Directed graph:
 - ♦ $G = (V, E)$ where V is set of nodes
 - ♦ E is a set containing (u,v) where u,v are nodes.
 - ♦ Example: $G = (\text{set}([1, 2, 3]), \text{set}([(1,2)]))$
- ♦ Note that graph are usually represented by matrices or adjacency lists.

Sets

- ♦ Sets are frequently implemented using balanced trees.
 - ♦ C++ `std::set`: trees
 - ♦ Python `set`: hashtables