HW10

Ch. 3.3: 16,18,20

16.) What is the largest n for which one can solve within a day using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 10^{-11} seconds, with these functions $f(n)$?

s = 84600sec/day, o = 10^{11} operations/sec

a) $\log n$
2(so) = 7.46496(10^{20})

b) $1000n$
$\frac{so}{1000} = 8.46(10^{12})$

c) n^2
$\sqrt{so} = 92,951,600$

d) $1000n^2$
$\sqrt{\frac{so}{1000}} = 2,939,387$

e) n^3
$\sqrt[3]{so} = 205,197$

f) 2^n
$\log(xy) = 52$

g) 2^{2n}
$\frac{\log(so()}{2} = 26$

h) 2^{2^n}
$\log(\log(so)) = 5$

18.) How much time does an algorithm take to solve a problem of size n if this algorithm uses $2n^2$ + 2^n operations, each requiring 10^{-9} seconds, with these values of n?

a) 10
$2(10)^2 + 2^{10} = 1224(10^{-9})$ seconds

b) 20
$2(20)^2 + 2^{20} = 1049376(10^{-9})$ seconds

c) 50
$2(50)^2 + 2^{50} = 1.12589907(10^{15})(10^{-9})$ seconds

d) 100
$2(100)^2 + 2^{100} = 1.2676506(10^{30})(10^{-9})$ seconds

20.) What is the effect in the time required to solve a problem when you double the size of the input from n to $2n$, assuming that the number of milliseconds the algorithm use problem with input size n is each of these functions?

Express your answer in the simplest form possible, either as a ratio or a difference. Your answer may be a function of n or a constant.

a) $\log \log n$
$\log \log 2n$

b) $\log n$
$\log 2n$

c) $100n$
$200n$

d) $n \log n$
$2n \log 2n$

e) n^2
$2n^2$

f) n^3
$2n^3$

g) 2^n
2^{2n}

Ch. 4.1: 4, 6, 8, 14, 26, 28, 32, 34, 44, 46

4.) Prove that part(*iii*) of Theorem 1 is true.

Part in question: If $a|b$ and $b|c$, then $a|c$.

Assume $a|b$ and $b|c$. Then, x, $y \in \mathbb{Z}$ s.t. c = ax and d = by
Then, cd = $axby$, so $(ab|cd)$ = $(ab|axby)$
$\therefore ab|cd$

6.) Show that if a, b, c and d are integers, where $a \neq 0$, s.t. $a|c$ and $b|d$, then $ab|cd$.

Assume $a|c \wedge b|c$. Then, $\exists x, y \in \mathbb{Z}$ s.t. $c = ax$ and $d = by$
Then, $cd = axby$, So $(ab|cd) = (axby)$. $\therefore ab|cd$

8.) Prove or disprove that if $a|bc$, where a, b, and c are positive integers and $a \neq 0$, then $a|b$ or $a|c$.
Let $a = 6$, $b = 2$, $c = 3$
In this case does $a|bc$?
$bc = 2 * 3 = 6$
$6|6 \therefore a|bc$ is true in this case.
However, $a \nmid b$ because $a = 6$ and thus $6 \nmid 2$.
Additionally, $a \nmid c$ because $a = 6$ and thus $6 \nmid 3$
$\therefore$ because $a|bc$ but $a \nmid b$ and $a \nmid c$ the overall statement is false.

14.) What are the quotient and remainder when:

a) **44 is divided by 8?**
$8|44 \equiv 8 * 5 + 4$

b) **777 is divided by 21?**
$21|777 \equiv 777 = 21 * 37 + 0$

c) **-123 is divided by 19?**
$19| - 123 \equiv -123 = 19 * -7 + 9$
d) **-1 is divided by 23?**
$23| - 1 \equiv -1 = 23 * -1 + 22$

e) **-2002 is divided by 87?**
$87| - 2002 \equiv -2002 = 87 * -24 + 86$

f) **0 is divided by 17?**
$17|0 \equiv 17 * 0 + 0$

g) **1,234,567 is divided by 1001**
$1001|1,234,567 = 1001 * 1233 + 334$

h) **-100 is divided by 101**
$101| - 100 \equiv -100 = 101 * -1 + 1$

26.) Evaluate these quantities

a) -17 mod 2
-1

b) 144 mod 7
4

c) -101 mod 13
-10

d) 199 mod 19

9

28.) Find a div m and a mod m when

a) $a = -111, m = 99$

$q = \frac{a}{m}$

$q = -\frac{111}{99} = -1.\overline{12}$

$|-2| * 99 = 198$

$198 - 111 = 87$

$0 \leq 87 < d$ ($m = d$ in this case)

$q = 2, r = 87$

b) $a = -9999, m = 101$

Following the same formula as above

$-\frac{9999}{101} \approx -99$

$-9999 = -99 * 101 + r$

$-9999 = 99 * 101 + 0$

$q = 99, r = 0$

c) $a = 10299, m = 999$

Following the same formula as above

$\frac{10299}{999} \approx 10$

$10299 = 10 * 999 + r$

$10299 = 10 * 999 + 9$

$q = 10, r = 9$

d) $a = 123456, m = 1001$

Following the same formula as above

$\frac{123456}{1001} \approx 123$

$123456 = 123 * 1001 + r$

$123456 = 123 * 1001 + 333$

$q = 123, r = 333$

32.) List five integers that are congruent to 4 modulo 12

4, 16, 28, 40, 52

Decide whether each of these integers is congruent to 3 modulo 7:

a) 37

False

b) 66

True

c) -17

False

d) -67
False

44.) Show that if n is an integer then $n^2 \equiv 0$ or 1 (mod 4)

Assume $n \in \mathbb{Z}$. In the case that n is odd $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$.
Then, $4k^2 + 4k + 1 \equiv 1 \vee 0$(mod 4)
$\therefore n^2 \equiv 0 \vee 1$(mod 4)

46.) Prove that if n is an odd positive integer, then $n^2 \equiv 1$(mod8)

Assume $n = 2k + 1$ by the definition of odd positive integer.

Then, $n^2 = 4k^2 + 4k + 1 = 4(k^2 + k)$.

In the case where k is odd, $k^2 = k$ has to be the sum of an odd square and any other odd number. This would mean that the sum would be even by nature of even and odds.

Let $2k = k^2 + k$ where $x \in \mathbb{Z}$.

So, $4(k^2 + k) + 1 = 4(2x) + 1 \rightarrow 8x + 1 \equiv 1(mod 8) \rightarrow 1 \equiv 1$
$\therefore n^2 \equiv 1(mod 8)$