

# Notes August 24, 2023

## Updated Aug25

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CAP4720

## Vectors

Defined using a magnitude and direction, possibly also with an origin

### Notations

- Ordered set notation  $\vec{v} = (x, y, z)$  or  $\vec{v} = \langle x, y, z \rangle$
- Unit vector notation  $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$
- Matrix notation  $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

### Magnitude

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad \|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$$

### Unit Vector

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

### Dot Product

Computes the “likeness” of two vectors. For two unit vectors dotted together,  $-1$  is facing opposite directions,  $0$  is orthogonal, and  $1$  is facing the same direction.

*Hunter side-note: Dot products with surface normals are going to be useful later, as they can be used in conjunction with a normalized vector pointing towards a scene's light source to calculate how much of that given pixel is facing the light.*

$$\vec{u} \cdot \vec{v} = \vec{u}_x \vec{v}_x + \vec{u}_y \vec{v}_y + \vec{u}_z \vec{v}_z = \|\vec{u}\| \|\vec{v}\| \cos(\theta)$$

### Triangle Cosine Law

Refresh on the Law of Cosines from Trig.

$$2ab \times \cos(\theta) = a^2 + b^2 - c^2$$

### Projection

$$\text{proj}_{\vec{w}} \vec{v} = \|\vec{v}\| \cos(\theta) \frac{\vec{w}}{\|\vec{w}\|}$$

$$\text{proj}_{\vec{w}} \vec{v} = \frac{(\vec{v} \cdot \vec{w}) \vec{w}}{\|\vec{w}\|^2}$$

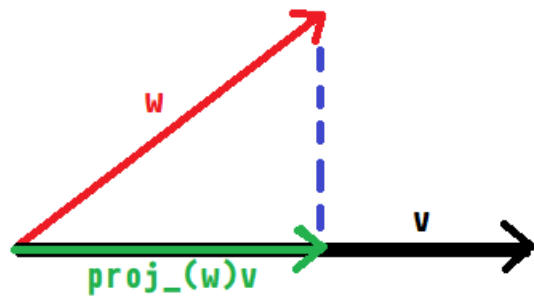


Diagram of  $\vec{w}$  being projected onto  $\vec{v}$