interpoláció:

Lagrange: $y = f(x) = \sqrt{x^7}$ ①

$[0,1]$

$x_0 = 0$

$x_1 = 0,5$

$x_2 = 1$

	x_0	x_1	x_2
x	0	0,5	1
y	0	$\sqrt{0,5^7}$	1.
	y_0	y_1	y_2

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0,5)(x-1)}{\underbrace{(0-0,5)(0-1)}_{(-0,5)\cdot(-1)}} = 2\cdot(x-0,5)\cdot(x-1)$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-1)}{\underbrace{(0,5-0)}_{0,5}\underbrace{(0,5-1)}_{(-0,5)}} = -4\cdot x\cdot(x-1)$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)\cdot(x-0,5)}{\underbrace{(1-0)\cdot(1-0,5)}_{(1)\cdot(0,5)}} = 2\cdot x\cdot(x-0,5)$$

$$L_2(x) = 0\cdot 2\cdot(x-0,5)\cdot(x-1) + \sqrt{0,5^7}\cdot(-4)\cdot x\cdot(x-1) + 1\cdot 2\cdot x\cdot(x-0,5)$$

hibabecslés:

$$|f(x) - L_2(x)| \leq \frac{M_3}{4\cdot 3}\cdot\left(\frac{1-0}{2}\right)^3 = \frac{\frac{105}{8}}{12}\cdot\left(\frac{1}{2}\right)^3 = 0,13672$$

$$f(x) = \sqrt{x^7} = x^{\frac{7}{2}} \rightarrow f'(x) = \frac{7}{2}\cdot x^{\frac{5}{2}} \rightarrow f''(x) = \frac{7}{2}\cdot\frac{5}{2}\cdot x^{\frac{3}{2}}$$

$$\rightarrow f'''(x) = \frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot x^{\frac{1}{2}} = \frac{105}{8}\cdot\sqrt{x}$$

$f'''(0) = 0$

$f'''(1) = \frac{105}{8}$

② $f(x) = \ln(4x)$ $\qquad x = \left(\overset{x_0}{\tfrac{1}{4}} ; \overset{x_1}{\tfrac{1}{2}} ; \overset{x_2}{1}\right)$

$$l_0(x) = \frac{\left(x-\frac{1}{2}\right)(x-1)}{\left(\frac{1}{4}-\frac{1}{2}\right)\left(\frac{1}{4}-1\right)} = \frac{16}{3}\cdot\left(x-\frac{1}{2}\right)(x-1)$$

$$l_1(x) = \frac{\left(x-\frac{1}{4}\right)(x-1)}{\left(\frac{1}{2}-\frac{1}{4}\right)\left(\frac{1}{2}-1\right)} = -8\cdot\left(x-\frac{1}{4}\right)(x-1)$$

$$l_2(x) = \frac{\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)}{\left(1-\frac{1}{4}\right)\left(1-\frac{1}{2}\right)} = \frac{8}{3}\cdot\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)$$

$$\underline{L_2(x) = 0\cdot\frac{16}{3}\cdot\left(x-\frac{1}{2}\right)(x-1) + \ln(2)\cdot(-8)\cdot\left(x-\frac{1}{4}\right)(x-1) +}$$
$$\underline{\underline{+\ln(4)\cdot\frac{8}{3}\cdot\left(x-\frac{1}{4}\right)\left(x-\frac{1}{2}\right)}}$$

<u>hibabecslés:</u>

$$|f(x) - L_2(x)| \leq \frac{M_3}{12}\cdot\left(\frac{1-\frac{1}{4}}{2}\right)^3$$

$$f'(x) = \frac{1}{x} \rightarrow f''(x) = \frac{-1}{x^2} \rightarrow \underline{f'''(x) = \frac{2}{x^3}}$$

$$f'''\left(\tfrac{1}{4}\right) = 128 \qquad f'''(1) = 2$$

$$|f(x) - L_2(x)| \leq \frac{128}{12}\cdot\left(\frac{3}{8}\right)^3 = \underline{\underline{0,5625}}$$

3.) $f(x) = \sin(-3x)$ $\qquad x = \left(\underset{x_0}{\frac{\pi}{12}} ; \underset{x_1}{\frac{\pi}{6}} ; \underset{x_2}{\frac{\pi}{3}}\right)$

$$l_0(x) = \frac{\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{3}\right)}{\underbrace{\underbrace{\left(\frac{\pi}{12}-\frac{\pi}{6}\right)}_{-\frac{\pi}{12}}\underbrace{\left(\frac{\pi}{12}-\frac{\pi}{3}\right)}_{-\frac{\pi}{4}}}_{\frac{\pi^2}{48}}} = \frac{48}{\pi^2}\cdot\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{3}\right)$$

$$l_1(x) = \frac{\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{3}\right)}{\left(\frac{\pi}{6}-\frac{\pi}{12}\right)\left(\frac{\pi}{6}-\frac{\pi}{3}\right)} = -\frac{72}{\pi^2}\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{3}\right)$$

$$l_2(x) = \frac{\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)}{\left(\frac{\pi}{3}-\frac{\pi}{12}\right)\left(\frac{\pi}{3}-\frac{\pi}{6}\right)} = \frac{24}{\pi^2}\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)$$

$$L_2(x) = -\frac{\sqrt{2}}{2}\cdot\frac{48}{\pi^2}\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{3}\right) + (-1)\cdot\frac{-72}{\pi^2}\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{3}\right) + 0\cdot\frac{24}{\pi^2}\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)$$

hibabecslés: $\left|f(x) - L_2(x)\right| \leq \frac{M_3}{12}\cdot\left(\frac{\pi}{8}\right)^3$

$$f'(x) = (-3)\cdot\cos(-3x) \to f''(x) = 9\cdot -\sin(-3x) \to f'''(x) = +27\cdot\cos(-3x)$$

$$f'''\left(\frac{\pi}{12}\right) = \frac{27\sqrt{2}}{2} \qquad f'''\left(\frac{\pi}{3}\right) = -27$$

$$\left|f(x) - L_2(x)\right| \leq \frac{27}{12}\cdot\left(\frac{\pi}{8}\right)^3 = 0{,}13626$$

Newton-féle alak:

$f(x) = \sqrt{x^5}$

$x_0 = 0{,}1$

$x_1 = 0{,}5$ $\qquad [0{,}1 ; 0{,}9]$

$x_2 = 0{,}9$

$x_0 = 0{,}1 \to f(x_0) = \sqrt{0{,}1^5}$

$x_1 = 0{,}5 \to f(x_1) = \sqrt{0{,}5^5}$

$x_2 = 0{,}9 \to f(x_2) = \sqrt{0{,}9^5}$

$\frac{f(x_1) - f(x_0)}{x_1 - x_0}$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

$\frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$

$f[x_0] = f(x_0) = \sqrt{0{,}1^5} = 3{,}1623 \cdot 10^{-3}$

$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{\sqrt{0{,}5^5} - \sqrt{0{,}1^5}}{0{,}5 - 0{,}1} = 0{,}434036$

$f[x_0, x_1, x_2] = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} =$

$= \frac{\frac{\sqrt{0{,}9^5} - \sqrt{0{,}5^5}}{0{,}9 - 0{,}5} - \frac{\sqrt{0{,}5^5} - \sqrt{0{,}1^5}}{0{,}5 - 0{,}1}}{0{,}9 - 0{,}1} = 1{,}3064$

$N_2(x) = f[x_0] + f[x_0, x_1] \cdot (x - x_0) + f[x_0, x_1, x_2] \cdot (x - x_0) \cdot$
$\cdot (x - x_1)$

$N_2(x) = 3{,}1623 \cdot 10^{-3} + 0{,}434036 \cdot (x - 0{,}1) +$
$+ 1{,}3064 \cdot (x - 0{,}1)(x - 0{,}5)$

Newton interpolációs

(1.) $f(x) = \cos(2x)$ $\qquad x = \begin{pmatrix} x_0 & x_1 & x_2 \\ \frac{\pi}{8}, & \frac{\pi}{4}, & \frac{\pi}{2} \end{pmatrix}$ (radiánban számolyon)

$x_0 \to f(x_0) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$x_1 \to f(x_1) = \cos\left(\frac{\pi}{2}\right) = 0$

$x_2 \to f(x_2) = \cos(\pi) = -1$

$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = f[x_0, x_1] = \frac{0 - \frac{\sqrt{2}}{2}}{\frac{\pi}{4} - \frac{\pi}{8}} = -\frac{\sqrt{2}}{2} \cdot \frac{8}{\pi}$

$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f[x_1, x_2] = \frac{-1 - 0}{\frac{\pi}{2} - \frac{\pi}{4}} = -\frac{4}{\pi}$

$\Rightarrow \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = f[x_0, x_1, x_2] = \frac{-\frac{4}{\pi} + \frac{4\sqrt{2}}{\pi}}{\frac{\pi}{2} - \frac{\pi}{8}} = \frac{\frac{4\sqrt{2} - 4}{\pi}}{\frac{3\pi}{8}} = \frac{32\sqrt{2} - 32}{3\pi^2}$

$N_2(x) = f[x_0] + f[x_0, x_1] \cdot (x - x_0) + f[x_0, x_1, x_2] \cdot (x - x_0)(x - x_1)$

$N_2(x) = \frac{\sqrt{2}}{2} - \frac{4\sqrt{2}}{\pi}\left(x - \frac{\pi}{8}\right) + \frac{32\sqrt{2} - 32}{3\pi^2} \cdot \left(x - \frac{\pi}{8}\right)\left(x - \frac{\pi}{4}\right)$

(2.) $f(x) = \sin\left(\frac{x}{2}\right)$ $\qquad x = \begin{pmatrix} x_0 & x_1 & x_2 \\ \frac{\pi}{2}, & \pi, & 2\pi \end{pmatrix}$ (radiánban számolyon)

$x_0 \to f(x_0) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$x_1 \to f(x_1) = \sin\left(\frac{\pi}{2}\right) = 1$

$x_2 \to f(x_2) = \sin(\pi) = 0$

$\frac{1 - \frac{\sqrt{2}}{2}}{\pi - \frac{\pi}{2}} = \frac{2 - \sqrt{2}}{\pi}$

$\frac{0 - 1}{2\pi - \pi} = -\frac{1}{\pi}$

$\frac{-\frac{1}{\pi} - \frac{2 - \sqrt{2}}{\pi}}{\underbrace{2\pi - \frac{\pi}{2}}_{\frac{3\pi}{2}}} = \frac{-3 + \sqrt{2}}{\pi} \cdot \frac{2}{3\pi} = \frac{-6 + 2\sqrt{2}}{3\pi^2}$

$N_2(x) = \frac{\sqrt{2}}{2} + \frac{2 - \sqrt{2}}{\pi} \cdot \left(x - \frac{\pi}{2}\right) + \frac{-6 + 2\sqrt{2}}{3\pi^2} \cdot \left(x - \frac{\pi}{2}\right)(x - \pi)$

(3.) $f(x) = e^{2x}$ $\qquad x = \begin{pmatrix} x_0 & x_1 & x_2 \\ 0, & \frac{1}{2}, & 1 \end{pmatrix}$

$x_0 \to f(x_0) = 1$

$x_1 \to f(x_1) = e^1$

$x_2 \to f(x_2) = e^2$

$\frac{e - 1}{\frac{1}{2} - 0} = \frac{e - 1}{\frac{1}{2}}$

$\frac{e^2 - e^1}{1 - \frac{1}{2}} = \frac{e(e - 1)}{\frac{1}{2}}$

$\frac{\frac{e(e-1)}{\frac{1}{2}} - \frac{e - 1}{\frac{1}{2}}}{1 - 0} = \frac{e(e-1) - (e-1)}{\frac{1}{2}} = \frac{(e-1)(e-1)}{\frac{1}{2}}$

$N_2(x) = 1 + \frac{e - 1}{\frac{1}{2}} \cdot (x - 0) + \frac{(e-1)^2}{\frac{1}{2}} \cdot (x - 0)\left(x - \frac{1}{2}\right)$

Hermite: $f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$

$x_0 = \frac{\pi}{2}$ $\quad\left[\frac{\pi}{2};\pi\right]$

$x_1 = \pi$

$$\left.\begin{array}{l} x_0 = \frac{\pi}{2} \rightarrow f(x_0) = \sin\left(\frac{\pi}{2}\right) = 1 \\ x_0 = \frac{\pi}{2} \rightarrow f(x_0) = 1 \\ x_1 = \pi \rightarrow f(x_1) = \sin(\pi) = 0 \\ x_1 = \pi \rightarrow f(x_1) = 0 \end{array}\right\rangle \begin{array}{l} f[x_0,x_0] = f'(x_0) = 0 \\ f[x_0,x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{0-1}{\pi-\frac{\pi}{2}} \\ f[x_1,x_1] = f'(x_1) = -1 \end{array}$$

$$\left.\begin{array}{l} f'(x_0) = 0 \\ \frac{f(x_1)-f(x_0)}{x_1-x_0} = -\frac{2}{\pi} \\ f'(x_1) = -1 \end{array}\right\rangle \left.\begin{array}{l} \frac{-\frac{2}{\pi}-0}{\pi-\frac{\pi}{2}} = \frac{-\frac{2}{\pi}}{\frac{\pi}{2}} = -\frac{4}{\pi^2} \\ \frac{-1-\left(-\frac{2}{\pi}\right)}{\pi-\frac{\pi}{2}} = \frac{-\frac{\pi}{\pi}+\frac{2}{\pi}}{\frac{\pi}{2}} = \frac{\frac{2-\pi}{\pi}}{\frac{\pi}{2}} \end{array}\right\rangle \Rightarrow$$

$$\Rightarrow \frac{\frac{\frac{2-\pi}{\pi}}{\frac{\pi}{2}} - \frac{-4}{\pi^2}}{\pi - \frac{\pi}{2}} = \underline{0,11074}$$

$$H_3(x) = f[x_0] + f[x_0,x_0]\cdot(x-x_0) + f[x_0,x_0,x_1]\cdot(x-x_0)^2 + f[x_0,x_0,x_1,x_1]\cdot(x-x_0)^2\cdot(x-x_1)$$

$$\underline{\underline{H_3(x) = 1 + \cancel{0\cdot\left(x-\frac{\pi}{2}\right)} - \frac{4}{\pi^2}\cdot\left(x-\frac{\pi}{2}\right)^2 + 0,11074\cdot\left(x-\frac{\pi}{2}\right)^2\left(x-\pi\right)}}$$

hibabecslés:

$$|f(x) - H_3(x)| \le \frac{M_4}{4!} \cdot \left|\left(x - \frac{\pi}{2}\right)^2 \cdot (x - \pi)^2\right|$$

$f(x) = \sin(x) \rightarrow f'(x) = \cos(x) \rightarrow f''(x) = -\sin(x) \rightarrow f'''(x) = -\cos(x)$

$f^{IV}(x) = \sin(x)$

$$\Longrightarrow \quad f^{IV}\left(\frac{\pi}{2}\right) = 1$$

$$f^{IV}(\pi) = 0$$

$$|f(x) - H_3(x)| \le \frac{1}{4!} \cdot \left|\left(x - \frac{\pi}{2}\right)^2 \cdot (x - \pi)^2\right|$$

Hermite interpoláció:

1. $f(x) = \cos(4x)$ $\qquad x = \left(\overset{x_0}{\frac{\pi}{8}}, \overset{x_1}{\frac{\pi}{4}}\right)$ (radiánban!)

$$x_0 \to f(x_0) = \cos\left(\frac{\pi}{2}\right) = 0$$
$$x_0 \to f(x_0) = \cos\left(\frac{\pi}{2}\right) = 0 \quad > \quad f'(x_0) = f[x_0,x_0] = -4$$
$$x_1 \to f(x_1) = \cos(\pi) = -1 \quad > \quad \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f[x_0,x_1] = \frac{-1-0}{\frac{\pi}{4} - \frac{\pi}{8}} = -\frac{8}{\pi}$$
$$x_1 \to f(x_1) = \cos(\pi) = -1 \quad > \quad f'(x_1) = f[x_1,x_1] = 0$$

$$f'(x) = -4 \cdot \sin(4x)$$

$$\Rightarrow \frac{f[x_0,x_1] - f[x_0,x_0]}{x_1 - x_0} = \frac{-\frac{8}{\pi} + 4}{\frac{\pi}{4} - \frac{\pi}{8}} = \frac{4\pi - 8}{\pi} \cdot \frac{8}{\pi} = \frac{32\pi - 64}{\pi^2} = f[x_0,x_0,x_1]$$

$$\Rightarrow \frac{f[x_1,x_1] - f[x_0,x_1]}{x_1 - x_0} = \frac{0 + \frac{8}{\pi}}{\frac{\pi}{4} - \frac{\pi}{8}} = \frac{16}{\pi^2} = f[x_0,x_1,x_1]$$

$$\Rightarrow \frac{f[x_0,x_1,x_1] - f[x_0,x_0,x_1]}{x_1 - x_0} = \frac{\frac{16}{\pi^2} - \frac{32\pi - 64}{\pi^2}}{\frac{\pi}{4} - \frac{\pi}{8}} = \frac{80 - 32\pi}{\pi^2} \cdot \frac{8}{\pi} =$$

$$= \frac{640 - 256\pi}{\pi^3} = f[x_0,x_0,x_1,x_1]$$

$$H_3(x) = f[x_0] + f[x_0,x_0] \cdot (x - x_0) + f[x_0,x_0,x_1] \cdot (x-x_0)^2 + f[x_0,x_0,x_1,x_1] \cdot (x-x_0)^2 (x - x_1)$$

$$H_3(x) = 0 + (-4) \cdot \left(x - \frac{\pi}{8}\right) + \frac{32\pi - 64}{\pi^2} \cdot \left(x - \frac{\pi}{8}\right)^2 + \frac{640 - 256\pi}{\pi^3} \cdot \left(x - \frac{\pi}{8}\right)^2 \left(x - \frac{\pi}{4}\right)$$

hibabecslés: $\left| f(x) - H_3(x) \right| \le \frac{M_4}{4!} \cdot \left| \left(x - \frac{\pi}{8}\right)^2 \left(x - \frac{\pi}{4}\right)^2 \right|$

$$f(x) = \cos(4x) \to f'(x) = -4 \cdot \sin(4x) \to f''(x) = -16 \cdot \cos(4x) \to f'''(x) = 4^3 \cdot \sin(4x)$$
$$\to f^{IV}(x) = 4^4 \cdot \cos(4x)$$

$$f^{IV}\left(\frac{\pi}{8}\right) = 0$$

$$f^{IV}\left(\frac{\pi}{4}\right) = -256$$

$$\left| f(x) - H_3(x) \right| \le \frac{256}{4!} \cdot \left| \left(x - \frac{\pi}{8}\right)^2 \cdot \left(x - \frac{\pi}{4}\right)^2 \right| = \underline{\underline{10{,}667 \left| \left(x - \frac{\pi}{8}\right)^2 \left(x - \frac{\pi}{4}\right)^2 \right|}}$$

2. $f(x) = \sin\left(\frac{3}{2}x\right)$ $\quad x = \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ (radiánban!)

$x_0 \quad x_1$

$x_0 \to f(x_0) = \frac{\sqrt{2}}{2}$

$x_0 \to f(x_0) = \frac{\sqrt{2}}{2}$

$x_1 \to f(x_1) = 1$

$x_1 \to f(x_1) = 1$

$$\frac{3\sqrt{2}}{4}, \qquad \frac{1-\frac{\sqrt{2}}{2}}{\frac{\pi}{6}}, \qquad 0$$

$$\frac{\frac{6(2-\sqrt{2})}{2\pi} - \frac{3\sqrt{2}}{4}}{\frac{\pi}{6}} = \frac{12-6\sqrt{2}-\frac{3}{2}\pi\sqrt{2}}{2\pi}\cdot\frac{6}{\pi} \Rightarrow$$

$$\frac{0 - \frac{12-6\sqrt{2}}{2\pi}}{\frac{\pi}{6}} = \frac{12+6\sqrt{2}}{2\pi}\cdot\frac{6}{\pi} = \frac{36+18\sqrt{2}}{\pi^2}$$

$f'(x) = \frac{3}{2}\cos\left(\frac{3}{2}x\right)$

$$\Rightarrow \frac{\left(12-6\sqrt{2}-\frac{3}{2}\pi\sqrt{2}\right)\cdot 3}{\pi^2} \qquad \frac{\frac{36+18\sqrt{2}}{\pi^2} - \frac{36-18\sqrt{2}-\frac{9}{2}\sqrt{2}\pi}{\pi^2}}{\frac{\pi}{6}} =$$

$$\Rightarrow \frac{36+18\sqrt{2}}{\pi^2}$$

$$= \frac{\cancel{36}+18\sqrt{2}-\cancel{36}+18\sqrt{2}+\frac{9}{2}\sqrt{2}\pi}{\pi^2}\cdot\frac{6}{\pi} = \frac{6\cdot\left(36\sqrt{2}+\frac{9}{2}\sqrt{2}\pi\right)}{\pi^3}$$

$$H_3(x) = \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}}{4}\cdot\left(x-\frac{\pi}{6}\right) + \frac{3\left(12-6\sqrt{2}-\frac{3}{2}\sqrt{2}\pi\right)}{\pi^2}\left(x-\frac{\pi}{6}\right)^2 +$$

$$+ \frac{6\cdot\left(36\sqrt{2}+\frac{9}{2}\sqrt{2}\pi\right)}{\pi^3}\cdot\left(x-\frac{\pi}{6}\right)^2\left(x-\frac{\pi}{3}\right)$$

hibabecslés: $\left|f(x) - H_3(x)\right| \le \frac{M_4}{4!}\left|\left(x-\frac{\pi}{6}\right)^2\left(x-\frac{\pi}{3}\right)^2\right|$

$$f^{IV}(x) = \left(\frac{3}{2}\right)^4\cdot\sin\left(\frac{3}{2}x\right) \to f^{IV}\left(\frac{\pi}{6}\right) = \left(\frac{3}{2}\right)^4\cdot\frac{\sqrt{2}}{2} \approx 3{,}5797$$

$$\to f^{IV}\left(\frac{\pi}{3}\right) = \left(\frac{3}{2}\right)^4 = 5{,}0625$$

$$\left|f(x) - H_3(x)\right| \le \frac{5{,}0625}{24}\cdot\left|\left(x-\frac{\pi}{6}\right)^2\left(x-\frac{\pi}{3}\right)^2\right| = 0{,}2194\cdot\left|\left(x-\frac{\pi}{6}\right)^2\left(x-\frac{\pi}{3}\right)^2\right|$$

3. $f(x) = e^{5x}$ $\qquad x = (\underset{x_0}{0}, \underset{x_1}{\tfrac{1}{5}})$ $\qquad f'(x) = 5 \cdot e^{5x}$

$x_0 \to f(x_0) = 1$

$x_0 \to f(x_0) = 1$

$x_1 \to f(x_1) = e$

$x_1 \to f(x_1) = e$

$f'(x_0) = 5$

$f[x_0, x_1] = \dfrac{e-1}{\frac{1}{5}}$

$f'(x_1) = 5e$

$f[x_0, x_0, x_1] = \dfrac{\frac{e-1}{\frac{1}{5}} - 5}{\frac{1}{5}} = \dfrac{5e-5-5}{\frac{1}{5}} = 25e - 50$

$f[x_0, x_1, x_1] = \dfrac{5e - (5e-5)}{\frac{1}{5}} = 25$

$\Rightarrow f[x_0, x_0, x_1, x_1] = \dfrac{25 - (25e - 50)}{\frac{1}{5}} = 5 \cdot (25 - 25e + 50) = 375 - 125e$

$H_3(x) = 1 + 5 \cdot (x-0) + (25e-50) \cdot (x-0)^2 + (375-125e)(x-0)^2\left(x-\tfrac{1}{5}\right)$

hibabecslés: $|f(x) - H_3(x)| \leq \dfrac{M_4}{4!} \cdot \left|(x-0)^2\left(x-\tfrac{1}{5}\right)^2\right|$

$f^{IV}(x) = 5^4 \cdot e^{5x} \to f^{IV}(0) = 5^4 = 625$

$\to f^{IV}(\tfrac{1}{5}) = 5^4 \cdot e = 1698,925$

$|f(x) - H_3(x)| \leq \dfrac{1698,925}{24} \cdot \left|x^2\left(x-\tfrac{1}{5}\right)^2\right| = \underline{70,7885 \cdot \left|x^2\left(x-\tfrac{1}{5}\right)^2\right|}$