Adaptive Signal Processing

Homework

Homework 2

- 1. Passing through a linear filter
  - Input: white noise, mean value 0, variance 0.1
  - Low-pass filter, bandwidth 10 Hz
  - Q1: Output variance ?
  - Q2: if Gaussian white noise, output PDF ?

Home Work

Q1：Assume u(t) represents the input white noise, *h*(t) is the lowpass filter with bandwidth 10Hz, y(t) = u(t) **h*(t) is the output.

Then $D[y(t)] = E[y^2(t)] - E^2[y(t)]$

$$E[y(t)] = E[u(t) * h(y)] = E[\int_{-\infty}^{+\infty} u(\tau) \cdot h(t-\tau) d\tau]$$

$$= \int_{-\infty}^{+\infty} E[u(\tau) \cdot h(t-\tau)] d\tau$$

Because u(t) is the white noise with mean value 0 , so:

$$E[y(t)] = 0$$

$$D[y(t)] = E[y^2(t)] = \mathrm{r}_y(0)$$

According to Parseval theorem:

$$\mathrm{r}_y(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_y(\Omega)d\Omega$$

Because *h*(t) is a low-pass filter:

$$S_y(\Omega) = \begin{cases} S_u(\Omega), |\Omega| < 10\pi \\ 0, else \end{cases}$$

$$S_u(\Omega) = 0.1$$

So:

$$D[y(t)] = E[y^2(t)] = r_y(0) = \frac{1}{2\pi}\int_{-10\pi}^{10\pi} S_u(\Omega)d\Omega$$

$$= 0.1 \times \frac{20\pi}{2\pi} = 1$$

Q2：According to Q1, because :

$$y(t)=u(t)*h(t)=\int_{-\infty}^{+\infty}h(\tau)\cdot u(t-\tau)d\tau=\lim_{\tau\to 0}\sum_{-\infty}^{+\infty}h(\tau_k)\cdot u(t-\tau_k)\cdot\tau$$

and u(t) is gaussian-distributed, for any time moment t , the sum of several gaussian variables is still a gaussian variable($h(\tau k)$ and τ could be seen as two factors for constant time t). Thus, $y(t)$ is a gaussian-distributed variable, too. In Q1, we have calculated the mean value and the variance of y(t) , which are 0 and 1，so the PDF of y(t) is :

$$f_Y(y)=\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2\cdot 1}}=\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}$$

- 2. All Joint cumulants of 3^{rd} and higher are **zero** for multivariate Gaussian distributions. Why?

Assignment 2:

Assume one n dimension Gaussian vector:

$$\vec{x} = [x_1, \ldots, x_n]^T$$

The mean value vector is:

$$\vec{a} = [a_1, \ldots, a_n]^T$$

And the covariance matrix is:

$$c = \begin{bmatrix} c_{11} & \ldots & c_{1n} \\ \ldots & \ldots & \ldots \\ c_{n1} & \ldots & c_{nn} \end{bmatrix}$$

where

$$c_{ik} = E[(x_i - a_i)(x_k - a_k)](i, k = 1, 2, \ldots, n)$$

Joint probability density function is

$$f(\vec{x})\frac{1}{(2\pi)^{n/2}|\mathbb{C}|^{1/2}}\exp\left\{-\frac{1}{2}(\vec{x}-\vec{a})\mathbb{C}^{-1}(\vec{x}-\vec{a})\right\}$$

Joint eigenfunction of $\vec{\chi}$ is:

$$\phi(\overrightarrow{w}) = \exp\left\{j\vec{a}^{T}\overrightarrow{w} - \tfrac{1}{2}\overrightarrow{w}\mathbb{C}\overrightarrow{w}\right\}$$

where $\overrightarrow{w} = [w_1, \ldots, w_n]^T$

The second eigenfunction of $\vec{x}$

$$\psi(\vec{w}) = \ln\phi(\vec{w}) = j\vec{a}^T\vec{w} - \tfrac{1}{2}\vec{w}^T\mathbb{C}w = j\sum_{i=1}^{n} a_i w_i - \tfrac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n} c_{ik} w_i w_k$$

For high order cumulant $r = k_1 +, \ldots, + k_n$

$r = 1,\ k_i = 1$ and other $k_j = 0$

$$c_{0\ldots1\ldots0} = (-j)\frac{\partial\Psi(\vec{\varpi})}{\partial\omega_i}\Big|_{\omega_1=\ldots=\omega_n} = a_i = E[x_i]$$

$r = 2$

$k_i = k_l = 1\ (i \neq l)$ and other $k_j = 0$

$$c_{0\ldots1\ldots1\ldots0} = (-j)^2\frac{\partial^2\Psi(\vec{\varpi})}{\partial\omega_i\partial\omega_l}\Big|_{\omega_1=\ldots=\omega_n} = c_{il}$$

$k_i = 2$ and other $k_j = 0$

$$c_{0\ldots2\ldots0} = (-j)^2\frac{\partial^2\Psi(\vec{\varpi})}{\partial^2\omega_i}\Big|_{\omega_1=\ldots=\omega_n} = c_{ii}$$

$r \geq 3,\ k_1 + \ldots + k_n \geq 3,\ c_{k_1\ldots k_n} = 0$

Homework 3

- Tap input vector

$$\mathbf{U}(n) = \alpha(n)\mathbf{s}(n) + v(n)$$

where $v(n) = \left[v(n), v(n-1), \ldots, v(n-M+1)\right]^T$

$$\mathbf{s}(\omega) = \left[1, e^{-j\omega}, \ldots, e^{-j\omega(M-1)}\right]^T \qquad \mathrm{E}\left[\alpha(n)\right] = 0 \qquad \sigma_\alpha^2 = \mathrm{E}\left[\left|\alpha(n)\right|^2\right]$$

- Q1： correlation matrix of $\mathbf{U}(n)$

- Q2: tap weight of Wiener filter when the desired response $d(n)$ is uncorrelated with $\mathbf{U}(n)$

- Q3: tap weight of Wiener filter, when

$$d(n) = v(n-k) \quad 0 \le k \le M-1$$

$$\sigma_{\alpha}^{2} = 0$$

- Q4: tap weight of Wiener filter, when

$$d(n) = \alpha(n) e^{j\omega\tau}$$

τ prescribed delay

Q1：correlation matrix of $U(n)$

- The correlation coefficient r can be expressed as:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{x\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

- Correlation matrix of U(n):

$$R = \begin{bmatrix} r(0) & r(1) & \cdots & r(M-1) \\ r(-1) & r(0) & \cdots & r(M-2) \\ \vdots & \vdots & \ddots & \cdots \\ r(-M+1) & r(-M+2) & \cdots & r(0) \end{bmatrix}$$

Q1：correlation matrix of $U(n)$

- Since $u(i) = \alpha(n)exp[-jw(n-i)] + v(i)$, $i \in [n-M+1, n]$, and considering that $v(n)$ is white noise (zero-mean, σ_v^2):

$$r(k) = \begin{cases} \sigma_\alpha^2 + \sigma_v^2 & k = 0 \\ \sigma_\alpha^2 \exp(jwk) & k \neq 0 \end{cases}$$

$$R = \sigma_\alpha^2 \begin{bmatrix} 1 + \frac{1}{\rho} & \exp(jw) & \cdots & \exp[jw(M-1)] \\ \exp(-jw) & 1 + \frac{1}{\rho} & \cdots & \exp[jw(M-1)] \\ \vdots & \vdots & \ddots & \vdots \\ \exp[-jw(M-1)] & \exp[-jw(M-2)] & \cdots & 1 + \frac{1}{\rho} \end{bmatrix}$$

Where $\rho = \sigma_\alpha^2 / \sigma_v^2$

Q2: tap weight of Wiener filter when the desired response $d(n)$ is uncorrelated with $U(n)$

- The estimated error $e(n)$:

$$e(n) = d(n) - \hat{d}(n) = d(n) - w^H U(n)$$

- The average power of $e(n)$:

$$J(w) = E\{|e(n)|^2\} = E\{e(n)e^*(n)\} = \sigma_d^2 - p^H W - w^H p + w^H R w$$

where $\sigma_d^2 = E\{|d(n)|^2\}$, R is correlation matrix,

$$p = E\{U(n)d^*(n)\} = \begin{bmatrix} E\{U(n)d^*(n)\} \\ E\{U(n-1)d^*(n)\} \\ \vdots \\ E\{U(n-M+1)d^*(n)\} \end{bmatrix}$$

Q2: tap weight of Wiener filter when the desired response $d(n)$ is uncorrelated with $U(n)$

- The gradient of $J(w)$:

$$\nabla J(w) = 2\frac{\partial}{\partial w^*}[J(w)] = -2p + 2Rw = 0$$

$$Rw_0 = p$$

$$w_0 = R^{-1}p$$

Q3: tap weight of Wiener filter, when $d(n)=v(n-k) \quad 0 \le k \le M-1 \qquad \sigma_\alpha^2=0$

- $\sigma_\alpha^2 = 0, E[\alpha(n)] = 0 \ \rightarrow \alpha(n) = 0$
- $U(n) = \alpha(n)s(n) + v(n) \ \rightarrow U(n) = v(n)$
- Thus,

$$p = E\{U(n)d^*(n)\} = \begin{bmatrix} E\{v(n)v^*(n-k)\} \\ E\{v(n-1)v^*(n-k)\} \\ \vdots \\ E\{v(n-M+1)v^*(n-k)\} \end{bmatrix} = \begin{bmatrix} r_v(k) \\ r_v(k-1) \\ \vdots \\ r_v(k-M+1) \end{bmatrix} = R_v \begin{bmatrix} \underbrace{0,\cdots,0}_{k},1,\underbrace{0,\cdots,0}_{M-k-1} \end{bmatrix}$$

$$w = R^{-1}P = R_v^{-1}P = R_v^{-1}R_v \begin{bmatrix} \underbrace{0,\cdots,0}_{k},1,\underbrace{0,\cdots,0}_{M-k-1} \end{bmatrix} = \begin{bmatrix} \underbrace{0,\cdots,0}_{k},1,\underbrace{0,\cdots,0}_{M-k-1} \end{bmatrix}$$

Q4: tap weight of Wiener filter, when $d(n)=\alpha(n)e^{j\omega\tau}$ τ prescribed delay

$$P = E\{U(n)d^*(n)\} = E\begin{bmatrix} u(n)\alpha^*(n)\exp(-jw\tau) \\ u(n-\alpha)\alpha^*(n)\exp(-jw\tau) \\ \vdots \\ u(n-M+\alpha)\alpha^*(n)\exp(-jw\tau) \end{bmatrix}$$

- Where

$$\begin{aligned} E[u(n)\alpha^*(n)\exp(-jw\tau)] &= \exp(-jw\tau)E\{\alpha(n)\alpha^*(n)\exp[-jw(n-i)] + v(i)\alpha^*(n)\} \\ &= \exp[-jw(\tau+n-i)]E[\alpha(n)\alpha^*(n)] + \exp(-jw\tau)E[v(i)]E[\alpha^*(n)] \\ &= \sigma_\alpha^2\exp[-jw(\tau+n-i)] \end{aligned}$$

- Thus

$$P = \begin{bmatrix} \sigma_\alpha^2\exp(-jw\tau) \\ \sigma_\alpha^2\exp[-jw(\tau+1)] \\ \vdots \\ \sigma_\alpha^2\exp[-jw(\tau+M-1)] \end{bmatrix} = \sigma_\alpha^2\exp(-jw\tau)\begin{bmatrix} 1 \\ \exp(-jw) \\ \vdots \\ \exp[-jw(M-1)] \end{bmatrix}$$

Q4: tap weight of Wiener filter, when $d(n)=\alpha(n)e^{j\omega\tau}$ τ prescribed delay

- The weight vector:

$$w = R^{-1}P = \sigma_\alpha^2 \exp(-jw\tau) R^{-1} \begin{bmatrix} 1 \\ \exp(-jw) \\ \vdots \\ \exp[-jw(M-1)] \end{bmatrix}$$

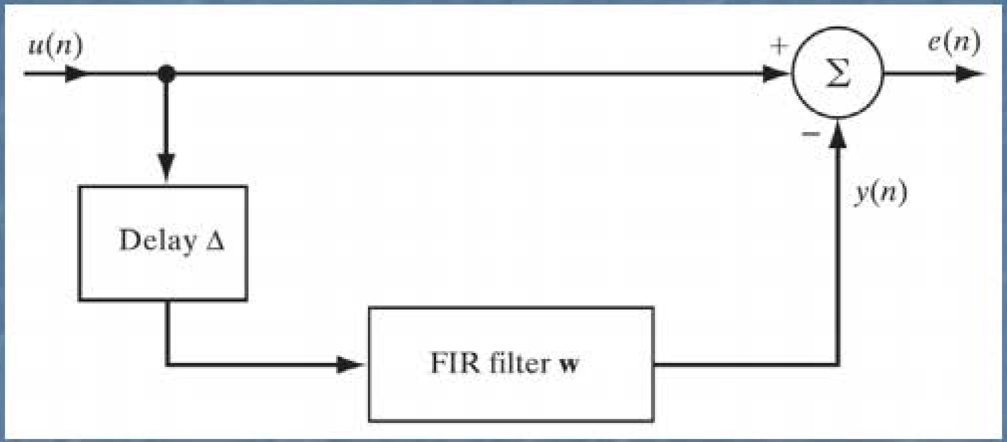
- Considering that $v(n)$ is white noise (zero-mean, σ_v^2):

$$P = \sigma_\alpha^2 \exp(-jw\tau) \begin{bmatrix} 1 \\ \exp(-jw) \\ \vdots \\ \exp[-jw(M-1)] \end{bmatrix} = \exp(-jw\tau) \left\{ R \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} \sigma_v^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right\}$$

$$w = R^{-1}P = \exp(-jw\tau) \left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} - \sigma_v^2 R^{-1} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right\}$$

Homework 4

Multi-Step FLP



- Multi-Step forward linear predictor, integer $\Delta > 1$
- Measure: mean square of prediction error $e(n)$
- Q: optimum of $\mathbf{w}(n)$

Homework 4

- The estimation error $e(n)$ is

$$e(n) = u(n) - w^H x(n)$$

Where $x(n) = u(n-\Delta)$

$$= [u(n-\Delta), u(n-1-\Delta), \ldots, u(n-M-\Delta)]^T$$

The mean square value of the estimation error is

$$
\begin{aligned}
J &= E[|e(n)|^2] \\
&= E[u(n) - w^H x(n)(u^*(n) - x^H(n)w)] \\
&= E[|u(n)|^2] - w^H E[x(n)u^*(n)] - E[u(n)x^H(n)]w + w^H E[x(n)x^H(n)]w \\
&= P_0 - w^H E[u(n-\Delta)u^*(n)] - E[u(n)u^H(n-\Delta)]w + w^H E[u(n-\Delta)u^H(n-\Delta)]w
\end{aligned}
$$

- We now note the following:

$$E[u(n-\Delta)u^*(n)] = E\left\{\begin{bmatrix} u(n-\Delta) \\ u(n-1-\Delta) \\ \dots \\ u(n-M-\Delta) \end{bmatrix} u^*(n)\right\}$$

$$= \begin{bmatrix} r(-\Delta) \\ r(-1-\Delta) \\ \dots \\ r(-M-\Delta) \end{bmatrix} = r_\Delta$$

$$E[u(n)u^H(n-\Delta)] = \begin{bmatrix} r(-\Delta) \\ r(-1-\Delta) \\ \dots \\ r(-M-\Delta) \end{bmatrix}^H = r_\Delta^H$$

$$E[u(n-\Delta)u^H(n-\Delta)] = R$$

- We may thus rewritten Eq.(1) as

$$J = P_0 - w^H r_\Delta - r_\Delta^H w + R$$

- The optimum value of the weight vector is

$$w_0 = R^{-1} r_\Delta$$

Where R^{-1} is the inverse of the correlation matrix R .

Homework 5

Homework

- Experiment: the eigenvalue spread increases, the input process becomes more correlated.
- Observation:

$$u(n) = \sum_i A_i \cos(\omega_i n + \varphi_i) + v(n)$$

- Q1: less correlated $\omega_i \neq \omega_j, i \neq j$, calculate correlation matrix R, eigenvalue spread
- Q2: more correlated $\omega_i = \omega_j, i \neq j$, calculate correlation matrix R, eigenvalue spread

Home Work

Assume that denotes white noise

$$E[v(n)v^*(n)] = \begin{cases} \sigma_v^2 & k=0 \\ 0 & k \neq 0 \end{cases}$$

When $w_i = w_j, i \neq j$

$$r(k) = E[u(n)u^*(n-k)]$$

$$= \begin{cases} \frac{1}{2}\sum_i A_i^2 + \sigma_v^2 & ,k=0 \\ \frac{1}{2}\sum_i A_i^2 \cos(w_i k) & ,k \neq 0 \end{cases}$$

$$R = \begin{bmatrix} r(0) & r(1) & \dots & r(M-1) \\ r^*(1) & r(0) & \dots & r(M-2) \\ \dots & \dots & \dots & \dots \\ r^*(M-1) & r^*(M-2) & \dots & r(0) \end{bmatrix}$$

Home Work

Assume M= 2, the eigenvalues of are as follow

$$\lambda_1 = \tfrac{1}{2}\sum_i A_i^2 \cos(w_i) + \tfrac{1}{2}\sum_i A_i^2 + \sigma_i^2$$

$$\lambda_2 = -\tfrac{1}{2}\sum_i A_i^2 \cos(w_i) + \tfrac{1}{2}\sum_i A_i^2 + \sigma_i^2$$

$$\chi_1(R) = \frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\tfrac{1}{2}\sum_i A_i^2 \cos(w_i) + \tfrac{1}{2}\sum_i A_i^2 + \sigma_v^2}{-\tfrac{1}{2}\sum_i A_i^2 \cos(w_i) - \tfrac{1}{2}\sum_i A_i^2 + \sigma_v^2}$$

Home Work

When $w_i = w_j, i \neq j$

$$r(k) = E[u(n)u^*(n-k)]$$

$$= \begin{cases} \frac{1}{2}\sum_i A_i^2 + \sigma_v^2 & ,k = 0 \\ \frac{1}{2}\sum_i A_i^2 \cos(w_i k) + \frac{1}{2}\sum_i \sum_j A_i A_j \cos(w_i k + (\varphi_i - \varphi_j)), & k \neq 0 \end{cases}$$

$$R = \begin{bmatrix} r(0) & r(1) & \dots & r(M-1) \\ r^*(1) & r(0) & \dots & r(M-2) \\ \dots & \dots & \dots & \dots \\ r^*(M-1) & r^*(M-2) & \dots & r(0) \end{bmatrix}$$

Assume M = 2, the two eigenvalues of $\boldsymbol{R}$ are

$$\lambda_1 = \frac{1}{2}\sum_i A_i^2 \cos(w_i k) + \frac{1}{2}\sum_i \sum_j A_i A_j \cos(w_i k + (\varphi_i - \varphi_j)) + \frac{1}{2}\sum_i A_i^2 + \sigma_v^2$$

$$\lambda_2 = -\frac{1}{2}\sum_i A_i^2 \cos(w_i k) + \frac{1}{2}\sum_i \sum_j A_i A_j \cos(w_i k + (\varphi_i - \varphi_j)) + \frac{1}{2}\sum_i A_i^2 + \sigma_v^2$$

Hence, the eigenvalue spread equals

$$\chi_2(R)=\frac{\lambda_{\max}}{\lambda_{\min}}=\frac{\frac{1}{2}\sum_i A_i^2\cos(w_i)+\frac{1}{2}\sum_i\sum_j A_iA_j\cos(w_i+(\varphi_i-\varphi_j))+\frac{1}{2}\sum_i A_i^2+\sigma_v^2}{-\frac{1}{2}\sum_i A_i^2\cos(w_i)-\frac{1}{2}\sum_i\sum_j A_iA_j\cos(w_i+(\varphi_i-\varphi_j))+\frac{1}{2}\sum_i A_i^2+\sigma_v^2}$$

Obviously

$$\chi_1(R)<\chi_2(R)$$

Hence, the eigenvalue spread increases, the input process becomes more correlated.

Homework 6

Homework

- AR process $u(n)$

$$u(n) = -a_1 u(n-1) - a_2 u(n-2) + v(n)$$

- $v(n)$ zero mean, σ_v^2 white noise
- a_1=0.1, a_2=-0.8
- Q1：calculate the noise variance σ_v^2 such that $u(n)$ has unit variance. Give different realizations of $u(n)$

Home Work

- Q1: The characteristic equation:

$$1+\mathrm{a}_1 z^{-1}+\mathrm{a}_2 z^{-2}=0$$

- Two roots of the equation are calculated as follows:

$$p_1, p_2=\tfrac{1}{2}(-a_1 \pm \sqrt{a_1^2-4a_2})=\tfrac{1}{2}(-0.1 \pm \sqrt{3.21})<1$$

So the AR process u(n) is an asymptotically stationary process

Home Work

Use Yule-Walker equations to find the relationship between σ_v^2 and σ_u^2

$$\begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} \cdot \begin{bmatrix} -a_1 \\ -a_2 \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \end{bmatrix}$$

$$w_1 = -a_1 = \frac{r(1)[r(0) - r(2)]}{r^2(0) - r^2(1)}$$

$$w_2 = -a_2 = \frac{r(0)r(2) - r^2(1)}{r^2(0) - r^2(1)}$$

Home Work

$$r(0)=\sigma_u^2 \qquad r(1)=(\frac{-a_1}{1+a_2})\sigma_u^2 \qquad r(2)=(-a_2+\frac{a_1^2}{1+a_2})\sigma_u^2$$

From the equations above,we can get

$$\sigma_u^2=(\frac{1+a_2}{1-a_2})\frac{\sigma_v^2}{(1+a_2)^2-a_1^2}$$

$$\sigma_u^2=\frac{100}{27}\sigma_v^2$$

$$\sigma_v^2=0.27$$

```
sample_variance_u =

    1.0008


var_v =

    0.2784


k =

    90
```

- Q2: Given the input $u(n)$,an LMS algorithm of length M=2 is used to estimate the unknown AR parameters a1 and a2, the step $\mu=0.05$. Justify the use of this design value in the application of the small step-size theory.

Home Work

Q2:Use LMS to calculate a1 and a2

$$y(n) = \hat{w}^H(n) \cdot u(n)$$

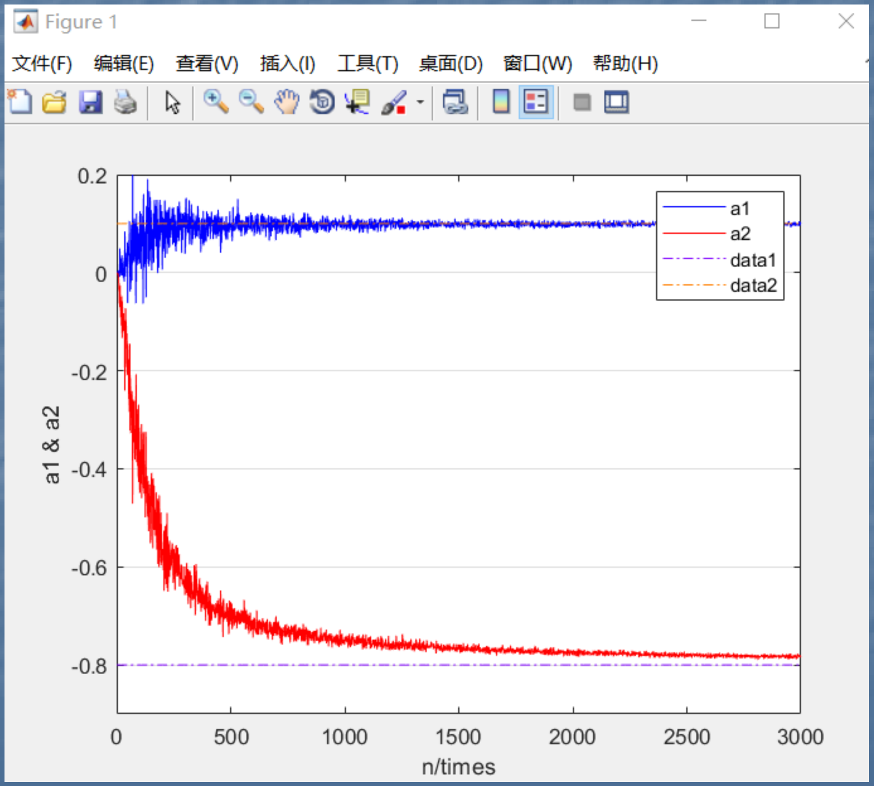
$$e(n) = d(n) - y(n)$$

$$\hat{w}^H(n+1) = \hat{w}^H(n) + \mu \cdot u(n) \cdot e(n)$$

We use this method to iteratively figure out the value of a1 and a2.

Home Work

- Result



Home Work

- And from Q1, we can get

$$R = \begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

- we can get eigenvalues of R, $\lambda_1 = 0.5, \lambda_2 = 1.5$
- So $\lambda_{\max} = 1.5 \quad \mu = 0.05$
- we have $0 < \mu < \dfrac{2}{\lambda_{\max}} = \dfrac{4}{3}$
- Here, μ meets the requirement of small step theory.

- Q3: For one realization of LMS, compute the prediction error $f(n)=u(n)-\hat{u}(n)$, two tap-weight errors $\varepsilon_1(n)=-a_1-\hat{w}_1(n)$ and $\varepsilon_2(n)=-a_2-\hat{w}_2(n)$. Give the power spectral plots of $f(n)$, $\varepsilon_1(n)$ and $\varepsilon_2(n)$, justify that $f(n)$ behaves as white noise, $\varepsilon_1(n)$ and $\varepsilon_2(n)$ behave as low pass processes.

Home Work

- Q3
- Because $f(n)=u(n)-\sum_{k=1}^{2}\hat{w}(k)u(n-k)$

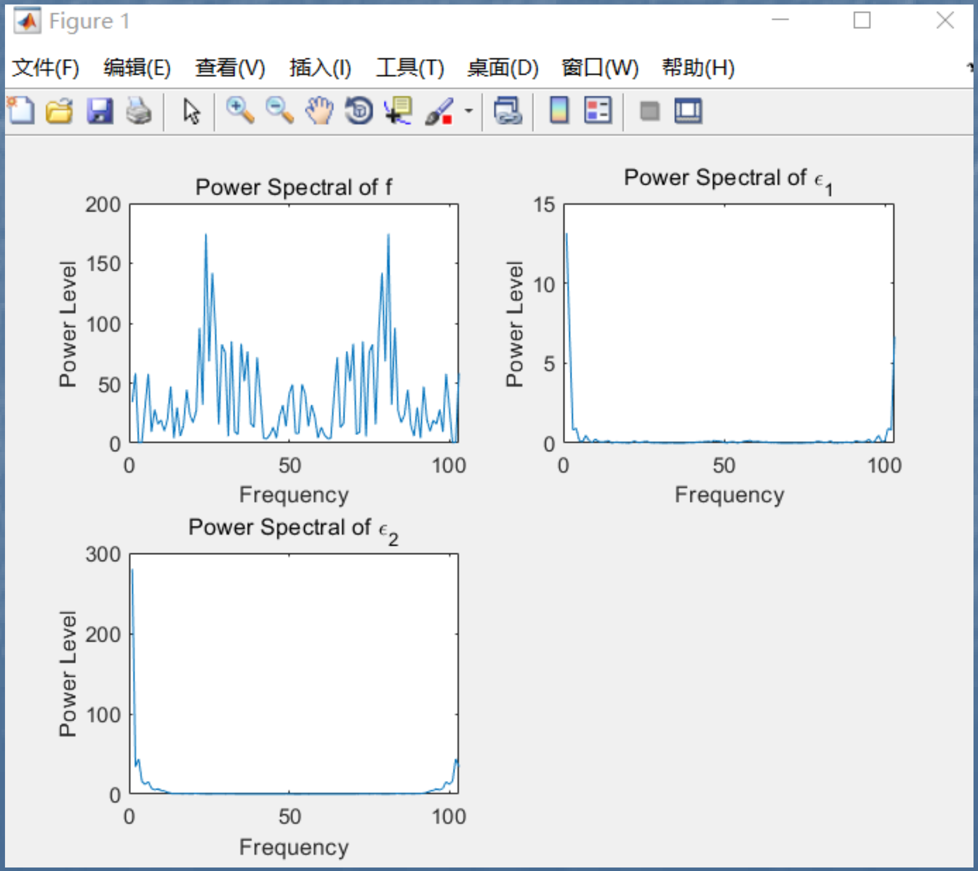
$$\varepsilon_1(n)=-a_1-\widehat{w_1}(n)$$

$$\varepsilon_2(n)=-a_2-\widehat{w_2}(n)$$

$$\hat{w}^H(n+1)=\hat{w}^H(n)+\mu\cdot f(n)\cdot u(n)$$

Home Work

- Matlab $f(n), \varepsilon_1(n), \varepsilon_2(n)$



- Q4: Compute the ensemble-average learning curve of the LMS algorithm by averaging the squared value of the prediction error f(n) over an ensemble of 100 different realizations.
- Q5: Using the small step-size theory, compute the theoretical learning curve of the LMS algorithm and compare your result against the measured result of Q4.

Home Work

- Q4&Q5： From Q3,we can get

$$f(n)=u(n)-\sum_{k=1}^{2}\widehat{w_k}(n)u(n-k)$$

- In LMS prosess,accumulate squared error of estimation can be expressed as

$$g=\sum_{n=3} f^2(n)$$

Home Work

- And theoretical cost function J can be expressed as

$$J(n) = (1 - \sigma_v^2 \cdot (1 + \tfrac{\mu}{2})) \cdot (1 - \mu)^{2n} + \sigma_v^2 \cdot (1 + \tfrac{\mu}{2})$$

Home Work

- Result of two cost functions