补充题 给定数据条件如下

x_i	0	1	2	3
y_i	0	0	0	0

试求三次样条函数满足 ① $y_0'=1,\ y_3'=0$ ② $y_0''=1,\ y_3''=0$.

解：①
$$S(x)=\begin{cases} 0\cdot\alpha_0(x)+0\cdot\alpha_{01}(x)+m_0\beta_0(x)+m_1\beta_{01}(x) & 0\le x\le 1 \\ 0\cdot\alpha_1(x)+0\cdot\alpha_{11}(x)+m_1\beta_1(x)+m_2\beta_{11}(x) & 1\le x\le 2 \\ 0\cdot\alpha_2(x)+0\cdot\alpha_{21}(x)+m_2\beta_2(x)+m_3\beta_{21}(x) & 2\le x\le 3 \end{cases}$$

$h_0=h_1=h_2=1$, $\lambda_1=\frac{h_0}{h_0+h_1}=\frac{1}{2}$, $\mu_1=\frac{h_0}{h_0+h_1}=\frac{1}{2}$, $\lambda_2=\frac{1}{2}$, $\mu_2=\frac{1}{2}$

$g_1=3\times(\lambda_1 f[x_1,x_2]+\mu_1 f[x_0,x_1])=\frac{3}{2}\times\left(\frac{0-0}{2-1}+\frac{0-0}{1-0}\right)=0$

$g_2=3\times(\lambda_2 f[x_2,x_3]+\mu_2 f[x_1,x_2])=\frac{3}{2}\times\left(\frac{0-0}{3-2}+\frac{0-0}{2-1}\right)=0$

$$\therefore\begin{cases}\frac{1}{2}m_0+2m_1+\frac{1}{2}m_2=0\\ \frac{1}{2}m_1+2m_2+\frac{1}{2}m_3=0\\ m_0=1\\ m_3=0\end{cases}\quad 解得\begin{cases}m_0=1\\ m_1=-\frac{4}{15}\\ m_2=\frac{1}{15}\\ m_3=0\end{cases}$$

$\beta_0(x)=(x-x_0)\left(\frac{x-x_1}{x_0-x_1}\right)^2=x(x-1)^2=x^3-2x^2+x$

$\beta_{01}(x)=(x-x_1)\left(\frac{x-x_0}{x_1-x_0}\right)^2=(x-1)x^2=x^3-x^2$

$\beta_1(x)=(x-x_1)\left(\frac{x-x_2}{x_1-x_2}\right)^2=(x-1)(x-2)^2=x^3-5x^2+8x-4$

$\beta_{11}(x)=(x-x_2)\left(\frac{x-x_1}{x_2-x_1}\right)^2=(x-2)(x-1)^2=x^3-4x^2+5x-2$

$\beta_2(x)=(x-x_2)\left(\frac{x-x_3}{x_2-x_3}\right)^2=(x-2)(x-3)^2=x^3-8x^2+21x-18$

$\beta_{21}(x)=(x-x_3)\left(\frac{x-x_2}{x_3-x_2}\right)^2=(x-3)(x-2)^2=x^3-7x^2+16x-12$.

综上，$S(x)=\begin{cases} \frac{11}{15}x^3-\frac{26}{15}x^2+x, & 0\le x\le 1 \\ -\frac{1}{5}x^3+\frac{16}{15}x^2-\frac{27}{15}x+\frac{14}{15} & 1\le x\le 2, \\ \frac{1}{15}x^3-\frac{8}{15}x^2+\frac{7}{5}x-\frac{6}{5}, & 2\le x\le 3. \end{cases}$

② 当 $y_0''=1$、$y_3''=0$ 时，

可增加方程 $\begin{cases} 2m_0+m_1=3\cdot f[x_0,x_1]-\frac{1}{2}\times 1 \\ m_2+2m_3=3\cdot f[x_2,x_3]+\frac{1}{2}\times 0 \end{cases}$ 可得：$\begin{cases} m_0=-\frac{13}{45} \\ m_1=\frac{7}{90} \\ m_2=-\frac{1}{45} \\ m_3=\frac{1}{90} \end{cases}$

$\beta_0(x)$、$\beta_{01}(x)$、$\beta_1(x)$、$\beta_{11}(x)$、$\beta_2(x)$、$\beta_{21}(x)$ 和①一样。

综上，$S(x)=\begin{cases} -\frac{19}{90}x^3+\frac{1}{2}x^2-\frac{13}{45}x & 0\le x\le 1 \\ \frac{1}{18}x^3-\frac{3}{10}x^2+\frac{23}{45}x-\frac{4}{15} & 1\le x\le 2. \\ -\frac{1}{90}x^3+\frac{1}{10}x^2-\frac{13}{45}x+\frac{4}{15} & 2\le x\le 3 \end{cases}$