

Communication Efficient Federated Learning With Energy Awareness Over Wireless Networks

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Abstract—In federated learning (FL), reducing the communication overhead is one of the most critical challenges since the parameter server and the mobile devices share the training parameters over wireless links. With such consideration, we adopt the idea of SignSGD in which only the signs of the gradients are exchanged. Moreover, most of the existing works assume Channel State Information (CSI) available at both the mobile devices and the parameter server, and thus the mobile devices can adopt fixed transmission rates dictated by the channel capacity. In this work, only the parameter server side CSI is assumed, and channel capacity with outage is considered. In this case, an essential problem for the mobile devices is to select appropriate local processing and communication parameters (including the transmission rates) to achieve a desired balance between the overall learning performance and their energy consumption. Two optimization problems are formulated and solved, which optimize the learning performance given the energy consumption requirement, and vice versa. Furthermore, considering that the data may be distributed across the mobile devices in a highly uneven fashion in FL, a stochastic sign-based algorithm is proposed. Extensive simulations are performed to demonstrate the effectiveness of the proposed methods.

Index Terms—Federated learning, wireless communications, communication efficiency, data heterogeneity.

I. INTRODUCTION

TO TRAIN a machine learning model, traditionally a centralized approach is adopted in which the training data are aggregated on a single machine. On the one hand, such a centralized training approach is privacy-intrusive, especially when the data are collected by mobile devices and contain the owners' sensitive information (e.g., locations, user preference on websites, social media, etc.). On the other hand, transmitting all the collected data from mobile devices may be impractical due to communication resource limitations. With such consideration, the concept of federated learning (FL), which enables training on a large corpus of decentralized data residing on mobile devices, is proposed in [1].

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As a distributed training approach, FL adopts the parameter server paradigm in which most of the computation is offloaded to the mobile devices in a parallel manner. During each communication round, after receiving the learning model parameters from the server, the workers (i.e., mobile devices) train their local learning models using their local data and transmit the parameter updates back to the server, which will aggregate the information from all the workers and start the next round by broadcasting the updated model parameters. In wireless networks, since all the communications between the workers and the server are over wireless links, the learning performance depends on the wireless environments as well as the workers' communication resource and energy constraints. There have been some works that study the communication aspects of FL [2]–[25]. Nonetheless, they either do not consider the existing strategies that have shown promising improvement in communication efficiency (e.g., gradient quantization [26]) or ignore the energy consumption of the workers, the impact of transmission errors, and data heterogeneity. In addition, most of these works assume channel-state information (CSI) at both the server side and the workers side. As a result, the workers are assumed to adopt fixed transmission rates dictated by the channel capacity. In this work, the idea of SignSGD with majority vote [26] is adopted to improve the communication efficiency of the FL algorithm, in which only the signs of the parameter updates are shared between the server and the workers. The workers are assumed to transmit their parameter updates over flat-fading channels and CSI is only available at the server side. Channel capacity with outage is considered and each worker is supposed to determine its transmission rate and transmission power.

It is worth mentioning that in real-world FL applications over wireless networks, the communication time between the server and the workers is not negligible. Therefore, it becomes more critical to improve the learning performance with respect to the total training time instead of the number of communication rounds. With such consideration, the implementation of the FL algorithms given a fixed total training time is considered in this work. In such a case, the learning performance depends on the number of communication rounds that the FL algorithm can be run and the outage probabilities of the workers for each communication round. Intuitively, increasing the transmission power and decreasing the transmission rate of a worker both decrease its outage probability. However, increasing the transmission power results in higher energy consumption for communication while decreasing the transmission rate

requires faster local computation (i.e., training the local FL model) given fixed time duration for each communication round, which leads to higher energy consumption for local computation. Considering that mobile devices usually have limited batteries, it is essential to minimize their energy consumption by appropriately configuring the local computation and communication parameters while satisfying the learning performance requirement (or the other way around). More specifically, our main contributions are summarized as follows.

- The impact of wireless communications on SignSGD, in particular, the probability of correct aggregation for each communication round, is analyzed. Combining our analysis on the performance improvement of each communication round and the convergence rate which captures the speed that the improvement accumulates over communication rounds, a new metric is proposed to more accurately measure the learning performance of SignSGD in practice.
- Two optimization problems are formulated and solved. The first problem optimizes the learning performance given the energy consumption constraint, while the second problem minimizes the energy consumption of the workers given the learning performance requirement.
- The scenario with heterogeneous data distribution across the workers is considered and a stochastic sign based algorithm that can deal with data heterogeneity is proposed.
- Extensive simulations are performed to demonstrate the effectiveness of the proposed method. Particularly, compared with SignSGD and FedAVG [27], the proposed stochastic sign based algorithm achieves better learning performance while reducing the energy consumption of the workers.

The remainder of this work is organized as follows. Section II discusses related works. Section III introduces the system model and the performance analysis of SignSGD over wireless networks. The optimization problems are formulated in Section IV and the corresponding solutions are presented in Section V. Section VI extends the proposed method to the scenario with heterogeneous data distribution across the workers. Section VII presents the simulation results. Conclusions are presented in Section VIII.

II. RELATED WORKS

To improve the communication efficiency of the distributed learning algorithms, various methods have been proposed, including quantization [26], [28]–[31], sparsification [32]–[34] and subsampling [1], [35]. However, most of these works ignore the impact of wireless environments and the resource constraints of the mobile devices, which are of vital importance in the implementation of FL algorithms over real-world wireless networks.

Recently, there have been a number of existing works that study the communication aspects of FL algorithms. In [2], [3], the weighted sum of the training time and the energy consumption is optimized by properly selecting the local computation and the communication parameters. [4]

considers the minimization of the training time by properly configuring the local computation and communication parameters and the bandwidth allocation. The energy consumption of the communications between the mobile devices and the server is also considered in [5] and the goal is to minimize the weighted sum of the energy consumption and the number of participated mobile devices by mobile device scheduling and effective bandwidth allocation. [6] considers the minimization of the total energy consumption of all the workers under a latency constraint. [7] considers a joint mobile device scheduling and bandwidth allocation problem to minimize the expected FL training time, while [8] proposes a joint device scheduling and resource allocation policy to maximize the model accuracy within a given total training time budget for latency constrained wireless FL. To further reduce the FL convergence time, [9] incorporates artificial neural networks (ANNs) to estimate the local FL models of the devices that are not scheduled for transmission. [10] proposes a joint device scheduling and resource block allocation policy for FL under imperfect CSI. [11] considers a cell-free massive MIMO scenario and the training time is minimized by jointly optimizing the local computation and the communication parameters. [12] empirically proposes a learning efficiency metric which is a function of the mini-batch size and the time of each communication round. Resource allocation and the mini-batch size are jointly optimized to maximize the learning efficiency. [13] takes the effect of packet transmission errors into consideration and analyzes its impact on the performance of FL. A joint bandwidth allocation and mobile device selection problem is formulated and solved to minimize a FL loss function that captures the performance of the FL algorithm. However, in these works, the aforementioned effective strategies for improving communication efficiency are not considered. In this work, we adopt the idea of SignSGD with majority vote [26], which improves the communication efficiency by a factor of 32.

To reduce the communication overhead, [14]–[17] incorporate the quantization, sparsification and error compensation schemes. However, these works do not consider the energy consumption and communication errors. Another line of works [18]–[20] adopt the over-the-air computation scheme for model updates aggregation to improve the communication efficiency. [21]–[23] further combine model updates compression schemes with over-the-air aggregation. While over-the-air computation mechanisms demonstrate advantages in terms of bandwidth efficiency, they usually require tight synchronization and CSI information at the workers side for power equalization. Besides, none of these works consider the energy consumption of the workers. A summary of the main differences between this work and the current state of the art can be found in Table I.

III. SYSTEM MODEL

In this work, a wireless multi-user system consisting of one parameter server and a set of M workers is considered. In particular, each worker $m \in \mathcal{M}$ stores a local dataset \mathcal{D}_m , which will be used for local training. The local dataset can be

TABLE I
SUMMARY OF THE CURRENT STATE OF THE ART

	Multiple Access Method	Compressed Model Updates?	Client Energy Consumption?	Impact of Communication Errors?
[18]–[20]	Over-the-air computation	✗	✗	analog communication with high SNR
[21]–[23]	Over-the-air computation	✓	✗	analog communication with high SNR
[4], [7]–[10], [12]	TDMA/FDMA	✗	✗	✗
[2], [3], [5], [6], [11]	TDMA/FDMA	✗	✓	✗
[14]–[17]	TDMA/FDMA	✓	✗	✗
[13]	TDMA/FDMA	✗	✓	✓
This work	TDMA/FDMA	✓	✓	✓

locally generated or collected through each worker's usage of mobile devices. Considering that the training of a prediction model, especially in deep learning, usually requires a large dataset, the goal of the workers is to cooperatively learn a machine learning model while keeping the local training data on their mobile devices.

A. Machine Learning Model

A typical federated learning problem with M normal workers is considered. Formally, the goal is to minimize a finite-sum objective of the form

$$\min_{\mathbf{w} \in \mathbb{R}^d} F(\mathbf{w}) \quad \text{where} \quad F(\mathbf{w}) \stackrel{\text{def}}{=} \frac{1}{M} \sum_{m=1}^M F_m(\mathbf{w}). \quad (1)$$

For a machine learning problem, we have a sample space $I = X \times Y$, where X is a space of feature vectors and Y is a label space. Given the hypothesis space $\mathcal{W} \subseteq \mathbb{R}^d$, we define a loss function $l: \mathcal{W} \times I \rightarrow \mathbb{R}$ which measures the loss of prediction on the data point $(\mathbf{x}, y) \in I$ made with the hypothesis vector $\mathbf{w} \in \mathcal{W}$. In such a case, $F_m(\mathbf{w})$ is a local function defined by the local dataset of worker m and the hypothesis \mathbf{w} . More specifically,

$$F_m(\mathbf{w}) = \frac{1}{|\mathcal{D}_m|} \sum_{(\mathbf{x}_n, y_n) \in \mathcal{D}_m} l(\mathbf{w}; (\mathbf{x}_n, y_n)), \quad (2)$$

where $|\mathcal{D}_m|$ is the size of worker m 's local dataset \mathcal{D}_m . The loss function $l(\mathbf{w}; (\mathbf{x}_n, y_n))$ depends on the learning tasks and the machine learning models. In this work, $F_m(\mathbf{w})$'s are not necessarily convex.

To accommodate the requirement of communication efficiency in FL, we adopt the popular idea of gradient quantization as in SignSGD with majority vote [26], which is presented in Algorithm 1. At the k -th communication round, each worker m computes the gradient $\mathbf{g}_m^{(k)}$ based on its locally stored model weights $\mathbf{w}^{(k)}$ and the local datasets \mathcal{D}_m .¹ Then, instead of transmitting the gradient $\mathbf{g}_m^{(k)}$ directly, worker m transmits $\text{sign}(\mathbf{g}_m^{(k)})$ to the parameter server, in which $\text{sign}(\cdot)$ is the sign function. After receiving the shared signs of the gradients from the workers (prone to channel errors), the parameter server performs aggregation using the majority vote rule and sends the aggregated result back to the workers. Finally, the workers update their local model weights using the aggregated result.

¹In this work, it is assumed that each worker evaluates the gradients over its whole local dataset for simplicity (i.e., $\mathbf{g}_m^{(k)} = \nabla F_m(\mathbf{w}^{(k)})$, $\forall 1 \leq m \leq M$). The analysis and the proposed method can also be applied in the scenario where each worker computes its local gradients over a mini-batch of its local dataset.

Algorithm 1 SignSGD With Majority Vote [26] Over Wireless Networks

- 1: Input: initial weight: $\mathbf{w}^{(0)}$; number of workers: M ; learning rate: η .
- 2: **for** $k = 0, 1, \dots, K$ **do**
- 3: Each worker m transmits $\text{sign}(\mathbf{g}_m^{(k)})$ to the parameter server over wireless links, where $\mathbf{g}_m^{(k)} = \nabla F_m(\mathbf{w}^{(k)})$ is the local gradient.
- 4: The parameter server obtains a noisy estimate (denoted by $\hat{\mathbf{g}}_m^{(k)}$) of the transmitted information $\text{sign}(\mathbf{g}_m^{(k)})$ from each worker m and sends the aggregated result $\hat{\mathbf{g}}^{(k)} = \text{sign}(\sum_{m=1}^M \hat{\mathbf{g}}_m^{(k)})$ back to the workers.
- 5: The workers update their local models

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \eta \hat{\mathbf{g}}^{(k)}. \quad (3)$$

6: **end for**

B. Local Computation Model

In this work, we consider a similar local computation model as those in [2] and [13]. Let c_m and f_m denote the number of CPU cycles required for worker m to process per bit data and its CPU cycle frequency, respectively, which are assumed known to the parameter server. We note that the frequency essentially measures the computation speed of a CPU. In practice, the users usually adjust the CPU frequency of the devices to reduce energy consumption and prevent overheating. Then, the CPU energy consumption of worker m for the local computation of one communication round is given by [36]

$$E_m^{\text{comp}} = \frac{\alpha_m}{2} c_m D_m f_m^2, \quad (4)$$

in which $\frac{\alpha_m}{2}$ is the effective capacitance coefficient of worker m 's computing chip, D_m is the size of worker m 's training data for each communication round (in bits). In addition, the computation time for each communication round of worker m is given by

$$T_m^{\text{comp}} = \frac{c_m D_m}{f_m}. \quad (5)$$

C. Transmission Model

In this work, it is assumed that the workers transmit their local updates (i.e., the signs of the gradients) to the parameter server via the orthogonal frequency division multiple access (OFDMA), and does not interfere with each other. Given

that the parameter server has more power and bandwidth compared to the mobile devices, the downlink transmission time is ignored in this work.² Moreover, similar to most of the existing literature (e.g., [2], [13]), we assume that the downlink transmissions are error-free for simplicity.

For the uplink transmission, different from the existing works that consider CSI at both the transmitter and the receiver sides, we consider flat-fading channels with receiver only CSI and the capacity with outage. Capacity with outage is defined as the maximum rate that can be transmitted over a channel with a certain outage probability, which corresponds to the probability that an outage happens and the transmission cannot be decoded correctly [37]. For each worker m , we assume a discrete-time channel with stationary and ergodic time-varying normalized gain $\sqrt{h_m}$ following Rayleigh distribution, and additive white Gaussian noise (AWGN).³ Suppose that worker m transmits at a rate of $r_m = \log_2(1 + \gamma_{\min})$, in which γ_{\min} is some fixed minimum received SNR, the data can be correctly received if the instantaneous received SNR $\gamma_m = \frac{P_m h_m}{N_0 B_m}$ is greater than or equal to γ_{\min} , in which P_m is the transmission power of worker m ; N_0 is the noise power spectral density and B_m is the corresponding bandwidth. The probability of outage is thus $p_{\text{out}} = P(\gamma_m < \gamma_{\min})$. Particularly, for Rayleigh fading channel, we have

$$p_{\text{out}}(r_m) = 1 - e^{-\frac{(2^{r_m} - 1)N_0 B_m}{P_m}}. \quad (6)$$

The corresponding communication time and energy consumption are given by

$$T_m^{\text{com}} = \frac{s_m}{r_m B_m}, \quad E_m^{\text{com}} = \frac{P_m s_m}{r_m B_m}, \quad (7)$$

in which s_m is the size of the transmitted data (in bits).⁴

For simplicity, the wireless link between each worker m and the parameter server for each entry of the transmitted gradients is assumed to be a binary symmetric channel with crossover probability $p_{\text{out}}(r_m)$. In this sense, we have

$$\hat{\mathbf{g}}_m^{(k)} = \begin{cases} -\text{sign}(\mathbf{g}_m^{(k)}), & \text{with probability } p_{\text{out}}(r_m), \\ \text{sign}(\mathbf{g}_m^{(k)}), & \text{with probability } 1 - p_{\text{out}}(r_m). \end{cases} \quad (8)$$

Remark 1: In (8), it is assumed that for each worker m , $\text{sign}(\mathbf{g}_m^{(k)})$ is transmitted as a single packet in the uplink and all entries of $\text{sign}(\mathbf{g}_m^{(k)})$ are incorrectly decoded when an outage happens. This is considered to be the worst case scenario. One may also assume that the parameter server can detect the outage and discard the corresponding packets. In this case,

$$\hat{\mathbf{g}}_m^{(k)} = \begin{cases} 0, & \text{with probability } p_{\text{out}}(r_m), \\ \text{sign}(\mathbf{g}_m^{(k)}), & \text{with probability } 1 - p_{\text{out}}(r_m). \end{cases} \quad (9)$$

²Note that given a fixed transmission rate for the parameter server, the downlink transmission time is a constant that can be readily integrated to the first and the second constraints of the optimization problems (19) and (17), respectively, if needed.

³Note that we assume Rayleigh fading for simplicity, and the study can be easily generalized to other channel models.

⁴Note that in the schemes where full precision gradients are transmitted, each worker is supposed to transmit 32 bits for each element in the gradient vectors. Therefore, Algorithm 1 leads to a 32-fold improvement in communication time and communication energy consumption.

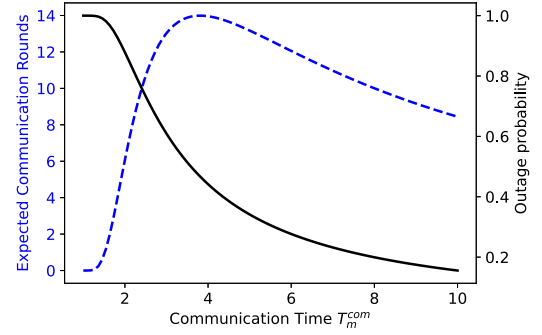


Fig. 1. The blue curve shows the expected successful communication rounds for worker m per 100 seconds. The black curve shows the outage probability of worker m . We set $s_m = 10^6$ bits, $N_0 = 10^{-8}$ W/Hz; $P_m = 0.005$ W; $B_m = 180$ kHz.

Besides, the extension to the scenarios where partial bits of the packet may be recovered is also straightforward, and won't change the fundamental tradeoffs revealed in this study. In the following performance analysis of the learning algorithms, we show the lower bound of the learning performance in the worst case scenario. That being said, our analysis on learning performance is conservative, and the derived results can be applied in the other aforementioned scenarios as well.

According to (6) and (7), given a total communication time, there exists a tradeoff between the number of communication rounds $K \propto 1/T_m^{\text{com}}$ and the outage probability $p_{\text{out}}(r_m)$ for each communication round. Particularly, the communication time T_m^{com} for each communication round should be large enough to guarantee reliable transmission. Meanwhile, a large T_m^{com} results in fewer communication rounds in total. In FL, during each communication round, the parameter server sends the current global model to the workers and waits for the workers to report updates. If enough workers report within a given communication round, the server will update its global model and ignore the updates from the workers that fail to report in time [38]. That being said, different from many existing applications in wireless communications, the goal of FL is not to reliably receive the information from all the workers. Instead, the parameter server intends to maximize the number of successful global model updates. Fig. 1 shows the outage probability and the expected communication rounds that worker m is not in outage with respect to different communication time T_m^{com} , given a total communication time of 100 seconds.⁵ It can be observed that the expected successful communication rounds achieves the highest when $T_m^{\text{com}} = 3.82$ s and the correspond outage probability is around 46.6%. This indicates that if the negative impact of unsuccessful communication is small (e.g., the packets in outage can be detected and discarded), the parameter server can tolerate a relatively large outage probability while achieving good learning performance.

⁵We note that the choice of the total communication time does not change the fundamental tradeoff revealed in Fig. 1.

D. Performance Analysis of Algorithm 1 Over Wireless Networks

Before diving into the details of the system design, we first analyze how wireless communications affect the performance of Algorithm 1. To facilitate the analysis, the following assumptions are made.

Assumption 1: (Smoothness) $\forall \mathbf{w}_1, \mathbf{w}_2$, we require for some non-negative constant L

$$F(\mathbf{w}_1) \leq F(\mathbf{w}_2) + \langle \nabla F(\mathbf{w}_2), \mathbf{w}_1 - \mathbf{w}_2 \rangle + \frac{L}{2} \|\mathbf{w}_1 - \mathbf{w}_2\|_2^2, \quad (10)$$

where $\langle \cdot, \cdot \rangle$ is the standard inner product.

We note that the Assumption 1 is commonly adopted in the literature (e.g., [26]), given which the following result can be proved.

Theorem 1: Suppose that the model parameter at the beginning of k -th communication round is $\mathbf{w}^{(k)}$, then by performing one communication round of Algorithm 1, we have

$$\mathbb{E}[F(\mathbf{w}^{(k)}) - F(\mathbf{w}^{(k+1)})] \geq -\eta \|\nabla F(\mathbf{w}^{(k)})\|_1 - \frac{L\eta^2 d}{2} + 2\eta \times \sum_{i=1}^d |\nabla F(\mathbf{w}^{(k)})_i| P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i), \quad (11)$$

in which d is the dimension of the gradients; $\tilde{\mathbf{g}}_i^{(k)}$ and $\nabla F(\mathbf{w}^{(k)})_i$ are the i -th entry of the aggregated result $\tilde{\mathbf{g}}^{(k)}$ and the gradient vector $\nabla F(\mathbf{w}^{(k)})$, respectively. $\text{sign}(\cdot)_i$ is the i -th entry of the vector after taking the sign operation. The expectation and the probability are over the dynamics of the wireless channels.

Proof: Please see Appendix A. \square

Remark 2: Theorem 1 lower bounds the expected improvement of the learning objective during the k -th communication round (i.e., $\mathbb{E}[F(\mathbf{w}^{(k)}) - F(\mathbf{w}^{(k+1)})]$). Intuitively, the learning performance depends on two quantities: (1) the improvement of the learning objective during each communication round; (2) the number of communication rounds. When the data are homogeneously distributed across the workers, SignSGD converges with a rate of $O(1/\sqrt{K})$ [26], in which K is the number of communication rounds.

Note that given $\mathbf{w}^{(k)}$, $F(\mathbf{w}^{(k)})$ and $\nabla F(\mathbf{w}^{(k)})$ are constants. Therefore, maximizing the lower bound of the expected improvement of the learning objective (i.e., the right-hand side of (11)) is equivalent to maximizing the probabilities of correct aggregation $P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i)$, $1 \leq i \leq d$. For the ease of discussion, we consider the i -th entry of the gradient⁶ and define a series of random variables $\{X_{m,i}\}_{m=1}^M$ given by

$$X_{m,i} = \begin{cases} 1, & \text{if } \text{sign}(\tilde{\mathbf{g}}_m^{(k)})_i \neq \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i, \\ 0, & \text{if } \text{sign}(\tilde{\mathbf{g}}_m^{(k)})_i = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i. \end{cases} \quad (12)$$

$X_{m,i}$ can be considered as the outcome of one Bernoulli trial (which is unknown to the parameter server) indicating whether the m th worker shares the wrong sign (due to

data incompleteness or channel errors) with the server. Let $Z_i = \sum_{m=1}^M X_{m,i}$, then it can be verified that⁷

$$P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i) = P\left(Z_i < \frac{M}{2}\right). \quad (13)$$

In addition, Z_i follows the Poisson binomial distribution with mean $\mathbb{E}[Z_i] = \sum_{m=1}^M P(X_{m,i})$. Since Z_i is non-negative, the Markov's inequality indicates

$$P(Z_i \geq \frac{M}{2}) \leq \frac{2\mathbb{E}[Z_i]}{M}, \quad (14)$$

and therefore

$$\begin{aligned} P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i) &= 1 - P(Z_i \geq \frac{M}{2}) \\ &\geq \frac{M - 2\mathbb{E}[Z_i]}{M}. \end{aligned} \quad (15)$$

Note that $\mathbb{E}[Z_i]$ and $M - \mathbb{E}[Z_i]$ are the expected number of workers that share wrong and correct signs, respectively. The lower bound in (15) represents the difference between the ratios of workers that share the correct signs (i.e., $(M - \mathbb{E}[Z_i])/M$) and that share the wrong signs (i.e., $\mathbb{E}[Z_i]/M$).

Remark 3: Note that the closed form of the expected improvement of the learning objective during the k -th communication round (i.e., $\mathbb{E}[F(\mathbf{w}^{(k)}) - F(\mathbf{w}^{(k+1)})]$) and the probabilities of correct aggregation $P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i)$, $1 \leq i \leq d$ are difficult to obtain, especially when the objective function $F(\cdot)$ is unknown. Therefore, the bound derived in (15) is used to measure the expected improvement during each communication round. In this sense, in order to optimize the learning performance, we need to: (1) maximize $(M - 2\mathbb{E}[Z_i])/M$; (2) increase the total number of communication rounds given a fixed total training time (until convergence).⁸

Given any M , maximizing the right-hand side of (15) is equivalent to minimizing $\mathbb{E}[Z_i] = \sum_{m=1}^M P(X_{m,i} = 1)$. Let $p_{m,i}^{(k)}$ denote the probability of $\text{sign}(\tilde{\mathbf{g}}_m^{(k)})_i = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i$ (i.e., the i -th entry of the local gradient of worker m has the same sign as that of the true gradient $\nabla F(\mathbf{w}^{(k)})$), it can be shown that

$$P(X_{m,i} = 1) = p_{m,i}^{(k)} p_{\text{out}}(r_m) + (1 - p_{m,i}^{(k)})(1 - p_{\text{out}}(r_m)). \quad (16)$$

When $p_{m,i}^{(k)} > 0.5$,⁹ $P(X_{m,i} = 1)$ is minimized if $p_{\text{out}}(r_m)$ is minimized.

⁷Note that the scenario in which $F(\mathbf{w}^{(k)})_i = 0$ is not considered in our study for simplicity.

⁸Theoretically, it can be shown that $\mathbb{E}[\frac{1}{T} \sum_{t=1}^T \|\nabla F(\mathbf{w}^{(k)})\|_1] \leq O(1/\sqrt{K})$. In practice, it is usually not expected that the gradients be reduced to 0. The machine learning algorithms are said to converge when the performance stops improving subject to a given threshold, which usually takes a finite number of communication rounds.

⁹We note that $p_{m,i}^{(k)}$'s depend on the distribution of the local training datasets. As a result, it is impossible to theoretically obtain $p_{m,i}^{(k)}$'s. As a special case, when all the workers have the same dataset, $\mathbf{g}_m^{(k)} = \nabla F(\mathbf{w}^{(k)})$ and therefore $p_{m,i}^{(k)} = 1, \forall m, t, i$. In the homogeneous data distribution setting, $\mathbf{g}_m^{(k)}$ can be considered as a noisy version of $\nabla F(\mathbf{w}^{(k)})$. Following the assumption in [26] that the noise is symmetric with zero mean, $p_{m,i}^{(k)} > 0.5$ always holds. In addition, we note that $p_{m,i}^{(k)} > 0.5$ still holds if each worker computes its gradients over a mini-batch of its local dataset instead of the whole local dataset.

⁶Note that the following analysis is universal for any $1 \leq i \leq d$.

IV. PROBLEM FORMULATION

In this section, the scenario with homogeneous data distribution across the workers is considered. According to our discussion in Section III-D, to optimize the learning performance, it is desired to minimize the outage probabilities of the workers and maximize the number of communication rounds. In this work, the implementation of the FL algorithm given a fixed total training time is considered. In this case, the number of communication rounds K is inversely proportional to the time duration for each communication round i.e., $T_l \triangleq T_m^{cmp} + T_m^{com}$. Considering that the workers (i.e., the mobile devices) have limited batteries, two optimization problems are formulated. The first optimization problem is of more interests to the parameter server, whose goal is to optimize the learning performance without consuming excessive energy for the workers. The second optimization problem addresses the needs of battery-constrained workers while satisfying the learning performance requirement (which may be dictated by the server).

In the first optimization problem, the learning performance is optimized given the energy consumption constraint for the workers. As we discussed in Remark 3, in order to optimize the learning performance, we need to maximize $(M - 2\mathbb{E}[Z_i])/M$ and increase the total number of communication rounds given a fixed total training time. Considering that SignSGD converges with a rate of $O(1/\sqrt{K})$, the metric $\sqrt{K}(M - 2\mathbb{E}[Z_i])/M$ is proposed to measure the learning performance. Given a fixed total training time, K is inversely proportional to the time duration for each communication round T_l . Essentially, $(M - 2\mathbb{E}[Z_i])/M$ captures the improvement of the learning objective at each communication round (c.f. (15)) and the impact of communication outages, and \sqrt{K} (and therefore $1/\sqrt{T_l}$) captures the impact of the number of communication rounds. As a result, the product of these two factors characterizes the overall performance improvement. In particular, to obtain the convergence rate of $O(1/\sqrt{K})$, the learning rate in (11) is set as $\eta \propto 1/\sqrt{K}$ [26]. Accumulating it over K communication rounds yields the factor \sqrt{K} .

In the second optimization problem, the energy consumption is minimized given the learning performance constraint (i.e., the outage probabilities of the workers and the time duration for each communication round). It can be seen from (6) that given fixed bandwidth B_m and noise power spectral density N_0 , the transmission rate r_m and the transmission power P_m determine the outage probability of worker m . Increasing the transmission power P_m and decreasing the transmission rate r_m both decrease the outage probability. However, according to (7), a larger P_m and a smaller r_m result in higher communication energy consumption. In addition, given a fixed time duration for each communication round (i.e., $T_m^{cmp} + T_m^{com}$), decreasing r_m increases the communication time T_m^{com} and therefore requires worker m to increase the CPU frequency f_m such that the local computation time can be reduced. As a result, the local computation energy consumption of worker m also increases. By solving the second optimization problem, each worker m minimizes its energy consumption by selecting appropriate local

computation parameter f_m , communication parameters P_m and r_m while satisfying the learning performance constraint.

A. Learning Performance Optimization Given Energy Consumption Constraint

In the first optimization problem, it is assumed that each worker m first reports its local processing CPU frequency f_m , transmission power P_m and energy consumption limit E_m to the parameter server. Then, the parameter server determines the time duration for each communication round T_l and the transmission rate r_m for each worker. According to the discussion in Section III-D, $\mathbb{E}[Z_i] = \sum_{m=1}^M p_{m,i}^{(k)} p_{out}(r_m) + (1 - p_{m,i}^{(k)})(1 - p_{out}(r_m))$, in which $p_{m,i}^{(k)}$ is determined by the local dataset of worker m and therefore unknown to the server. Moreover, in the homogeneous data distribution scenario, the local datasets (and therefore the local gradients) of the workers share the same distribution. In this case, $p_{m,i}^{(k)}$'s are supposed to be the same across the workers, and minimizing $\sum_{m=1}^M p_{out}(r_m)$ also minimizes $\mathbb{E}[Z_i]$. The optimization problem is formulated as follows.

$$\begin{aligned} \max_{T_l, r_m} & \frac{M - 2 \sum_{m=1}^M p_{out}(r_m)}{\sqrt{T_l}} \\ \text{s.t.} & \frac{\alpha_m}{2} c_m D_m f_m^2 + \frac{P_m s_m}{r_m B_m} \leq E_m, \quad \forall m, \\ \max_m & \left\{ \frac{c_m D_m}{f_m} + \frac{s_m}{r_m B_m} \right\} \leq T_l, \end{aligned} \quad (17)$$

in which the time duration for each communication round T_l and the transmission rate r_m are the parameters to be optimized. E_m is the energy consumption upper limit for worker m . The first constraint captures the energy consumption requirement for each worker m and the second constraint captures the time duration requirement for each communication round. Furthermore, we assume that the workers transmit with high SNR and therefore we have¹⁰

$$p_{out}(r_m) \approx \frac{(2^{r_m} - 1)N_0 B_m}{P_m}. \quad (18)$$

Remark 4: According to (18), given fixed transmission power P_m , $p_{out}(r_m)$ is determined by r_m . In this sense, by solving the first optimization problem, the parameter server can obtain and send the learning performance requirements T_l and $p_{out}(r_m)$ to each worker m .

B. Energy Minimization Given Learning Performance Constraint

In this subsection, the energy minimization problem given the requirements for the time duration for each communication round and the outage probability of each worker is considered. Given a constraint $p_{out,m}$ on the outage probability and a constraint T_l on the time duration for each communication round, the goal of worker m is to minimize its energy consumption.

¹⁰Note that this assumption is made so that (17) can be solved more efficiently (see Section V-A). Its effectiveness is verified through our simulations.

The corresponding optimization problem is formulated as follows.

$$\begin{aligned}
\min_{f_m, r_m, P_m} \quad & \frac{\alpha_m}{2} c_m D_m f_m^2 + \frac{P_m s_m}{r_m B_m} \\
\text{s.t.} \quad & \frac{c_m D_m}{f_m} + \frac{s_m}{r_m B_m} \leq T_l, \\
& 1 - e^{-\frac{(2^{r_m} - 1) N_0 B_m}{P_m}} \leq p_{out, m}, \\
& f_{min, m} \leq f_m \leq f_{max, m}, \\
& P_{min, m} \leq P_m \leq P_{max, m},
\end{aligned} \quad (19)$$

The CPU frequency for local computation f_m , the transmission rate r_m and the transmission power P_m are the parameters to be optimized. The feasible regions of CPU frequency and transmission power of worker m are imposed by the third and the fourth constraints, respectively. Considering that the time duration for each communication round is determined by the slowest worker (the straggler), T_l is set the same for all the workers.

Remark 5: Given a fixed total training time, the time duration requirement imposes a lower bound on the number of communication rounds, while the outage probability requirement imposes a lower bound on $(M - 2\mathbb{E}[Z_i])/M$. Considering that the learning performance improves as the number of communication rounds and $(M - 2\mathbb{E}[Z_i])/M$ increase, T_l and $p_{out, m}$ specify the worst learning performance that the system will achieve in the considered scenario. The selection of T_l and $p_{out, m}$ in this case will be left as our future work.

V. SYSTEM PARAMETERS CONFIGURATION FOR SIGNSGD OVER WIRELESS NETWORKS

A. Learning Performance Optimization Given Energy Consumption Constraint

Lemma 1: In the optimization problem (17), given any fixed T_l , the optimal transmission rate of worker m is given by

$$\begin{aligned}
r_m^* = \max & \\
& \times \left\{ \frac{P_m s_m}{B_m (E_m - \frac{\alpha_m}{2} c_m D_m f_m^2)}, \frac{s_m f_m}{B_m f_m T_l - B_m c_m D_m} \right\}.
\end{aligned} \quad (20)$$

Proof: Please see Appendix C. \square

Remark 6: Note that (17) is infeasible when $\frac{\alpha_m}{2} c_m D_m f_m^2 \geq E_m$. In this case, the energy consumption of local computation for worker m exceeds its energy consumption limit. As a result, worker m will not be able to participate in the learning process.

Let $\mathcal{U} = \{m | \frac{P_m s_m}{B_m (E_m - \frac{\alpha_m}{2} c_m D_m f_m^2)} \geq \frac{s_m f_m}{B_m f_m T_l - B_m c_m D_m}\}$. According to Lemma 1, the workers can be divided into two groups. The optimal transmission rates of the workers in the first group (i.e., \mathcal{U}) is limited by their energy consumption upper limit E_m (i.e., further decreasing the transmission rate results in the violation of the energy consumption constraint), while those of the workers in the second group is limited by the time duration for each communication round T_l (i.e., further decreasing the transmission rate results in the violation of the time duration requirement), which is subject to design. Further

define the following two functions:

$$\begin{aligned}
g(x) = & \frac{2 \sum_{m \in \mathcal{U}} (2^{\frac{P_m s_m}{B_m (E_m - \frac{\alpha_m}{2} c_m D_m f_m^2)}} - 1) N_0 B_m}{P_m \sqrt{x}} \\
& + \frac{2 \sum_{m \notin \mathcal{U}} (2^{\frac{s_m f_m}{B_m f_m T_l - B_m c_m D_m}} - 1) N_0 B_m}{P_m \sqrt{x}},
\end{aligned} \quad (21)$$

$$h(x) = \frac{M}{\sqrt{x}}. \quad (22)$$

Based on Lemma 1, the optimization problem (17) can be reformulated as follows.

$$\begin{aligned}
\min_{T_l} \quad & g(T_l) - h(T_l) \\
\text{s.t.} \quad & T_l \geq \max_m \left\{ \frac{c_m D_m}{f_m} \right\}.
\end{aligned} \quad (23)$$

It can be verified that both $g(x)$ and $h(x)$ are convex functions of x . Therefore, (23) is a difference of convex programming problem, which can be solved by the DCA algorithm [39].

B. Energy Minimization Given Outage Probability Constraint

We note that the optimization problem (19) is not always feasible. In particular, according to the time duration requirement T_l , it is required that $r_m \geq \frac{s_m}{(T_l - \frac{c_m D_m}{f_{max, m}}) B_m}$. Combining it with the power constraint and plugging them into (6) yields

$$p_{out}(r_m) \geq 1 - e^{-\frac{(2^{\frac{s_m}{(T_l - \frac{c_m D_m}{f_{max, m}}) B_m}} - 1) N_0 B_m}{P_{max, m}}}, \quad (24)$$

which may violate the given outage constraint $p_{out, m}$ at least for some worker m . Therefore, two scenarios are considered.

1) *The Optimization Problem (19) Is Infeasible:* In this case, the corresponding parameters for worker m are set as $P_m = P_{max, m}$, $f_m = f_{max, m}$ and $r_m = \frac{s_m}{(T_l - \frac{c_m D_m}{f_{max, m}}) B_m}$.

Remark 7: We note that T_l and $p_{out}(r_m)$ are the two most important parameters that determine the performance of the FL algorithm. When the optimization problem (19) is infeasible for worker m , i.e., the outage probability given in (24) exceeds $p_{out, m}$, the delay requirement and the outage probability requirement cannot be satisfied simultaneously. Since the time duration for each communication round is supposed to be determined by the slowest worker (the straggler), we assume that worker m accommodates the time duration requirement while reducing the outage probability as best it can.

2) *The Optimization Problem (19) Is Feasible:* (19) requires a joint optimization of f_m , r_m and P_m , in which the second condition on the outage probability is not necessarily convex. In the following, we explore the inherent structure of (19) and reduce it to a convex optimization problem over single variable. Particularly, let $r_m^{(1)} = \log_2 \left(-\frac{P_{min, m} \ln(1 - p_{out, m})}{N_0 B_m} + 1 \right)$, $r_m^{(2)} = \log_2 \left(-\frac{P_{max, m} \ln(1 - p_{out, m})}{N_0 B_m} + 1 \right)$, and $r_m^{(3)} = \frac{s_m}{B_m (T_l - \frac{c_m D_m}{f_{max, m}})}$, the following lemma can be proved.

Lemma 2: Given any $\max\{r_m^{(1)}, r_m^{(3)}\} \leq r_m \leq r_m^{(2)}$, the optimal transmission power of worker m is

$$P_m^* = -\frac{N_0 B_m (2^{r_m} - 1)}{\ln(1 - p_{out,m})}, \quad (25)$$

and the optimal CPU frequency for local computation is given by

$$f_m^* = \max \left\{ \frac{c_m D_m}{T_l - \frac{s_m}{r_m B_m}}, f_{min,m} \right\}. \quad (26)$$

Proof: Please see Appendix B. \square

Remark 8: Note that $[r_m^{(3)}, r_m^{(2)}]$ defines the feasible region of r_m . If $r_m < r_m^{(3)}$, the time duration requirement cannot be satisfied even with the maximum f_m . Similarly, the outage probability requirement cannot be satisfied even with the maximum P_m if $r_m > r_m^{(2)}$. $r_m^{(1)}$ denotes the minimum transmission rate that worker m will select. For any feasible transmission power, the outage probability requirement in (19) is satisfied (i.e., $p_{out}(r_m) \leq p_{out,m}$) for any $r_m \leq r_m^{(1)}$. When $r_m < r_m^{(1)}$, worker m can increase r_m and decrease f_m accordingly to accommodate the time duration requirement T_l . Considering that the objective function of the optimization problem (19) is a decreasing (increasing) function of r_m (f_m), a smaller objective in (19) can be achieved. As a result, we have the optimal transmission rate $r_m^* \geq r_m^{(1)}$.

With Lemma 2 at hand, the optimization problem (19) can be reformulated as follows.

$$\begin{aligned} \min_{r_m} \quad & \frac{\alpha_m c_m D_m}{2} z_m^2(r_m) - \frac{N_0 s_m (2^{r_m} - 1)}{\ln(1 - p_{out,m}) r_m} \\ \text{s.t.} \quad & \max\{r_m^{(1)}, r_m^{(3)}\} \leq r_m \leq r_m^{(2)}, \end{aligned} \quad (27)$$

in which $z_m(r_m) = \max\{\frac{c_m D_m}{T_l - \frac{s_m}{r_m B_m}}, f_{min,m}\}$. It can be verified that the objective in (27) is convex and therefore, the subgradient methods [40] can be adopted to solve the optimization problem (27).

VI. EXTENSION TO THE HETEROGENEOUS DATA DISTRIBUTION SCENARIO

The discussions in the previous sections consider the scenario with homogeneous data distribution across the workers. It has been shown that SignSGD fails to converge when the data are heterogeneously distributed across the workers even when the workers can deliver their information without any error [41]. The following example is provided for further illustration.

Example 1: Suppose that the i -th entry of worker m 's gradient is given as follows¹¹

$$\nabla F_m(\mathbf{w}^{(k)})_i = \begin{cases} -1, & \text{if } 1 \leq m \leq M-1, \\ M, & \text{if } m = M. \end{cases} \quad (28)$$

¹¹Recall that in the homogeneous data distribution setting, the gradients of the workers are considered noisy versions of $\nabla F(\mathbf{w}^{(k)})$. As a result, such a scenario as (28) happens with a small probability. In the heterogeneous data distribution setting, $p_{m,i}^{(k)}$'s depend on the local datasets of the workers and may be very different from those in the homogeneous data distribution setting.

In this case, we have

$$\begin{aligned} \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i &= \text{sign}\left(\frac{1}{M} \sum_{m=1}^M \nabla F_m(\mathbf{w}^{(k)})\right)_i \\ &= \text{sign}\left(\frac{1}{M}\right) = 1 \end{aligned} \quad (29)$$

It can be easily verified that (c.f. (12))

$$\begin{aligned} P(X_{m,i} = 1) &= P(\text{sign}(\hat{\mathbf{g}}_m^{(k)})_i \neq \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i) \\ &= \begin{cases} 1 - p_{out}(r_m), & \text{if } 1 \leq m \leq M-1, \\ p_{out}(r_m), & \text{if } m = M. \end{cases} \end{aligned} \quad (30)$$

Essentially, it can be observed that the aggregation result is wrong even free of channel errors.¹²

In Example 1, it can be observed that for worker $m \in \{1, 2, \dots, M-1\}$, a smaller $p_{out}(r_m)$ results in a larger $P(X_{m,i} = 1)$ and smaller $(M - 2\mathbb{E}[Z_i])/M$, in sharp contrast with the homogeneous case. In this case, the parameter server obtains wrong aggregation results even if all the workers deliver their information without any error. In addition, when the data are heterogeneously distributed across the workers, the probability of such scenarios that lead to wrong aggregation is unknown since neither the parameter server nor the workers have knowledge about the global objective function $F(\cdot)$. As a result, the convergence of Algorithm 1 cannot be guaranteed. Therefore, it is of vital importance to develop an algorithm that can deal with heterogeneous data distribution across the workers.

With such consideration, a stochastic sign based algorithm (i.e., Algorithm 2), termed as Stochastic SignSGD with majority vote, is proposed. In particular, compared to Algorithm 1, there is a pre-processing step (i.e., step 3) in Algorithm 2, in which each worker projects each entry of the locally obtained gradient to -1 and $+1$ with certain probabilities, respectively. More specifically, each worker m computes its gradient $\nabla F_m(\mathbf{w}^{(k)})$ and obtains its expected transmission rate r_m (either provided by the parameter server after solving (17) or obtained by solving (19)), after which the outage probability $p_{out}(r_m)$ can be estimated through (6), and the pre-processing parameter p_m^i 's can be calculated accordingly. The aforementioned issue is alleviated by the stochasticity of the projection. Taking $M = 3$ in Example 1 as an example, it can be verified that

$$\begin{aligned} p_{m,i}^{(k)} &= P(\text{sign}(\mathbf{g}_m^{(k)})_i = \text{sign}(\nabla F(\mathbf{w}^{(k)}))_i) \\ &= \begin{cases} \frac{\frac{1}{2} - p_{out}(r_m) - b|\nabla F_m(\mathbf{w}^{(k)})_i|}{1 - 2p_{out}(r_m)}, & \text{if } 1 \leq m \leq 2, \\ \frac{\frac{1}{2} - p_{out}(r_m) + b|\nabla F_m(\mathbf{w}^{(k)})_i|}{1 - 2p_{out}(r_m)}, & \text{if } m = 3. \end{cases} \end{aligned} \quad (31)$$

Plugging (31) into (16) yields

$$P(X_{m,i} = 1) = \begin{cases} \frac{1}{2} + b, & \text{if } 1 \leq m \leq 2, \\ \frac{1}{2} - 3b, & \text{if } m = 3. \end{cases} \quad (32)$$

¹²Note that (30) is based on our worst case scenario analysis given by (8). The same conclusion holds in general for other scenario (e.g., (9)) as well.

Therefore, the probability of correct aggregation (c.f. (13)) is given by

$$\begin{aligned}
 P\left(Z_i < \frac{3}{2}\right) &= P\left(\sum_{m=1}^3 X_{m,i} = 1\right) + P\left(\sum_{m=1}^3 X_{m,i} = 0\right) \\
 &= 2\left(\frac{1}{2} + b\right)\left(\frac{1}{2} - b\right)\left(\frac{1}{2} + 3b\right) \\
 &\quad + \left(\frac{1}{2} - b\right)^2\left(\frac{1}{2} - 3b\right) \\
 &\quad + \left(\frac{1}{2} - b\right)^2\left(\frac{1}{2} + 3b\right) \\
 &= \frac{1}{2} + \frac{1}{2}b - 6b^3.
 \end{aligned} \tag{33}$$

It can be verified that when $0 < b < \frac{1}{\sqrt{12}}$, $P(Z_i < 3/2) > \frac{1}{2}$. That being said, the probability of correct aggregation is strictly larger than $\frac{1}{2}$ when b is small enough, based on which the convergence of Algorithm 2 can be established [42]. For more general scenarios, the following lemma can be proved.

Lemma 3: When $p_{out}(r_m) \leq \min_i \{\frac{1}{2} - b|\nabla F_m(\mathbf{w}^{(k)})_i|\}$, in which $\nabla F_m(\mathbf{w}^{(k)})_i$ is the i -th entry of the gradient $\nabla F_m(\mathbf{w}^{(k)})$, we have

$$P(\tilde{\mathbf{g}}_i^{(k)} = 1)$$

Algorithm 2 Stochastic SignSGD With Majority Vote Over Wireless Networks

- 1: Input: initial weight: \mathbf{w}_0 ; number of workers: M ; learning rate: η .
- 2: **for** $k = 0, 1, \dots, K$ **do**
- 3: Each worker m transmits $\text{sign}(\mathbf{g}_m^{(k)})$ to the parameter server over wireless links, where $\mathbf{g}_m^{(k)}$ is obtained as follows: each worker m first obtains its gradient $\nabla F_m(\mathbf{w}^{(k)})$. Then, it estimates its outage probability $p_{out}(r_m)$ and does the following pre-processing

$$(\mathbf{g}_m^{(k)})_i = \begin{cases} -\text{sign}(\nabla F_m(\mathbf{w}^{(k)}))_i, & \text{with probability } p_m^i, \\ \text{sign}(\nabla F_m(\mathbf{w}^{(k)}))_i, & \text{with probability } 1 - p_m^i, \end{cases} \tag{34}$$

where $p_m^i = \frac{\frac{1}{2} - p_{out}(r_m) - b|\nabla F_m(\mathbf{w}^{(k)})_i|}{1 - 2p_{out}(r_m)}$ and b is a parameter subject to design. Particularly, $0 < b < \frac{1 - 2p_{out}(r_m)}{2|\nabla F_m(\mathbf{w}^{(k)})_i|}$ such that $p_m^i \in (0, \frac{1}{2})$.

- 4: The parameter server obtains a noisy estimate (denoted by $\hat{\mathbf{g}}_i^{(k)}$) of the transmitted information $\text{sign}(\mathbf{g}_m^{(k)})$ from each worker m and sends the aggregated result $\tilde{\mathbf{g}}^{(k)} = \text{sign}(\frac{1}{M} \sum_{m=1}^M \hat{\mathbf{g}}_i^{(k)})$ back to the workers.
- 5: The workers update their local models

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \eta \tilde{\mathbf{g}}^{(k)}. \tag{35}$$

6: **end for**

$$= \begin{cases} \frac{1}{2} + \frac{\left(\frac{M-1}{2}\right) \sum_{m=1}^M b \nabla F_m(\mathbf{w}^{(k)})_i}{2^M} + O\left(\frac{b^2}{2^M}\right), & \text{when } M \text{ is odd,} \\ \frac{1}{2} + \frac{\left[\left(\frac{M-1}{2}\right) + \left(\frac{M-1}{2}\right)\right] \sum_{m=1}^M b \nabla F_m(\mathbf{w}^{(k)})_i}{2^{M+1}} + O\left(\frac{b^2}{2^M}\right), & \text{when } M \text{ is even.} \end{cases} \tag{36}$$

where $\tilde{\mathbf{g}}_i^{(k)}$ is the i -th entry of the aggregated result at the parameter server's side, and the parameter server breaks the tie when there is no winner in the majority vote by selecting -1 and $+1$ uniformly at random.

Proof: Please see Appendix D. \square

Remark 9: (36) measures the probability of the i -th entry of the aggregated result being 1. If the second term in (36) dominates (i.e., b is sufficiently small), $P(\tilde{\mathbf{g}}_i^{(k)} = 1) > \frac{1}{2}$ when $\sum_{m=1}^M \nabla F_m(\mathbf{w}^{(k)})_i > 0$; $P(\tilde{\mathbf{g}}_i^{(k)} = 1) < \frac{1}{2}$ when $\sum_{m=1}^M \nabla F_m(\mathbf{w}^{(k)})_i < 0$. That being said, the probability of wrong aggregation is always smaller than $1/2$. As a result, the convergence of Algorithm 2 can be established.

When the workers compute their local gradients (denoted by $\nabla \tilde{F}_m(\mathbf{w}^{(k)})$'s) over a mini-batch of their local training data, $\nabla F_m(\mathbf{w}^{(k)})_i$ in Lemma 3 will be replaced by $\nabla \tilde{F}_m(\mathbf{w}^{(k)})_i$. That being said, it can be shown that $P(\text{sign}(\tilde{\mathbf{g}}_i^{(k)}) \neq \text{sign}(\sum_{m=1}^M \nabla \tilde{F}_m(\mathbf{w}^{(k)}))_i) < \frac{1}{2}$, when b is sufficiently small. In this case, $\sum_{m=1}^M \nabla \tilde{F}_m(\mathbf{w}^{(k)})$ can be understood as a noisy version of $\sum_{m=1}^M \nabla F_m(\mathbf{w}^{(k)})$ due to the training data sampling. Following the same assumption in [26] that the sampling noise is symmetric with zero mean, $P(\text{sign}(\sum_{m=1}^M \nabla \tilde{F}_m(\mathbf{w}^{(k)}))_i \neq \text{sign}(\sum_{m=1}^M \nabla F_m(\mathbf{w}^{(k)}))_i) < \frac{1}{2}$ always holds. As a result, it can also be shown that the probability of wrong aggregation is always smaller than $1/2$.

Given Lemma 3, the following theorem can be proved according to [42].

Theorem 2: Suppose Assumption 1 is satisfied and the learning rate is set as $\eta = \frac{1}{\sqrt{dT}}$, then by running Algorithm 2 for K iterations, we have

$$\frac{1}{T} \sum_{t=1}^T c \|\nabla F(\mathbf{w}^{(k)})\|_1 \leq \frac{\mathbb{E}[F(\mathbf{w}^{(0)}) - F(\mathbf{w}^{(k+1)})] \sqrt{d}}{\sqrt{K}}, \tag{37}$$

where c is some positive constant.

It is worth mentioning that according to Lemma 3 and Theorem 2, the convergence of Algorithm 2 is based on the condition $p_{out}(r_m) \leq \min_i \{\frac{1}{2} - b|\nabla F_m(\mathbf{w}^{(k)})_i|\}$. Therefore, in order to optimize the learning performance, the corresponding constraint needs to be added to the optimization problem. In the meantime, b should also be optimized. However, since $\nabla F_m(\mathbf{w}^{(k)})$'s are unknown to the server, optimizing b and the learning performance of Algorithm 2 is highly non-trivial and left as our future work.

With such consideration, in this work, we mainly consider the energy consumption minimization problem given

predetermined b ,¹³ time duration requirement T_l and the outage probability requirement $p_{out,m}$ for each worker. It can be observed from (36) that the probability of correct aggregation is independent of the outage probability $p_{out}(r_m)$. In the meantime, according to (19), the feasible region of the energy consumption minimization problem with a smaller $p_{out,m}$ is a subset of that with a larger $p_{out,m}$. As a result, it is optimal to select $p_{out,m} = \min_i \{\frac{1}{2} - b|\nabla F_m(\mathbf{w}^{(k)})_i|\}$ at the k -th communication round. In this case, worker m has to obtain $\nabla F_m(\mathbf{w}^{(k)})$ before computing $p_{out,m}$. That being said, it has to finish the local computation of the gradients before solving the energy consumption minimization problem. To handle this challenge, we consider a pre-determined local computation CPU frequency f_m for each worker m 's energy consumption minimization problem (which can be realized by setting $f_{min,m} = f_{max,m} = f_m$ in (19)).

To this end, during each communication round, each worker m first computes its gradient $\nabla F_m(\mathbf{w}^{(k)})$ and determines the optimal outage probability requirement $p_{out,m}$ as above. By solving the energy consumption minimization problem (19), it obtains the communication parameters P_m and r_m . Then, each worker m estimates its outage probability $p_{out}(r_m)$ (e.g., through (6)) and performs the pre-processing step, after which the processed information is transmitted to the parameter server over wireless links.

VII. SIMULATION RESULTS

In this section, we examine the performance of the proposed methods through extensive simulations. We implement the proposed method with a two-layer fully connected neural network with 101,770 trainable parameters on the well-known MNIST dataset that consists of 10 categories ranging from digit "0" to "9" and a total of 60,000 training samples and 10,000 testing samples. In this case, the size of updates for each worker is $s_m = 101,770$ bits for each communication round. It is assumed that there are 31 workers that collaboratively train a global model. For all the workers, similar to [2] and [13], we set $\alpha_m = 2 \times 10^{-28}$; $c_m = 20$ cycles/bit; $D_m = 5 \times 10^7$ bits; $N_0 = 10^{-8}$ W/Hz; $B_m = 180$ kHz. In the training process, all the workers use a mini-batch of size 16 to compute their local gradients.¹⁴ In the scenario with homogeneous data distribution across the workers, each worker randomly samples 2000 training samples from the training dataset. In the scenario with heterogeneous data distribution across the workers, two heterogeneous data distribution schemes are considered. In the first scheme, we consider the extreme case where the whole training dataset is divided into 31 subsets (each assigned to one worker), and each subset contains training data with one label only. In the second scheme, we adopt the Dirichlet distribution $\text{Dir}(\alpha)$ scheme [43] to synthesize label distribution

skew in experiments, where $\text{Dir}(\alpha)$ is the symmetric Dirichlet distribution with the concentration parameter α and a smaller α indicates more severe heterogeneity. The data allocation procedure consists of the following two steps: (1) for each worker m , a random vector $\mathbf{q}_m \sim \text{Dir}(\alpha)$ is drawn where $\mathbf{q}_m = [q_{m,1}, \dots, q_{m,N}]^\top$ and $q_{m,n}$ indicates the ratio of worker m 's local training dataset from the n -th class; (2) the training data are assigned to each worker m , with the number of training samples from the n -th class proportional to $q_{m,n}$.

We compare the proposed method with a commonly adopted baseline for federated learning algorithms: FedAVG [27] in which the model updates are transmitted in full precision. In FedAVG, the workers run τ local SGD steps before sharing their model updates with the parameter server. In our simulations, we examine the performance of FedAVG for $\tau \in \{1, 5, 10, 20\}$ and present the results with the best testing accuracy. Note that our aforementioned analyses on SignSGD consider the worst case scenario, i.e., all entries of the transmitted updates are incorrectly decoded in an outage. However, the worst case scenario for FedAVG is not easy to define, and it may fail to converge if the workers in outage share arbitrary information with the parameter server [44]. With such consideration, we assume in this section that the parameter server can detect the outage and discard the packages in outage.

A. Learning Performance Optimization Given Energy Consumption Constraint: Homogeneous

In this subsection, we examine the impact of the transmission power P_m and the time duration for each communication round T_l . The CPU frequencies of all the workers are set to 2 GHz. For SignSGD, the energy consumption upper limit is set as $E_m = 100$ J for each communication round. For FedAVG, the workers are assumed to have unlimited battery (i.e., $E_m = \infty$) such that the outage probabilities of the workers are not limited by their energy consumption constraints. Fig. 2 show the testing accuracy of Algorithm 1 with different P_m and T_l , given a total training time $T_{total} = 100$ s. For SignSGD, the transmission rates r_m 's are given by (20), while the configurations of "SignSGD-Proposed" are given by the solution of (17). For FedAVG, the transmission rates are selected such that the outage probabilities are minimized given the time duration for each communication round.

It can be seen from Fig. 2 that the proposed method for SignSGD works close to the optimal operation point for all the examined scenarios, which validates its effectiveness. Besides, it is shown that as T_l increases, the learning performance of both algorithms first increases and then decreases. According to (20), when T_l increases, r_m decreases and therefore the outage probability $p_{out}(r_m)$ also decreases. However, in the meantime, as T_l increases, the number of communication rounds decreases given the fixed training time. As a result, when the outage probability has a larger impact on the learning performance, increasing T_l results in better performance. When T_l is larger than a certain critical value, the number of communication rounds plays a more important role and therefore increasing T_l leads to worse performance.

¹³In the implementation of Algorithm 2, it is possible that the predetermined $b \geq \frac{1-2p_{out}(r_m)}{2|\nabla F_m(\mathbf{w}^{(k)})_i|}$ for the i -th entry of worker m 's gradient such that $p_m^i \leq 0$. In this case, we round p_m^i to 0, and it can be verified that (34) is reduced to $(\mathbf{g}_m^{(k)})_i = \text{sign}(\nabla F_m(\mathbf{w}^{(k)}))_i$. That being said, Algorithm 1 is a special case of Algorithm 2 where b is large enough.

¹⁴Note that similar results are obtained for the full batch scenario where each worker evaluates its local gradient over the whole local dataset, which are omitted in the interest of space.

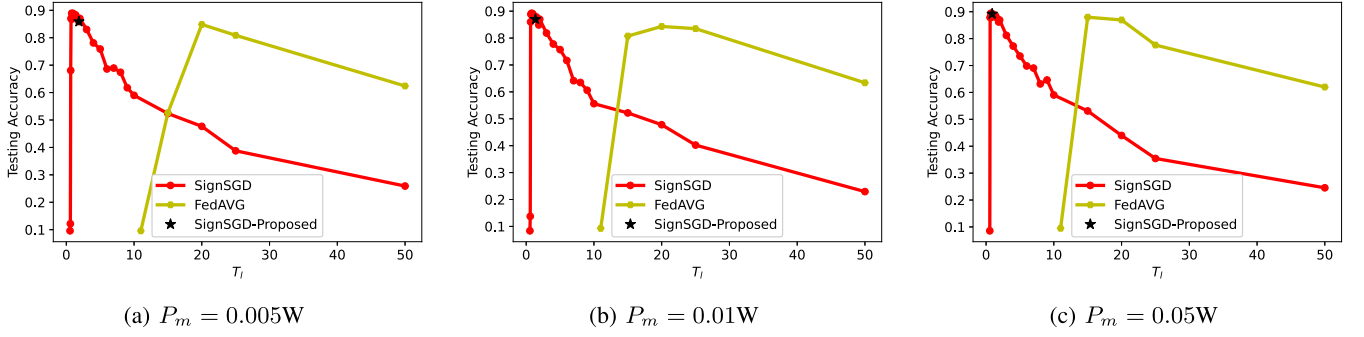


Fig. 2. The impact of communication round duration T_l on the learning algorithms given different transmission power P_m .

TABLE II
TESTING ACCURACY OF THE LEARNING ALGORITHMS

P_m	0.005W	0.01W	0.05W
SignSGD-Proposed	85.89%	87.01%	89.19%
FedAVG (optimal T_l)	84.85%	84.31%	87.96%

TABLE III
ENERGY CONSUMPTION OF THE LEARNING ALGORITHMS

P_m	0.005W	0.01W	0.05W
SignSGD-Proposed	21.56J	29.11J	44.62J
FedAVG (optimal T_l)	40.25J	40.5J	49.5J

Table II and Table III show the testing accuracy and energy consumption of SignSGD with the proposed operation point and FedAVG with the optimal T_l . It can be observed that SignSGD outperforms FedAVG in both testing accuracy and energy consumption. Moreover, we note that there is no existing method in literature that can obtain the optimal T_l and the number of local iterations τ for FedAVG.

B. Energy Minimization Given Learning Performance Constraint: Homogeneous

In this subsection, the performance of SignSGD with energy minimization is examined. We set $f_{min,m} = 0.2$ GHz; $f_{max,m} = 2$ GHz; $P_{min,m} = 0$ W; $P_{max,m} = 0.01$ W. For FedAVG, the CPU frequency f_m and the transmission power P_m are set to 2 GHz and 0.01 W, respectively, for all the workers such that the minimum outage probability can be achieved. For SignSGD and the proposed method, we set $T_l = 1.5$ s; while for FedAVG, we examine the performance of different $T_l \in \{3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 25, 50\}$ and present the results with the best testing accuracy. For the proposed method, we set the same outage probability constraints for all the workers, i.e., $p_{out,m} = p_{out}, \forall m$.

Fig. 3-4 show the testing accuracy and energy consumption of the learning algorithms. It can be observed that the proposed method with $p_{out} \in \{0.1, 0.5\}$ outperforms FedAVG in both testing accuracy and energy consumption. Moreover, it is shown that increasing the outage probability of SignSGD does

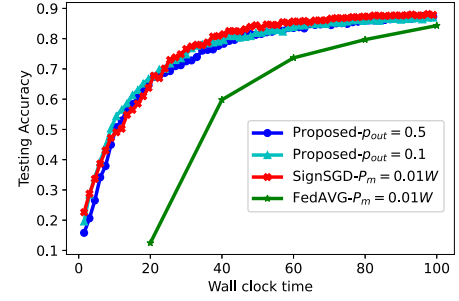


Fig. 3. Testing accuracy of the learning algorithms.

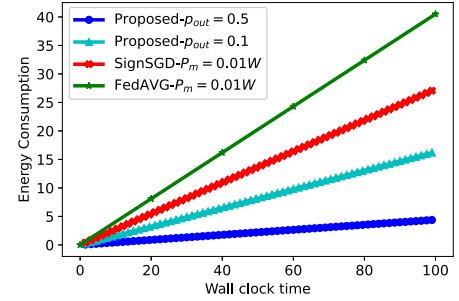


Fig. 4. Energy consumption of the learning algorithms.

not change the testing accuracy much. This is because most of the workers share the correct signs in the homogeneous data distribution scenario, and therefore, the parameter server can update the model in the correct direction even when some workers are in the outage. Fig. 5 shows the energy consumption of the proposed method for different p_{out} and T_l . It can be observed that as p_{out} and T_l increase, the energy consumption decreases. This is because the feasible region of (19) corresponding to a smaller p_{out} and T_l is a subset of that of (19) corresponding to a larger p_{out} and T_l .

C. Energy Minimization Given Learning Performance Constraint: Heterogeneous

In this subsection, the performance of Algorithm 2 is examined. We first consider the extreme case where each worker stores only training data from one label. We set $P_{min,m} = 0$ W and $P_{max,m} = 0.05$ W for all the workers.

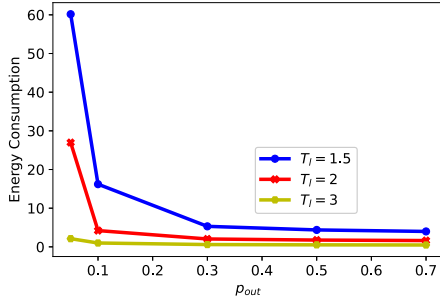


Fig. 5. Energy consumption of the proposed method for different p_{out} and T_l .

TABLE IV

TESTING ACCURACY OF THE LEARNING ALGORITHMS

CPU Frequency f	1GHz	2GHz	3GHz
Algorithm 2- $b = 100$, $T_{total} = 250s$	86.86%	88.32%	87.23%
SignSGD, $T_{total} = 300s$	64.07%	60.5%	60.70%
FedAVG (optimal T_l), $T_{total} = 300s$	82.10%	82.95%	86.19%
SignSGD, $p_{out} = 0.1$, $T_{total} = 300s$	61.23%		
Algorithm 2- $b = 100$, $T_{total} = 250s$, optimal f , $p_{out} = 0.1$	86.89%		

TABLE V

ENERGY CONSUMPTION OF THE LEARNING ALGORITHMS

CPU Frequency	1GHz	2GHz	3GHz
Algorithm 2- $b = 100$, $T_{total} = 250s$	20.65J	74.47J	158.78J
SignSGD, $T_{total} = 300s$	25.0J	90.0J	191.67J
FedAVG (optimal T_l), $T_{total} = 300s$	23.1J	93.45J	235.83J
SignSGD, $p_{out} = 0.1$, $T_{total} = 300s$	16.45J		
Algorithm 2- $b = 100$, $T_{total} = 250s$, optimal f , $p_{out} = 0.1$	13.65J		

Table IV and Table V show the testing accuracy and energy consumption of the learning algorithms, respectively. The time duration for each communication round is set to $T_l = 1.5s$ for both Algorithm 2 and SignSGD. For FedAVG, we examine the performance of different T_l 's and present the results with the best testing accuracy. For SignSGD and FedAVG, we set the total training time $T_{total} = 300s$, while for Algorithm 2, we present the results with a total training time of $T_{total} = 250s$. For SignSGD and FedAVG, we first set the transmission power $P_m = 0.05W$ for all the workers (such that the minimum outage probability can be achieved) and examine their performance given different CPU frequencies (i.e., 1 GHz, 2 GHz and 3 GHz). In addition to the pre-determined CPU frequency cases where the workers obtain their gradients before computing their outage probabilities, we also examined the scenario where each worker optimizes the CPU frequency given a pre-determined outage probability requirement $p_{out,m} = 0.1$ for Algorithm 2 with $f_{min,m} = 0.2$ GHz and $f_{max,m} = 3$ GHz. Correspondingly, the performance of SignSGD is examined in the same setting. For

TABLE VI

TESTING ACCURACY OF THE LEARNING ALGORITHMS

α	0.01	0.1	1	10
Algorithm 2- $b = 100$, $T_{total} = 250s$	93.02%	92.35%	92.31%	92.63%
SignSGD, $T_{total} = 300s$	82.99%	86.83%	87.64%	88.11%
FedAVG (optimal T_l), $T_{total} = 300s$	87.50%	89.38%	91.81%	91.88%

TABLE VII

ENERGY CONSUMPTION OF THE LEARNING ALGORITHMS

α	0.01	0.1	1	10
Algorithm 2- $b = 100$, $T_{total} = 250s$	74.70J	74.70J	74.68J	74.66J
SignSGD, $T_{total} = 300s$	90.00J	90.00J	90.00J	90.00J
FedAVG (optimal T_l), $T_{total} = 300s$	142.50J	138.75J	165.00J	150.00J

FedAVG with optimal T_l , we note that the outage probabilities of the workers are larger than 0.1 for all the examined CPU frequencies. In this sense, adding the outage probability requirement essentially leads to the same results as $P_m = P_{max,m} = 0.05$ W and $f_m = f_{max,m} = 3$ GHz, given that the workers try their best to minimize their outage probabilities. It can be observed from Table IV and Table V that Algorithm 2 outperforms FedAVG and SignSGD in both testing accuracy and energy consumption given a shorter training time. That being said, Algorithm 2 improves the learning performance and reduces the energy consumption simultaneously, which demonstrates its effectiveness. Moreover, we note that the condition $p_{out}(r_m) \leq \min_i \{\frac{1}{2} - b|\nabla F_m(\mathbf{w}^{(k)})_i|\}$ may be violated for a pre-determined $p_{out,m} = 0.1$. In this case, the pre-processing probabilities p_m^i and $1 - p_m^i$ in (34) may fall out of the range $[0, 1]$. We round them to 1 if they are positive and 0 otherwise. Table IV and Table V show that the energy consumption of the workers can be further reduced by optimizing the CPU frequency f_m 's with a negligible loss in testing accuracy.

We also examine the impact of data heterogeneity (i.e., α) in the Dirichlet distribution scheme. Table VI and Table VII compare the testing accuracy and the energy consumption of the learning algorithms, respectively, given $f_m = 2$ GHz for all the workers. It can be observed that the performance of SignSGD and FedAVG improves as α increases, which corresponds to more homogeneous data distribution. Meanwhile, Algorithm 2 outperforms SignSGD and FedAVG in both testing accuracy and energy consumption in all the examined scenarios, which demonstrates its effectiveness in the presence of different levels of data heterogeneity. Moreover, it is worth mentioning that the optimal T_l and number of local iterations for FedAvg are different given different α , which leads to different energy consumption.

D. The Impact of Outage Probability Estimation Errors

In the previous subsections, we assume that the channel gain follows Rayleigh distribution and the outage probability can be captured by (6). In practice, however, the fading over the

TABLE VIII
TESTING ACCURACY OF SIGNSGD: HOMOGENEOUS

P_m	0.005W	0.01W	0.05W
$\Delta = 0$	85.89%	87.01%	89.19%
$\Delta = 10\%$	85.65%	88.26%	89.29%
$\Delta = 30\%$	86.08%	88.47%	89.17%
$\Delta = 50\%$	86.42%	88.56%	89.25%

TABLE IX
TESTING ACCURACY OF ALGORITHM 2: HETEROGENEOUS

CPU Frequency	1GHz	2GHz	3GHz
$\Delta = 0$	88.11%	88.85%	88.61%
$\Delta = 10\%$	88.39%	87.23%	88.55%
$\Delta = 30\%$	88.34%	87.02%	88.60%
$\Delta = 50\%$	88.55%	87.35%	88.74%
$\Delta = \pm 10\%$	87.87%	87.64%	88.74%
$\Delta = \pm 30\%$	87.90%	88.11%	88.62%
$\Delta = \pm 50\%$	87.79%	88.18%	87.60%

wireless channels is usually more complicated than Rayleigh fading. In this subsection, we demonstrate the robustness of the proposed method against outage probability estimation errors by examining the scenario where the actual outage probability is different from (6).

For the homogeneous data distribution scenario, given the theoretically derived outage probability $p_{out}(r_m)$, we assume that the actual outage probability is uniformly distributed in the interval $[p_{out}(r_m), p_{out}(r_m) \times (1 + \Delta)]$. That being said, the channel conditions are worse than the ones that each worker uses for parameter configuration. We note that this is conservative since the performance of the learning algorithms is supposed to improve as the probability of outage decreases. Table VIII shows the testing accuracy of SignSGD with the configurations given by the solution of (17). The other parameters are the same as those in Section VII-A. It can be observed that the SignSGD with $\Delta \in \{10\%, 30\%, 50\%\}$ achieves comparable performance in testing accuracy to the $\Delta = 0$ case. We note that, according to (19), the energy consumption is independent of Δ . Therefore, the energy consumption remains the same for different Δ .

For the heterogeneous data distribution scenario, it is shown in Section VII-C that decreasing the outage probability is not necessarily better for SignSGD. With such consideration, we also examine the scenario that the actual outage probability is uniformly distributed in the interval $[p_{out}(r_m) \times (1 - \Delta), p_{out}(r_m) \times (1 + \Delta)]$. Table IX shows the testing accuracy of Algorithm 2 with different Δ , given a total training time of $T_{total} = 300$ s. The other parameters are the same as those in Section VII-C. It can be observed that Algorithm 2 achieves

similar performance in testing accuracy for different Δ , which demonstrates its robustness against outage probability estimation errors.

VIII. CONCLUSION

In this work, the implementation of FL algorithms over wireless networks is studied. In particular, considering that the workers have limited batteries, two optimization problems concerning the learning performance and the energy consumption of the workers are formulated and solved for appropriate local processing and communication parameter configuration. Furthermore, since SignSGD fails to converge in the scenario with heterogeneous data distribution across the workers, a stochastic sign based algorithm that can deal with data heterogeneity across the workers is proposed and the corresponding energy minimization problem is solved. It is shown that the proposed algorithm improves the learning performance with less energy consumption for the workers. The simulation results demonstrate the effectiveness of the proposed method.

APPENDIX A PROOF OF THEOREM 1

Proof: According to Assumption 1, we have

$$\begin{aligned}
 & F(\mathbf{w}^{(k+1)}) - F(\mathbf{w}^{(k)}) \\
 & \leq \nabla F(\mathbf{w}^{(k)}), \mathbf{w}^{(k+1)} - \mathbf{w}^{(k)} > + \frac{L}{2} \|\mathbf{w}^{(k+1)} - \mathbf{w}^{(k)}\|^2 \\
 & = -\eta < \nabla F(\mathbf{w}^{(k)}), \text{sign}(\sum_{m=1}^M \hat{\mathbf{g}}_m^{(k)}) > \\
 & \quad + \frac{L}{2} \|\eta \text{sign}(\sum_{m=1}^M \hat{\mathbf{g}}_m^{(k)})\|^2 \\
 & = -\eta < \nabla F(\mathbf{w}^{(k)}), \text{sign}(\sum_{m=1}^M \hat{\mathbf{g}}_m^{(k)}) > + \frac{L\eta^2 d}{2} \\
 & = \eta \|\nabla F(\mathbf{w}^{(k)})\|_1 + \frac{L\eta^2 d}{2} - 2\eta \sum_{i=1}^d |\nabla F(\mathbf{w}^{(k)})_i| \\
 & \quad \times \mathbb{1}_{\text{sign}(\sum_{m=1}^M \hat{\mathbf{g}}_m^{(k)})_i = \text{sign}(\nabla F(\mathbf{w}^{(k)})_i)}, \tag{38}
 \end{aligned}$$

in which $\nabla F(\mathbf{w}^{(k)})_i$ is the i -th entry of the vector $\nabla F(\mathbf{w}^{(k)})$. Taking expectation on both sides yields

$$\begin{aligned}
 & \mathbb{E}[F(\mathbf{w}^{(k)}) - F(\mathbf{w}^{(k+1)})] \geq -\eta \|\nabla F(\mathbf{w}^{(k)})\|_1 - \frac{L\eta^2 d}{2} + 2\eta \\
 & \quad \times \sum_{i=1}^d |\nabla F(\mathbf{w}^{(k)})_i| P(\tilde{\mathbf{g}}_i^{(k)} = \text{sign}(\nabla F(\mathbf{w}^{(k)})_i)). \tag{39}
 \end{aligned}$$

□

APPENDIX B PROOF OF LEMMA 2

Proof: According to the constraint $1 - e^{-\frac{(2^{r_m} - 1)N_0 B_m}{P_m}} \leq p_{out,m}$, when $\max\{r_m^{(1)}, r_m^{(3)}\} \leq r_m \leq r_m^{(2)}$, it can be obtained that

$$P_m \geq -\frac{N_0 B_m (2^{r_m} - 1)}{\ln(1 - p_{out,m})}. \tag{40}$$

Since the objective function $\frac{\alpha_m}{2}c_m D_m f_m^2 + \frac{P_m s_m}{r_m B_m}$ is an increasing function of P_m , we have

$$P_m^* = -\frac{N_0 B_m (2^{r_m} - 1)}{\ln(1 - p_{out,m})}. \quad (41)$$

According to the constraint $\frac{c_m D_m}{f_m} + \frac{s_m}{r_m B_m} \leq T_l$, we have

$$f_m \geq \frac{c_m D_m}{T_l - \frac{s_m}{r_m B_m}}. \quad (42)$$

In addition, the objective function is an increasing function of f_m . Therefore,

$$f_m^* = \max \left\{ \frac{c_m D_m}{T_l - \frac{s_m}{r_m B_m}}, f_{min,m} \right\} \quad (43)$$

□

APPENDIX C PROOF OF LEMMA 1

Proof: According to the constraint $\frac{\alpha_m}{2}c_m D_m f_m^2 + \frac{P_m s_m}{r_m B_m} \leq E_m$, we have

$$r_m \geq \frac{P_m s_m}{B_m (E_m - \frac{\alpha_m}{2}c_m D_m f_m^2)}. \quad (44)$$

According to the constraint $\frac{c_m D_m}{f_m} + \frac{s_m}{r_m B_m} \leq T_l$, we have

$$r_m \geq \frac{s_m f_m}{B_m f_m T_l - B_m c_m D_m}. \quad (45)$$

In addition, it can be shown that the objective function $\frac{M-2 \sum_{m=1}^M p_{out}(r_m)}{\sqrt{T_l}}$ is a decreasing function of r_m . Therefore,

$$r_m^* = \max \left\{ \frac{P_m s_m}{B_m (E_m - \frac{\alpha_m}{2}c_m D_m f_m^2)}, \frac{s_m f_m}{B_m f_m T_l - B_m c_m D_m} \right\}. \quad (46)$$

□

APPENDIX D PROOF OF LEMMA 3

In the interest of space, we present the proof of Lemma 3 when M is odd. The corresponding result for even M can be obtained following a similar strategy.

Proof: Define a series of random variables $\{\hat{X}_{m,i}\}_{m=1}^M$ given by

$$\hat{X}_{m,i} = \begin{cases} 1, & \text{if } \text{sign}(\hat{\mathbf{g}}_m^{(k)})_i \neq \text{sign}(\nabla F_m(\mathbf{w}^{(k)}))_i, \\ 0, & \text{if } \text{sign}(\hat{\mathbf{g}}_m^{(k)})_i = \text{sign}(\nabla F_m(\mathbf{w}^{(k)}))_i. \end{cases} \quad (47)$$

It can be verified that

$$P(\hat{X}_{m,i} = 1) = (1 - p_m^i) p_{out}(r_m) + p_m^i (1 - p_{out}(r_m)) \\ = \frac{1}{2} - b |\nabla F_m(\mathbf{w}^{(k)})_i|. \quad (48)$$

$$\text{sign}(\hat{\mathbf{g}}_m^{(k)})_i = \begin{cases} 1, & \text{with probability } \frac{1+b|\nabla F_m(\mathbf{w}^{(k)})_i|}{2}, \\ -1, & \text{with probability } \frac{1-b|\nabla F_m(\mathbf{w}^{(k)})_i|}{2}, \end{cases} \quad (49)$$

Further define a series of random variables $\{\hat{Z}_{m,i}\}_{m=1}^M$ given by

$$\hat{Z}_{m,i} = \begin{cases} 1, & \text{if } \text{sign}(\hat{\mathbf{g}}_m^{(k)})_i = 1, \\ 0, & \text{if } \text{sign}(\hat{\mathbf{g}}_m^{(k)})_i = -1. \end{cases} \quad (50)$$

Let $\hat{Z}_i = \sum_{m=1}^M \hat{Z}_{m,i}$, then

$$P\left(\text{sign}\left(\frac{1}{M} \sum_{m=1}^M \text{sign}(\hat{\mathbf{g}}_m^{(k)})_i\right) = 1\right) = P\left(\hat{Z}_i \geq \frac{M}{2}\right) \\ = \sum_{H=\lceil \frac{M+1}{2} \rceil}^M P(\hat{Z}_i = H). \quad (51)$$

In addition, let $u_m = \nabla F_m(\mathbf{w}^{(k)})_i$, we have

$$P(\hat{Z}_i = H) = \frac{\sum_{A \in F_H} \prod_{i \in A} (1 + bu_i) \prod_{j \in A^c} (1 - bu_j)}{2^M} \\ = \frac{a_{M,H} b^M + a_{M-1,H} b^{M-1} + \dots + a_{0,H} b^0}{2^M}, \quad (52)$$

in which F_H is the set of all subsets of H integers that can be selected from $\{1, 2, 3, \dots, M\}$; $a_{m,H}, \forall 0 \leq m \leq M$ is some constant. It can be easily verified that $a_{0,H} = \binom{M}{H}$.

When b is sufficiently small, $P(\hat{Z}_i = H)$ is dominated by the last two terms in (52). In particular, $\forall m$, we have

$$\sum_{A \in F_H} \prod_{i \in A} (1 + bu_i) \prod_{j \in A^c} (1 - bu_j) \\ = (1 + bu_m) \sum_{A \in F_H, m \in A} \prod_{i \in A/\{m\}} (1 + bu_i) \prod_{j \in A^c} (1 - bu_j) \\ + (1 - bu_m) \sum_{A \in F_H, m \notin A} \prod_{i \in A} (1 + bu_i) \prod_{j \in A^c/\{m\}} (1 - bu_j). \quad (53)$$

As a result, when $\lceil \frac{M+1}{2} \rceil \leq H \leq M-1$, the u_m related term in $a_{1,H}$ is given by $[\binom{M-1}{H-1} - \binom{M-1}{H}]u_m$; when $H = M$, the u_m related term in $a_{M-1,H}$ is given by $[\binom{M-1}{H-1}]u_m$. By summing over m , we have

$$a_{1,H} = \left[\binom{M-1}{H-1} - \binom{M-1}{H} \right] \sum_{m=1}^M u_m, \\ \text{if } \left\lceil \frac{M+1}{2} \right\rceil \leq H \leq M-1, \quad (54)$$

and

$$a_{1,H} = \left[\binom{M-1}{H-1} \right] \sum_{m=1}^M u_m, \quad \text{if } H = M. \quad (55)$$

By summing over H , we have

$$\sum_{H=\lceil \frac{M+1}{2} \rceil}^M a_{0,H} = \sum_{H=\lceil \frac{M+1}{2} \rceil}^M \binom{M}{H} = 2^{M-1}, \quad (56)$$

$$\sum_{H=\lceil \frac{M+1}{2} \rceil}^M a_{1,H} = \left(\binom{M-1}{\lceil \frac{M+1}{2} \rceil - 1} \right) \sum_{m=1}^M u_m \quad (57)$$

As a result,

$$\begin{aligned}
 P\left(\hat{Z}_i \geq \frac{M}{2}\right) &= \sum_{H=\lceil \frac{M+1}{2} \rceil}^M P(\hat{Z}_i = H) \\
 &= \frac{2^{M-1} + \left(\lceil \frac{M+1}{2} \rceil - 1\right) \sum_{m=1}^M u_m b}{2^M} + O\left(\frac{b^2}{2^M}\right) \\
 &= \frac{1}{2} + \frac{\left(\lceil \frac{M+1}{2} \rceil - 1\right)}{2^M} \sum_{m=1}^M u_m b + O\left(\frac{b^2}{2^M}\right),
 \end{aligned} \tag{58}$$

which completes the proof. \square

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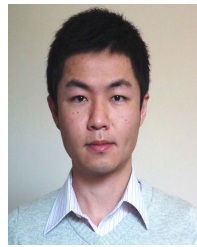
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