Robust and nonlinear control EEN050

Assignment 4

Linear Parameter Varying Models and Analysis

Before submission pre-approval of solution is mandatory

Problem formulation and aim

Consider the problem of SISO LPV system for a virus epidemiology model. Observability and quadratic (parameter independent and dependent) analysis. Some time domain analysis will be analyzed.

Administration

- The assignment is solved in a group of three students, documented in an electronic solution report (plots, explanation, reply to questions).
- Pre-approval of solution by TAs is mandatory before submission (during tutorial sessions)
- Electronic submission to CANVAS, upload 1 solution per group using the filename: Group#-Assignment#.pdf after getting a preliminary oral approval from the TA (tutorial session).
- Results, no later than 1 week after submission (with constructive feedback). If the solution is not approved, one week time is given for correction (one extra chance only).
- Deadline: check the course PM.

Questions

Given a control oriented modified SEIR (Susceptible-Exposed-Infected-Recovered) epidemiological compartmental model [8,7] as

$$\begin{aligned}
\dot{x}_1(t) &= -u(t)x_1(t)x_3(t) + c_1x_4(t)\\
\dot{x}_2(t) &= u(t)x_1(t)x_3(t) - c_2x_2(t)\\
\dot{x}_3(t) &= -c_3x_3(t) + c_2x_2(t)\\
\dot{x}_4(t) &= c_3x_3(t) - c_1x_4(t)
\end{aligned}$$

where $x \in \mathbb{R}^4$ x_1, x_2, x_3 and x_4 are the susceptible, exposed, infected and recovered fraction of the population. Parameters are given as $c_1 = 1/60$,$c_2 = 0.2$,$c_3 = 0.1$ (coefficients with unit $1/day$). $u(t)$ is kept *constant* control input describing the effect of quarantining/self isolation/social distancing in unit $1/(people \cdot day)$, $0 \leq u(t) \leq 1$. The traditional SEIR model is expanded; recovered people can be re-infected. Only the infected amount of people can be measured.

Exercise 1 Transform the SEIR model into Linear Parameter Varying form under constant control input actions $\forall t$. Use a minimum number of time varying parameters $p(t)$ while creating an LPV model. Classify the obtained model (exo or endo, linear, polynomial or rational in the parameter, exact or non-exact).

Exercise 2 Simulate the LPV model with constant control inputs $u(t) = 0$ (complete lockdown), $u(t) = 0.3$ (mild distancing), $u(t) = 0.8$ (no distancing). Initial state for all scenarios is the same: $x_0 = \begin{bmatrix}0.6 & 0.3 & 0.1 & 0\end{bmatrix}^T$. How does $x(t)$ behave in time domain $t \in [0\ T]$ with a preselected terminal time started from $x(0)$? What is the relation between $\sum_{i=1}^4 x_i(T)$ and $\sum_{i=1}^4 x_i(0)$ and why?

Exercise 3 Is the LPV model structurally observable? Find the condition that helps to decide such property, and describe what this means in practice.

Exercise 4 Show why there does not exist any control law, u, such that all the states of SEIR converges to zero if $x(0) \neq 0$.

Before submission pre-approval of solution is mandatory

References

[1] S. Skogestad, I. Postletwaite *Multivariable feedback control: analysis and design*. Wiley, New York, 2nd edition, 2001

[2] HK Khalil *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ, 1996

[3] Jeff Moehlis `https://sites.me.ucsb.edu/~moehlis/APC514/tutorials/tutorial_seasonal/node4.html`.

[4] G Horvath, G Szederkenyi *Identifiability analysis and parameter estimation of a SEIR epidemic model*. In Proc of PCIT, 2020 `https://pcit2020.mik.uni-pannon.hu/images/program/kiadvany.pdf`.