Robust and nonlinear control EEN050

Assignment $1-3$

Robust flight control system design

Before submission pre-approval of solution is mandatory

Problem formulation and aim

We consider the problem of robust stabilization and tracking for MIMO systems, aiming to understand the concept of 2 DoF robust control design by minimizing the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms. We analyze and numerically solve these type of problems (with Matlab or alike).

Administration

- The assignment is solved in a group of three students, documented in an electronic solution report (plots, explanation, reply to questions).
- Pre-approval of solution by TAs is mandatory before submission (during tutorial sessions)
- Electronic submission to CANVAS, upload 1 solution per group using the filename: Group#-Assignment#.pdf after getting a preliminary oral approval from the TA (tutorial session).
- Results, no later than 1 week after submission (with constructive feedback). If the solution is not approved, a single week time is given for correction (one extra chance only).
- Deadline: check the course PM.

Software preliminaries

The assignment is made in MATLAB with Robust Control Toolbox and a script **EEN050_Assignment_1_2_and_3_template.m** (hereby referred to as *the m-file* is provided with system initialization. Key MATLAB commands: *ss*, *connect*, *ureal*, *uss*, *usample*,*sigma*, *makeweight*, *ucover*, *hinfsyn*, *h2syn*, *sigma*, *norm*, *ultidyn*, *lsim*, *randn*, *bodemag*. See MATLAB online documentation for more information about the commands.

System description

The mfile contains a linearized longitudinal aircraft model F-16 [1] for a straight and level flight condition. The linearized model is denoted by G_n and is considered as a nominal model, and an additional actuator dynamics model, denoted by G_a, is also provided. The schematic is depicted in Fig. 1 and nominal system state, input, and output are given in Table 1. We aim at tracking a reference signal r for the angle of attack α in a disturbance and uncertainty-heavy environment.

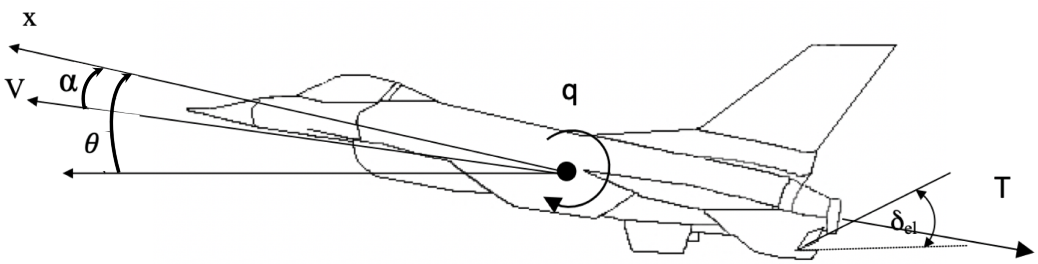

Figure 1: Schematic of longitudinal aircraft dynamics

System state	Input signals	Measured signals
Total velocity (true airspeed), V [ft/s]	Thrust, τ [N]	Angle of attack, α [deg]
Angle of attack, α [rad]	Elevator deflection, δ_{el} [rad]	Pitch rate, q [rad/s]
Pitch angle, θ [rad]		Normal acceleration of
Pitch rate, q [rad/s]		centre of gravity, a_n [ft/s^2]
Engine power, T [lb]		

Table 1: System state, input and output of the nominal F-16 model.

Fig. 2 shows the block diagram we will use for designing a controller for the F-16 aircraft. The block diagram shows a 2-DoF controller design, meaning that the controller $K = \begin{bmatrix} K_r & K_y \end{bmatrix}$ consists of a pre-filter, K_r, and a feedback, K_y. The signals r, n, and d are the reference, measurement noise, and input noise signals, respectively. The signals z_e, z_p, and z_u are performance signals; weighted tracking error, weighted output, and weighted input, respectively.

We will minimize $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms corresponding to the weighted transfer function matrix (between the augmented systems inputs and outputs). Weightings have dynamism. The transfer functions W_r, W_m, W_d, W_n, W_e, W_p, W_u are weights used to augment the nominal model information and support robust controller design. Δ_m is assumed to be a stable transfer function having a bounded magnitude, $\|\Delta_m\|_\infty < 1$, but Δ_M is otherwise unknown. The weight W_r is the desired transfer function of the closed-loop system, specifying the desired relationship between the reference and the output. The weights W_d and W_n emphasize which components and/or frequency content of the disturbances d and n, respectively, influence the flight the most. The transfer functions W_m and Δ_M constitute an input multiplicative uncertainty, describing uncertainty about the nominal plant G_n. The weights W_e, W_p, and W_u emphasize which components and/or frequency content in z_e, z_p, and z_u, respectively, are most important to minimize. For example, the gain of W_e is chosen as large in frequency ranges where small errors are desired and small where larger errors are tolerated.

Now, we proceed to formulate a standard robust design problem as illustrated in Fig. 3 and determine K such that the $\mathcal{H}_\infty$ norm, or $\mathcal{H}_2$ norm, of the transfer function matrix from $\tilde{w}$ to $\tilde{z}$ is minimized

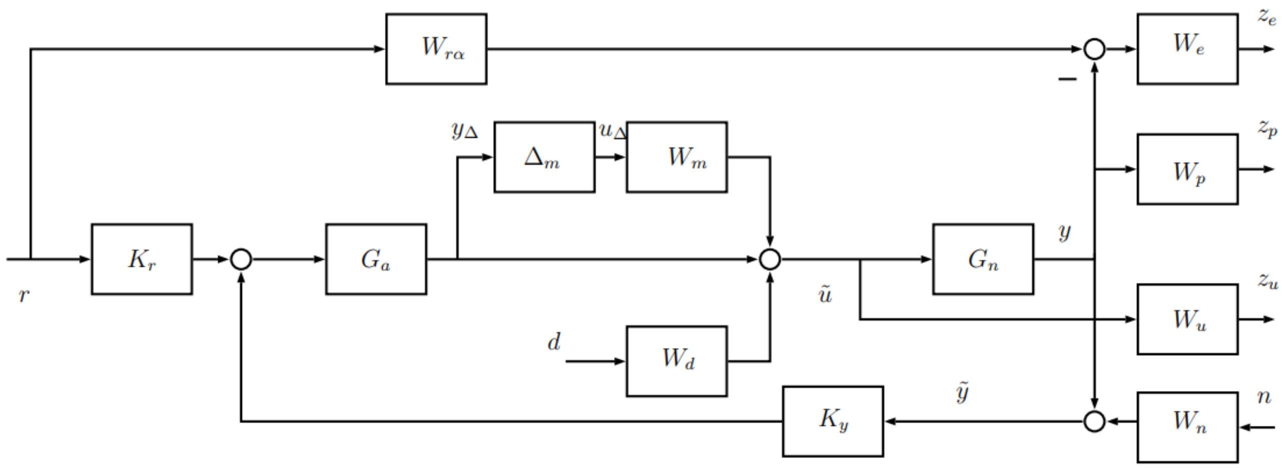

Figure 2: Augmented closed loop layout

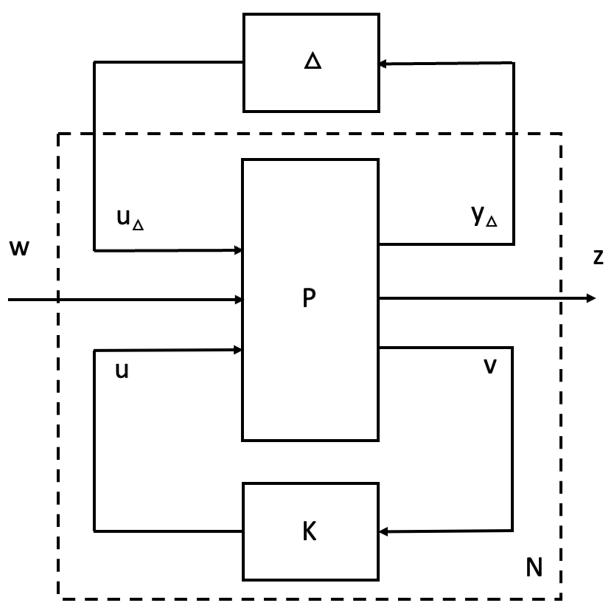

Figure 3: Generalized $P - K$ structure

Assignment 1

A1/ Ex1 *Open loop analysis.* In the given m-file you will find a state space object 'G_F16' for the airplane model $G_n(s)G_a(s)$. Answer the following questions with a brief motivation.

- Is the airplane model stable? and
- Is the airplane model minimal order?
- Plot the singular values of the airplane model.
- Can you determine the $\mathcal{H}_\infty$ -norm from this plot?
- Compute the $\mathcal{H}_\infty$ -norm and $\mathcal{H}_2$ -norm of the airplane model.

If you try to compute (with MATLAB) the $\mathcal{H}_2$ -norm of $G_n(s)$ you will get an error message. Motivate, using the frequency domain definition of the $\mathcal{H}_2$ -norm, why the $\mathcal{H}_2$ -norm of G_n is unbounded/not defined. (Run 'tf(Gn)' in the MATLAB to obtain the transfer function representation of G_n.)

A1/ Ex2 *Uncertain actuator dynamics.* The actuator dynamics $G_a(s)$ is given by

$$G_a(s) = \begin{bmatrix} G_T(s) & 0 \\ 0 & G_e(s) \end{bmatrix} \tag{1}$$

where

$$G_T(s) = \frac{g_T}{\frac{s^2}{t_1 t_2} + (\frac{1}{t_1} + \frac{1}{t_2})s + 1}, \quad G_e(s) = \frac{g_e}{\frac{s}{t_3} + 1}. \tag{2}$$

We consider the (scalar and real-valued) parameters g_T, g_e and t_1, t_2, t_3 to be *uncertain* with nominal values and uncertainty provided in Table 2.

It is your task to create an (parametric) uncertain state-space ('uss'-object in MATLAB) representation of G_T, G_e, and G_a. (To do this, declare the uncertainty parameters with 'ureal' and define the appropriate transfer functions with 'tf' as you would normally do.)

Take a hundred samples with 'usample' of the uncertain representation of $G_T(s)$ and $G_e(s)$ each, and plot their step responses and singular values. (A plotting routine is created for you in the given m-file.) Comment your observations in the report.

Parameter	Nominal value	Uncertainty
t_1	2.5	±30%
t_2	10.0	±40%
t_3	25	±25%
g_T	1.0	±50%
g_e	1.0	±10%

Table 2: Uncertain parameters in the actuator dynamics

A1/ Ex3 *Baseline LQG controller with integral action.* An LQG controller with integral action has been designed for the open-loop system

$$\tilde{y}(s) = G_n(s)G_a(s)u(s) + G_n(s)W_d(s)d(s) + W_n(s)n(s). \tag{3}$$

The controller is designed such that it tracks the angle of attack α a reference signal r.

Your task is to:

- Read the documentation for the MATLAB commands 'connect' and 'lsim'. The latter is used to simulate LTI systems and the former is used to connect multiple LTI systems and define inputs/outputs.
- Run the simulation in the code section 'A1/Ex3 - (1)'. Here you can see the performance of the LQG controller. Feel free to change the variables 'flag_x0' and 'flag_noise' to 1 or 0.
- Repeat the simulation you studied in the code section 'A1/Ex3 -(1)', but replace the actuator dynamics G_a with a random sample of the uncertain state space representation of G_a. A skeleton for the code you need to write is provided in the code section 'A1/Ex3 - (2)'. Comment on your observations in the report.

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Assignment 2

A2/ Ex1 *Weight selection, uncertainty.*

To compute the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ controllers we need to define the 'P' matrix in Figure 2-3 as a state space object in MATLAB. The components of each weighting matrix are either zero or of the form $k\frac{s+z}{s+p}$, unless specified otherwise. It is your task to define the 'P' matrix with the following specification of the weighting matrices (make sure that the dimension of each weighting matrix makes sense according to the block diagram!):

- $W_{r\alpha}(s)$: The reference dynamics for the angle of attack is modeled as a critically damped second-order system with natural frequency 6.25 Hz, $\frac{6.25^2}{s^2+2\cdot 6.25\cdot s+6.25^2}$. (Note output dimension of weighting).
- $W_e(s)$: Tracking is only considered for the angle of attack. The tracking error weight has DC gain 400, crossover frequency 4.3 rad/sec and high frequency gain 0.4.
- $W_p(s)$: The performance weight put emphasis on angle of attack α and the normal acceleration α_n. The element corresponding to α has DC gain 2.5, crossover frequency 0.45 rad/sec, and high-frequency gain 0.015 and the element corresponding to α_n has DC gain 2.5, crossover frequency 0.7 rad/sec, and high-frequency gain 0.0063.
- W_u: Large elevator deflections δ_{el} are penalized by the inverse of the maximally allowed control input, 35 deg.
- W_n(s): All components of y are exposed to noises with magnitudes 0.001 rad, 0.001 rad/sec, and 0.001 m/s^2, for α, q, and a_n, respectively. These are modeled as static weights. (already defined in the given m-file.)

- W_d(s): The disturbance weight defines Dryden gust, a wind turbulence model, which acts as a disturbance on the elevator deflection (not on thrust). The Dryden gust model is given as $\frac{0.9751s+0.2491}{s^2+0.885s+0.1958}$. (already defined in the given m-file.)
- W_m(s): In A1/Ex3, you generated 100 parametric uncertainty samples and commented on the effect of them. $W_m(s)$ is found such that it covers those 100 samples but with an unstructured uncertainty form. Therefore, we convert a parametric uncertainty region into a non-parametric structure. We pick a second-order output multiplicative uncertainty structure and your task is to find W_m (hint: *ucover*). When converting the uncertainty from parametric regions to output multiplicative form, perform it independently for the throttle and the elevator channels of G_a, so that the resulting output W_m becomes block-diagonal as

$$W_m(s) = \begin{bmatrix} W_{mT}(s) & 0 \\ 0 & W_{me}(s) \end{bmatrix},$$

where $W_{mT}(s)$ and $W_{me}(s)$ are obtained channel-wise as second order cover approximations.

Define the 'P' matrix using the command 'connect'.

A2/ Ex2 $\mathcal{H}_\infty$ *controller design.* Compute the $\mathcal{H}_\infty$ controller with 'hinfsyn'. Provide a singular value plot of the controller.

A2/ Ex3 $\mathcal{H}_2$ *controller design.* Compute the $\mathcal{H}_2$ controller with 'h2syn' and provide a singular value plot of the controller.

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Assignment 3

A3/ Ex1 *Closed loop analysis.* For the $\mathcal{H}_\infty$ controller, check NS and find the transfer functions used to check the conditions NP, RS, and RP, and plot their singular value diagrams. Are these conditions satisfied for the frequencies $\omega \in [0.01, 1000]$?

A3/ Ex2 *Closed loop simulation.* Close the loop between the $\mathcal{H}_\infty$, $\mathcal{H}_2$, and LQG controllers and the open loop system (3) with a random sample of the uncertain actuator dynamics G_a. That makes three individual closed-loop systems.

Simulate the three closed-loop systems for 50 seconds and with

- n and d normally distributed, and
- the reference signal r cylcling though the values -0.5, 1, 5, -2, and 0 every 10-th second. A plotting routine is provided for your convenience.

Include the plot(s) in the report.

A3/ Ex3 *Conclusion.* Compare the performance of the three controllers. Based upon the simulation experiments, do you think the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ approach could better handle uncertain actuator dynamics compared with LQG? Do you think this assignment is a fair comparison between the three?

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References

[1] BL Stevens and FL Lewis and N Johnson *Aircraft Control and Simulation*. John Wiley and Sons Ltd, 1989.

[2] JG Balchen, NA Jenssen, E Mathisen, and S Saelid *A dynamic positioning system based on Kalman fitering and optimal control.*. Modeling and Identification and Control vol 1, pp 135-163, 1980.

[3] OM Faltinsen *Sea Loads on Ships and Offshore Structures*. John Wiley and Sons Ltd, 1994.

[4] E A Johannessen *Dynamic Positioning of Surface Vessel*. 1996.

[5] S. Skogestad, I. Postletwaite *Multivariable feedback control: analysis and design*. Wiley, New York, 2nd edition, 2001