Robust and nonlinear control EEN050

Assignment 6

Feedback linearization

Before submission pre-approval of solution is mandatory

Problem formulation and aim

Consider the problem of nonlinear stabilization and tracking of SISO SEIR model. Synthesise and numerically solve these type of problems (with Matlab or alike).

Administration

- The assignment is solved in a group of three students, documented in an electronic solution report (plots, explanation, reply to questions).
- Pre-approval of solution by TAs is mandatory before submission (during tutorial sessions)
- Electronic submission to CANVAS, upload 1 solution per group using the filename: Group#-Assignment#.pdf after getting a preliminary oral approval from the TA (tutorial session).
- Results, no later than 1 week after submission (with constructive feedback). If the solution is not approved, one week time is given for correction (one extra chance only).
- Deadline: check the course PM.

Questions

Given a control oriented modified SEIR (Susceptible-Exposed-Infected-Recovered) epidemiological compartmental model [8,7] as

$$\begin{aligned}
\dot{x}_1(t) &= -u(t)x_1(t)x_3(t) + c_1x_4(t) \\
\dot{x}_2(t) &= u(t)x_1(t)x_3(t) - c_2x_2(t) \\
\dot{x}_3(t) &= -c_3x_3(t) + c_2x_2(t) \\
\dot{x}_4(t) &= c_3x_3(t) - c_1x_4(t)
\end{aligned}$$

where $x \in \mathbb{R}^4$ x_1, x_2, x_3 and x_4 are the susceptible, exposed, infected and recovered fraction of the population. Parameters are given as $c_1 = 1/60$,$c_2 = 0.2$,$c_3 = 0.1$ (coefficients with unit $1/day$). The traditional SEIR model is expanded; recovered people can be re-infected. In this assignment we will

- Analyze the SEIR model:
  - Find the relative degree
- Characterize the zero dynamics, is the SEIR model minimum phase with $y = x_3 - \bar{y}$ ($\bar{y}$ standing for a reference signal)?
- Design input to output linearizing controller
- Simulations
  - Simulate the control system with various reference signals
  - Discover qualitative properties of the SEIR model

Question 1 What is the relative degree r? Define the domain $D_0 \subset \mathbb{R}^4$ for which the relative degree is valid.

Question 2 Presume we successfully drive the output and all the derivatives to zero. What can we expect to happen to the rest of the states?

Question 3 Design a feedback law $u = \beta(x)v + \alpha(x)$ that will drive the output to zero

Question 4 With the software of your choosing: Do simulations with the designed controller with initial state

$$x_0 = \begin{bmatrix} 60 & 15 & 20 & 5 \end{bmatrix}^T \tag{1}$$

and with the following references

$$\bar{y}_1 = 10, \quad \bar{y}_2 = 50, \quad \bar{y}_3 = 110 \tag{2}$$

Do you notice anything peculiar? (no correct answer).

Before submission pre-approval of solution is mandatory

References

[1] S. Skogestad, I. Postletwaite *Multivariable feedback control: analysis and design*. Wiley, New York, 2nd edition, 2001

[2] HK Khalil *Nonlinear Systems*. Prentice Hall, Upper Saddle River, NJ, 1996

[3] Jeff Moehlis `https://sites.me.ucsb.edu/~moehlis/APC514/tutorials/tutorial_seasonal/node4.html`.

[4] G Horvath, G Szederkenyi *Identifiability analysis and parameter estimation of a SEIR epidemic model*. In Proc of PCIT, 2020 `https://pcit2020.mik.uni-pannon.hu/images/program/kiadvany.pdf`.