Introduction to Quantum Computing (IN2381) – WS020/2021 **Sheet 2**

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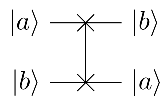
Tutorial 2 (The no-cloning theorem[1])
The *no-cloning theorem* states that, surprisingly, one cannot make a copy of an unknown quantum state. In more detail, we consider a system of two qubits (source and target): the first qubit is the source state $|\psi\rangle$, and the second qubit starts out in some standard state $|s\rangle$, for example $|s\rangle = |0\rangle$. Thus the initial state is $|\psi\rangle \otimes |s\rangle \equiv |\psi\rangle|s\rangle$. One would like to copy $|\psi\rangle$ into $|s\rangle$, that is, find some unitary transformation $U \in \mathbb{C}^{4\times 4}$ such that

$$|\psi\rangle \otimes |s\rangle \mapsto U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle \otimes |\psi\rangle.$$

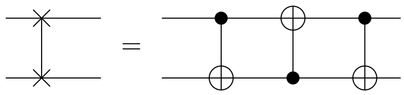
Show that such a copying procedure is impossible: the equation cannot hold for arbitrary source qubits $|\psi\rangle$.

Exercise 2.1 (Basic quantum circuits)

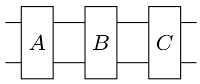
(a) Find the matrix representation (with respect to the computational basis states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$) of the *swap-gate* $|a, b\rangle \mapsto |b, a\rangle$, which is written in circuit form as



Also show that the swap operation is equivalent to the following sequence of three CNOT gates:

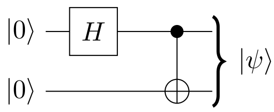


Hint: You can either work directly with basis states, e.g. $|a, b\rangle \overset{\text{CNOT}}{\mapsto} |a, a \oplus b\rangle$, or use matrix representations. In the latter case, note that a sequence of gates like



(with A, B, C unitary 4×4 matrices) corresponds to the matrix product CBA since the circuit is read from left to right, but the input vector in the matrix representation is multiplied from the right.

(b) Compute the output $|\psi\rangle$ of the following "entanglement circuit" applied to the input $|00\rangle$:



with $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ denoting the Hadamard gate.

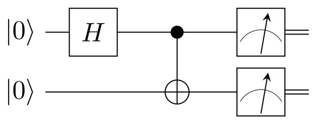
(c) Build the CNOT gate from the controlled-Z gate and two Hadamard gates, and verify your construction.

[1] M. A. Nielsen, I. L. Chuang: *Quantum Computation and Quantum Information*. Cambridge University Press (2010), page 532

Exercise 2.2 (IBM Q and Qiskit)
IBM Q Experience (`https://quantum-computing.ibm.com`) is a quantum cloud service and software platform, which allows users to run experiments even on real quantum computing hardware. For this exercise you should create a personal account and familiarize yourself with the service.

IBM Q offers a graphical *Circuit Composer* and *Qiskit Notebooks* as interface, which are Jupyter Python notebooks using the Qiskit open-source framework (`https://qiskit.org`). Both can be conveniently accessed online via a web browser; alternatively, you can also install Qiskit locally via `pip install qiskit`. An introduction to Qiskit is available at `https://qiskit.org/documentation/getting_started.html`.

(a) Use the Circuit Composer to construct the quantum circuit from exercise 3.1(b) together with measurement operations:



Note that the Circuit Composer stores the measurement results in a classical register. You can view the corresponding `OPENQASM` code in the Circuit editor. Now run your circuit using 1024 "shots" (repetitions) to collect a statistical distribution of measurements, and compare these with the state $|\psi\rangle$ computed in exercise 3.1(b).

Please hand in a picture of your circuit and a histogram of the measurement results, which you can obtain via the online interface.

(b) Construct the circuit again using Qiskit, and execute the circuit via Aer's qasm_simulator (1024 shots as above).

Please hand in your code together with the measurement counts.