- This is an 80 minute exam. There are 6 problems, worth a total of 57 points. - You may use three pages of personal notes and a standard scientific calculator. (You may not share these or any other items with anyone else.) - Write all ansuers in the spaces provided. If you require more space to write your answer, you may use the back side of the page. You are not allowed to communicate with anyone except the instructor or proctors before you submit this exam. Useful Abbreviations: CI = confidence interval PI = prediction interval

se = standard error
E = expected value

var = variance (or variance-covariance) cov = covariance SLR = simple linear regression BLUE = best linear unbiased estimate/estimator GLS = generalized least WLS = weighted least squares

MSE = mean square error
RSS = residual sum of squares

VIF = variance inflation factor VST = variance stabilizing transformation H_0 = the null hypothesis of a test H_a = hypothesis of a test Some selected formulas:

$$C_p = \frac{RSS_p}{\hat{\sigma}^2} + 2p' - n$$

1. In the process of using R to select a regression model of Y on X1 and X2 based on $\underline{n=16}$. the following results are obtained for the current model (having X only): observations, current.model ¡- 1 m(Y $\sim$ X1) $>$ add1 (current.model, $\sim$ X1 + x2, test="F") Single term additions $\therefore$ drop1(current.model, test="F") Single term deletions (a) If this is forward selection based on an F -statistic threshold of $F_{\text{in}} = 3$, what would be the next step? Why? X_2, because its "add1" F-statistic

$$is \approx 204 > 3 = F_{\text{in}}$$

(b) If this is backward elimination based on an F -statistic threshold of $F_{\text{out}} = 3$, what would be the next step? Why? [2pts] Drop X_1, because its "drop $1''$ F-statistic is $\approx 1.3 < 3 =$ Fout (c) Compute Mallows' C_p for the full model Y x1 + x2 and the reduced model Y $\sim$ X1. Of these two models, which is better according to C_p? [5pts] Full model: $\frac{5.75}{5.75/(16-3)} + 2 \cdot 3 - 16 = 3$

$$\begin{array}{c} \text{Model} \\ Y \sim x1 \end{array} \frac{96}{5.75/(16-3)} + 2 \cdot 2 - 16 \approx 205$$

Cp prefers the full model $Y \sim x1 + X2$

2. For each part below, CIRCLE the ONE BEST answer. [1 pt each] (a) A variance stabilizing transformation is intended to remedy the problem of heteroscedasticity curvature in the mean non-normality none of these (b) According to the principle of hierarchy, any sub-model of the two-factor ANOVA model $y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}$ that contains the term α_i <u>must</u> also contain the term β_i $\alpha\beta_{ij}$ both of these neither of these (c) In a randomized complete block design (with no missing values), the number of experimental units is evenly divisible by the number of treatments blocks (both of these neither of these (d) If CI_1 and CI_2 are (random) Tukey 95% simultaneous confidence intervals for mean differences δ_1 and δ_2, respectively, then the probability that $\delta_1 \in \mathrm{CI}_1$ is 0.95 ≥ 0.95 ≤ 0.95 0.95^2 (e) In a balanced completely randomized design with 3 treatments, the probability that a particular experimental unit ends up in the first treatment group is 1/9 is 1/2 is 1/3 cannot be determined (f) Which of these cannot be a problem for a one-way ANOVA model ($y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$) ? non-normality heteroscedasticity curvature in the mean none of these

3. Briefly answer the following: (a) List one advantage and one disadvantage of using a randomization test. [2pts] Advantage: no need for the usual assumptions

Disadvantage: requires special software or programming (b) Consider homogeneity-of-regressions model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$, where X_1 is quantitative and X_2 is a dummy (0/1) variable. Draw a Y -versus- X_1 least-squares fitted regression plot for the case $\hat{\beta}_0 = 1, \hat{\beta}_1 = -1, \hat{\beta}_2 = 0, \hat{\beta}_3 = 2$. Label which line corresponds to $X_2 = 0$ and which to $X_2 = 1$. [3pts]

4. An experiment is designed to compare the mean quarter-mile times of three racehorses (coded A, B, and C). Three jockeys are recruited to ride the horses. Three separate quarter-mile races take place. In each race, all three horses run (once), and each of the three jockeys rides one horse. The assignments of the horses are randomized with the restriction that each horse has a different jockey in each race. (a) Name the type of design for this experiment. Then draw a diagram that shows one possible outcome of the randomization. race [4pts]

1	2	3

(b) What are the treatments in this experiment? How many? [2pts] the racehorses - three (c) What are the experimental units in this experiment? How many? jockey-race combinatios - 9 (d) What are the blocks in this experiment (if any)? How many (in total)? [2 pts] jockeys races(3) = 6 (e) Write out a full model equation for the usual analysis, and identify each term. (Start with $y_{ijk} = \cdots$ where y_{ijk} is the finish time of horse i ridden by jockey j in race k..) (f) Analysis of the finish times (seconds) using an appropriate model yields this ANOVA: What conclusion should the experimenters make? (There should be only one!) Why? [2pts] No evidence of differences in mean quarter-mile time among the horses, since the p-value = 0.136364 > 0.05.

5. Consider these two models for data with two (crossed) factors:

$$
\begin{aligned}
y_{ijk} &= \mu_{ij} + \varepsilon_{ijk} \\
y_{ijk} &= \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}
\end{aligned}
$$

where:

$$
i = 1, 2 \quad j = 1, 2 \quad k = 1, \ldots, K
$$

(a) In model (2), which parameters are main effects, and which are interaction effects? main: $\alpha_1, \alpha_2, \beta_1, \beta_2$

[2pts]

interaction: $\alpha\beta_{11}, \alpha\beta_{12}, \alpha\beta_{21}, \alpha\beta_{22}$ (b) How many levels do the first factor and the second factor have? both have turo (c) What is the apparent (not effective) total number of mean-related parameters for model (1)? For model (2)? [2pts] model(1) : $2 \times 2 = 4$ model (2): $1 + 2 + 2 + 4 = 9$ (d) What is the error degrees of freedom (that is, $n - p'$) for model (1)? For model (2)? (Your answers should be in terms of K.) [2pts] model (1): $4k - 4 = 4(K - 1)$ model(2): Same (e) Suppose least squares estimates for model (1) are $\hat{\mu}_{11} = 5, \hat{\mu}_{12} = 7, \hat{\mu}_{21} = 3$, and $\hat{\mu}_{22} = 3$. Draw an interaction plot with the first factor (corresponding to i) on the horizontal axis. [4pts]

6. For each data set plotted above (A, B, C, and D), we seek transformations of X and/or Y so that a linear model satisfying the usual assumptions is appropriate. Briefly answer: (a) Which (if any) do not need any transformation o C only, because it has both a straight line trend and a homogeneous, roughly normal, vertical spread. (b) For which (if any) might using a polynomial model (in X) be enough by itself? Why? A (and perhaps C) because it [2 pts has only the problem of curvature (and not unequal spread or non-nommality) (c) For which (if any) might a single monotone transformation of X be enough by itself? None (except perhaps C) because B and D also have heteroscedasticity problems, and A (and D) have a non-monotone trend. (d) Which one (if any) would certainly require transformations of both X and Y ? [1pt] D (e) If the Box-Cox method (based on a SLR) were used for data set C, what would you expect the estimated power $\hat{\lambda}$ to be, approximately? Why? [2 pts] $\hat{\lambda} \approx 1$ because that corresponds to the identity transformation (ie no transformation at all)

STAT 425− Section D1U, D1G - Spring 2014 Midterm Exam II April 23, 2014 Full Name: Key - This is a 50 minute exam. There are 4 problems for everyone, and 2 additional problems for graduate students only. students. - You may use three pages of personal notes and a standard scientific calculator. (You may not share these items with anyone else.) - Write all answers in the spaces provided. If you require more space to write your answer, you may use the back side of the page.

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var = variance (or variance-covariance) cov = covariance SLR = simple linear regression BLUE = best linear unbiased estimate/estimator GLS = generalized least WLS = weighted least squares MSE = mean square error TSS = total sum of squares RSS = residual sum of squares df = degrees of freedom VIF = variance inflation factor VST = variance stabilizing transformation H_0 = the null hypothesis of a test H_a = the alternative hypothesis of a test.

1. For each part below, CIRCLE the ONE BEST answer. [1 pt each] (a) How many blocking factors are in a Latin Square design? 0 3 (b) A Box-Cox transformation requires the dependent variable to be positive negative nonzero none of these (c) Initially, all variables are in the model in

forward selection
backward elimination
stepwise selection
none of these

(d) The C_p statistic may be used to – a model. (select test H_0 for estimate design (e) In a randomized complete block design, the number of blocks must be – the number of treatments.

greater than less than equal to
none of these

(f) In a homogeneity-of-regressions ("ANCOVA") model, a significant interaction between the X variable and the "dummy" variable(s) would indicate that the regression lines are

parallel not parallel perpendicular
none of these

(g) A VST involves transformation of the independent variable(s) dependent variable both neither (h) Both adding and dropping variables may be performed in forward selection backward elimination selection none of these 2. You intend to compare two brands of eyedrops (liquid solutions applied to the eyes to treat redness) using a randomized experiment with 20 human subjects. Think of a designed experiment that uses blocking. Answer the following: (a) What are the treatments? the two brands of eyedrops (b) What are the blocks? the human subjects pt] (c) What are the experimental units? eyes (d) Briefly describe the structure of the randomization. [2pts] For each human subject, independently, one eye is randomly chosen to receive brand 1, and the other eye receives brand 2 .

3. A linear regression of a variable Y on variables X_1 and X_2, including some polynomial terms, is performed and summarized in R as follows:

$$> \text{summary}\left(\text{lm}\left(y^{-} x1 + x2 + I\left(x2^{\wedge}2\right) + x1 * x2\right)\right)$$

Coefficients:

	Estimate	Std. Error	t value	Pr(> |t|)
(Intercept)	9.8981	0.2879	34.383	$< 2\mathrm{e}-16 * **$
x1	0.8269	0.3178	2.602	0.0143*
x2	0.1883	0.3219	0.585	0.5631
I (x2-2)	0.7890	0.6353	1.242	0.2239
x1: x2	2.2568	0.5666	3.983	$0.0004 * **$

(a) Write an expression for the full model equation. (Do not substitute any estimates!)

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 X_2 + \beta_{22} x_2^2 + \beta_{12} X_1 X_2 + \varepsilon^{[3pts]}$$

(b) Test for interaction between X_1 and X_2. (Give the test statistic, p -value, and conclusion.) [3pts]

$$t = 3.983$$
$$p = 0.0004$$

Reject the null hypothesis of no interaction. (c) What is the estimated expected change in Y if X_1 increases by one unit? (Hint: It depends on X_2.)

[2pts]

$$\hat{\beta}_1 + \hat{\beta}_{12} x_2 \approx 0.8269 + 2.2568 X_2$$

(d) What polynomial term appears to be "missing" from this model? [1pt] the term for X_1^2 (e) Suppose you are going to perform backward elimination on this model, based on a p -value threshold of $p = 0.05$. Which term (if any) would be removed first? Why? (Hint: Remember hierarchy!) [2pts] By hievarchy, only X_2^2 and $X_1 X_2$ are eligible for removal, at the first stage. Since X_2^2 has the larger p-value (0.2239), and it exceeds 0.05, remove X_2^2 first.

4. An experiment is conducted with a completely randomized design and two treatment factors, A and B, each with two levels. The design is balanced, with 5 replications. An ANOVA table from R is > anova(lm(response A $*$ B)) Analysis of Variance Table Response: Response: response

	Df	Sum Sq	Mean Sq	F value	Pr(> F)
A	1	11.25	11.25	1.125	0.304594
B	1	101.25	101.25	10.125	0.005793 $**$
A : B	1	1.25	1.25	0.125	0.728289
Residuals	16	160.00	10.00		

(a) How many observations are there? 20 (b) How many treatments are there? (c) How many blocks are there? none (d) Summarize the conclusions you would draw concerning the interactions and main effects. [3pts] Interaction: No evidence for it ($p > 0.05$). Factor A: No evidence for it ($p > 0.05$). Factor B: There is evidence of effects differing ($p < 0.05$) (e) Let α_1 and α_2 be the main effects of Factor A. Perform a test of $H_0 : \alpha_1 = \alpha_2$ versus $H_a : \alpha_1 \neq \alpha_2$. (Give test statistic, p -value, and conclusion.) [2pts]

$$F = 1.125 \quad p = 0.305$$

No evidonce against $H_0 : \alpha_1 = \alpha_2$. (f) Would you need the Tukey multiple comparisons method for comparisons between the levels of Factor A? Explain briefly. [2pts] No. Factor A has only two levels, so there would be only one comparison.

GRADUATE STUDENTS ONLY 5. In the previous problem, the estimated mean responses are

$$\hat{\mu}_{11} = 63 \quad \hat{\mu}_{12} = 59 \quad \hat{\mu}_{21} = 65 \quad \hat{\mu}_{22} = 60$$

where $\hat{\mu}_{ij} = \bar{y}_{ij}$ is the average response for the observations at level i of Factor $A(i = 1, 2)$, and level j of Factor B$(j = 1, 2)$.

Draw an interaction plot for this situation, with Factor A on the horizontal axis. Make sure it is fully labeled!

[4pts]

6. A scientific model proposes the following relationship between variables X and Y :

$$Y = \gamma X^{\alpha} e^{-\beta X} \epsilon$$

where γ, α, and β are unknown constants ($\gamma > 0$) and $\epsilon > 0$ is a multiplicative error whose distribution is the same for all X.

Transform this to a linear regression model: Write the transformed model equation. What are the (transformed) dependent and independent variables? What are the regression parameters? [4pts]

1. A study examined the impact of two methods for teaching sight-singing in 40 thh grade students. Students were evenly randomly assigned to control (Treatment = 0) and experimental (Treatment = 1) groups. Sight-singing test scores were collected before (Pretest) and after the experimental intervention (Posttest). Fitting a linear model with the Posttest score as response yields the following:

	Estimate	Std. Error	t value	$\Pr(>\lvert t\rvert)$
(Intercept)	4.1780	3.448	1.212	0.233510
Pretest	1.0061	0.2457	4.094	0.000229 $***$
Treatment	15.0953	4.7309	3.191	0.002939 $**$
Pretest:Treatment	-0.6383	0.3048	-2.094	0.043349$*$

(a) Write out the linear model equation ($Y = \cdots$) that was apparently used. Let Y be the Posttest score, X_1 the Pretest score, and X_2 the treatment indicator.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + e$$

(b) Is there evidence of an interaction effect? Support your answer using the output above, Yes, the Pretest: Treatment tem [2 pts has $p = 0.043349 < 0.05$ indicating mild evidence for interaction. (c) Compute the estimated intercepts of the Posttest-versus-Pretest relationship: one for the control group and one for the experimental group. [2pts] control: $\hat{\beta}_0 = 4.1780$ experimenta $\hat{\beta}_0 + \hat{\beta}_2 = 4.1780 + 15.0953 = 19.2733$ (d) Compute the estimated slopes of the Posttest-versus-Pretest relationship: one for the control group and one for the experimental group. [2pts] control: $\hat{\beta}_1 = 1.0061$ experimental: $\hat{\beta}_1 + \hat{\beta}_{12} = 1.0061 + (-0.6383) = 0.3678$ (e) What is the predicted Posttest score for a student in the experimental group who had Pretest score of 10? [2pts]

$$\begin{aligned}\hat{Y} &= 4.1780 + 1.0061 \times 10 + 15.0953 + (-0.6383) \times 10 \\ &= 22.9513\end{aligned}$$

2. For each part below, CIRCLE the ONE BEST answer. [1pt each] (a) Which design necessarily has overlapping blocks? Latin square randomized complete block split-plot none of these (b) In a single 4×4 Latin square design, the number of treatments is (4) 8 16 none of these (c) A randomized paired comparison experiment (like the shoes example from lecture) is a special case of which design? split-plot randomized complete block Latin square none of these (d) Methods for estimating the variance components in a random effects model include maximum likelihood) least squares both neither (e) In an interaction plot, the horizontal axis represents different factor levels treatment means treatments blocks (f) In a split-plot design, the number of levels of the whole-plot factor must be = the number of levels of the split-plot factor. less than more than the same as none of these (g) In a one-factor random effects model, the random effects generally sum to zero (have expected value zero both neither (h) Selection of a subset of independent variables may be based on stepwise algorithms theoretical considerations variable selection criteria (any of these 3. Briefly answer the following: (a) In the context of variable selection for a linear model with many observations, which criterion would tend to choose a model with fewer variables: AIC or BIC? Explain. [2pts BIC, because it has the same penalty for lack of fit asAIC (n $\ln(RSS/n)$) but has larger penalty (coefficient $\ln(n)$ versus 2 for AIC) for number of variables. (b) Briefly describe how variable selection tends to affect the bias and the variance of the coefficient estimators for the retained variables. Generally, variable selectwn increases the absolute bias of regression coefficient estimatars, but decreases their variance.

4. An experiment involving nine cyclists was conducted to study the effect of caffeine on cycling endurance. On each of four days, each cyclist completed an endur ance test. Each day, a cyclist would receive a different dose of caffeine $(0, 5, 9,$ or 13mg) before the test. The order in which the doses were assigned was randomly determined, independently between cyclists. The response was endurance time (minutes) until exhaustion.

Df Sum Sq Mean Sq	value	$\Pr(> F)$
factor (Cyclist)	3.2159	$4.174\mathrm{e}-07 ***$
85558.0		

Residuals (a) Name the design of this experiment. What role do cyclists play in this design? Randamized complete block design with cyclists as blocks (b) In this particular experiment, what advantage might this design offer over complete randomization? If cyclists have inherently different endurance levels, using them as blocks helps to cancel out those effects, making treatment comparisons more precise. (c) What are the experimental units? How many are there? The individual endurance tests Cor cyclist/day combinations), of vhich there are $9 \times 4 = 36$. (d) From the ANOVA, make a conclusion about the treatments. (State a p-value.) [2pt s $p = 0.003591 < 0.05$ so there is evidence for different levels of endurance due to different caffeine levels (e) Based on the following 95% Tukey intervals, draw a general conclusion about the evidence for how different caffeine doses affect endurance. diff

	diff	1wr	upr	padj
			on 8030	
		20.665303	0.015329185	
$5-0$	11.2366667	1.808030	20.665303	0.0.07661564
3-0	11.7088889	2.280252	21.137526	0.011092908
$9-5$	1.0044444	-8.424192	10.433081	0.990936901
$13-5$	0.4722222	-8.956414	9.900859	0.999031318

-9.960859 0.998616188 Every positive caffeine dose $(5, 9, 13\text{mg})$ leads to greater endurance than no caffeine (Omg), but there is insufficient evidence of any This study The positive doses.

5. Consider the following means model (e.g. for data from a CRD:

$$Y_{ij} = \mu_i + e_{ij} \quad i = 1, 2, 3 \quad j = 1, \ldots, 5$$

Suppose the least squares estimates are $\hat{\mu}_1 = 5, \quad \hat{\mu}_2 = 3, \quad \hat{\mu}_3 = 4$. (a) Write out the row of the matrix X (in the matrix-vector formulation) that corresponds to the observation $Y_3, 2$. Also, to which parameter does each column correspond? [2 ptss

$$\left(\begin{array}{ccc} 0 & 0 & 1 \\ \mu_1 & \mu_2 & \mu_3 \end{array} \right.$$

(b) Write the usual equation ($Y_{ij} = \cdots$) for the corresponding (treatment) efffects model. pts

$$Y_{ij} = \mu + \tau_i + e_{ij}$$

(c) Compute the least squares estimates of all mean-related parameters in the (treatment) effects model, under the (unweighted) sum-to-zero restriction. [3pts]

$$\hat{\mu} = \frac{\hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3}{3} = \frac{5 + 3 + 4}{3} = 4$$

$$\hat{\tau}_1 = \hat{\mu}_1 - \hat{\mu} = 5 - 4 = 1 \quad \overset{a}{*}= 3 - 4 = -1 \quad \hat{\tau}_3 = 4 - 4 = 0$$

(d) Give least squares estimates of all pairwise (mean) differences.

$$\begin{aligned} \hat{\mu}_1 - \hat{\mu}_2 &= 5 - 3 = 2 \\ \hat{\mu}_1 - \hat{\mu}_3 &= 5 - 4 = 1 \\ \hat{\mu}_2 - \hat{\mu}_3 &= 3 - 4 = -1 \end{aligned}$$

(e) For this situation, consider instead a random-effects model with a single random factor. Write out an appropriate model equation ($Y_{ij} = \cdots$), along with <u>all</u> of the usual conditions that the terms satisfy. [4pts]

$$\begin{aligned} Y_{ij} &= \mu + \alpha_i + e_{ij} \\ \alpha_i &\sim N\left(0, \sigma_\alpha^2\right) \\ e_{ij} &\sim N\left(0, \sigma^2\right) \sum \text{ all independent} \end{aligned}$$

6. An experiment is conducted to determine the effects of three laundry detergent brands (A. B, C) and also of whether or not a pre-treatment is applied (yes or no). Twelve identical white T-shirts that have been soiled are completely randomized, such that exactly two are assigned each brand/pre-treatment combination. Each T-shirt is washed separately in its own wash load. The response, photometric brightness of the garments, is measured after washing, with the following ANOVA results: (a) Is the design balanced? How do you know? Yes. Exactly two T-shirts are assigned each treatiment (same number fom all treatments). (b) How many treatments are there? (c) Is there evidence that detergent brand and pre-treatment interact? State the null and alternative hypotheses, p -value, and conclusion. [4pts] Ho: no interaction Ha: factors interact since $p = 0.017388 < 0.05$ there is evidence for interaction. (d) If appropriate, draw conclusions regarding the presence of the main effects. If not, explain why not. [2pts] Not appropnate, since main eftects are meaningless in the presence of interaction. (e) [GRADUATE SECTION ONLY] Use the other side of this page to answer: Describe an alternative design that uses 12 T-shirts, but only needs 6 wash loads. Name the design, and describe the roles of the two factors, and the T-shirts, and the wash loads. Make sure to describe the randomization. (Hint: T-shirts in the same wash load must receive the same brand of detergent, but may be differently pre-treated.) [5pts]

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-9.960859 0.998616188 Every positive caffeine dose $(5, 9, 13\text{mg})$ leads to greater endurance than no caffeine (Omg), but there is insufficient evidence of any This study The positive doses.

5. Consider the following means model (e.g. for data from a CRD:

$$Y_{ij} = \mu_i + e_{ij} \quad i = 1,2,3 \quad j = 1,\ldots,5$$

Suppose the least squares estimates are $\hat{\mu}_1 = 5, \quad \hat{\mu}_2 = 3, \quad \hat{\mu}_3 = 4$. (a) Write out the row of the matrix X (in the matrix-vector formulation) that corresponds to the observation $Y_3, 2$. Also, to which parameter does each column correspond? [2 ptss

$$\left(\begin{array}{ccc} 0 & 0 & 1 \\ \mu_1 & \mu_2 & \mu_3 \end{array} \right.$$

(b) Write the usual equation ($Y_{ij} = \cdots$) for the corresponding (treatment) efffects model. pts

$$Y_{ij} = \mu + \tau_i + e_{ij}$$

(c) Compute the least squares estimates of all mean-related parameters in the (treatment) effects model, under the (unweighted) sum-to-zero restriction. [3pts]

$$\hat{\mu} = \frac{\hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3}{3} = \frac{5+3+4}{3} = 4$$
$$\hat{\tau}_1 = \hat{\mu}_1 - \hat{\mu} = 5 - 4 = 1 \quad \overset{a}{*}= 3 - 4 = -1 \quad \hat{\tau}_3 = 4 - 4 = 0$$

(d) Give least squares estimates of all pairwise (mean) differences.

$$\hat{\mu}_1 - \hat{\mu}_2 = 5 - 3 = 2$$
$$\hat{\mu}_1 - \hat{\mu}_3 = 5 - 4 = 1$$
$$\hat{\mu}_2 - \hat{\mu}_3 = 3 - 4 = -1$$

(e) For this situation, consider instead a random-effects model with a single random factor. Write out an appropriate model equation ($Y_{ij} = \cdots$), along with <u>all</u> of the usual conditions that the terms satisfy. [4pts]

$$\begin{aligned} Y_{ij} &= \mu + \alpha_i + e_{ij} \\ \alpha_i &\sim N\left(0, \sigma_\alpha^2\right) \\ e_{ij} &\sim N\left(0, \sigma^2\right) \sum \text{ all independent} \end{aligned}$$

6. An experiment is conducted to determine the effects of three laundry detergent brands (A. B, C) and also of whether or not a pre-treatment is applied (yes or no). Twelve identical white T-shirts that have been soiled are completely randomized, such that exactly two are assigned each brand/pre-treatment combination. Each T-shirt is washed separately in its own wash load. The response, photometric brightness of the garments, is measured after washing, with the following ANOVA results: (a) Is the design balanced? How do you know? Yes. Exactly two T-shirts are assigned each treatiment (same number fom all treatments). (b) How many treatments are there? (c) Is there evidence that detergent brand and pre-treatment interact? State the null and alternative hypotheses, p -value, and conclusion. [4pts] Ho: no interaction Ha: factors interact since $p = 0.017388 < 0.05$ there is evidence for interaction. (d) If appropriate, draw conclusions regarding the presence of the main effects. If not, explain why not. [2pts] Not appropnate, since main eftects are meaningless in the presence of interaction. (e) [GRADUATE SECTION ONLY] Use the other side of this page to answer: Describe an alternative design that uses 12 T-shirts, but only needs 6 wash loads. Name the design, and describe the roles of the two factors, and the T-shirts, and the wash loads. Make sure to describe the randomization. (Hint: T-shirts in the same wash load must receive the same brand of detergent, but may be differently pre-treated.) [5pts]

Practice Questions - Stat 425 , Spring 2017 1. Consider the linear model $Y = X\beta + \epsilon$ where $E(\epsilon) = 0$ and $\operatorname{Var}(\epsilon) = \sigma^2\Sigma$, for known and invertible Σ (a) Find $\operatorname{Var}(\hat{\beta})$ for the ordinary least squares estimator $\hat{\beta} = \left(X^T X\right)^{-1} X^T Y$. Is $\hat{\beta}$ unbiased

$$
\begin{aligned}
E(\tilde{\beta}) = E\left[\left(x'x\right)^{-1} x'y\right] &= \left(x'x\right)^{-1} x'\beta = \beta \cdot \text{ Unbiased.} \\
\operatorname{Var}(\hat{\beta}) = \operatorname{Var}\left[\left(x'x\right)^{-1} x'y\right] &= \left(x'x\right)^{-1} x^\top \operatorname{vav}(y) \cdot x\left(x'x\right)^{-1} \\
&= \sigma^2 \cdot \left(x'x\right)^{-1} x^\top \xi_1 x\left(x'x\right)^{-1}
\end{aligned}
$$

(b) If $\Sigma = CC^T$ for a known invertible matrix C, what is the covariance matrix of $\epsilon' = C^{-1}\epsilon$? (c) If $Y^* = C^{-1}Y$ and $X^* = C^{-1}X$. Does the linear model $Y^* = X^*\beta + \epsilon^*$ satisfy GaussMarkov conditions? Show that the least squares estimator for this model is same as the generalized least squares estimator $\hat{\beta}_G = \left(X^T\Sigma^{-1}X\right)^{-1} X^T\Sigma^{-1}Y$. YEs, because of (b) and $E\left(\varepsilon^x\right) = 0$.

2. A sample of 654 youths of ages 3 to 19 was collected in East Boston during the middle to late 1970s. Researchers measured the forced expiratory volume (FEV) of each youth as a measure of limg capacity. Using the response $Y = \log(FEV)$, a quadratic model in the variable Age (which has integer values only) was t by least squares with these results:

$$\hat{Y} = -055 + 0.21 \times Age - 0.0058 \times (Age)^2 \quad RSS = 26.036$$

(a) The transformation of FEV was suggested by the Box-Cox procedure. What λ value was apparently chosen? (b) Determine the vector $\hat{\beta}$ of least squares regression coefficients.

$$\hat{\beta} = \begin{pmatrix} -0.55 \\ 0.21 \\ -0.0058 \end{pmatrix}$$

(c) In the matrix-vector form $Y = X\beta + \epsilon$ for this regression, what would be the dimensions of X ? 654×3
(d) Compute the usual (unbiased) estimate of error variance. Show your work.

$$\sigma^2 = \frac{RSS}{n - p'} = \frac{26.036}{654 - 3} = 0.04$$

(e) Predict the FEV of a 10-year-old. (Note: log is the natural logarithm.)

$$\hat{y} = -0.55 + 0.21 \times 10 - 0.0058 \times 100$$
$$\approx 0.97$$

$$FE4 = e^{\hat{4}} = 2.64$$

(f) The simple linear regression of log(FEV) on Age has a residual sum of squares of 29.316. Compute the F -statistic for testing whether the quadratic term is needed. Also, state a conclusion based on the critical value $F_{0.05, m_1, m_2 \approx 3.856}$. Show your work.

$$F = \frac{(29.316 - 26 \cdot 036)(1}{26.036 - 1651} \approx 82 > 3.856$$

THERE IS EVIOENCE TO SUPPOR+ PRESENCE OF QOADRATIC TERM. 3. For each part below, CIRCLE the ONE BEST answer. (a) In a factorial experiment, which factor is used as a block?
(x) Treatment factor (xx)) Factor causing variability
(xxx) Any random factor
(b) Which one below is likely to produce exact zeroes for the linear regression estimates? (x) LASSO (xx) Ridge regression (xxx) Least Squares (c) In the classroom example of shoe experiment with boys, what are the blocks? (x) Right or Left foot (xx) Shoe material type (xxx)) Boys (d) Which of the following criteria provides a smaller model? (x) Mallow's C_P (xx)AIC BIC (e) How many blocking factors are present in a 4×4 Latin square design? (x) 1 (xx)2 (xxx)4
4. Briefly answer the following:

(a) When there are many predictors that are highly correlated, which estimator is preferred between least squares and lasso, and why? Which one has larger bias? Which one has larger variance? LASSO is PREFERRED BECAUSE IT INTRODUCES SHRINKAGE WHEREAS LEAST SQUARES SUFFERS WHEN (8 SPARSITU) MANY PREDICTORS. CASSO HAS IARGER THERE ARE MANY OLS IN GENEAAL HAS LARGER UARIANCE BIAS WHEREAS

$$= BE$$

(b) Describe the methods of best subset selection, backward elimination, and forward selection to perform variable selection. What are some pros and cons of each of these methods? BEST SUBSET SELECTION FINDS THE BEST COMBINATION Of VARIABLES FROM ACLPOSSIBLE COMBINATIONS. BE SEQUENTIALLY FINDS UARBLES To ELIMINATE STARTING FROMTHE

FULL MODEL. LIKEWISE FS STARTS FRM NULL BEST SUBSETS HAS GOOD PROPERTIES B/C IT EXPLORES ALL MOPELS BUT RESTRICTIVE FOR

IMPLEME NTING WHEN H of VARIABLES LARGE. BE & FS ARE MORE APPEALING IN THAT CASE. 5. We are interested in testing the wear of a rubber-covered fabric. There are three types of fabric materials of interest: A, B, and C. The tester used for the experiment has two factors (1) three positions of the tester, (2) threc different times for setting up the tester. We are intercsted in conducting the experiment on $n = 9$ experimental units. In each of the below cases, explain which design you recommend along with an example design. Also, specify a model that can be used to analyze the data produced from each design. (a) It is known that the position of the tester and the different times of testing do not matter for measuring the wear. WE WOULD SELECT A COMPLETELY RANDOM DESIGN SNCE THE FACTORS POSITIONS & TIMES ARE KNOWN BNOT TO HAVE ANY EFFECT. ASA MPLE DESIGN IS MODEL TREATMENT EFFECT MODEL?

$$y_{ij} = M + \tau_i + \varepsilon_{ij} \rightarrow \text{ errovs}$$

K. , ith treatment Obs. for ith trt, ith unit.

(b) It is believed that the wear measurement may vary depending on the position of the tester. ℕ THS CASE, WE USE A RANDUMIZED BLOCK DESIGN. POSITION OF THE TESTER IS BLOCKING FACTOR'
(c) It is believed that the wear measurement may vary based on both the position of the tester and the time of testing.

LATW SQUARE DESIGN AS BOTH MATTER. Eg: design Model: $y_{ijk} = \mu + \alpha_i + \beta_j + \hat{\gamma}_k + \varepsilon_{ij} k$ Positim Time error Trt 2 Blocking Factors Treatment facto

1. In the month of May (which always has 31 days), the number of days with at least one US tornado report is modeled as binomial, with probability π satisfying

$$\operatorname{logit}(\pi)=\alpha+\beta \cdot \text { Year }$$

where Year is the year number (e.g., 2018). For data from 2005 to 2018 (assuming independence between years), the ML estimates (with standard errors) are

$$\hat{\alpha} \approx 146.73(48.90) \quad \hat{\beta} \approx-0.07289(0.02431)$$

(a) Briefly state why these counts were not modeled with a Poisson distribution. $\left[\begin{array}{ll}2 & \text{pts}\end{array}\right]$ The counts have a maximum of 31 , whereas a Poisson random variable has no upper bound. (b) What is the link function? Is it canonical? logit, which is canonical (for a binomial) (c) Perform a Wald test (level 0.05) for whether there is a year effect. Interpret.

$$z \approx \frac{-0.07289}{0.02431} \approx-3.0 \quad|z|>z_{0.025} \approx 1.96$$

There appears to be evidence for a year effect: The number of May days with a report appears to decrease, on average. (d) Estimate the mean of the number of days in May 2019 that will have at least one US [5pts] $\log_1 t\left(\hat{\pi}_{2019}\right) \approx 146.73+(-0.07289) \cdot 2019$

$$\approx-0.435$$

$$\hat{\pi}_{2019} \approx \frac{e^{-0.435}}{1+e^{-0.435}} \approx 0.393$$

So the mean number of May 2019 days is estimated to be $31 \cdot \hat{\pi}_{20100500} \approx 12.2$ iby 100

2. Let response Y be the number of far-right extremism terrorism incidents in a given year and region. Consider loglinear models having the following linear predictors and deviances: Model

$$D(\boldsymbol{y}; \hat{\mu}_1) \approx 46.782$$
$$D(\boldsymbol{y}; \hat{\mu}_2) \approx 49.792$$

Model where X_1 is year number (e.g., 2018), and X_2 indicates region (0 = Western Europe, 1 = North America). The data are from 2008 to 2017 , for a total of 20 observations. (a) Which of these two models, if any, is saturated? Justify your answer. [2 pts Neither, since the saturated model would need 20 parameters (one for each obs.). (b) Assuming Model 1 is correct, perform a likelihood ratio test (level 0.05) for whether the annual multiplicative change in the mean number of incidents depends on the region. Be sure to state H_0 and your conclusion. $\left[\chi_1^2(0.05) \approx (1.96)^2\right]$ [5pts] $H_0 : \beta_{12} = 0$ (annual change does not depend on region)

$$\begin{aligned} D(7; \hat{\mu}_2) - D(7; \hat{\mu}_1) &\approx 49.792 - 46.782 \\ &= 3.01 < 3.84 \approx x_1^2(0.05) \end{aligned}$$

So we fail to reject H_0. There is not sufficient evidence that the annual mult. change depends on region. (c) For a deviance-based goodness of fit test for Model 1, give the values of the chi-squared statistic and the degrees of freedom.

[2pts]

stat: 46.782 $df : 20 - 4 = 16$ (d) The sum of the squared Pearson residuals for Model 1 is about 41.4. Estimate the dispersion parameter (in the quasi-likelihood). What does its value suggest? [2pts]

$$\hat{\phi} \approx \frac{41.4}{16} \approx 2.59$$

which exceeds 1 , suggesting overdispersion

3. A simple binary logistic regression with (success) probability $\pi(x)$ has linear predictor

$$\operatorname{logit}(\pi(x)) = \alpha + \beta x$$

Suppose $\pi(2) = 0.5$ and the odds at $x = 3$ is 0.25 times the odds at $x = 2$. (a) Determine the ratio of the odds at $x = 4$ to the odds at $x = 3$. [2pts

$$\frac{\text{odds } (x=4)}{\text{odds } (x=3)} = e^{\beta} = \frac{\text{odds } (x=3)}{\text{odds}(x=2)} = 0.25$$

(b) Compute the slope of $\pi(x)$ at $x = 2$. 2pts]

$$\begin{aligned}\frac{d}{dx}\pi(2) = \beta\pi(2)(1-\pi(2)) &= \ln(0.25)\cdot 0.5 \cdot (1-0.5) \\ &\approx -0.35\end{aligned}$$

(c) Sketch $\pi(x)$ versus x (with reasonable accuracy). [3pts (d) Determine the median effective level (of x). [1pt]

$$x = 2, \text{ since } \pi(2) = 1/2$$

(e) Compute the distance (in x units) between the x values for which $\pi(x) = 0.25$ and $\pi(x) = 0.75$. [3pts Say $\pi\left(x_1\right) = 0.25$ and $\pi\left(x_2\right) = 0.75$

at he proporttional to observed positive constants $t_1, \ldots, t_n$, respectively. (a) Derive a simplified expression for the log-likelihood. (You may drop terms without

$$\begin{aligned}
E\left(Y_i\right) &= \mu_i = t_i \lambda_i = t_i e^{\alpha} \quad \text{[5pts]} \\
L(\alpha; 7) &= \ln \left(\prod_{i=1}^{n} \frac{\mu_i^{y_i}}{y_i!} e^{-\mu_i} \right) \\
&= \sum_{i=1}^{n} \left(y_i \ln \mu_i - \mu_i - \ln y_i! \right) \\
&= \sum_{i=1}^{n} \left(y_i \ln \left(t_i e^{\alpha} \right) - t_i e^{\alpha} - \ln y_i! \right) \\
&= \sum_{i=1}^{n} \left(y_i \alpha - t_i e^{\alpha} \right) + \underset{\uparrow}{\text{constants}}
\end{aligned}$$

(not depending on α) (b) Write an expression for the likelihood equation

$$\frac{\partial}{\partial \alpha} L(\alpha; 7) = \sum_{i=1}^{n} \left(y_i - t_i e^{\alpha} \right)$$

so the likelihood equation is this set to zero:

$$\left(\sum_{i=1}^{n} y_i \right) - \left(\sum_{i=1}^{n} t_i \right) e^{\alpha} = 0$$

the MI When does it exist?

$$\hat{\alpha} = \ln \left(\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} t_i} \right)$$

which exists when $\sum_{i=1}^{n} y_i > 0$.

STAT 425− Section D1U, D1G - Spring 2014 Midterm Exam II April 23, 2014 Full Name: Key - This is a 50 minute exam. There are 4 problems for everyone, and 2 additional problems for graduate students only. students. - You may use three pages of personal notes and a standard scientific calculator. (You may not share these items with anyone else.) - Write all answers in the spaces provided. If you require more space to write your answer, you may use the back side of the page.

You are not allowed to communicate with anyone except the instructor or proctors before you submit this exam. Useful Abbreviations: CI = confidence interval PI = prediction interval se = standard error

$$\mathrm{E} =$$

var = variance (or variance-covariance) cov = covariance SLR = simple linear regression BLUE = best linear unbiased estimate/estimator GLS = generalized least WLS = weighted least squares MSE = mean square error TSS = total sum of squares RSS = residual sum of squares df = degrees of freedom VIF = variance inflation factor VST = variance stabilizing transformation H_0 = the null hypothesis of a test H_a = the alternative hypothesis of a test.

1. For each part below, CIRCLE the ONE BEST answer. [1 pt each] (a) How many blocking factors are in a Latin Square design? 0 3 (b) A Box-Cox transformation requires the dependent variable to be positive negative nonzero none of these (c) Initially, all variables are in the model in

forward selection
backward elimination
stepwise selection
none of these

(d) The C_p statistic may be used to – a model. (select test H_0 for estimate design (e) In a randomized complete block design, the number of blocks must be – the number of treatments.

greater than less than equal to
none of these

(f) In a homogeneity-of-regressions ("ANCOVA") model, a significant interaction between the X variable and the "dummy" variable(s) would indicate that the regression lines are

parallel not parallel perpendicular
none of these

(g) A VST involves transformation of the independent variable(s) dependent variable both neither (h) Both adding and dropping variables may be performed in forward selection backward elimination selection none of these 2. You intend to compare two brands of eyedrops (liquid solutions applied to the eyes to treat redness) using a randomized experiment with 20 human subjects. Think of a designed experiment that uses blocking. Answer the following: (a) What are the treatments? the two brands of eyedrops (b) What are the blocks? the human subjects pt] (c) What are the experimental units? eyes (d) Briefly describe the structure of the randomization. [2pts] For each human subject, independently, one eye is randomly chosen to receive brand 1, and the other eye receives brand 2 .

3. A linear regression of a variable Y on variables X_1 and X_2, including some polynomial terms, is performed and summarized in R as follows:

$$> \text{summary}\left(\text{lm}\left(y^{-}x1 + x2 + I\left(x2^{\wedge}2\right) + x1 * x2\right)\right)$$

Coefficients:

	Estimate	Std. Error	t value	Pr(> |t|)
(Intercept)	9.8981	0.2879	34.383	$< 2\mathrm{e}-16 * **$
x1	0.8269	0.3178	2.602	0.0143*
x2	0.1883	0.3219	0.585	0.5631
I (x2-2)	0.7890	0.6353	1.242	0.2239
x1: x2	2.2568	0.5666	3.983	$0.0004 * **$

(a) Write an expression for the full model equation. (Do not substitute any estimates!)

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 X_2 + \beta_{22} x_2^2 + \beta_{12} X_1 X_2 + \varepsilon^{[3pts]}$$

(b) Test for interaction between X_1 and X_2. (Give the test statistic, p -value, and conclusion.) [3pts]

$$t = 3.983$$
$$p = 0.0004$$

Reject the null hypothesis of no interaction. (c) What is the estimated expected change in Y if X_1 increases by one unit? (Hint: It depends on X_2.)

[2pts]

$$\hat{\beta}_1 + \hat{\beta}_{12} x_2 \approx 0.8269 + 2.2568 X_2$$

(d) What polynomial term appears to be "missing" from this model? [1pt] the term for X_1^2 (e) Suppose you are going to perform backward elimination on this model, based on a p -value threshold of $p = 0.05$. Which term (if any) would be removed first? Why? (Hint: Remember hierarchy!) [2pts] By hievarchy, only X_2^2 and $X_1 X_2$ are eligible for removal, at the first stage. Since X_2^2 has the larger p-value (0.2239), and it exceeds 0.05, remove X_2^2 first.

4. An experiment is conducted with a completely randomized design and two treatment factors, A and B, each with two levels. The design is balanced, with 5 replications. An ANOVA table from R is > anova(lm(response A $*$ B)) Analysis of Variance Table Response:

Response: response

	Df	Sum Sq	Mean Sq	F value	Pr(> F)
A	1	11.25	11.25	1.125	0.304594
B	1	101.25	101.25	10.125	0.005793 $**$
A : B	1	1.25	1.25	0.125	0.728289
Residuals	16	160.00	10.00		

(a) How many observations are there? 20 (b) How many treatments are there? (c) How many blocks are there? none (d) Summarize the conclusions you would draw concerning the interactions and main effects. [3pts] Interaction: No evidence for it ($p > 0.05$). Factor A: No evidence for it ($p > 0.05$). Factor B: There is evidence of effects differing ($p < 0.05$) (e) Let α_1 and α_2 be the main effects of Factor A. Perform a test of $H_0 : \alpha_1 = \alpha_2$ versus $H_a : \alpha_1 \neq \alpha_2$. (Give test statistic, p -value, and conclusion.) [2pts]

$$F = 1.125 \quad p = 0.305$$

No evidonce against $H_0 : \alpha_1 = \alpha_2$. (f) Would you need the Tukey multiple comparisons method for comparisons between the levels of Factor A? Explain briefly. [2pts] No. Factor A has only two levels, so there would be only one comparison.

GRADUATE STUDENTS ONLY 5. In the previous problem, the estimated mean responses are

$$\hat{\mu}_{11} = 63 \quad \hat{\mu}_{12} = 59 \quad \hat{\mu}_{21} = 65 \quad \hat{\mu}_{22} = 60$$

where $\hat{\mu}_{ij} = \bar{y}_{ij}$ is the average response for the observations at level i of Factor $A(i = 1, 2)$, and level j of Factor B$(j = 1, 2)$.

Draw an interaction plot for this situation, with Factor A on the horizontal axis. Make sure it is fully labeled!

[4pts]

6. A scientific model proposes the following relationship between variables X and Y :

$$Y = \gamma X^{\alpha} e^{-\beta X} \epsilon$$

where γ, α, and β are unknown constants ($\gamma > 0$) and $\epsilon > 0$ is a multiplicative error whose distribution is the same for all X.

Transform this to a linear regression model: Write the transformed model equation. What are the (transformed) dependent and independent variables? What are the regression parameters? [4pts]