3. We aim to minimize the function

$$f(\beta) = \sum_{i=1}^{n} [y_i - \beta x_i]^2$$

Take the first order derivative of $f(\beta)$, we get

$$\begin{aligned} f'(\beta) &= \sum_{i=1}^{n} 2 (y_i - \beta x_i) (-x_i) \\ &= -2 \sum_{i=1}^{n} x_i y_i + 2\beta \sum_{i=1}^{n} x_i^2 \end{aligned}$$

And the second order derivative is

$$f''(\beta) = 2 \sum_{i=1}^{n} x_i^2 \geq 0$$

Which indicates this function is a convex function. We can then find the optimal $\hat{\beta}$ by setting $f'(\beta) = 0$. Finally we get

$$\hat{\beta} = \sum_{i=1}^{n} x_i y_i / \sum_{i=1}^{n} x_i^2$$

4.a)

$$\begin{aligned}\hat{\beta} &= \sum_{i=1}^{n} x_i y_i / \sum_{i=1}^{n} x_i^2 \\ &= 2.5\end{aligned}$$