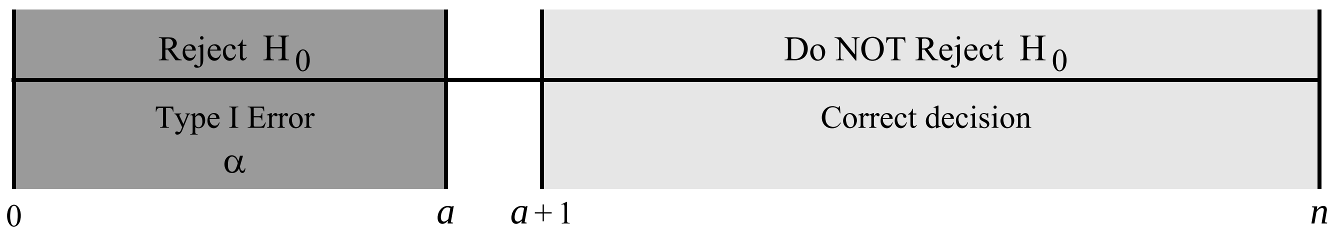
Left – tailed test

$H_0: p = p_0$ vs. $H_1: p < p_0$

If H_0 is TRUE: Use p_0.



Rejection Rule for a Left – tailed test:

Find a such that $P(X \le a) = \text{CDF @ } a \approx \alpha$. (using Binomial$(n,\ p_0)$)

Then the Rejection Rule is "Reject H_0 if $X \le a$."

If Rejection Rule is "Reject H_0 if $X \le a$,"

$P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \le a \mid p = p_0)$

$= \text{CDF @ } a$ (using Binomial$(n,\ p_0)$)

If H_0 is FALSE: Use new (given) p.

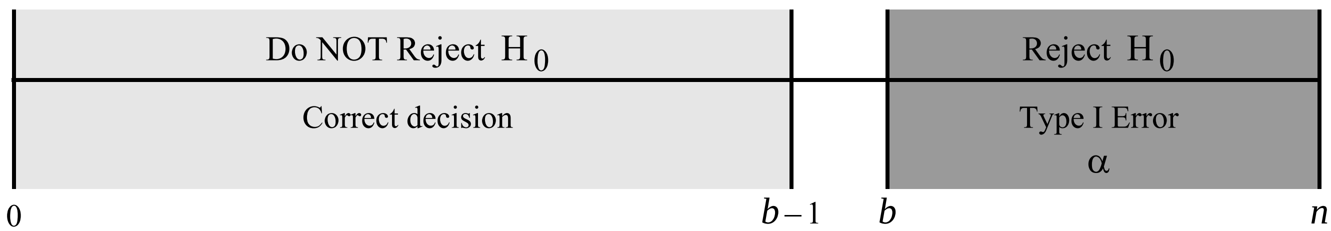
Power $= P(\text{Reject } H_0) = P(X \le a) = \text{CDF @ } a$ (using Binomial$(n,\ \text{new } p)$)

p-value $= P(\text{value of X as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true})$

$= P(X \le x_{\text{observed}} \mid p = p_0) = \text{CDF @ } x_{\text{observed}}$ (using Binomial$(n,\ p_0)$)

Right – tailed test $\quad H_0 : p = p_0 \quad \text{vs.} \quad H_1 : p > p_0$

If H_0 is TRUE: Use p_0.



Rejection Rule for a Right – tailed test:

Find b such that $P(X \le b-1) = \text{CDF @ } (b-1) \approx 1-\alpha.$ $\quad$ (using Binomial$(n,\ p_0)$)

(Then $P(X \ge b) \approx \alpha.$)

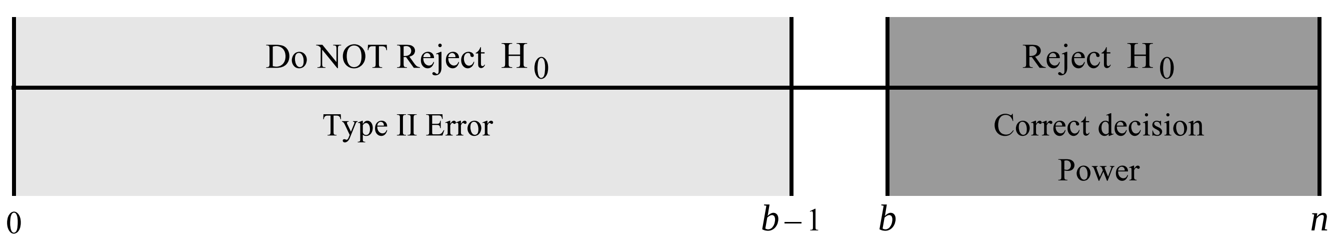
Then the Rejection Rule is "Reject H_0 if $X \ge b$."

If Rejection Rule is "Reject H_0 if $X \ge b$,"

$P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \ge b \mid p = p_0)$

$= (1 - \text{CDF @ } b-1)$ $\quad$ (using Binomial$(n,\ p_0)$)

If H_0 is FALSE: Use new (given) p.



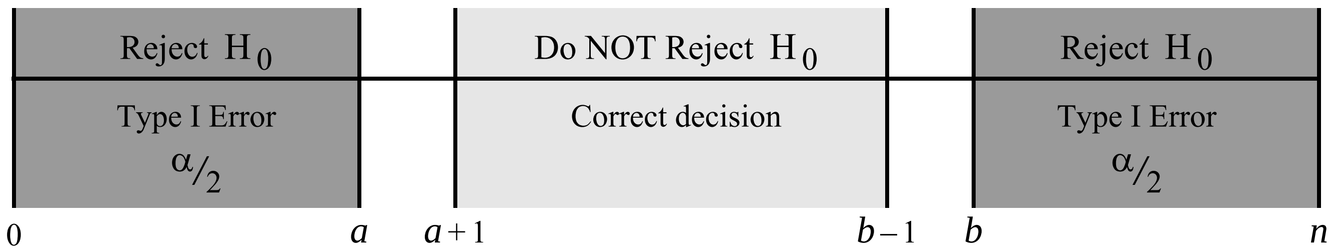
Power $= P(\text{Reject } H_0) = P(X \ge b) = (1 - \text{CDF @ } (b-1))$ $\quad$ (using Binomial$(n,\ \text{new } p)$)

p-value = $P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true})$

$= P(X \ge x_{\text{observed}} \mid p = p_0) = (1 - \text{CDF @ } (x_{\text{observed}} - 1))$ $\quad$ (using Binomial$(n,\ p_0)$)

Two – tailed test $\quad H_0 : p = p_0$ vs. $H_1 : p \neq p_0$

If H_0 is TRUE: Use p_0.



Rejection Rule for a Two – tailed test:

Find a such that $P(X \le a) = \text{CDF} \ @ \ a \approx \alpha/2$. $\quad$ (using Binomial(n, p_0))

Find b such that $P(X \le b-1) = \text{CDF} \ @ \ (b-1) \approx 1 - \alpha/2$. $\quad$ (Then $P(X \ge b) \approx \alpha/2$.)

(If $p = 0.50$, then $b = n - a$.)

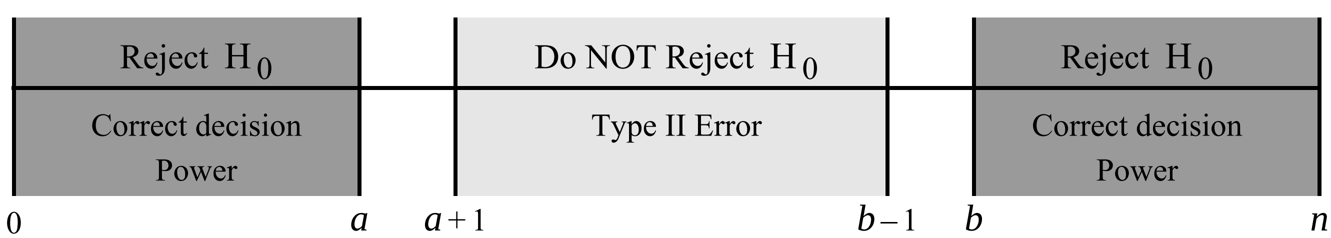
Then the Rejection Rule is "Reject H_0 if $X \le a$ or $X \ge b$."

If Rejection Rule is "Reject H_0 if $X \le a$ or $X \ge b$,"

$P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \le a \text{ or } X \ge b \mid p = p_0)$

$= \text{CDF} \ @ \ a + \big(1 - \text{CDF} \ @ \ (b-1)\big)$ $\quad$ (using Binomial(n, p_0))

If H_0 is FALSE: Use new (given) p.



$\text{Power} = P(\text{Reject } H_0) = P(X \le a \text{ or } X \ge b) = \text{CDF} \ @ \ a + \big(1 - \text{CDF} \ @ \ (b-1)\big)$

(using Binomial(n, new p))

$$H_0 : p = \frac{1}{2} \quad \text{vs.} \quad H_1 : p \neq \frac{1}{2}$$

If x_{observed} is the left tail $\left(\text{i.e., if } x_{\text{observed}} < n \cdot \frac{1}{2}\right)$,

$$\begin{aligned}
\text{p-value} &= \text{P}(\text{ value of X as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) \\
&= 2 \cdot \text{P}(X \leq x_{\text{observed}} \mid p = \tfrac{1}{2}) && \text{(since it is a two-tail test)} \\
&= 2 \cdot \text{CDF @ } x_{\text{observed}} && \left(\text{using Binomial}\left(n, \tfrac{1}{2}\right)\right)
\end{aligned}$$

If x_{observed} is the right tail $\left(\text{i.e., if } x_{\text{observed}} > n \cdot \frac{1}{2}\right)$,

$$\begin{aligned}
\text{p-value} &= \text{P}(\text{ value of X as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) \\
&= 2 \cdot \text{P}(X \geq x_{\text{observed}} \mid p = \tfrac{1}{2}) && \text{(since it is a two-tail test)} \\
&= 2 \cdot \left(1 - \text{CDF @ } (x_{\text{observed}} - 1)\right) && \left(\text{using Binomial}\left(n, \tfrac{1}{2}\right)\right)
\end{aligned}$$

If $x_{\text{observed}} = n \cdot \frac{1}{2}$,

$$\text{p-value} = \text{P}(\text{ value of X as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) = 1.$$