STAT 410 Fall 2020
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Examples for 09/28/2020 (3)

0. Grades on the last STAT 410 exam were not very good*. Graphed, their distribution had a shape similar to the p.d.f.

$$f_X(x) = \frac{2x+5}{10{,}000}, \qquad 20 \le x \le 100, \qquad \text{zero elsewhere.}$$

Recall: Examples for 08/26/2020 (3) Problem **7**

$$F_X(x) = \frac{x^2+5x-500}{10{,}000} = \frac{(x-20)(x+25)}{10{,}000}, \qquad 20 \le x \le 100.$$

Ten exam papers were selected at random. That is, let $X_1, X_2, X_3, \ldots, X_9, X_{10}$ be a random sample (i.i.d.) from the above population.

a) Find the probability that the lowest score of these 10 papers is below 40.

$$P(\min X_i < 40) = 1 - P(\min X_i \ge 40) = 1 - [\,P(X \ge 40)\,]^{10}$$

$$= 1 - [\,1 - F_X(40)\,]^{10} = 1 - [\,1 - 0.13\,]^{10} \approx 0.751577.$$

b) Find the probability that the largest score of these 10 papers is above 90.

$$P(\max X_i > 90) = 1 - P(\max X_i \le 90) = 1 - [\,P(X \le 90)\,]^{10}$$

$$= 1 - [\,F_X(90)\,]^{10} = 1 - [\,0.805\,]^{10} \approx 0.885723.$$

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* ☹ Unfortunately, this is pretty close to the actual Fall 2017 STAT 410 Exam 1 grades.

c) Find the probability that the second largest score of these 10 papers is above 90.

Hint: There are 10 independent scores, each one is either above 90 or below 90.
{ the second largest score is above 90 } = { at least two scores are above 90 }.

$$\text{P}(\text{ the second largest score is above 90 }) = \text{P}(\text{ at least two scores are above 90 })$$

$$= 1 - \text{P}(\text{ no scores above 90 }) - \text{P}(\text{ one score above 90 })$$

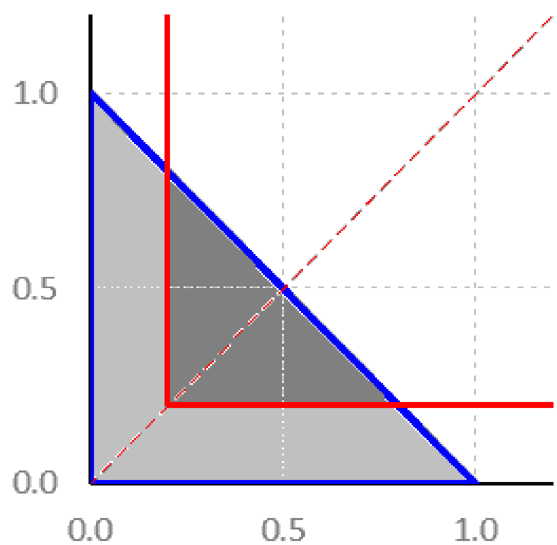
$$= 1 - [\,0.805\,]^{10} - 10\,[\,0.805\,]^{9}\,[\,0.195\,]^{1}$$

$$\approx 0.608903.$$

1. Consider two continuous random variables X and Y with joint p.d.f.

$$f_{\text{X,Y}}(x,y) = 60\,x^2 y, \qquad x > 0,\ \ y > 0,\ \ x + y < 1, \qquad \text{zero elsewhere.}$$

a) Let $\text{S} = \min(\text{X}, \text{Y})$. Find the probability distribution of S.



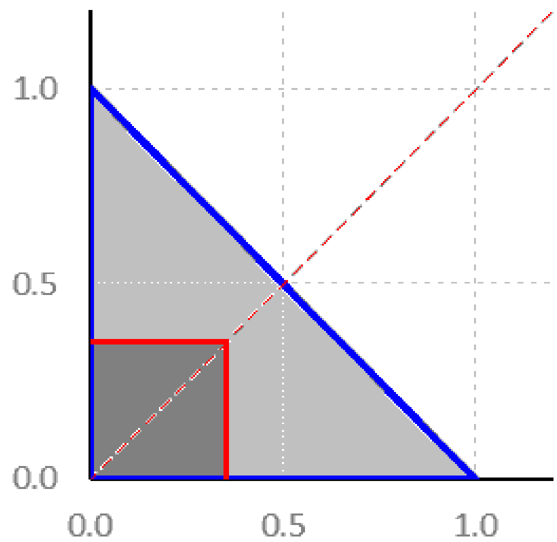
$$\text{F}_{\text{S}}(s) = 1 - \text{P}(\text{X} > s, \text{Y} > s)$$

$$= 1 - \int_{s}^{1-s} \left(\int_{s}^{1-x} 60\,x^2 y\,dy \right) dx$$

$$= 10\,s^2 - 10\,s^3 - 8\,s^5,$$

$$0 \le s < 0.5.$$

b) Let $T = \max(X, Y)$. Find the probability distribution of T.

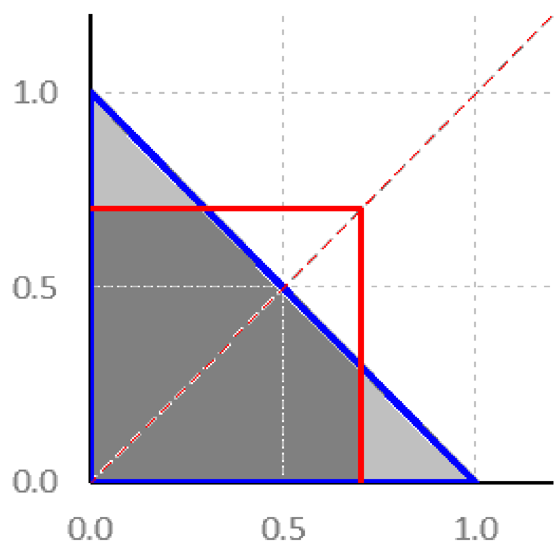


Case 1: $0 < t < 0.5$.

$$F_T(t) = P(X \le t, Y \le t)$$

$$= \int_0^t \left(\int_0^t 60\, x^2 y\, dy \right) dx$$

$$= 10\, t^5, \qquad 0 \le t < 0.5.$$



Case 2: $0.5 < t < 1$.

$$F_T(t) = P(X \le t, Y \le t)$$

$$= \int_0^{1-t} \left(\int_0^t 60\, x^2 y\, dy \right) dx$$

$$+ \int_{1-t}^t \left(\int_0^{1-x} 60\, x^2 y\, dy \right) dx$$

$$= 2\, t^5 - 10\, t^3 + 10\, t^2 - 1,$$

$$0.5 \le t < 1.$$