STAT 410

Fall 2020

Practice Problems 2

A. Stepanov

1. **1.9.18** (7th edition) **1.9.17** (6th edition)

Find the mean and the variance of the distribution that has the cdf

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x}{8} & 0 \le x < 2 \\ \dfrac{x^2}{16} & 2 \le x < 4 \\ 1 & 4 \le x. \end{cases}$$

2. **1.9.23** (7th edition) **1.9.22** (6th edition)

Let X have the c.d.f. $F(x)$ that is a mixture of the continuous and discrete types, namely

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x+1}{4} & 0 \le x < 1 \\ 1 & 1 \le x. \end{cases}$$

Determine reasonable definitions of $\mu = E(X)$ and $\sigma^2 = Var(X)$ and compute each.

Hint: Determine the parts of the p.m.f. and the p.d.f. associated with each of the discrete and continuous parts, and them sum for the discrete part and integrate for the continuous part.

3. The probability that a loss will occur is 0.15. If the loss occurs, the amount of the loss has the density function

$$f(x) = c\,x^{-5}, \quad x > 1, \qquad \text{zero otherwise.}$$

(That is, loss is a mixed random variable with $p(0) = 1 - 0.15 = 0.85$.)
An insurance company will pay the entire amount of loss.

a) Find the value of c that would make this a valid probability distribution.

b) Calculate the expected value of the payment.

c) Calculate the variance of the payment.

4. Consider a random variable X with the **p**.d.f.

$$f(x) = \begin{cases} 0 & x < 2 \\ 0.12\,x & 2 \le x < 3 \\ 0.20\,x & 3 \le x < 4 \\ \mathbf{0} & x \ge 4 \end{cases}$$

a) Find $\mu = \mathrm{E}(\mathrm{X})$.

b) Find $\sigma^2 = \mathrm{Var}(\mathrm{X})$.

c) Find $\mathrm{P}(2.5 < \mathrm{X} < 3)$, $\mathrm{P}(2.5 \le \mathrm{X} < 3)$, $\mathrm{P}(2.5 < \mathrm{X} \le 3)$, $\mathrm{P}(2.5 \le \mathrm{X} \le 3)$.

5. Consider a random variable X with the **c**.d.f.

$$\mathbf{F}(x) = \begin{cases} 0 & x < 2 \\ 0.12\,x & 2 \le x < 3 \\ 0.20\,x & 3 \le x < 4 \\ \mathbf{1} & x \ge 4 \end{cases}$$

a) Find $\mu = \mathrm{E}(\mathrm{X})$.

b) Find $\sigma^2 = \mathrm{Var}(\mathrm{X})$.

c) Find $\mathrm{P}(2.5 < \mathrm{X} < 3)$, $\mathrm{P}(2.5 \le \mathrm{X} < 3)$, $\mathrm{P}(2.5 < \mathrm{X} \le 3)$, $\mathrm{P}(2.5 \le \mathrm{X} \le 3)$.

6.* The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \dfrac{2}{y^3} & \text{if } y > 1 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the expected value and the variance of the policyholder's loss?

b) An insurance policy reimburses a loss up to a benefit limit of 10. What is the expected value and the variance of the benefit paid under the insurance policy?

c) An insurance policy has a deductible of 2. What is the expected value and the variance of the benefit paid under the insurance policy?

7. Find $E(X)$ for a mixed random variable with c.d.f.

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x}{5} & 1 \le x < 1.5 \\ \ln x & 1.5 \le x < 2.25 \\ 1 & x \ge 2.25 \end{cases}$$

8. Find $E(X)$ for a mixed random variable with c.d.f.

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{100} & 0 \le x < 5 \\ \dfrac{x}{20} & 5 \le x < 9 \\ \dfrac{\sqrt{x}}{5} & 9 \le x < 16 \\ 1 & x \ge 16 \end{cases}$$

9. Consider a mixed random variable X with the cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 \le x < 0.5 \\ \dfrac{8x+7}{32} & 0.5 \le x < 2 \\ 1 - \dfrac{4}{x^5} & x \ge 2 \end{cases}$$

a) Identify the discrete portion of the probability distribution.

b) Identify the continuous portion of the probability distribution.

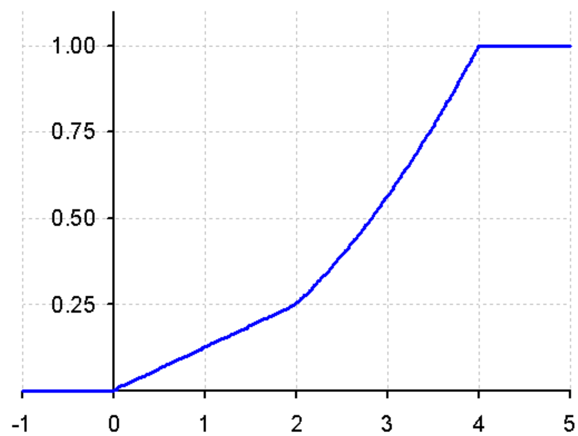
c) Find $E(X)$.

Answers:

1. **1.9.18** (7th edition) **1.9.17** (6th edition)

Find the mean and the variance of the distribution that has the cdf

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x}{8} & 0 \le x < 2 \\ \dfrac{x^2}{16} & 2 \le x < 4 \\ 1 & 4 \le x. \end{cases}$$



$$f(x) = F'(x) = \begin{cases} \dfrac{1}{8} & 0 < x < 2 \\ \dfrac{x}{8} & 2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = E(X) = \int_0^2 x \cdot \frac{1}{8}\, dx + \int_2^4 x \cdot \frac{x}{8}\, dx = \left(\frac{x^2}{16}\right)\Big|_0^2 + \left(\frac{x^3}{24}\right)\Big|_2^4 = \mathbf{31/12}.$$

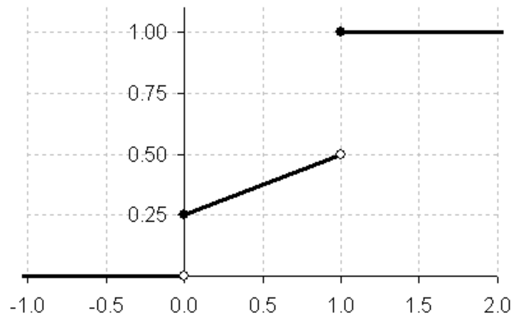
$$E(X^2) = \int_0^2 x^2 \cdot \frac{1}{8}\, dx + \int_0^2 x^2 \cdot \frac{x}{8}\, dx = \left(\frac{x^3}{24}\right)\Big|_0^2 + \left(\frac{x^4}{32}\right)\Big|_2^4 = 94/12.$$

$$\sigma^2 = \mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 94/12 - \left(31/12\right)^2 = \mathbf{167/144}.$$

2. **1.9.23** (7th edition) **1.9.22** (6th edition)

Let X have the c.d.f. $F(x)$ that is a mixture of the continuous and discrete types, namely

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x+1}{4} & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$$



Determine reasonable definitions of $\mu = E(X)$ and $\sigma^2 = \mathrm{Var}(X)$ and compute each.

Hint: Determine the parts of the p.m.f. and the p.d.f. associated with each of the discrete and continuous parts, and them sum for the discrete part and integrate for the continuous part.

Discrete portion of the probability distribution of X:

$$p(0) = {}^1\!/_4, \qquad p(1) = {}^1\!/_2.$$

Continuous portion of the probability distribution of X:

$$f(x) = F'(x) = \begin{cases} \dfrac{1}{4} & 0 < x < 1 \\ 0 & \text{o.w.} \end{cases}.$$

$$\mu = E(X) = 0 \cdot {}^1\!/_4 + 1 \cdot {}^1\!/_2 + \int_0^1 x \cdot \frac{1}{4}\,dx = \mathbf{{}^5\!/_8}.$$

$$E(X^2) = 0^2 \cdot {}^1\!/_4 + 1^2 \cdot {}^1\!/_2 + \int_0^1 x^2 \cdot \frac{1}{4}\,dx = {}^7\!/_{12}.$$

$$\sigma^2 = \mathrm{Var}(X) = E(X^2) - [E(X)]^2 = {}^7\!/_{12} - \left({}^5\!/_8\right)^2 = \mathbf{{}^{37}\!/_{192}}.$$

3. The probability that a loss will occur is 0.15. If the loss occurs, the amount of the loss has the density function

$$f(x) = c\,x^{-5}, \quad x > 1, \qquad \text{zero otherwise.}$$

(That is, loss is a mixed random variable with $p(0) = 1 - 0.15 = 0.85$.)
An insurance company will pay the entire amount of loss.

Loss is a mixed random variable.

Discrete portion: $p(0) = 0.85$.

Continuous portion: $f(x) = c\,x^{-5}, \quad x > 1$.

a) Find the value of c that would make this a valid probability distribution.

$$0.85 + \int_1^{\infty} c\,x^{-5}\,dx = 1. \qquad \Rightarrow \qquad 0.85 + \frac{c}{4} = 1. \qquad \Rightarrow \qquad c = \mathbf{0.60}.$$

b) Calculate the expected value of the payment.

$$\mathrm{E}(\text{Payment}) = \mathrm{E}(\text{Loss}) = 0 \times 0.85 + \int_1^{\infty} x \cdot 0.60\,x^{-5}\,dx = \mathbf{0.20}.$$

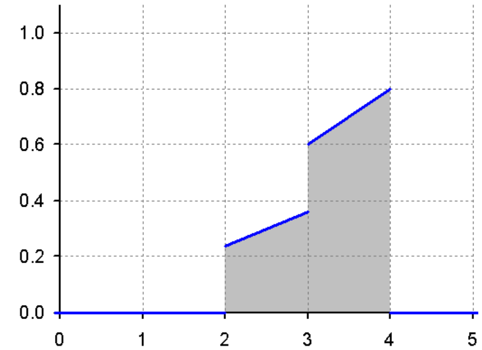
c) Calculate the variance of the payment.

$$\mathrm{E}(\text{Payment}^2) = \mathrm{E}(\text{Loss}^2) = 0^2 \times 0.85 + \int_1^{\infty} x^2 \cdot 0.60\,x^{-5}\,dx = 0.30.$$

$$\mathrm{Var}(\text{Payment}) = \mathrm{Var}(\text{Loss}) = 0.30 - 0.20^2 = \mathbf{0.26}.$$

4. Consider a random variable X with the **p**.d.f.

$$\boldsymbol{f}(x) = \begin{cases} 0 & x < 2 \\ 0.12\,x & 2 \le x < 3 \\ 0.20\,x & 3 \le x < 4 \\ \mathbf{0} & x \ge 4 \end{cases}$$



a) Find $\mu = \mathrm{E}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}) = \int_2^3 x \cdot 0.12\,x\,dx + \int_3^4 x \cdot 0.20\,x\,dx = \left(0.04\,x^3\right)\Big|_2^3 + \left(\frac{0.20}{3}x^3\right)\Big|_3^4$$

$$= \frac{19}{25} + \frac{37}{15} = \boldsymbol{\frac{242}{75}} \approx 3.226667.$$

b) Find $\sigma^2 = \mathrm{Var}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}^2) = \int_2^3 x^2 \cdot 0.12\,x\,dx + \int_3^4 x^2 \cdot 0.20\,x\,dx = \left(0.03\,x^4\right)\Big|_2^3 + \left(0.05\,x^4\right)\Big|_3^4$$

$$= 1.95 + 8.75 = 10.7.$$

$$\mathrm{Var}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) - [\mathrm{E}(\mathrm{X})]^2 = 10.7 - \left(\frac{242}{75}\right)^2 = \boldsymbol{\frac{3247}{11250}} \approx 0.288622.$$

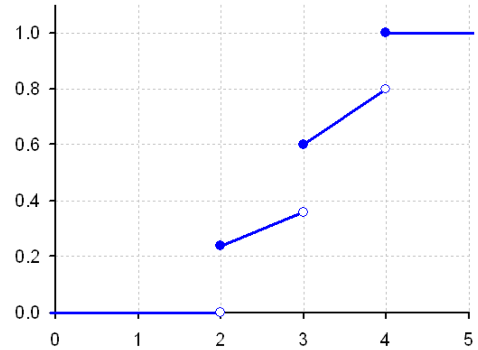
c) Find $\mathrm{P}(2.5 < \mathrm{X} < 3)$, $\mathrm{P}(2.5 \le \mathrm{X} < 3)$, $\mathrm{P}(2.5 < \mathrm{X} \le 3)$, $\mathrm{P}(2.5 \le \mathrm{X} \le 3)$.

Since $\mathrm{P}(\mathrm{X} = 2.5) = \mathrm{P}(\mathrm{X} = 3) = 0$, all four probabilities are equal.

$$\mathrm{P}(2.5 < \mathrm{X} < 3) = \int_{2.5}^3 0.12\,x\,dx = \left(0.06\,x^2\right)\Big|_{2.5}^3 = \mathbf{0.165}.$$

5. Consider a random variable X with the **c**.d.f.

$$\mathbf{F}(x) = \begin{cases} 0 & x < 2 \\ 0.12\,x & 2 \le x < 3 \\ 0.20\,x & 3 \le x < 4 \\ \mathbf{1} & x \ge 4 \end{cases}$$



$\mathrm{F}(x)$ "jumps" at $x = 2$ from 0 to 0.24, size of the "jump" = 0.24 – 0 = 0.24,
at $x = 3$ from 0.36 to 0.6, size of the "jump" = 0.6 – 0.36 = 0.24,
at $x = 4$ from 0.80 to 1.0, size of the "jump" = 1.0 – 0.8 = 0.20.

$\Rightarrow$ Discrete portion of the probability distribution of X:

$$p(2) = 0.24, \qquad p(3) = 0.24, \qquad p(4) = 0.20.$$

Continuous portion of the probability distribution of X:

$$f(x) = \mathrm{F}'(x) = \begin{cases} 0 & x < 2 \\ 0.12 & 2 < x < 3 \\ 0.20 & 3 < x < 4 \\ 0 & x \ge 4 \end{cases}.$$

a) Find $\mu = \mathrm{E}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}) = 2 \cdot 0.24 + 3 \cdot 0.24 + 4 \cdot 0.20 + \int_2^3 x \cdot 0.12\,dx + \int_3^4 x \cdot 0.20\,dx$$

$$= 0.48 + 0.72 + 0.80 + 0.30 + 0.70 = \mathbf{3}.$$

OR

Since X is a nonnegative random variable,

$$\mathrm{E}(\mathrm{X}) = \int_0^\infty (1 - \mathrm{F}(x))\,dx = \int_0^2 1\,dx + \int_2^3 (1 - 0.12\,x)\,dx + \int_3^4 (1 - 0.20\,x)\,dx$$

$$= 2 + (1 - 0.30) + (1 - 0.70) = \mathbf{3}.$$

b) Find $\sigma^2 = \mathrm{Var}(X)$.

$$E(X^2) = 2^2 \cdot 0.24 + 3^2 \cdot 0.24 + 4^2 \cdot 0.20 + \int_2^3 x^2 \cdot 0.12\,dx + \int_3^4 x^2 \cdot 0.20\,dx$$

$$= 0.96 + 2.16 + 3.20 + 0.76 + \frac{37}{15} = \mathbf{\frac{716}{75}} \approx 9.546667.$$

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \mathbf{\frac{41}{75}} \approx 0.546667.$$

c) Find $P(2.5 < X < 3)$, $P(2.5 \le X < 3)$, $P(2.5 < X \le 3)$, $P(2.5 \le X \le 3)$.

Since $P(X = 2.5) = 0$,

$$P(2.5 < X < 3) = P(2.5 \le X < 3)$$

and $$P(2.5 < X \le 3) = P(2.5 \le X \le 3).$$

$$P(2.5 \le X < 3) = P(2.5 < X < 3) = F(3-) - F(2.5) = 0.36 - 0.30 = \mathbf{0.06}.$$

$$P(2.5 \le X \le 3) = P(2.5 < X \le 3) = F(3) - F(2.5) = 0.60 - 0.30 = \mathbf{0.30}.$$

6.* The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \dfrac{2}{y^3} & \text{if } y > 1 \\ 0 & \text{otherwise} \end{cases}$$

a) What is the expected value and the variance of the policyholder's loss?

$$\mathrm{E}(\text{Loss}) = \int_1^\infty y \cdot \frac{2}{y^3}\, dy = -\frac{2}{y}\bigg|_1^\infty = \mathbf{2}.$$

$$\mathrm{E}(\text{Loss}^2) = \int_1^\infty y^2 \cdot \frac{2}{y^3}\, dy = 2 \ln y\bigg|_1^\infty \text{ is not finite. } \Rightarrow \mathrm{Var}(\text{Loss}) \text{ is not finite.}$$

b) An insurance policy reimburses a loss up to a benefit limit of 10. What is the expected value and the variance of the benefit paid under the insurance policy?

$$\text{The benefit paid under the insurance policy} = \begin{cases} y & \text{for } 1 < y \le 10 \\ 10 & \text{for } y \ge 10 \end{cases}$$

$$\mathrm{E}(\text{Benefit Paid}) = \int_1^{10} y \cdot \frac{2}{y^3}\, dy + \int_{10}^\infty 10 \cdot \frac{2}{y^3}\, dy = -\frac{2}{y}\bigg|_1^{10} - \frac{10}{y^2}\bigg|_{10}^\infty = \mathbf{1.9}.$$

$$\mathrm{E}(\text{Benefit Paid}^2) = \int_1^{10} y^2 \cdot \frac{2}{y^3}\, dy + \int_{10}^\infty 10^2 \cdot \frac{2}{y^3}\, dy = 2 \ln y\bigg|_1^{10} - \frac{100}{y^2}\bigg|_{10}^\infty$$

$$= 2 \ln 10 + 1.$$

$$\mathrm{Var}(\text{Benefit Paid}) = 2 \ln 10 + 1 - 1.9^2 = \mathbf{2 \ln 10 - 2.61} \approx \mathbf{1.99517}.$$

c) An insurance policy has a deductible of 2. What is the expected value and the variance of the benefit paid under the insurance policy?

$$\text{The benefit paid under the insurance policy} = \begin{cases} 0 & \text{for } 1 < y \le 2 \\ y-2 & \text{for } y \ge 2 \end{cases}$$

$$\mathrm{E}(\text{Benefit Paid}) = \int_2^\infty (y-2)\cdot\frac{2}{y^3}\,dy = \int_2^\infty \frac{2}{y^2}\,dy - \int_2^\infty \frac{4}{y^3}\,dy$$

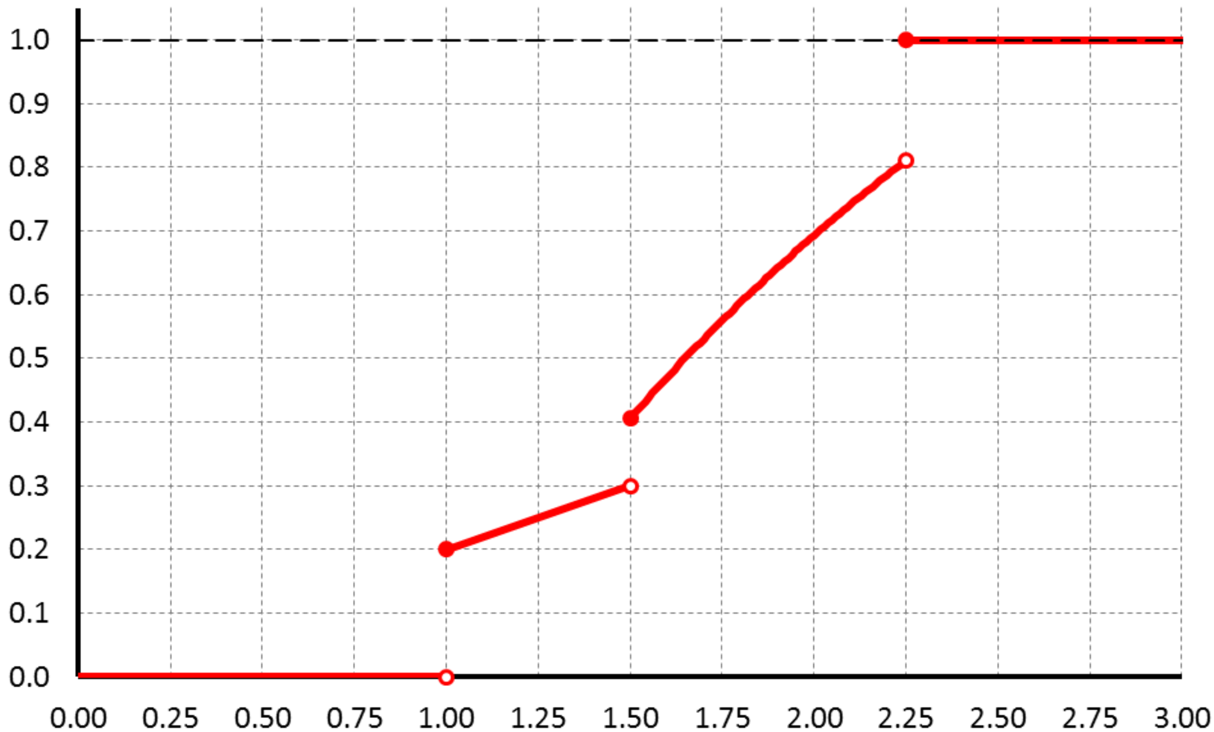
$$= -\frac{2}{y}\Big|_2^\infty + \frac{2}{y^2}\Big|_2^\infty = \mathbf{0.5}.$$

$$\mathrm{E}(\text{Benefit Paid}^2) = \int_2^\infty (y-2)^2\cdot\frac{2}{y^3}\,dy \quad \text{is not finite.}$$

$\Rightarrow$ $\mathrm{Var}(\text{Benefit Paid})$ is not finite.

7. Find $\mathrm{E}(\mathrm{X})$ for a mixed random variable with c.d.f.

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x}{5} & 1 \le x < 1.5 \\ \ln x & 1.5 \le x < 2.25 \\ 1 & x \ge 2.25 \end{cases}$$



Discrete portion of the probability distribution of X:

$$p(1) = 0.2 - 0 = 0.2, \qquad p(1.5) = \ln 1.5 - 0.3,$$

$$p(2.25) = 1 - \ln 2.25 = 1 - 2\ln 1.5.$$

Continuous portion of the probability distribution of X:

$$f(x) = \mathrm{F}'(x) = \begin{cases} \dfrac{1}{5} & 1 < x < 1.5 \\ \dfrac{1}{x} & 1.5 < x < 2.25 \\ 0 & \text{o.w.} \end{cases}$$

$$\mathrm{E}(\mathrm{X}) = 1 \cdot 0.2 + 1.5 \cdot (\ln 1.5 - 0.3) + 2.25 \cdot (1 - 2\ln 1.5) + \int_1^{1.5} x \cdot \frac{1}{5}\, dx + \int_{1.5}^{2.25} x \cdot \frac{1}{x}\, dx$$

$$= 1 \cdot 0.2 + 1.5 \cdot (\ln 1.5 - 0.3) + 2.25 \cdot (1 - 2\ln 1.5) + \frac{x^2}{10}\Bigg|_1^{1.5} + x\Bigg|_{1.5}^{2.25}$$

$$= 0.2 + 1.5\ln 1.5 - 0.45 + 2.25 - 4.5\ln 1.5 + \frac{(2.25-1)}{10} + (2.25 - 1.5)$$

$$= \mathbf{2.875 - 3\ln 1.5} \approx 1.6586.$$

OR

Since X is a non-negative random variable,

$$\mathrm{E}(\mathrm{X}) = \int_0^{\infty} (1 - \mathrm{F}(x))\, dx = \int_0^1 1\, dx + \int_1^{1.5} \left(1 - \frac{x}{5}\right) dx + \int_{1.5}^{2.25} (1 - \ln x)\, dx$$

$$= 1 + \left(x - \frac{x^2}{10}\right)\Bigg|_1^{1.5} + (2x - x\ln x)\Bigg|_{1.5}^{2.25}$$

$$= 1 + \left(1.5 - \frac{1.5^2}{10}\right) - \left(1 - \frac{1^2}{10}\right) + (4.5 - 2.25\ln 2.25) - (3 - 1.5\ln 1.5)$$

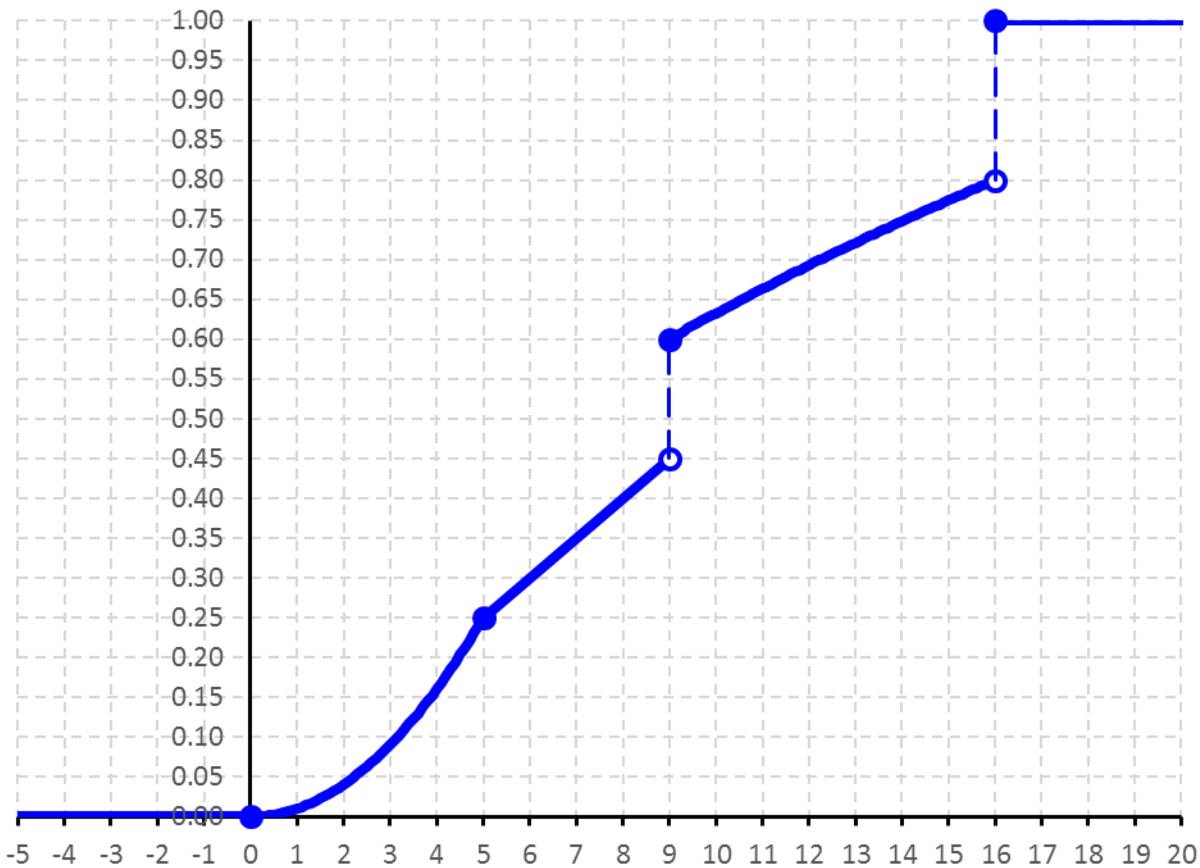
$$= 1 + 1.275 - 0.90 + (4.5 - 4.5\ln 1.5) - (3 - 1.5\ln 1.5)$$

$$= 1.375 + 1.5 - 3\ln 1.5$$

$$= \mathbf{2.875 - 3\ln 1.5} \approx 1.6586.$$

8. Find $E(X)$ for a mixed random variable with c.d.f.

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{100} & 0 \le x < 5 \\ \dfrac{x}{20} & 5 \le x < 9 \\ \dfrac{\sqrt{x}}{5} & 9 \le x < 16 \\ 1 & x \ge 16 \end{cases}$$



Discrete portion of the probability distribution of X:

$F_X(x)$ "jumps" at $x = 9$ from 0.45 to 0.60, size of the "jump" = 0.60 – 0.45 = 0.15,

at $x = 16$ from 0.80 to 1.00, size of the "jump" = 1.00 – 0.80 = 0.20.

$\Rightarrow$ Discrete portion of the probability distribution of X:

$p(9) = 0.15, \qquad p(16) = 0.20.$

Continuous portion of the probability distribution of X:

$$f(x) = \mathrm{F}'(x) = \begin{cases} \dfrac{x}{50} & 0 < x < 5 \\ \dfrac{1}{20} & 5 < x < 9 \\ \dfrac{1}{10\sqrt{x}} & 9 < x < 16 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{E}(\mathrm{X}) = 9 \cdot 0.15 + 16 \cdot 0.20 + \int_0^5 x \cdot \frac{x}{50}\,dx + \int_5^9 x \cdot \frac{1}{20}\,dx + \int_9^{16} x \cdot \frac{1}{10\sqrt{x}}\,dx$$

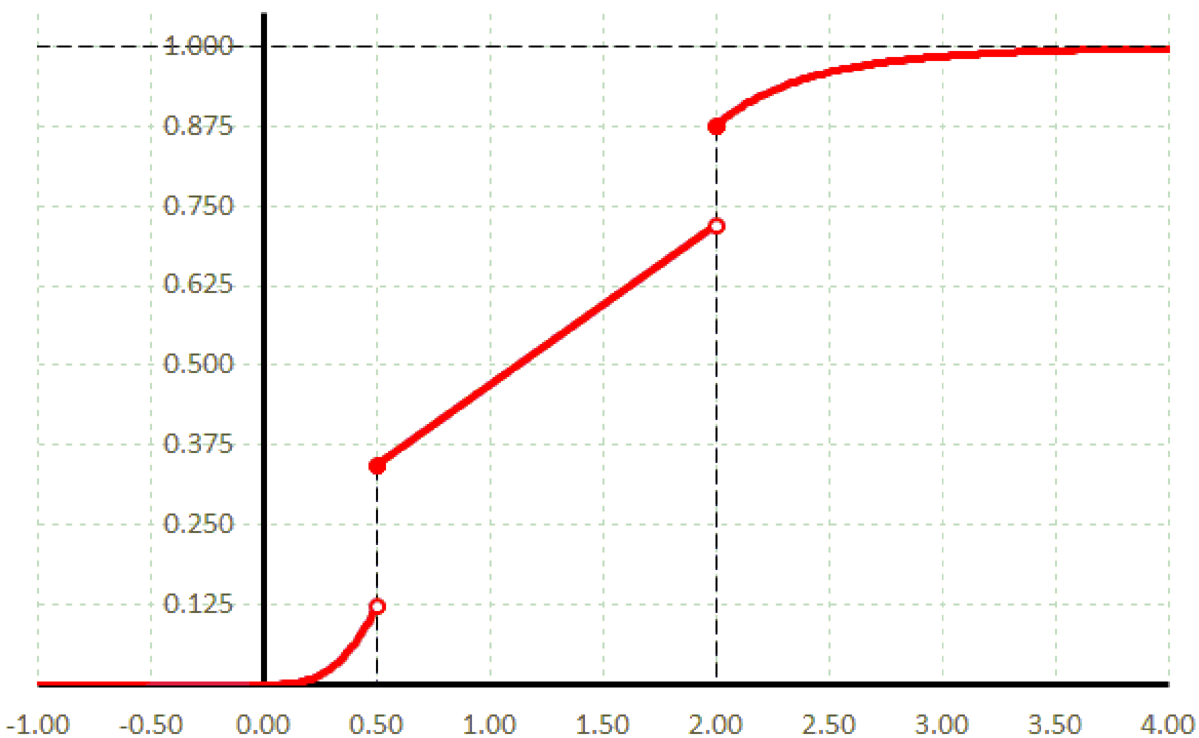
$$= 1.35 + 3.20 + \left.\frac{x^3}{150}\right|_0^5 + \left.\frac{x^2}{40}\right|_5^9 + \left.\frac{x^{1.5}}{15}\right|_9^{16}$$

$$= 4.55 + \frac{125-0}{150} + \frac{81-25}{40} + \frac{64-27}{15}$$

$= \mathbf{9.25}$.

9. Consider a mixed random variable X with the cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 \le x < 0.5 \\ \dfrac{8x+7}{32} & 0.5 \le x < 2 \\ 1 - \dfrac{4}{x^5} & x \ge 2 \end{cases}$$



a) Identify the discrete portion of the probability distribution.

$F_X(x)$ "jumps" at $x = 0.5$ from $\frac{1}{8}$ to $\frac{11}{32}$, size of the "jump" $= \frac{11}{32} - \frac{1}{8} = \frac{7}{32}$,

at $x = 2$ from $\frac{23}{32}$ to $\frac{7}{8}$, size of the "jump" $= \frac{7}{8} - \frac{23}{32} = \frac{5}{32}$.

$\Rightarrow$ Discrete portion of the probability distribution of X:

$$p(0.5) = \frac{7}{32} = 0.21875, \qquad p(2) = \frac{5}{32} = 0.15625.$$

b) Identify the continuous portion of the probability distribution.

Continuous portion of the probability distribution of X:

$$f(x) = \mathrm{F}'(x) = \begin{cases} 0 & x < 0 \\ 3x^2 & 0 < x < 0.5 \\ \frac{1}{4} & 0.5 < x < 2 \\ \frac{20}{x^6} & x > 2 \end{cases}$$

c) Find $\mathrm{E}(\mathrm{X})$.

$$\mathrm{E}(\mathrm{X}) = 0.5 \cdot \frac{7}{32} + 2 \cdot \frac{5}{32} + \int_0^{0.5} x \cdot 3x^2 \, dx + \int_{0.5}^{2} x \cdot \frac{1}{4} \, dx + \int_2^{\infty} x \cdot \frac{20}{x^6} \, dx$$

$$= \frac{7}{64} + \frac{20}{64} + \frac{3}{4} x^4 \Big|_0^{0.5} + \frac{1}{8} x^2 \Big|_{0.5}^{2} + \left(-\frac{5}{x^4} \right) \Big|_2^{\infty}$$

$$= \frac{27}{64} + \frac{3}{64} + \left(\frac{1}{2} - \frac{1}{32} \right) + \left(0 - \frac{5}{16} \right) = \frac{80}{64} = \mathbf{\frac{5}{4}} = \mathbf{1.25}.$$

OR

Since X is a non-negative random variable,

$$E(X) = \int_0^\infty (1 - F(x))\,dx$$

$$= \int_0^{0.5} \left(1 - x^3\right) dx + \int_{0.5}^{2} \left(1 - \frac{8x+7}{32}\right) dx + \int_2^\infty \frac{4}{x^5}\,dx$$

$$= \left(x - \frac{1}{4}x^4\right)\Bigg|_0^{0.5} + \left(\frac{25}{32}x - \frac{1}{8}x^2\right)\Bigg|_{0.5}^{2} + \left(-\frac{1}{x^4}\right)\Bigg|_2^\infty$$

$$= \frac{31}{64} + \frac{45}{64} + \frac{1}{16} = \frac{80}{64} = \mathbf{\frac{5}{4}} = \mathbf{1.25}.$$