Examples for 11/06/2020 (3)

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1 – 2. Let $\beta > 0$ and $\delta \in \mathbf{R}$. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x; \beta, \delta) = \beta e^{-\beta x + \beta\delta}, \qquad x > \delta, \qquad \text{zero otherwise.}$$

1. Suppose δ is known.

a) (i) Obtain a method of moments estimator of β, $\tilde{\beta}$.

(ii) Suppose $n = 4$, $\delta = 2$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$. Obtain a method of moments estimate of β, $\tilde{\beta}$.

b) (i) Obtain the maximum likelihood estimator of β, $\hat{\beta}$.

(ii) Suppose $n = 4$, $\delta = 2$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$. Obtain the maximum likelihood estimate of β, $\hat{\beta}$.

c) Is the maximum likelihood estimator $\hat{\beta}$ an unbiased estimator of β? If $\hat{\beta}$ is not an unbiased estimator of β, construct an unbiased estimator of β based on $\hat{\beta}$.

"Hint": $W = X - \delta$.

d) Find $\text{MSE}(\hat{\beta}) = (\text{bias}(\hat{\beta}))^2 + \text{Var}(\hat{\beta})$.

e) Find a sufficient statistic $Y = u(X_1, X_2, \ldots, X_n)$ for β.

f) Suggest a confidence interval for β with $(1 - \alpha)100\%$ confidence level.

"Hint": Use $\sum_{i=1}^{n}(X_i - \delta) = \sum_{i=1}^{n} W_i$.

g) Suppose $n = 4$, $\delta = 2$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$. Use part (f) to construct a 95% confidence interval for β.

2. Suppose β is known.

h) (i) Obtain a method of moments estimator of δ, $\tilde{\delta}$.

(ii) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.

Obtain a method of moments estimate of δ, $\tilde{\delta}$.

i) Is the method of moments estimator $\tilde{\delta}$ an unbiased estimator of δ?

If $\tilde{\delta}$ is not an unbiased estimator of δ, construct an unbiased estimator of δ based on $\tilde{\delta}$.

"Hint": $E(\overline{X}) = \mu$.

j) Find $\text{MSE}(\tilde{\delta})$. "Hint": $\text{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$.

k) (i) Obtain the maximum likelihood estimator of δ, $\hat{\delta}$.

(ii) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.

Obtain the maximum likelihood estimate of δ, $\hat{\delta}$.

"Hint": $x_1 > \delta$, $x_2 > \delta$, … $x_n > \delta$.

l) Is the maximum likelihood estimator $\hat{\delta}$ an unbiased estimator of δ?

If $\hat{\delta}$ is not an unbiased estimator of δ, construct an unbiased estimator of δ based on $\hat{\delta}$.

m) Find $\text{MSE}(\hat{\delta}) = (\text{bias}(\hat{\delta}))^2 + \text{Var}(\hat{\delta})$.

n) Find a sufficient statistic $Y = u(X_1, X_2, \ldots, X_n)$ for δ.

o) Find c such that $P(\delta < \min X_i < \delta + c) = 1 - \alpha$.

"Hint": c depends on α, β, n.

p) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.

Use part (o) to construct a 95% confidence interval for δ.

"Hint": $1 - \alpha = P(\delta < \min X_i < \delta + c) = P(\clubsuit < \delta < \spadesuit)$.

Then $(\clubsuit, \spadesuit)$ is a $(1 - \alpha)100\%$ confidence interval for δ.

Answers:

1 – 2. Let $\beta > 0$ and $\delta \in \mathbf{R}$. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with probability density function

$$f(x;\beta,\delta) = \beta e^{-\beta x + \beta\delta}, \qquad x > \delta, \qquad \text{zero otherwise.}$$

1. Suppose δ is known.

a) (i) Obtain a method of moments estimator of β, $\tilde{\beta}$.

(ii) Suppose $n = 4$, $\delta = 2$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$. Obtain a method of moments estimate of β, $\tilde{\beta}$.

$$f_X(x) = \beta e^{-\beta x + \beta\delta} = \beta e^{-\beta(x-\delta)}, \qquad x > \delta.$$

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y > 0. \qquad \Rightarrow \qquad E(Y) = \frac{1}{\lambda}.$$

$$f_Y(y) = \lambda e^{-\lambda(y-\delta)}, \quad y > \delta. \qquad \Rightarrow \qquad E(Y) = \delta + \frac{1}{\lambda}.$$

$$\Rightarrow \qquad E(X) = \delta + \frac{1}{\beta}.$$

OR $\qquad E(X) = \int\limits_{\delta}^{\infty} x \cdot \beta e^{-\beta(x-\delta)}\, dx = \ldots$ by parts $\ldots$

$$\overline{X} = \delta + \frac{1}{\tilde{\beta}}. \qquad \Rightarrow \qquad \tilde{\beta} = \frac{1}{\overline{X} - \delta}.$$

(ii) $\sum\limits_{i=1}^{n} x_i = 9. \qquad \bar{x} = 2.25. \qquad \hat{\beta} = \dfrac{1}{2.25 - 2} = \mathbf{4}.$

b) (i) Obtain the maximum likelihood estimator of β, $\hat{\beta}$.

(ii) Suppose $n = 4$, $\delta = 2$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.

Obtain the maximum likelihood estimate of β, $\hat{\beta}$.

(i)
$$L(\beta) = \prod_{i=1}^{n} f\left(x_i ; \beta, \delta\right) = \begin{cases} \prod_{i=1}^{n} \beta\, e^{-\beta x_i + \beta\delta} & x_1 > \delta,\ x_2 > \delta,\ \ldots,\ x_n > \delta \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \beta^n\, e^{-\beta \Sigma x_i + n\beta\delta} & \min x_i > \delta \\ 0 & \delta > \min x_i \end{cases}$$

$$\ln L(\beta) = n \ln\beta - \beta \sum_{i=1}^{n} x_i + n\beta\delta.$$

$$\frac{d}{d\beta} \ln L(\beta) = \frac{n}{\beta} - \sum_{i=1}^{n} x_i + n\delta = 0.$$

$$\Rightarrow \quad \hat{\beta} = \frac{n}{\sum_{i=1}^{n} X_i - n\delta} = \frac{n}{\sum_{i=1}^{n} \left(X_i - \delta\right)} = \frac{1}{\overline{X} - \delta}.$$

(ii)
$$\sum_{i=1}^{n} x_i = 9. \qquad \sum_{i=1}^{n} \left(x_i - 2\right) = 1. \qquad \overline{x} = 2.25.$$

$$\hat{\beta} = \frac{4}{9 - 4\cdot 2} = \frac{4}{1} = \frac{1}{2.25 - 2} = \mathbf{4}.$$

c) Is the maximum likelihood estimator $\hat{\beta}$ an unbiased estimator of β?
If $\hat{\beta}$ is not an unbiased estimator of β, construct an unbiased estimator of β based on $\hat{\beta}$.

"Hint": $\quad \mathrm{W} = \mathrm{X} - \delta.$

$$f_{\mathrm{X}}(x) = \beta\, e^{-\beta x + \beta\delta} = \beta\, e^{-\beta\,(x-\delta)}, \qquad x > \delta.$$

$$\mathrm{W} = \mathrm{X} - \delta \qquad x = w + \delta \qquad \frac{dx}{dw} = 1$$

$$f_{\mathrm{W}}(w) = \beta\, e^{-\beta w} \cdot |1| = \beta\, e^{-\beta w}, \qquad w > 0.$$

OR

$$\mathrm{F}_{\mathrm{X}}(x) = \int_{\delta}^{x} \beta\, e^{-\beta u + \beta\delta} du = 1 - e^{-\beta x + \beta\delta} = 1 - e^{-\beta\,(x-\delta)}, \qquad x > \delta.$$

$$\mathrm{F}_{\mathrm{W}}(w) = \mathrm{P}(\mathrm{X} - \delta \le w) = \mathrm{F}_{\mathrm{X}}(w+\delta) = 1 - e^{-\beta w}, \quad w > 0.$$

$\mathrm{W} = \mathrm{X} - \delta$ has an Exponential$\left(\theta = \frac{1}{\beta}\right)$ = Gamma$\left(\alpha = 1, \theta = \frac{1}{\beta}\right)$ distribution.

$\Rightarrow \quad \mathrm{Y} = \sum_{i=1}^{n} \mathrm{W}_i$ has a Gamma$\left(\alpha = n, \theta = \frac{1}{\beta}\right)$ distribution.

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n} \left(\mathrm{X}_i - \delta\right)} = \frac{n}{\sum_{i=1}^{n} \mathrm{W}_i} = \frac{n}{\mathrm{Y}}. \qquad \mathrm{E}(\mathrm{Y}^k) = \frac{\Gamma(n+k)}{\beta^k\,\Gamma(n)}.$$

$$\mathrm{E}(\hat{\beta}) = \mathrm{E}\left(\frac{n}{\mathrm{Y}}\right) = n\,\mathrm{E}\left(\frac{1}{\mathrm{Y}}\right) = n\,\mathrm{E}(\mathrm{Y}^{-1}) = n \cdot \frac{\beta}{n-1} = \frac{n}{n-1} \cdot \beta \ne \beta.$$

$\Rightarrow \quad \hat{\beta}$ is NOT an unbiased estimator for β.

Consider $\quad \hat{\hat{\beta}} = \dfrac{n-1}{n} \cdot \hat{\beta} = \dfrac{n-1}{\sum\limits_{i=1}^{n} X_i - n\delta} = \dfrac{n-1}{\sum\limits_{i=1}^{n} \left(X_i - \delta \right)}.$

Then $\quad E(\hat{\hat{\beta}}) = \dfrac{n-1}{n} \cdot E(\hat{\beta}) = \beta.$ $\qquad \hat{\hat{\beta}}$ is an unbiased estimator for β.

d) Find $\text{MSE}(\hat{\beta}) = (\text{bias}(\hat{\beta}))^2 + \text{Var}(\hat{\beta}).$

$$\text{bias}(\hat{\beta}) = E(\hat{\beta}) - \beta = \frac{\beta}{n-1}.$$

$$\text{Var}\left(\frac{1}{Y} \right) = E(Y^{-2}) - [E(Y^{-1})]^2 = \frac{\beta^2}{(n-1)(n-2)} - \left(\frac{\beta}{n-1} \right)^2$$

$$= \frac{\beta^2}{(n-1)^2 (n-2)}.$$

$$\text{Var}(\hat{\beta}) = \text{Var}\left(\frac{n}{Y} \right) = n^2 \, \text{Var}\left(\frac{1}{Y} \right) = \frac{n^2 \beta^2}{(n-1)^2 (n-2)}.$$

$$\text{MSE}(\hat{\beta}) = \left(\frac{\beta}{n-1} \right)^2 + \frac{n^2 \beta^2}{(n-1)^2 (n-2)} = \frac{\left(n^2 + n - 2 \right) \beta^2}{(n-1)^2 (n-2)} = \frac{(n+2)\beta^2}{(n-1)(n-2)}.$$

e) Find a sufficient statistic $Y = u(X_1, X_2, \ldots, X_n)$ for β.

$$\prod_{i=1}^{n} f(x_i\,;\,\beta,\delta) = \prod_{i=1}^{n} \beta\, e^{-\beta x_i + \beta\delta} \cdot \mathrm{I}_{\{x_i > \delta\}}$$

$$= \beta^n\, e^{-\beta \Sigma x_i + n\beta\delta} \cdot \mathrm{I}_{\{\min x_i > \delta\}}.$$

By Factorization Theorem,

$\sum_{i=1}^{n} X_i$ is a sufficient statistic for β.

[$\Rightarrow$ $\overline{X}$ is also a sufficient statistic for λ.]

[$\Rightarrow$ $\sum_{i=1}^{n} (X_i - \delta)$ is also a sufficient statistic for λ.]

f) Suggest a confidence interval for β with $(1-\alpha)100\%$ confidence level.

"Hint": Use $\sum_{i=1}^{n}\left(X_i-\delta\right)=\sum_{i=1}^{n} W_i$.

$$\sum_{i=1}^{n} W_i = \sum_{i=1}^{n}\left(X_i-\delta\right) \text{ has a } \operatorname{Gamma}\left(\alpha=n, \theta=\frac{1}{\beta}\right) \text{ distribution.}$$

$$\Rightarrow \quad 2\beta\sum_{i=1}^{n}\left(X_i-\delta\right) \text{ has a } \chi^2(2\alpha=2n) \text{ distribution.}$$

$$\Rightarrow \quad P\left(\chi^2_{1-\alpha/2}(2n) < 2\beta\sum_{i=1}^{n}\left(X_i-\delta\right) < \chi^2_{\alpha/2}(2n)\right) = 1-\alpha.$$

$$\Rightarrow \quad P\left(\frac{\chi^2_{1-\alpha/2}(2n)}{2\sum_{i=1}^{n}\left(X_i-\delta\right)} < \beta < \frac{\chi^2_{\alpha/2}(2n)}{2\sum_{i=1}^{n}\left(X_i-\delta\right)}\right) = 1-\alpha.$$

A $(1-\alpha)100\%$ confidence interval for β

$$\left(\frac{\chi^2_{1-\alpha/2}(2n)}{2\sum_{i=1}^{n}\left(X_i-\delta\right)}, \frac{\chi^2_{\alpha/2}(2n)}{2\sum_{i=1}^{n}\left(X_i-\delta\right)}\right)$$

g) Suppose $n=4$, $\delta=2$, and $x_1=2.15$, $x_2=2.55$, $x_3=2.10$, $x_4=2.20$.
Use part (f) to construct a 95% confidence interval for β.

$$\sum_{i=1}^{n}\left(x_i-2\right) = 1. \qquad \chi^2_{0.975}(8) = 2.180, \qquad \chi^2_{0.025}(8) = 17.54.$$

$$\left(\frac{2.180}{2\cdot 1}, \frac{17.54}{2\cdot 1}\right) \qquad \mathbf{(1.09, 8.77)}$$

2. Suppose β is known.

h) (i) Obtain a method of moments estimator of δ, $\tilde{\delta}$.

(ii) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.

Obtain a method of moments estimate of δ, $\tilde{\delta}$.

(i) $\mu = \mathrm{E}(\mathrm{X}) = \delta + \frac{1}{\beta}$.

$\overline{\mathrm{X}} = \tilde{\delta} + \frac{1}{\beta}$. $\quad\Rightarrow\quad$ $\tilde{\delta} = \overline{\mathrm{X}} - \frac{1}{\beta}$.

(ii) $\sum_{i=1}^{n} x_i = 9$. $\quad$ $\overline{x} = 2.25$.

$\tilde{\delta} = \overline{x} - \frac{1}{\beta} = 2.25 - \frac{1}{5} = \mathbf{2.05}$.

i) Is the method of moments estimator $\tilde{\delta}$ an unbiased estimator of δ?

If $\tilde{\delta}$ is not an unbiased estimator of δ, construct an unbiased estimator of δ based on $\tilde{\delta}$.

"Hint": $\mathrm{E}(\overline{\mathrm{X}}) = \mu$.

$\mathrm{E}(\tilde{\delta}) = \mathrm{E}(\overline{\mathrm{X}}) - \frac{1}{\beta} = \mu - \frac{1}{\beta} = \delta$.

$\Rightarrow$ $\tilde{\delta}$ is an unbiased estimator of δ.

j) Find $\text{MSE}(\tilde{\delta})$. "Hint": $\text{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$.

$$f_X(x) = \beta e^{-\beta x + \beta\delta} = \beta e^{-\beta(x-\delta)}, \qquad x > \delta.$$

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y > 0. \qquad \Rightarrow \qquad \text{Var}(Y) = \frac{1}{\lambda^2}.$$

$$f_Y(y) = \lambda e^{-\lambda(y-\delta)}, \quad y > \delta. \qquad \Rightarrow \qquad \text{Var}(Y) = \frac{1}{\lambda^2}.$$

$$\Rightarrow \qquad \sigma^2 = \text{Var}(X) = \frac{1}{\beta^2}.$$

OR

$$E(X^2) = \int_\delta^\infty x^2 \cdot \beta e^{-\beta(x-\delta)}\,dx = \dots \text{ by parts } \dots \text{ twice } \dots$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \dots$$

$$\text{Var}(\tilde{\delta}) = \text{Var}(\overline{X}) = \frac{\sigma^2}{n} = \frac{1}{n\beta^2}.$$

Since $\tilde{\delta}$ is an unbiased estimator of δ, and $\text{bias}(\tilde{\delta}) = 0$,

$$\text{MSE}(\tilde{\delta}) = \text{Var}(\tilde{\delta}) = \frac{1}{n\beta^2}.$$

k) (i) Obtain the maximum likelihood estimator of δ, $\hat{\delta}$.

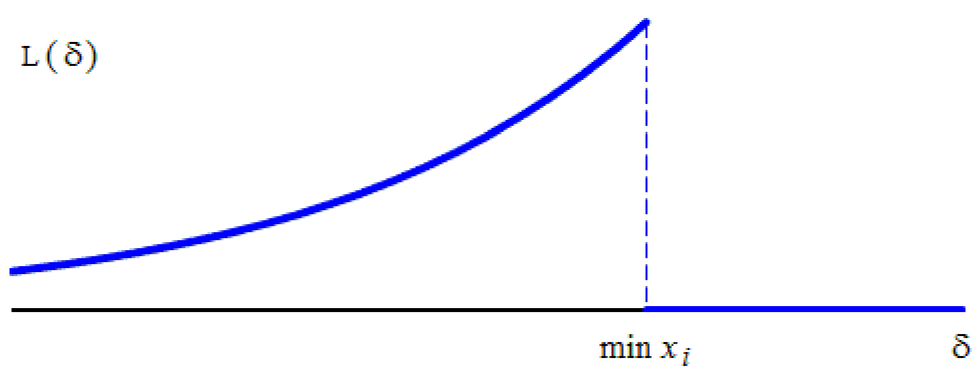
(ii) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$. Obtain the maximum likelihood estimate of δ, $\hat{\delta}$.

"Hint": $x_1 > \delta,\ x_2 > \delta,\ \ldots\ x_n > \delta.$

(i) $$L(\delta) = \prod_{i=1}^{n} f\left(x_i\,;\beta,\delta\right) = \begin{cases} \prod_{i=1}^{n} \beta\, e^{-\beta x_i + \beta\delta} & x_1 > \delta,\ x_2 > \delta,\ \ldots,\ x_n > \delta \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \beta^n\, e^{-\beta \Sigma x_i + n\beta\delta} & \min x_i > \delta \\ 0 & \delta > \min x_i \end{cases}$$

$$\ln L(\delta) = n \ln\beta - \beta \sum_{i=1}^{n} x_i + n\beta\delta. \qquad \frac{d}{d\delta} \ln L(\delta) = n\beta = 0 \ \ ???$$



$\Rightarrow \quad \hat{\delta} = \min X_i.$

(ii) $\hat{\delta} = \min x_i = \mathbf{2.10}.$

1) Is the maximum likelihood estimator $\hat{\delta}$ an unbiased estimator of δ?
If $\hat{\delta}$ is not an unbiased estimator of δ, construct an unbiased estimator of δ based on $\hat{\delta}$.

"Hint": $F_{\min X_i}(x) = 1 - (1 - F(x))^n$.

$$F_X(x) = \int_{\delta}^{x} \beta e^{-\beta u + \beta \delta} du = 1 - e^{-\beta x + \beta \delta} = 1 - e^{-\beta (x - \delta)}, \qquad x > \delta.$$

$$F_{\min X_i}(x) = 1 - (1 - F_X(x))^n = 1 - e^{-n\beta(x-\delta)}, \qquad x > \delta.$$

$$f_{\min X_i}(x) = n\beta e^{-n\beta(x-\delta)}, \qquad x > \delta.$$

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y > 0. \qquad \Rightarrow \qquad E(Y) = \frac{1}{\lambda}.$$

$$f_Y(y) = \lambda e^{-\lambda(y-\delta)}, \quad y > \delta. \qquad \Rightarrow \qquad E(Y) = \delta + \frac{1}{\lambda}.$$

$$\Rightarrow \qquad E(\min X_i) = \delta + \frac{1}{n\beta}.$$

OR $$E(\min X_i) = \int_{\delta}^{\infty} x \cdot n\beta e^{-n\beta(x-\delta)} dx = \ldots \text{ by parts } \ldots$$

$$E(\min X_i) = \delta + \frac{1}{n\beta} \neq \delta.$$

$\hat{\delta}$ is NOT an unbiased estimator of δ.

Consider $$\hat{\hat{\delta}} = \min X_i - \frac{1}{n\beta}.$$

$$E(\hat{\hat{\delta}}) = \delta.$$

$\hat{\hat{\delta}}$ is an unbiased estimator of δ.

m) Find $\mathrm{MSE}(\hat{\delta}) = (\mathrm{bias}(\hat{\delta}))^2 + \mathrm{Var}(\hat{\delta})$.

$$\mathrm{bias}(\hat{\delta}) = \mathrm{E}(\hat{\delta}) - \delta = \frac{1}{n\beta}.$$

$$f_{\min X_i}(x) = n\beta e^{-n\beta(x-\delta)}, \qquad x > \delta.$$

$$f_{Y}(y) = \lambda e^{-\lambda y}, \quad y > 0. \qquad \Rightarrow \qquad \mathrm{Var}(Y) = \frac{1}{\lambda^2}.$$

$$f_{Y}(y) = \lambda e^{-\lambda(y-\delta)}, \quad y > \delta. \qquad \Rightarrow \qquad \mathrm{Var}(Y) = \frac{1}{\lambda^2}.$$

$$\Rightarrow \qquad \mathrm{Var}(\min X_i) = \frac{1}{(n\beta)^2}.$$

OR

$$\mathrm{E}((\min X_i)^2) = \int_{\delta}^{\infty} x^2 \cdot n\beta e^{-n\beta(x-\delta)}\,dx = \ldots$$

$$\mathrm{Var}(\min X_i) = \mathrm{E}((\min X_i)^2) - [\mathrm{E}(\min X_i)]^2 = \ldots$$

$$\mathrm{MSE}(\hat{\delta}) = \left(\frac{1}{n\beta}\right)^2 + \frac{1}{(n\beta)^2} = \frac{2}{n^2\beta^2}.$$

Note that even though $\tilde{\delta}$ is an unbiased estimator of δ,

$\hat{\delta}$ is NOT an unbiased estimator of δ,

$$\mathrm{MSE}(\tilde{\delta}) = \frac{1}{n\beta^2} > \frac{2}{n^2\beta^2} = \mathrm{MSE}(\hat{\delta}) \qquad \text{for } n > 2.$$

For large n, $\quad \mathrm{MSE}(\tilde{\delta}) >> \mathrm{MSE}(\hat{\delta})$. $\quad \hat{\delta}$ is a better estimator.

n) Find a sufficient statistic $Y = u(X_1, X_2, \ldots, X_n)$ for δ.

$$\prod_{i=1}^{n} f(x_i; \beta, \delta) = \prod_{i=1}^{n} \beta e^{-\beta x_i + \beta\delta} \cdot I_{\{x_i > \delta\}}$$

$$= \beta^n e^{-\beta \Sigma x_i + n\beta\delta} \cdot I_{\{\min x_i > \delta\}}.$$

By Factorization Theorem,

$\min X_i$ is a sufficient statistic for δ.

o) Find c such that $P(\delta < \min X_i < \delta + c) = 1 - \alpha$.

"Hint": c depends on α, β, n.

Recall (part (l)): $F_{\min X_i}(x) = 1 - e^{-n\beta(x-\delta)}, \qquad x \ge \delta.$

$$1 - \alpha = P(\delta < \min X_i < \delta + c) = F_{\min X_i}(\delta + c) - F_{\min X_i}(\delta) = 1 - e^{-n\beta c}.$$

$$\alpha = e^{-n\beta c}. \qquad \Rightarrow \qquad c = -\frac{1}{n\beta}\ln\alpha = \frac{1}{n\beta}\ln\frac{1}{\alpha}$$

p) Suppose $n = 4$, $\beta = 5$, and $x_1 = 2.15$, $x_2 = 2.55$, $x_3 = 2.10$, $x_4 = 2.20$.
Use part (o) to construct a 95% confidence interval for δ.

"Hint": $1 - \alpha = P(\delta < \min X_i < \delta + c) = P(\clubsuit < \delta < \spadesuit)$.

Then $(\clubsuit, \spadesuit)$ is a $(1-\alpha)100\%$ confidence interval for δ.

$$1 - \alpha = P(\delta < \min X_i < \delta + c) = P(\min X_i - c < \delta < \min X_i).$$

A $(1-\alpha)\,100\,\%$ confidence interval for δ

$$(\min X_i - c,\ \min X_i).$$

$\alpha = 0.05$. $\qquad c = -\dfrac{1}{n\beta} \ln \alpha = -\dfrac{1}{20} \ln 0.05 \approx 0.15.$

$(2.10 - 0.15,\ 2.10)$ $\qquad$ **$(1.95, 2.10)$**