

Left – tailed test

$$H_0 : p = p_0 \quad \text{vs.} \quad H_1 : p < p_0$$

If H_0 is TRUE :

Use p_0 .

Reject H_0		Do NOT Reject H_0	
Type I Error α		Correct decision	
0	a	$a+1$	n

Rejection Rule for a Left – tailed test:

Find a such that $P(X \leq a) = \text{CDF @ } a \approx \alpha$. (using Binomial(n, p_0))

Then the Rejection Rule is “Reject H_0 if $X \leq a$.”

If Rejection Rule is “Reject H_0 if $X \leq a$,”

$$\begin{aligned}
 P(\text{Type I error}) &= P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \leq a \mid p = p_0) \\
 &= \text{CDF @ } a \quad \text{(using Binomial}(n, p_0))
 \end{aligned}$$

If H_0 is FALSE :

Use new (given) p .

Reject H_0		Do NOT Reject H_0	
Correct decision Power		Type II Error	
0	a	$a+1$	n

$$\text{Power} = P(\text{Reject } H_0) = P(X \leq a) = \text{CDF @ } a \quad \text{(using Binomial}(n, \text{new } p))$$

$$\text{p-value} = P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true})$$

$$= P(X \leq x_{\text{observed}} \mid p = p_0) = \text{CDF @ } x_{\text{observed}} \quad \text{(using Binomial}(n, p_0))$$

Right – tailed test

$$H_0 : p = p_0 \quad \text{vs.} \quad H_1 : p > p_0$$

If H_0 is TRUE :

Use p_0 .

Do NOT Reject H_0	Reject H_0
Correct decision	Type I Error α
0 $b-1$	b n

Rejection Rule for a Right – tailed test:

Find b such that $P(X \leq b-1) = \text{CDF @ } (b-1) \approx 1 - \alpha$. (using Binomial(n, p_0))

(Then $P(X \geq b) \approx \alpha$.)

Then the Rejection Rule is “Reject H_0 if $X \geq b$.”

If Rejection Rule is “Reject H_0 if $X \geq b$,”

$$P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \geq b \mid p = p_0)$$

$$= (1 - \text{CDF @ } b-1) \quad \text{(using Binomial}(n, p_0))$$

If H_0 is FALSE :

Use new (given) p .

Do NOT Reject H_0	Reject H_0
Type II Error	Correct decision Power
0 $b-1$	b n

$$\text{Power} = P(\text{Reject } H_0) = P(X \geq b) = (1 - \text{CDF @ } (b-1)) \quad \text{(using Binomial}(n, \text{new } p))$$

$$\text{p-value} = P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true})$$

$$= P(X \geq x_{\text{observed}} \mid p = p_0) = (1 - \text{CDF @ } (x_{\text{observed}} - 1)) \quad \text{(using Binomial}(n, p_0))$$

Two – tailed test

$$H_0 : p = p_0 \quad \text{vs.} \quad H_1 : p \neq p_0$$

If H_0 is TRUE :

Use p_0 .

Reject H_0	Do NOT Reject H_0	Reject H_0
Type I Error $\alpha/2$	Correct decision	Type I Error $\alpha/2$
0 a	a+1 b-1	b n

Rejection Rule for a Two – tailed test:

Find a such that $P(X \leq a) = \text{CDF @ } a \approx \alpha/2$. (using Binomial(n, p_0))

Find b such that $P(X \leq b-1) = \text{CDF @ } (b-1) \approx 1 - \alpha/2$. (Then $P(X \geq b) \approx \alpha/2$.)

(If $p = 0.50$, then $b = n - a$.)

Then the Rejection Rule is “Reject H_0 if $X \leq a$ or $X \geq b$.”

If Rejection Rule is “Reject H_0 if $X \leq a$ or $X \geq b$,”

$$\begin{aligned}
 P(\text{Type I error}) &= P(\text{Reject } H_0 \mid H_0 \text{ true}) = P(X \leq a \text{ or } X \geq b \mid p = p_0) \\
 &= \text{CDF @ } a + (1 - \text{CDF @ } (b-1)) \quad (\text{using Binomial}(n, p_0))
 \end{aligned}$$

If H_0 is FALSE :

Use new (given) p .

Reject H_0	Do NOT Reject H_0	Reject H_0
Correct decision Power	Type II Error	Correct decision Power
0 a	a+1 b-1	b n

$$\begin{aligned}
 \text{Power} &= P(\text{Reject } H_0) = P(X \leq a \text{ or } X \geq b) = \text{CDF @ } a + (1 - \text{CDF @ } (b-1)) \\
 &\quad (\text{using Binomial}(n, \text{new } p))
 \end{aligned}$$

$$H_0 : p = \frac{1}{2} \quad \text{vs.} \quad H_1 : p \neq \frac{1}{2}$$

If x_{observed} is the left tail (i.e., if $x_{\text{observed}} < n \cdot \frac{1}{2}$),

$$\begin{aligned} \text{p-value} &= P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) \\ &= 2 \cdot P(X \leq x_{\text{observed}} \mid p = \frac{1}{2}) \quad (\text{since it is a two-tail test}) \\ &= 2 \cdot \text{CDF @ } x_{\text{observed}} \quad (\text{using Binomial}(n, \frac{1}{2})) \end{aligned}$$

If x_{observed} is the right tail (i.e., if $x_{\text{observed}} > n \cdot \frac{1}{2}$),

$$\begin{aligned} \text{p-value} &= P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) \\ &= 2 \cdot P(X \geq x_{\text{observed}} \mid p = \frac{1}{2}) \quad (\text{since it is a two-tail test}) \\ &= 2 \cdot (1 - \text{CDF @ } (x_{\text{observed}} - 1)) \quad (\text{using Binomial}(n, \frac{1}{2})) \end{aligned}$$

If $x_{\text{observed}} = n \cdot \frac{1}{2}$,

$$\text{p-value} = P(\text{value of } X \text{ as extreme or more extreme than } X = x_{\text{observed}} \mid H_0 \text{ true}) = 1.$$