Examples for 09/28/2020 (2)

0. Consider a continuous random variable X with the probability density function

$$f_X(x) = \frac{x^3}{60}, \qquad 2 \le x \le 4, \qquad \text{zero elsewhere.}$$

Let Y follows a Uniform distribution on $(0, 5)$.

Suppose that X and Y are independent.

$$F_X(x) = \begin{cases} 0 & x < 2 \\ \dfrac{x^4 - 16}{240} & 2 \le x < 4 \\ 1 & x \ge 4 \end{cases} \qquad F_Y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y}{5} & 0 \le y < 5 \\ 1 & y \ge 5 \end{cases}$$

a) Find the probability distribution of $W = \max(X, Y)$.

$$F_W(x) = P(\max(X, Y) \le w) = P(X \le w, Y \le w)$$

$$= P(X \le w) \cdot P(Y \le w) = F_X(w) \cdot F_Y(w)$$

$$= \begin{cases} 0 \cdot 0 = 0 & w < 0 \\ 0 \cdot \dfrac{w}{5} = 0 & 0 \le w < 2 \\ \dfrac{w^4 - 16}{240} \cdot \dfrac{w}{5} & 2 \le w < 4 \\ 1 \cdot \dfrac{w}{5} = \dfrac{w}{5} & 4 \le w < 5 \\ 1 \cdot 1 = 1 & w \ge 5 \end{cases}$$

b) Find the probability distribution of $V = \min(X, Y)$.

$$F_V(x) = P(\min(X,Y) \le v) = 1 - P(\min(X,Y) > v) = 1 - P(X > v, Y > v)$$

$$= 1 - P(X > v)\cdot P(Y > v) = 1 - (1 - F_X(v))\cdot(1 - F_Y(v))$$

$$= \begin{cases} 1 - (1-0)\cdot(1-0) = 0 & v < 0 \\ 1 - (1-0)\cdot\left(1 - \dfrac{v}{5}\right) = \dfrac{v}{5} & 0 \le v < 2 \\ 1 - \left(1 - \dfrac{v^4 - 16}{240}\right)\cdot\left(1 - \dfrac{v}{5}\right) & 2 \le v < 4 \\ 1 - (1-1)\cdot\left(1 - \dfrac{v}{5}\right) = 1 & 4 \le v < 5 \\ 1 - (1-1)\cdot(1-1) = 1 & v \ge 5 \end{cases}$$

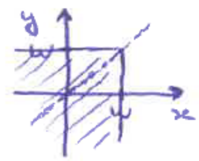
1. Consider two continuous random variables X and Y with joint p.d.f.

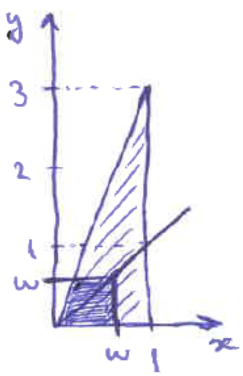
$$f_{X,Y}(x,y) = \frac{4}{3}x^3 y, \qquad 0 < x < 1, \quad 0 < y < 3x, \qquad \text{zero otherwise.}$$

o) Find the probability distribution of $W = \max(X, Y)$.

$W = \max(X, Y)$.

$F_W(w) = P(\max(X,Y) \le w) = P(X \le w,\ Y \le w) = \ldots$

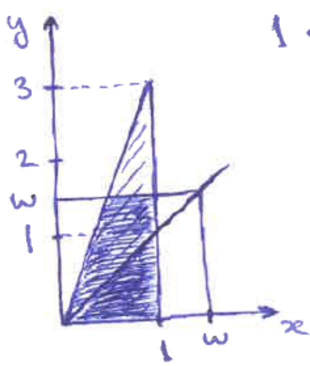




$0 < w < 1$

$$\ldots = \int_0^w \left(\int_{y/3}^w \frac{4}{3} x^3 y \, dx \right) dy$$

$$= \frac{w^6}{6} - \frac{w^6}{1458} = \frac{121}{729} w^6, \qquad 0 < w < 1.$$



$1 < w < 3$

$$\ldots = \int_0^w \left(\int_{y/3}^1 \frac{4}{3} x^3 y \, dx \right) dy$$

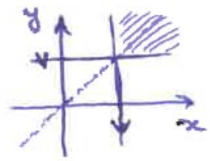
$$= \frac{w^2}{6} - \frac{w^6}{1458}, \qquad 1 < w < 3.$$

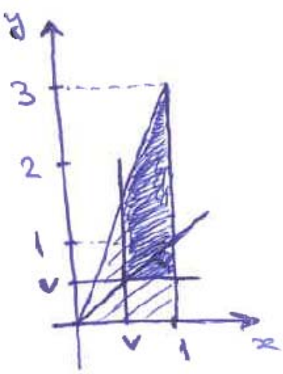
p) Find the probability distribution of $V = \min(X, Y)$.

$V = \min(X, Y)$

$F_V(v) = P(\min(X,Y) \le v) = 1 - P(\min(X,Y) > v)$

$= 1 - P(X > v, Y > v) = \ldots$





$0 < v < 1$

$$\ldots = 1 - \int_v^1 \left(\int_v^{3x} \frac{4}{3} x^3 y \, dy \right) dx$$

$$= \frac{v^2 + 5v^6}{6}, \qquad 0 < v < 1.$$