Examples for 10/07/2020   (1)

Let $\quad X_1, X_2, \ldots, X_n \quad$ be i.i.d. $\ \mathbf{N}\left(\mu, \sigma^2\right)$.

Let

$$\overline{X} = \frac{\sum X_i}{n} = \frac{X_1 + X_2 + \ldots + X_n}{n} \qquad (\text{ sample mean })$$

$$S^2 = \frac{\sum\left(X_i - \overline{X}\right)^2}{n-1} \qquad (\text{ sample variance })$$

Then

$\overline{X}$ and S^2 are independent;

$\overline{X}$ has $\mathbf{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ distribution;

$\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$ has $\mathbf{N}(0,1)$ distribution;

$\dfrac{\sum\left(X_i - \mu\right)^2}{\sigma^2}$ has $\chi^2(n)$ distribution;

$\dfrac{(n-1)\cdot S^2}{\sigma^2} = \dfrac{\sum\left(X_i - \overline{X}\right)^2}{\sigma^2}$ has $\chi^2(n-1)$ distribution;

$\dfrac{\overline{X} - \mu}{S / \sqrt{n}}$ has $t(n-1)$ distribution.

William Gosset
(1876-1937)

The t Distribution

$$f(t)=\frac{\Gamma\left(v+\frac{1}{2}\right)}{\sqrt{v\pi}\ \Gamma\left(\frac{v}{2}\right)}\left(1+\frac{t^2}{v}\right)^{-(v+1)/2}$$

$$-\infty<t<\infty$$

T has t distribution with v degrees of freedom $(v>2)$

$$E(T)=0$$

$$Var(T)=\frac{v}{v-2}$$

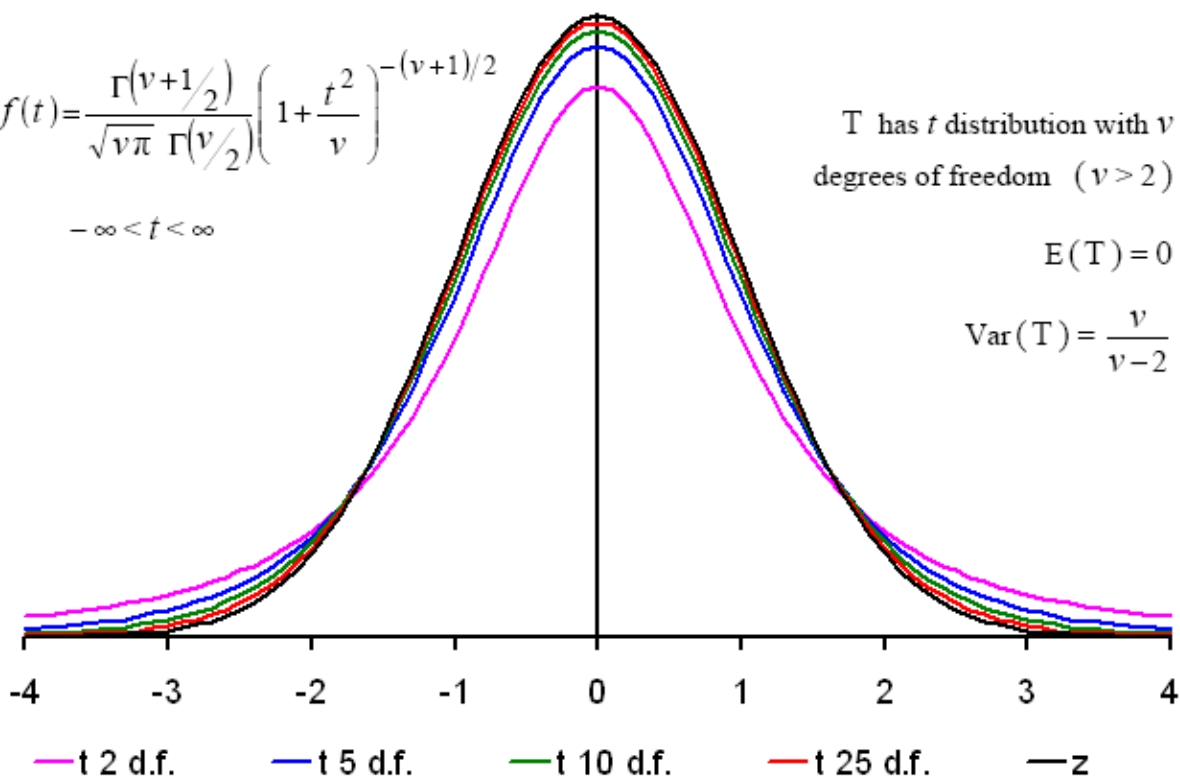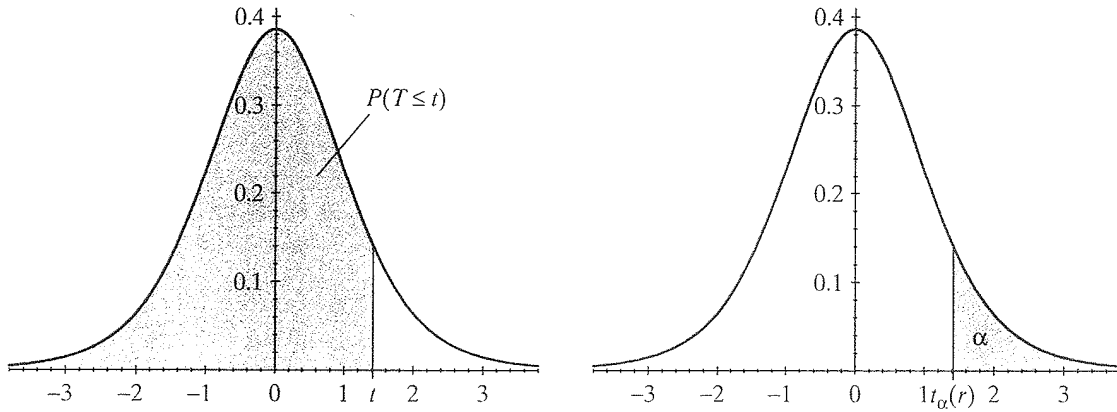|				
— t 2 d.f.	— t 5 d.f.	— t 10 d.f.	— t 25 d.f.	— z

TABLE VI: The t Distribution

$$P(T \leq t) = \int_{-\infty}^{t} \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\,\Gamma(r/2)(1 + w^2/r)^{(r+1)/2}}\, dw$$

$$P(T \leq -t) = 1 - P(T \leq t)$$

				$P(T \leq t)$			
	0.60	0.75	0.90	0.95	0.975	0.99	0.995
r	$t_{0.40}(r)$	$t_{0.25}(r)$	$t_{0.10}(r)$	$t_{0.05}(r)$	$t_{0.025}(r)$	$t_{0.01}(r)$	$t_{0.005}(r)$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576