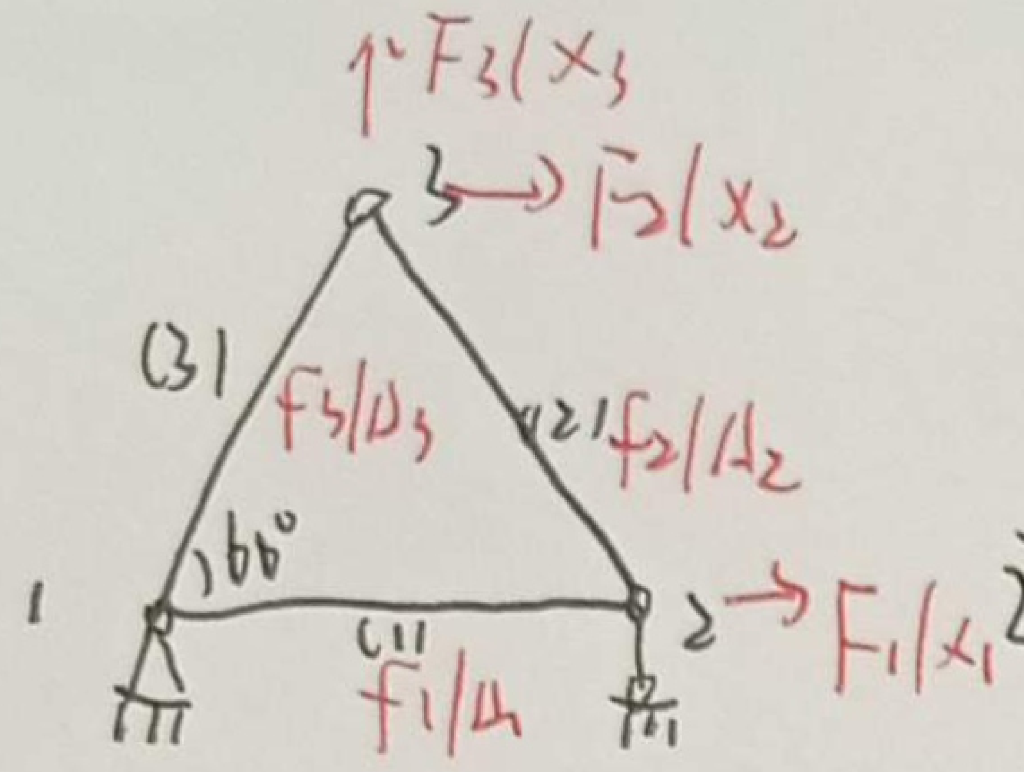


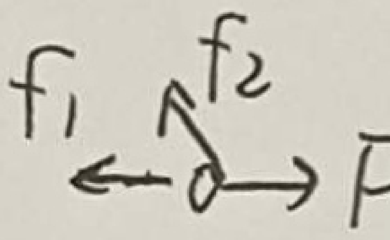
各杆拉压刚度均为EA，各杆均视为二力杆。

已知 $Af=F$，$BX=\Delta$，其中，f 为杆内力矩阵，F 为结点力矩阵，X 为结点位移矩阵，Δ 为杆变形矩阵。

试证：$A=B^T$。

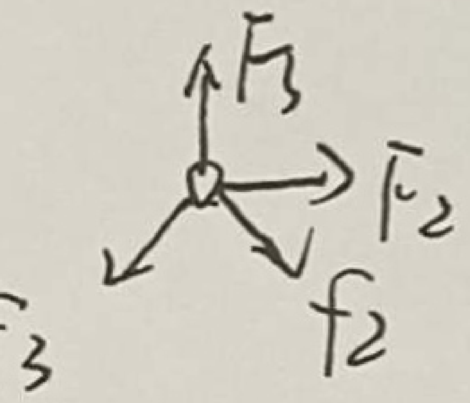
证：① 求A。由 $Af=F$，即 $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}=\begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$，$\Leftrightarrow \begin{cases} A_{11}f_1+A_{12}f_2+A_{13}f_3=F_1 \\ \vdots \\ A_{31}f_1+\cdots+A_{33}f_3=F_3 \end{cases}$

对结点2，列平衡方程，有：$\Sigma F_x=0$，$F_1-f_1-\frac{1}{2}f_2=0$



对结点3同理有：$\Sigma F_x=0$，$F_2+\frac{1}{2}f_2-\frac{1}{2}f_3=0$

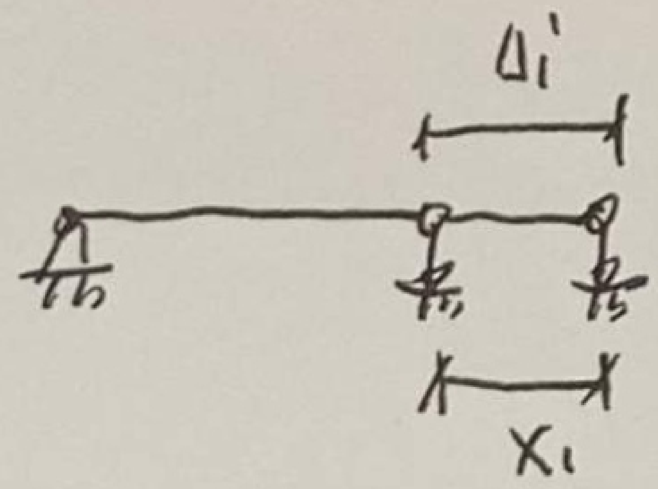
$\Sigma F_y=0$，$F_3-\frac{\sqrt{3}}{2}f_2-\frac{\sqrt{3}}{2}f_3=0$.



$$\Rightarrow A=\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

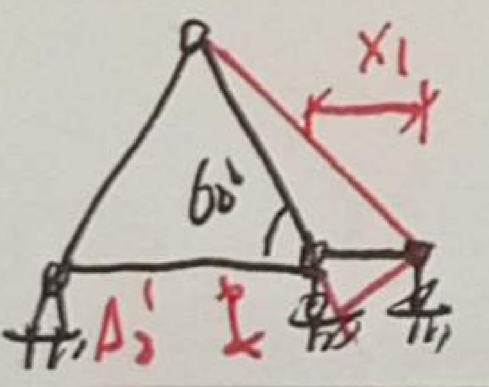
② 求B。由 $BX=\Delta$，同理有 $\begin{cases} B_{11}X_1+\cdots+B_{13}X_3=\Delta_1 \\ \vdots \\ B_{31}X_1+\cdots+B_{33}X_3=\Delta_3 \end{cases}$

对 Δ_1：

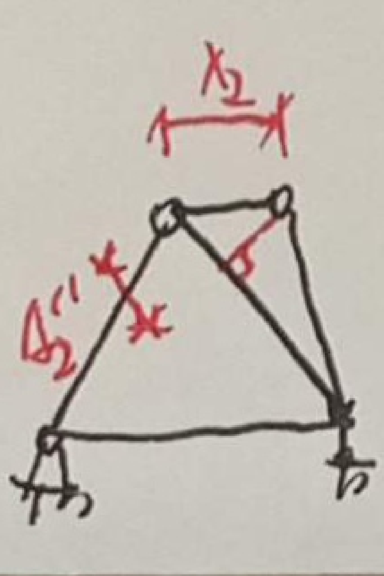


$\Delta_1=\Delta_1'=X_1$.

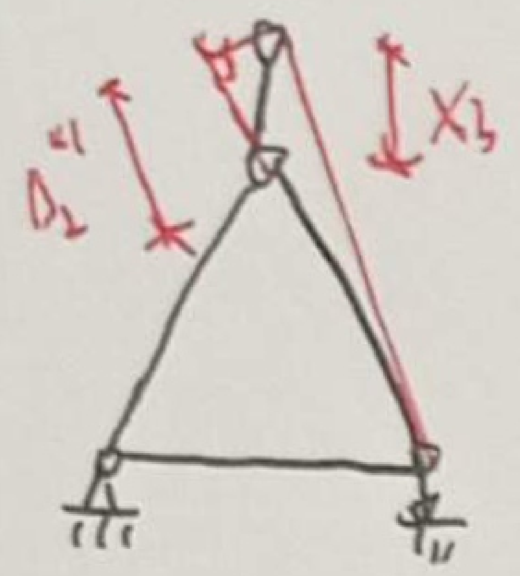
对 Δ_2：



$\Delta_2'=\frac{1}{2}X_1$.



$\Delta_2''=-\frac{1}{2}X_2$.



$\Delta_2''' = \frac{\sqrt{3}}{2} X_3$. $\Rightarrow \Delta_2 = \Delta_2' + \Delta_2'' + \Delta_2''' = \frac{1}{2} X_1 \cancel{+} - \frac{1}{2} X_2 + \frac{\sqrt{3}}{2} X_3$

同理,得 $\Delta_3 = \frac{1}{2} X_2 + \frac{\sqrt{3}}{2} X_3$.

$$\Rightarrow B = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

$$\text{而} A = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

$\Rightarrow A = B^T$. 证毕!