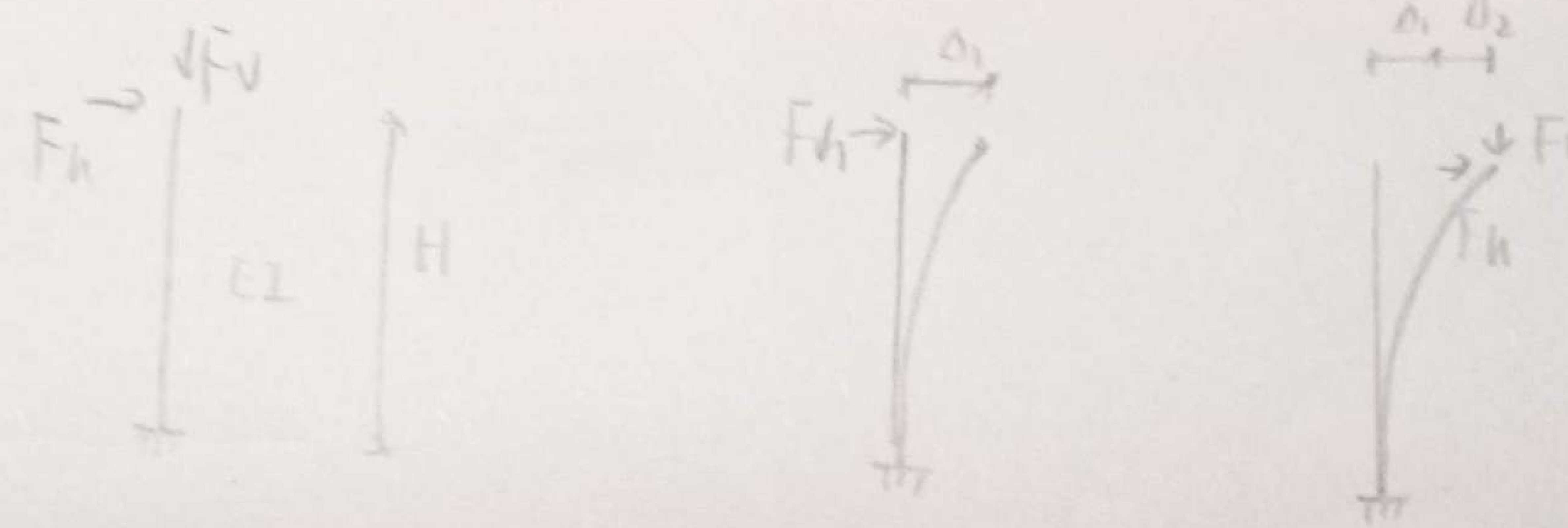
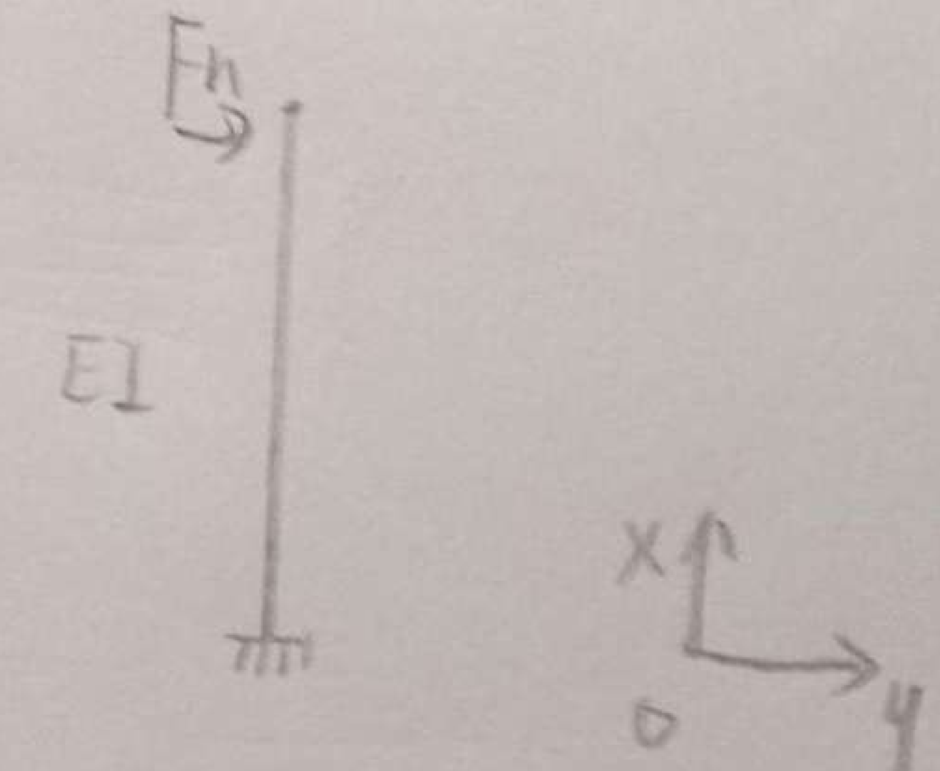
作业：



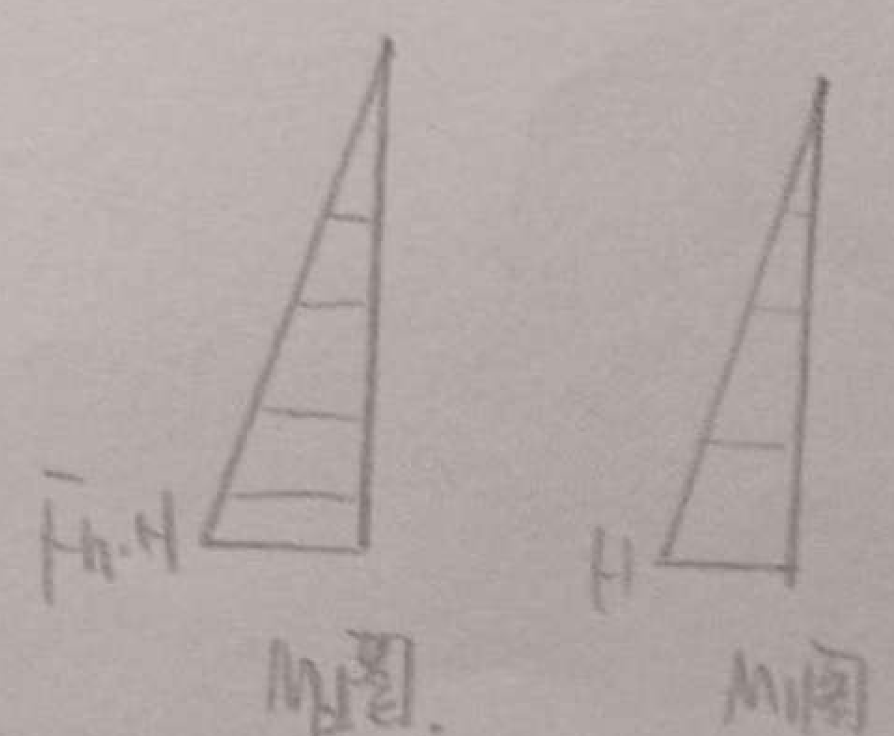
求Fv作用后引起杆端水平位移为Δ_2.



同理：在Fh作用下，杆端水平位移为Δ_1.

由图乘原理，得 $\Delta_1 = \frac{1}{EI}\int M_P M_1\, ds = \frac{1}{EI}\left(\frac{1}{2}\times F_h H \times H \times \frac{2}{3}H\right)$

$$= \frac{F_h H^3}{3EI}$$



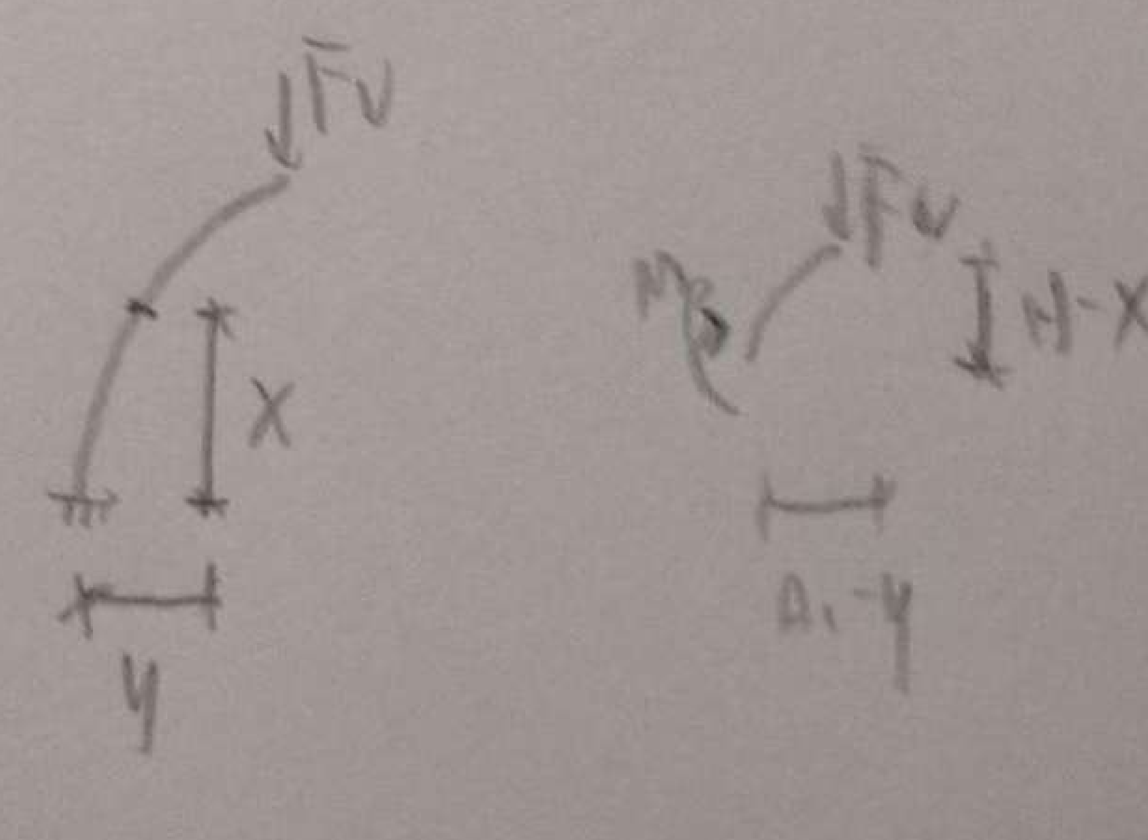
同时，由挠曲线微分方程 $M = -EIy''$.

$M = -F_h(H-x) \Rightarrow y'' = \frac{F_h}{EI}(H-x), \Rightarrow y = \frac{F_h}{EI}\left(\frac{1}{2}Hx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$.

由 $y|_{x=0} = 0$, $y|_{x=H} = \Delta_1$. $\Rightarrow C_2 = 0$, $C_1 = 0$.

故 $y = \frac{F_h}{EI}\left(\frac{1}{2}Hx^2 - \frac{1}{6}x^3\right)$.

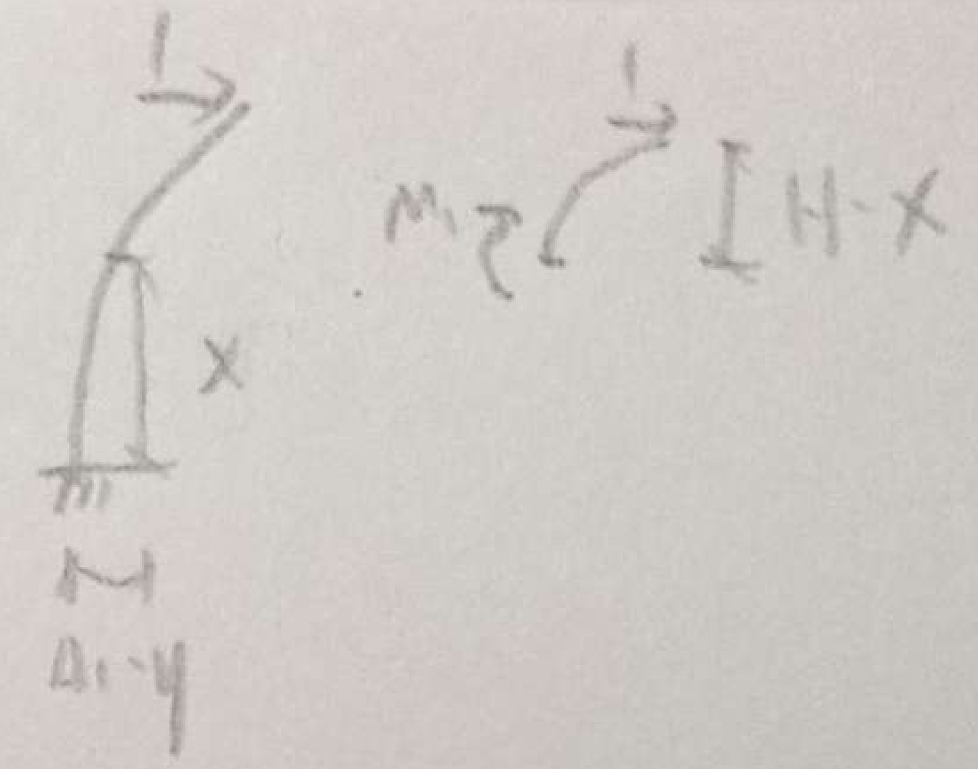
在Fv作用下，如右图所示.



$\Sigma M = 0$, $-M_p - F_v(\Delta_1 - y) = 0$,

$\Rightarrow M_p = -F_v\left[\frac{F_h H^3}{3EI} - \frac{F_h}{EI}\left(\frac{1}{2}Hx^2 - \frac{1}{6}x^3\right)\right]$.

在单位水平力作用下，$\Sigma M=0$，$-M_1-(H-x)=0$.



即 $M_1=-(H-x)$.

由虚功原理，得

$$\Delta_2=\int\frac{M_P M_1}{EI}ds=\frac{1}{EI}\int_0^H M_P M_1\sqrt{1+(y')^2}\,dx$$

$$=\frac{1}{EI}\int_0^H F_v\cdot\left[\frac{F_h H^3}{3EI}-\frac{F_h}{EI}\left(\frac{1}{2}Hx^2-\frac{1}{6}x^3\right)\right]\cdot(H-x)\cdot\sqrt{1+\left[\frac{F_h}{EI}\left(Hx-\frac{1}{2}x^2\right)\right]^2}\,dx$$

$$=\frac{F_v F_h}{(EI)^2}\int_0^H\left(\frac{1}{3}H^3-\frac{1}{2}Hx^2+\frac{1}{6}x^3\right)(H-x)\cdot\sqrt{1+\left[\frac{F_h}{EI}\left(Hx-\frac{1}{2}x^2\right)\right]^2}\,dx.$$

此项积分不易手算得到结果，当各参数明确，计算机可快速计算得到其值。

不过近似计算可运用泰勒展开：$\sqrt{1+\left[\frac{F_h}{EI}\left(Hx-\frac{1}{2}x^2\right)\right]^2}\approx 1+\frac{F_h}{2(EI)^2}\left(Hx-\frac{1}{2}x^2\right)^2$，仅仅是多项式的积分繁琐而已。