傅里叶变换.

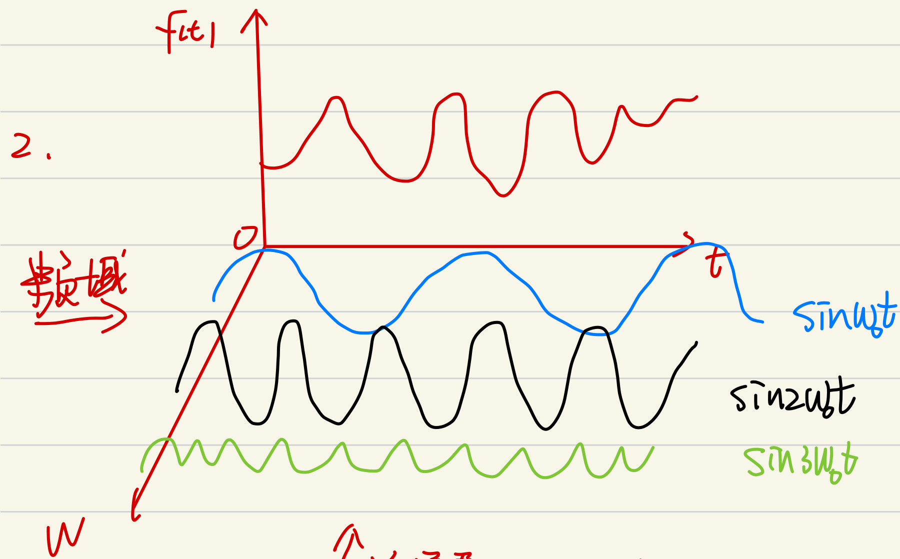
频率为w. 也称为基频率

1. 任何周期函数 $f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos(nwt) + b_n\sin(nwt)]$

（傅里叶级数）

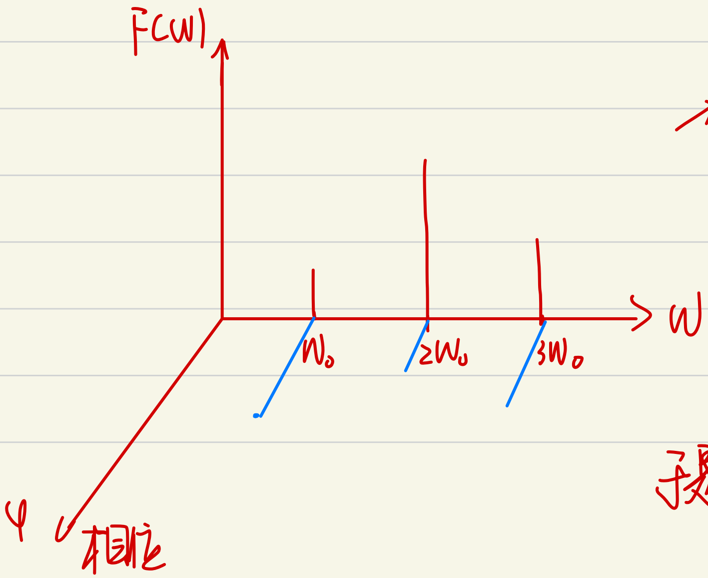
$T = \frac{2\pi}{w}$

2.



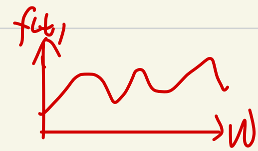
这样看即为时域

为了和坐标轴保持一致, W改为W0. 此外W是f(t)的周期反



见周期函数的频率是离散的点, 而对于非周期函数, 可看成是周期为无穷大.

于是 $W_0 \to 0$, ⇒



$1、\sin\omega t、\cos\omega t、\cdots\quad \sin n\omega t、\cos n\omega t\cdots$ 称为一组基.

是正交的. 但 $\overrightarrow{\sin\omega t} = (\sin\omega t_1, \sin\omega t_2, \cdots)$

故 $\overrightarrow{\sin\omega t}\cdot\overrightarrow{\cos\omega t} = \sum_{n=1}^{\infty} \sin\omega t_n \cos\omega t_n = \int_0^{2\pi} \sin\omega t\cos\omega t\,dt$

$= 0.$

(形象理解,不严谨)

其余同理.

对于 $f(t) = a_0 + \sum_{n=1}^{\infty} [a_n\cos(n\omega t) + b_n \sin(n\omega t)]$

$$\xlongequal[e^{ix}=\cos x+i\sin x]{a_0纳入其中} \sum_{n=0}^{\infty} \left[a_n \cdot \frac{e^{in\omega t}+e^{-in\omega t}}{2} + b_n\cdot\frac{e^{in\omega t}-e^{-in\omega t}}{2i}\right] \quad (b_0=0)$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{a_n}{2}+\frac{b_n}{2i}\right)e^{in\omega t} + \left(\frac{a_n}{2}-\frac{b_n}{2i}\right)e^{-in\omega t}\right]$$

$$\xlongequal[对称]{n\omega与-n\omega} \sum_{n=-\infty}^{\infty} C_n e^{in\omega t}. \qquad (例如\ C_{-1} = \frac{a_1}{2}-\frac{b_1}{2i}).$$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty} \underbrace{C_n}_{坐标} \underbrace{e^{in\omega t}}_{基}$$

$$C_m e^{im\omega t} = f(t) - \sum_{n\neq m} C_n e^{in\omega t},$$

$$\Rightarrow \quad C_m = f(t)\, e^{-im\omega t} - \left(\sum_{n\neq m} C_n e^{in\omega t}\right)\cdot e^{-im\omega t}$$

在$f(t)$周期上积分有

$$\int_{-\frac{T}{2}}^{\frac{T}{2}} C_m = \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-im\omega t}\, dt$$

$$- \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\sum_{n\neq m} C_n e^{in\omega t}\right)\cdot e^{-im\omega t}\, dt$$

三角函数在[0,T]上正交

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-im\omega t}\, dt$$

$$\Rightarrow \quad C_m = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-im\omega t}\, dt.$$

对于非周期函数. $\Delta\omega = (n+1)\omega - n\omega = \omega = \frac{2\pi}{T}$,

$$C_m = \frac{\Delta\omega}{2\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-im\omega t}\, dt$$ ，记 $\omega_n = n\cdot\omega$.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in w t} = \sum_{n=-\infty}^{\infty} \frac{\Delta w}{2\pi} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-i w_n t} dt \cdot e^{i w_n t}$$

当 $T \to \infty$ 时. $\Delta w = w = \frac{2\pi}{T} \to 0$. $\Delta w = dw$.

$$\Rightarrow f(t) = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-iwt} dt \, e^{iwt} dw.$$

$$\left(\lim_{\substack{n \to \infty \\ T \to \infty}} \sum_{j=1}^{n} \frac{\Delta w}{2\pi} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-i w_j t} dt \, e^{i w_j t} \right.$$

$w_j = j \cdot w \in (-\infty, +\infty)$

$\Delta w = w_{j+1} - w_j$

$$\left. \int_{-\infty}^{+\infty} \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-iwt} dt \, e^{iwt} dw \right)$$

常规例子: $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{n+k} = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{1+\frac{k}{n}} \frac{1}{n} = \int_0^1 \frac{1}{1+x} dx.$

$$\Rightarrow f(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{2\pi} f(t) e^{-iwt} dt \, e^{iwt} dw.$$

令 $F(w) = \int_{-\infty}^{+\infty} f(t) e^{-iwt} dt$. 此即为傅里叶变换.

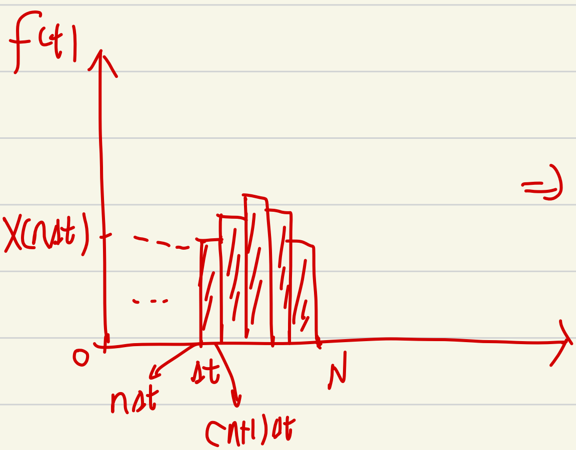
而 $f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(w) e^{iwt} dw$，称为傅里叶逆变换.

对于离散信号，在采样时间T内，有：（离散时间傅里叶变换DTFT）

$\Rightarrow t = n\Delta t$　　Δt为采样时间间隔. $\Delta t = \frac{1}{f_s}$，f_s为采样频率.

由 $F(w) = \int_{-\infty}^{+\infty} f(t) e^{-iwt} dt$,

$$\Rightarrow F(w_k) = \sum_{n=0}^{N-1} f(n\Delta t)\, e^{-iw_k \cdot n\Delta T} = \sum_{n=0}^{N-1} x(n)\, e^{-iknw\Delta T}$$



$$w = \frac{2\pi}{N\cdot\Delta t}$$

$$\Rightarrow F(w_k) = \sum_{n=0}^{N-1} x(n)\, e^{-ikn\cdot\frac{2\pi}{N}}$$

$x(n)$：时域信号第n个样本值.

共有N+1个采样点，总时间 $T = N\cdot\Delta t$.