

2.1 截面计算

面积: 2609000 1024.

解: ① 内力计算.

$$h_1 = 400 + 400 - \frac{1}{2} \times 20 - \frac{1}{2} \times 14 = 780 \text{ mm.}$$

$$h_2 = 2000 - \frac{1}{2} \times 20 = 1990 \text{ mm.}$$

$$\Rightarrow P_1 = \frac{N}{2} + \frac{M}{h_1} = \frac{500}{2} + \frac{650}{0.783} = 1080.1 \text{ kN.}$$

($h_1=780$ 时, $P_1=1083.3 \text{ kN}$, $P_2=-583.3 \text{ kN}$. 由于相差无几,

故后续仍取 $h_1=783 \text{ mm}$ 时内力计算, 误差忽略不计)

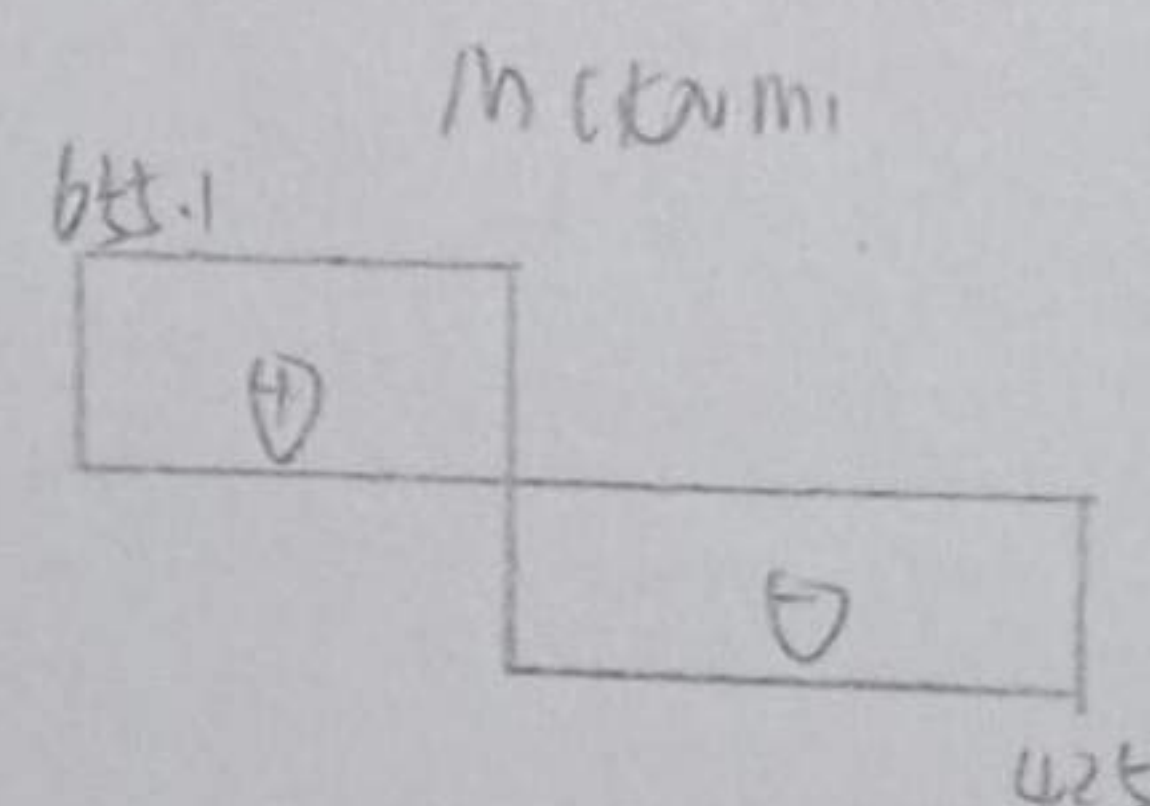
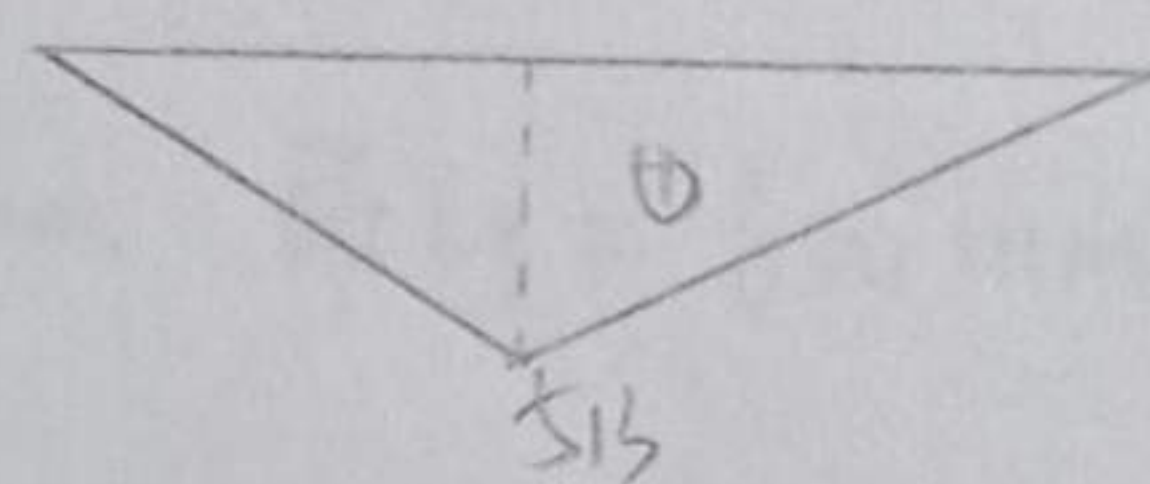
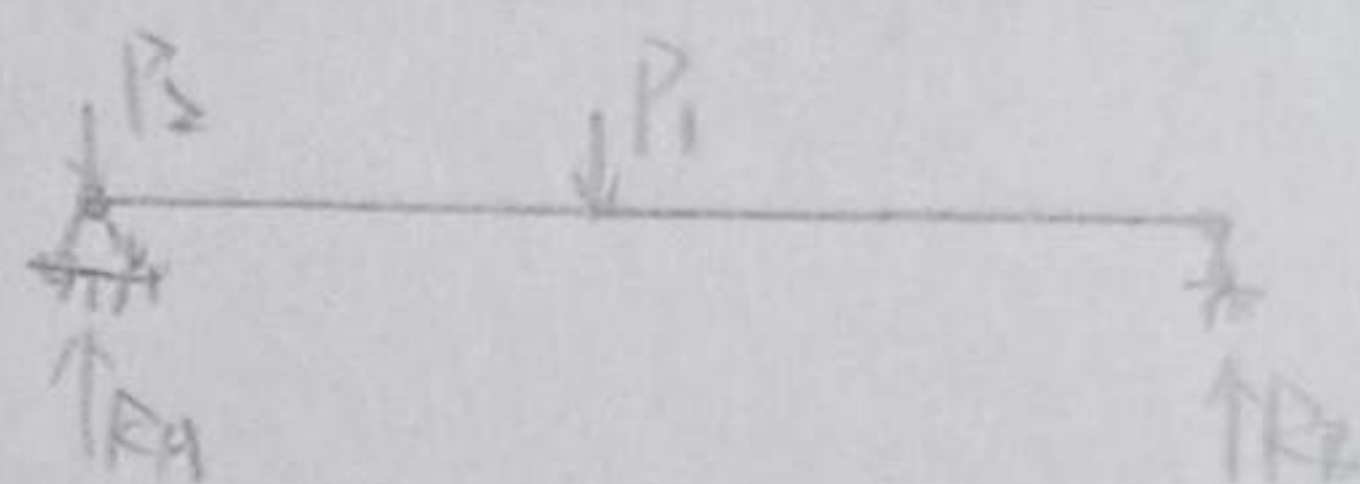
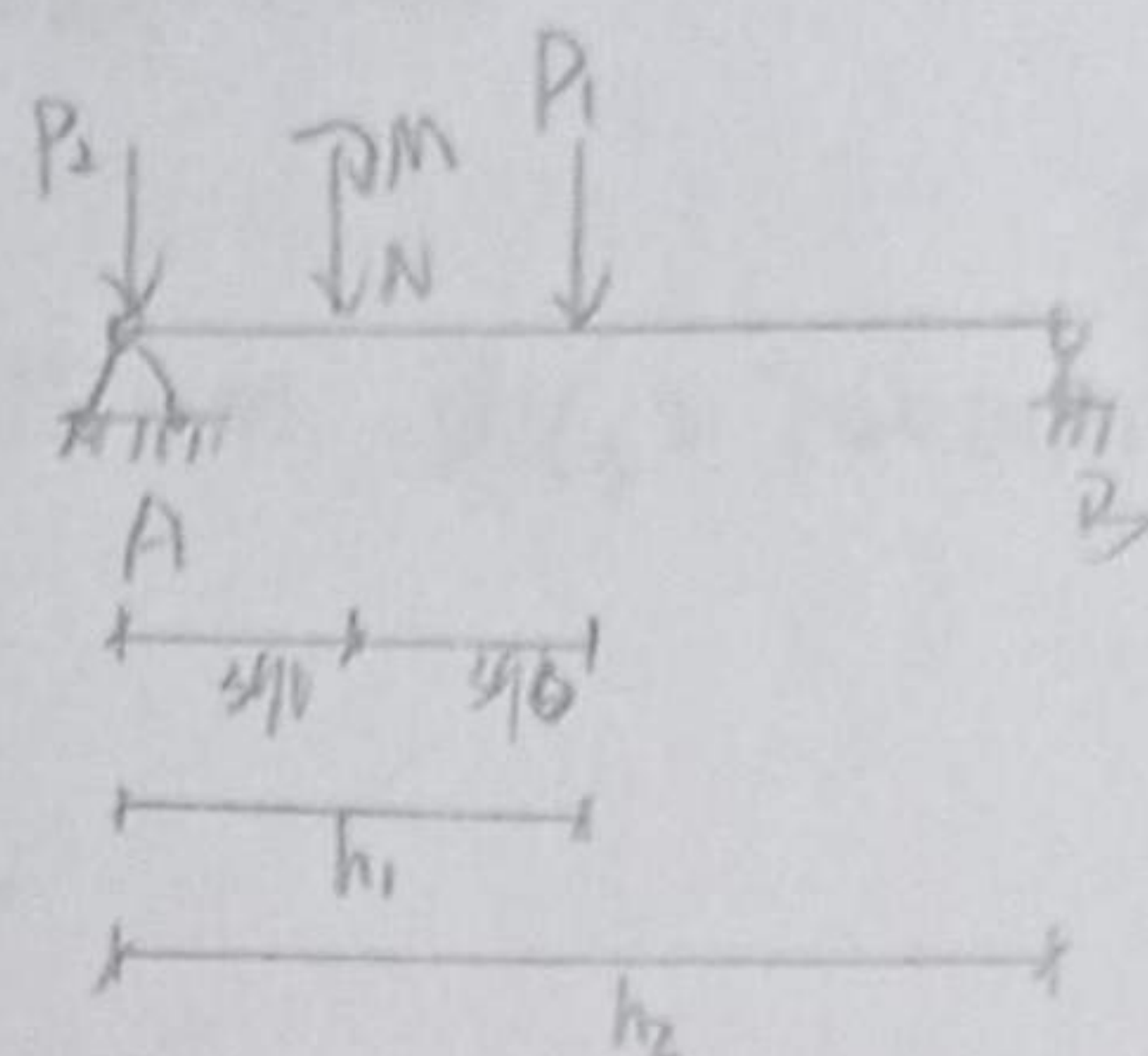
$$P_2 = \frac{N}{2} - \frac{M}{h_1} = \frac{500}{2} - \frac{650}{0.783} = -580.1 \text{ kN (受拉)}$$

$$R_B = \frac{h_1}{h_2} P_1 = \frac{783}{1990} \times 1080.1 = 425 \text{ kN.}$$

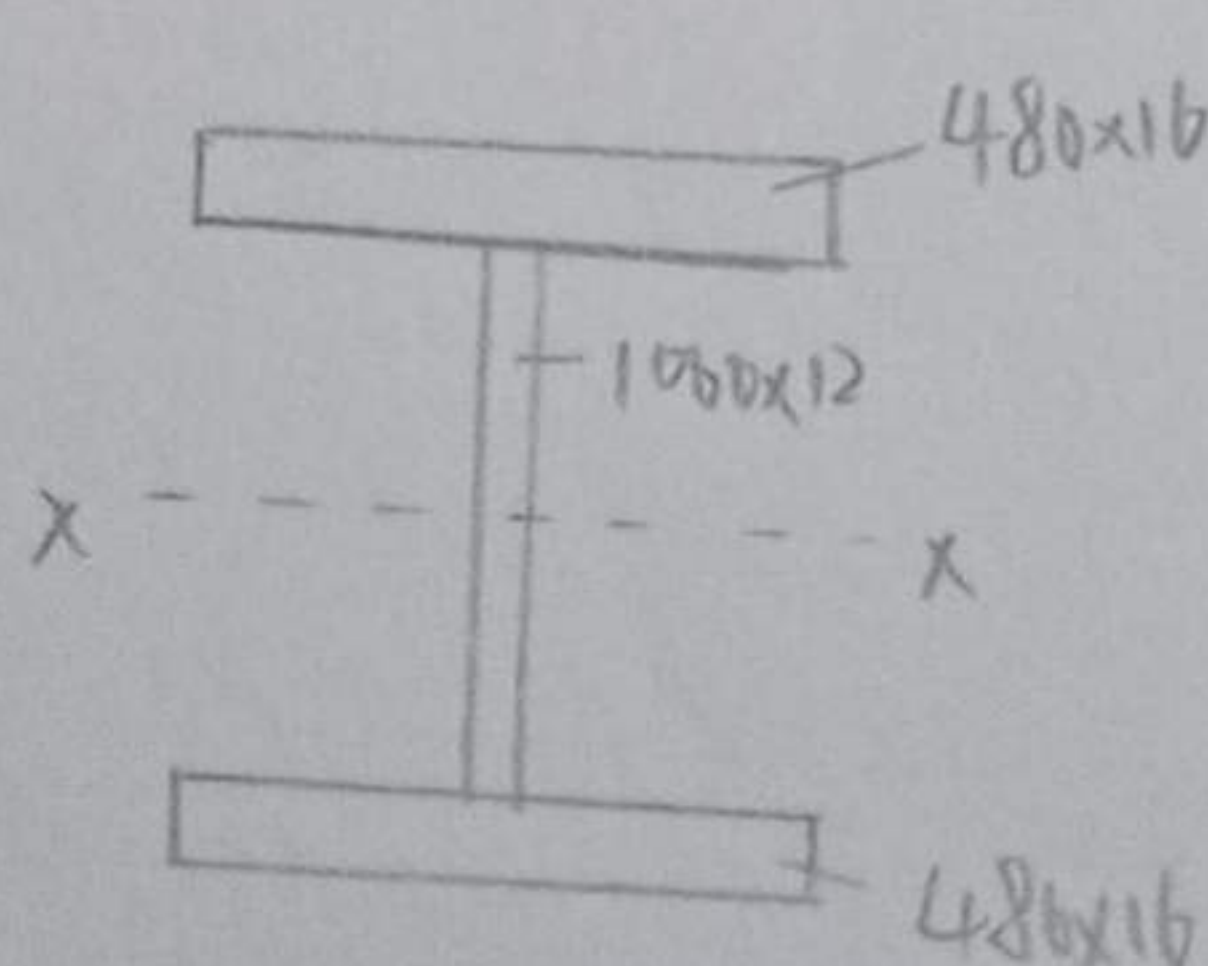
$$R_A = P_1 + P_2 - R_B = 1080.1 - 580.1 - 425 = 75 \text{ kN.}$$

$$M_{\max} = R_B \times h_2 = 425 \times (1.99 - 0.783) = 513 \text{ kN}\cdot\text{m.}$$

$$V_{\max} = 655.1 \text{ kN.}$$



V (kN)



② 截面特征计算.

$$I_x = \frac{1}{12} \times 12 \times 1000^3 + 2 \times 480 \times 16 \times 508^2 = 496 \times 10^9 \text{ mm}^4.$$

$$W_x = \frac{I_x}{500 + 16} = \frac{496 \times 10^9}{516} = 9.61 \times 10^6 \text{ mm}^3.$$

$$\frac{b}{t} = \frac{480 - 12}{2 \times 16} = 14.625 > 13 \varepsilon_k = 13, \text{ 但 } \frac{b}{t} = 15 \varepsilon_k = 15, \text{ 即翼缘全部有效.}$$

同理, 依《钢结构设计标准》表 3.5.1, 对腹板 $\frac{h_0}{t_w} = \frac{1000}{12} = 83.3 \leq 124 \varepsilon_k = 124$, 即腹板也全部有效.

3.2.1.1 例题

1° 上翼缘外边缘最大正应力

$$\sigma_{max} = \frac{M}{I_x W_x} = \frac{513 \times 10^6}{1.0 \times 9.61 \times 10^6} = 53.4 \text{ N/mm}^2 \leq f = 215 \text{ N/mm}^2 \quad (t = 16 \text{ mm} \leq 16 \text{ mm})$$

2° 腹板中点最大剪应力

$$\tau_{max} = \frac{V_{max}}{h \cdot t_w} = \frac{655.1 \times 10^3}{1000 \times 12} = 54.6 \text{ N/mm}^2 \leq f_v = 125 \text{ N/mm}^2$$

3° 腹板计算高度边缘局部压应力

$$a = 500 \text{ mm}, h_R = 0, h_y = 16 \text{ mm} \Rightarrow l_z = a + 5h_y + 2h_R = 500 + 5 \times 16 + 2 \times 0 = 580 \text{ mm}$$

$$\psi = 1.0, D_{max} = 1.5 \alpha D_{max} = 1.5 \times 1.05 \times 1600 = 2520 \text{ kN} \quad (\alpha \text{ 不妨取 } 1.05), t_w = 30 \text{ mm}$$

$$\Rightarrow \sigma_c = \frac{\psi F}{l_z t_w} = \frac{1.0 \times 2520 \times 10^3}{580 \times 30} = 144.83 \text{ N/mm}^2 \leq f = 205 \text{ N/mm}^2 \quad (t_w = 30 \text{ mm} > 16 \text{ mm})$$

4° 工字钢腹板与翼缘交接处的折算应力

$$\sigma = \frac{M}{I_x y} = \frac{513 \times 10^6 \times 500}{4.96 \times 10^9} = 51.7 \text{ N/mm}^2$$

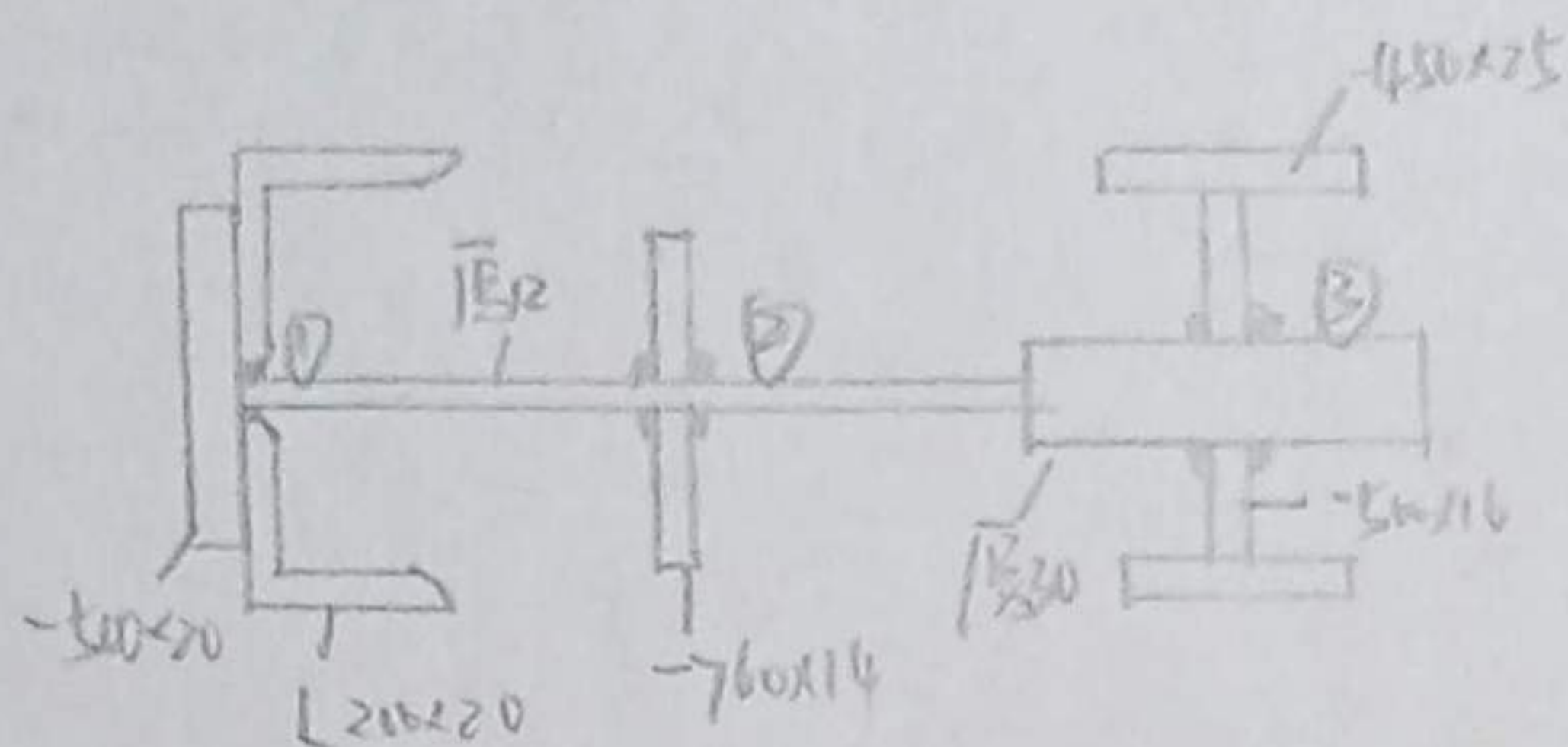
$$\tau = \frac{V S_x^*}{I_x t_w} = \frac{655.1 \times 10^3 \times 480 \times 16 \times 508}{4.96 \times 10^9 \times 12} = 42.94 \text{ N/mm}^2$$

$$\Rightarrow \sqrt{\sigma^2 + 3\tau^2} = \sqrt{51.7^2 + 3 \times 42.94^2} = 90.58 \text{ N/mm}^2 \leq f = 215 \text{ N/mm}^2$$

④ 焊接设计

连接处如图①、②、③处所示。

对①处焊缝，其承受 $V = 655.1 \text{ kN}$ 。



由 $h_f \geq 1.5\sqrt{t_{较厚}} = 1.5 \times \sqrt{20} = 6.7 \text{ mm}$ ，且 $h_f \leq 1.2t_{较薄} = 1.2 \times 20 = 24 \text{ mm}$ ，取 $h_f = 7 \text{ mm}$ 。

$\Rightarrow h_e = 0.7h_f = 4.9 \text{ mm}$ ， $f_f^w = 160 \text{ N/mm}^2$ 。

$\Rightarrow \Sigma l_w = \frac{V}{h_e f_f^w} = \frac{655.1 \times 10^3}{4.9 \times 160} = 835.6 \text{ mm}$ 。则 $l_{w1} = \frac{\Sigma l_w}{2} = 417.8 \text{ mm}$ 。

$\Rightarrow l_1 = l_w + 2h_f = 431.8 \text{ mm}$ ，取 $l_1 = 500 \text{ mm}$ 。此时 $l_{w1} = l_1 - 2h_f = 486 \text{ mm} > 60h_f = 420 \text{ mm}$ 。

故需再验算， $f_f^w = \alpha_f f_f^w$ ， $\alpha_f = 1.5 - \frac{l_w}{120h_f} = 1.5 - \frac{486}{120 \times 7} = 0.92$ 。

$\Rightarrow \sigma = \frac{V}{h_e \Sigma l_w} = \frac{655.1 \times 10^3}{4.9 \times 486 \times 2} = 137.5 \text{ N/mm}^2 < \alpha_f f_f^w = 0.92 \times 160 = 147.2 \text{ N/mm}^2$ ，可行。

对②处焊缝，其承受 $P_1 = 1080.1 \text{ kN}$ 。

$h_f \geq 1.5\sqrt{t_{较厚}} = 1.5 \times \sqrt{14} = 5.6 \text{ mm}$ ， $h_f \leq 1.2t_{较薄} = 1.2 \times 12 = 14.4 \text{ mm}$ ，取 $h_f = 7 \text{ mm}$ 。

$h_e = 0.7h_f = 4.9 \text{ mm}$ ， $\Rightarrow \Sigma l_w = \frac{P_1}{h_e f_f^w} = \frac{1080.1 \times 10^3}{4.9 \times 160} = 1377.7 \text{ mm}$ 。

共4条焊缝， $\Rightarrow l_{w2} = \frac{\Sigma l_w}{4} = 344.425 \text{ mm}$ ， $l_2 = l_w + 2h_f = 358.425 \text{ mm}$ ，取 $l_2 = 400 \text{ mm}$ 。

$l_{w2} = l_2 - 2h_f = 386 \text{ mm} < 60h_f = 420 \text{ mm}$ ，不需重新验算。

对③处焊缝，其需抗弯 D_{max} 和 $V = 425 \text{ kN}$ 。则 $N = D_{max} + V = 2520 + 425 = 2945 \text{ kN}$ 。

$$h_f > 1.5\sqrt{t_{p3}} = 1.5 \times \sqrt{30} = 8.2 \text{ mm}, \quad h_f \leq 1.2t_{p3} = 1.2 \times 16 = 19.2 \text{ mm}. \quad \text{取 } h_f = 10 \text{ mm}$$

$$\Rightarrow h_e = 0.7h_f = 7 \text{ mm}, \quad \Rightarrow \Sigma l_w = \frac{N}{h_e f_f^w} = \frac{2945 \times 10^3}{7 \times 160} = 2629.5 \text{ mm}.$$

$$l_{w3} = \frac{\Sigma l_w}{4} = 657.375 \text{ mm}. \quad \Rightarrow l_3 = l_{w3} + 2h_f = 677.375 \text{ mm}. \quad \text{取 } l_3 = 700 \text{ mm}.$$

$$l_w = 700 - 2h_f = 680 \text{ mm} > 60h_f = 600 \text{ mm}. \quad \text{取 } l_w = 680 \text{ mm}.$$

$$\sigma_f = 1.5 - \frac{l_w}{120h_f} = 1.5 - \frac{680}{120 \times 10} = 0.933.$$

$$\text{则 } \sigma = \frac{N}{h_e \Sigma l_w} = \frac{2945 \times 10^3}{7 \times 680 \times 4} = 154.7 \text{ N/mm}^2 > \sigma_{ff}^w = 148.8 \text{ N/mm}^2. \quad \text{再取 } h_f = 12 \text{ mm}.$$

$$h_e = 0.7h_f = 8.4 \text{ mm}, \quad \Rightarrow \Sigma l_w = \frac{N}{h_e f_f^w} = 2191.2 \text{ mm}.$$

$$\Rightarrow l_{w3} = \frac{\Sigma l_w}{4} = 547.8 \text{ mm}. \quad \Rightarrow l_3 = l_{w3} + 2h_f = 571.8 \text{ mm}. \quad \text{取 } l_3 = 600 \text{ mm}.$$

$$l_{w3} = l_3 - 2h_f = 576 \text{ mm} \leq 60h_f = 720 \text{ mm}. \quad \text{可行}.$$

综上，角钢截面强度满足要求，连接焊缝已设计。

2.2 吊杆设计算-1

解: ① 内力计算.

验算吊杆强度时, 考虑两台吊杆满载时的作用, 验算吊杆疲劳强度时, 只考虑起重量最大的一台吊杆的荷载作用.

(1) 两台吊杆满载时的内力

ca) 竖向荷载作用

编写python程序得梁的弯矩最大值 $M_{k,max}$ 和剪力 $V_{k,max}$ (位于梁内力边缘), (1号吊杆作用)

$$M_{k,max} = 3138.95 \text{ kN}\cdot\text{m}, \quad V_{k,max} = 1281.64 \text{ kN}.$$

cb) 横向荷载作用

每个吊压处的水平荷载:

由起重机的吊杆, 取 $T_k = 2P_{max,k} = 0.1 \times 324 = 32.4 \text{ kN}$.

其作用位置与竖向荷载相同, 因此, 横向荷载作用下产生 M_{ky} 和反弯点剪力 H_k .

$$M_{ky} = 0.1 M_{k,max} = 313.895 \text{ kN}\cdot\text{m}, \quad H_k = 0.1 V_{k,max} = 128.164 \text{ kN}.$$

(2) 一台吊杆满载时的内力

ca) 竖向荷载作用

风荷载, 即得

$$M_{k,max} = 2060.54 \text{ kN}\cdot\text{m}, \quad V_{k,max} = 911.84 \text{ kN}.$$

根据以上计算, 汇总内力如右所示, 其中1.1为动力系数. 吊杆自重以5%的活荷载标准值计入.
(A7级吊杆)

吊杆数量	荷载	M_{kmax} (kN·m)	M_{max} (kN·m)	M_{ky} (kN·m)	M_y (kN·m)	V_{kmax} (kN)	V (kN)
两台	吊杆	3138.95	$1.1 \times 1.5 \times 3138.95$ $= 5179.27$	3138.95	1.5×3138.95 $= 4708.8$	1281.64	$1.1 \times 1.5 \times 1281.64$ $= 2141.7$
	自重	0.65×3138.95 $= 156.95$	1.2×156.95 $= 204.04$			0.65×1281.64 $= 64.1$	1.2×64.1 $= 83.32$
	Σ	3295.4	5383.31			1345.74	2198.02
一台	吊杆	2060.54				911.84	

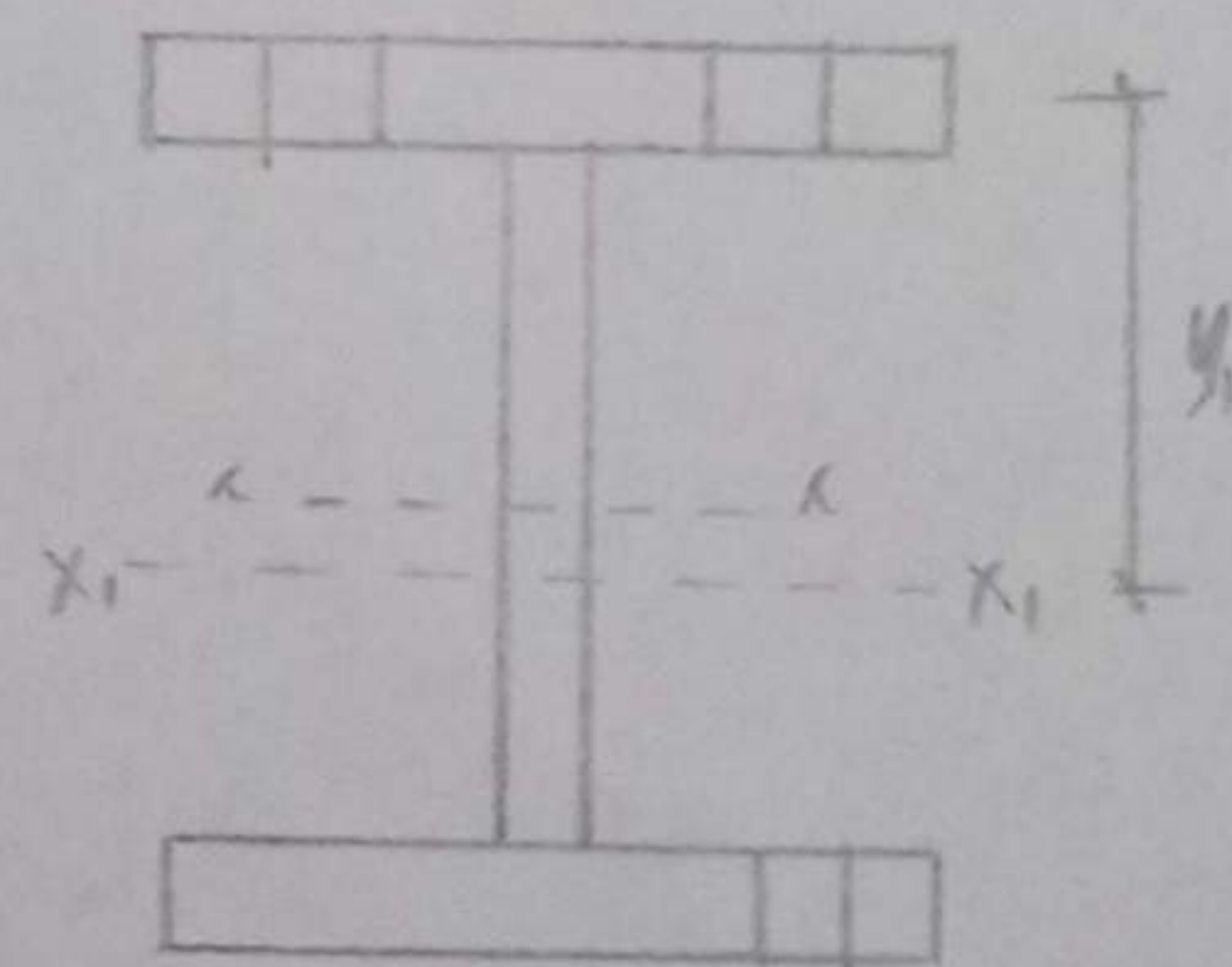
② 截面抗弯计算

III 吊杆梁

毛截面惯性矩 $I_x = \frac{1}{12} \times 14 \times 1600^3 + 2 \times 24 \times 500 \times (800 + 12)^2 = 2.06 \times 10^{10} \text{ mm}^4$

净截面面积 $A_n = [500 - 22 \times 21 \times 24 + (500 - 22 \times 21 \times 24 + 1600 \times 14] = 10944 + 11472 + 22400$
 $= 44816 \text{ mm}^2$

净截面形心位置 $y_1 = \frac{11472 \times 1624 + 22400 \times 812}{44816} = 821.6 \text{ mm}$



净截面惯性矩 $I_{nx1} = \frac{1}{12} \times 14 \times 1600^3 + 22400 \times 9.6^2 + 10944 \times 821.6^2$
 $+ 11472 \times (1624 - 821.6)^2 = 1.955 \times 10^{10} \text{ mm}^4$

上翼缘右侧外边缘净截面模量 $W_{nx1} = \frac{I_{nx1}}{y_1 + \frac{t}{2}} = \frac{1.955 \times 10^{10}}{821.6 + 12} = 2.35 \times 10^7 \text{ mm}^3$

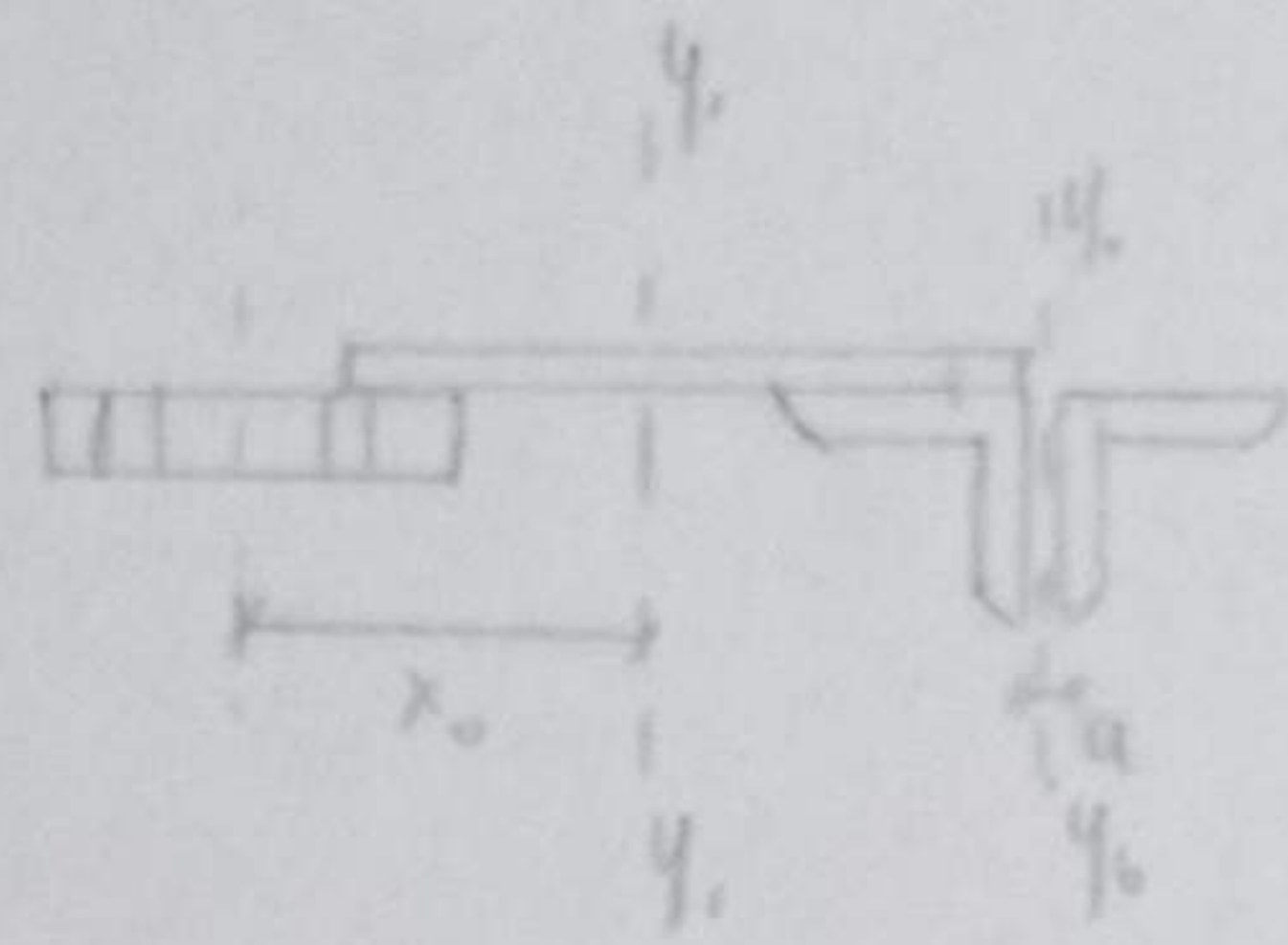
半毛截面对 x 轴的惯性矩 $S_x = 500 \times 24 \times 812 + 800 \times 14 \times 400 = 1.42 \times 10^7 \text{ mm}^3$

IV 吊杆梁

净截面面积 $A_{n2} = 24 \times (500 - 22 \times 21) + 860 \times 8 + 1930 \times 2 = 10944 + 6880 + 3860$
 $= 21684 \text{ mm}^2$

截面形心至腹板中心线之间的距离

$$x_0 = \frac{6880 \times (430 + 200) + 3860 \times 1100}{21684} = 395.7 \text{ mm}$$



不取 \$a=10 \text{ mm}\$.

$$I_{y0} = A \cdot i_{y0}^2$$

$$\begin{aligned} \text{净截面惯性矩 } I_{ny1} &= \frac{1}{12} \times 24 \times 500^3 - 2 \times 24 \times 22 \times 120^2 + 10944 \times 395.7^2 + \frac{1}{12} \times 8 \times 860^3 + 6880 \times (630 - 395.7)^2 + 3860 \times 452^2 \\ &+ 3860 \times 1100 - 395.7^2 = \frac{4.67 \times 10^9}{467} \text{ mm}^4 \end{aligned}$$

对 \$y_1\$ 轴的净截面模量 (距上翼缘外边缘距离)

$$W_{ny1} = \frac{I_{ny1}}{x + 250} = \frac{4.67 \times 10^9}{395.7 + 250} = 7.23 \times 10^6 \text{ mm}^3$$

③ 截面验算

(1) 强度验算

$$\text{上翼缘最大正应力 } \sigma_{\max} = \frac{M_x}{W_{nx1}} + \frac{M_y}{W_{ny1}} = \frac{5383.3 \times 10^6}{2.35 \times 10^7} + \frac{470.8 \times 10^6}{7.23 \times 10^6} = 229.1 + \frac{65.1}{1} = 294.2 \text{ N/mm}^2$$

$$\leq f = 295 \text{ N/mm}^2 \quad (t = 24 \text{ mm} > 16 \text{ mm})$$

$$\text{腹板最大剪力 } \tau_{\max} = \frac{V_{\max}}{I_x t_w} = \frac{2198.03 \times 10^3 \times 1.42 \times 10^7}{2.06 \times 10^{10} \times 14} = 128.24 \text{ N/mm}^2 \leq f_v = 175 \text{ N/mm}^2$$

腹板边缘局部压应力

$$\sigma_c = \frac{\psi F}{l_z t_w} \quad l_z = a + 2h_R + \psi h_y \quad a = 50 \text{ mm} \quad h_R = 150 \text{ mm} \quad h_y = 24 \text{ mm}$$

$$l_z = 50 + 2 \times 150 + 1.35 \times 24 = 470 \text{ mm} \quad \text{A7 为 Ⅱ级 4 吊钩, 取 } \psi = 1.35$$

$$\Rightarrow \sigma_c = \frac{1.35 \times 1.1 \times 1.5 \times 324 \times 10^3}{470 \times 14} = 109.7 \text{ N/mm}^2 \leq f = 305 \text{ N/mm}^2 \quad (t = 14 \text{ mm} \leq 16 \text{ mm})$$

(2) 疲劳验算

下翼缘同高螺栓连接处承受水平反力处的应力金属

$$\sigma = \frac{M_x}{I_{x1}} y = \frac{2060.5 \times 10^6}{1.955 \times 10^{10}} \times (1624 - 821.6 + 121) = 85.84 \text{ N/mm}^2.$$

根据旁计算的轴力和弯矩, 此处为2类, 查表2-11. $\Rightarrow [\sigma]_{2 \times 10^6} = 144 \text{ N/mm}^2$.

吊钩系数查表2-12, 得 $\alpha_f = 0.8$.

$$\Rightarrow \alpha_f \sigma = 0.8 \times 85.84 = 68.672 \text{ N/mm}^2 \leq [\sigma]_{2 \times 10^6} = 144 \text{ N/mm}^2, \text{ 满足要求.}$$

受拉翼缘连接焊缝处:

$$\sigma = \frac{M_x}{I_{x1}} y = \frac{2060.5 \times 10^6}{1.955 \times 10^{10}} \times (1624 - 821.6 - 121) = 83.31 \text{ N/mm}^2.$$

同理, 查表2-11, 由于题目未给出具体条件, 此处按最不利取为2类.

$$\Rightarrow [\sigma]_{2 \times 10^6} = 100 \text{ N/mm}^2, \text{ 且 } \alpha_f = 0.8.$$

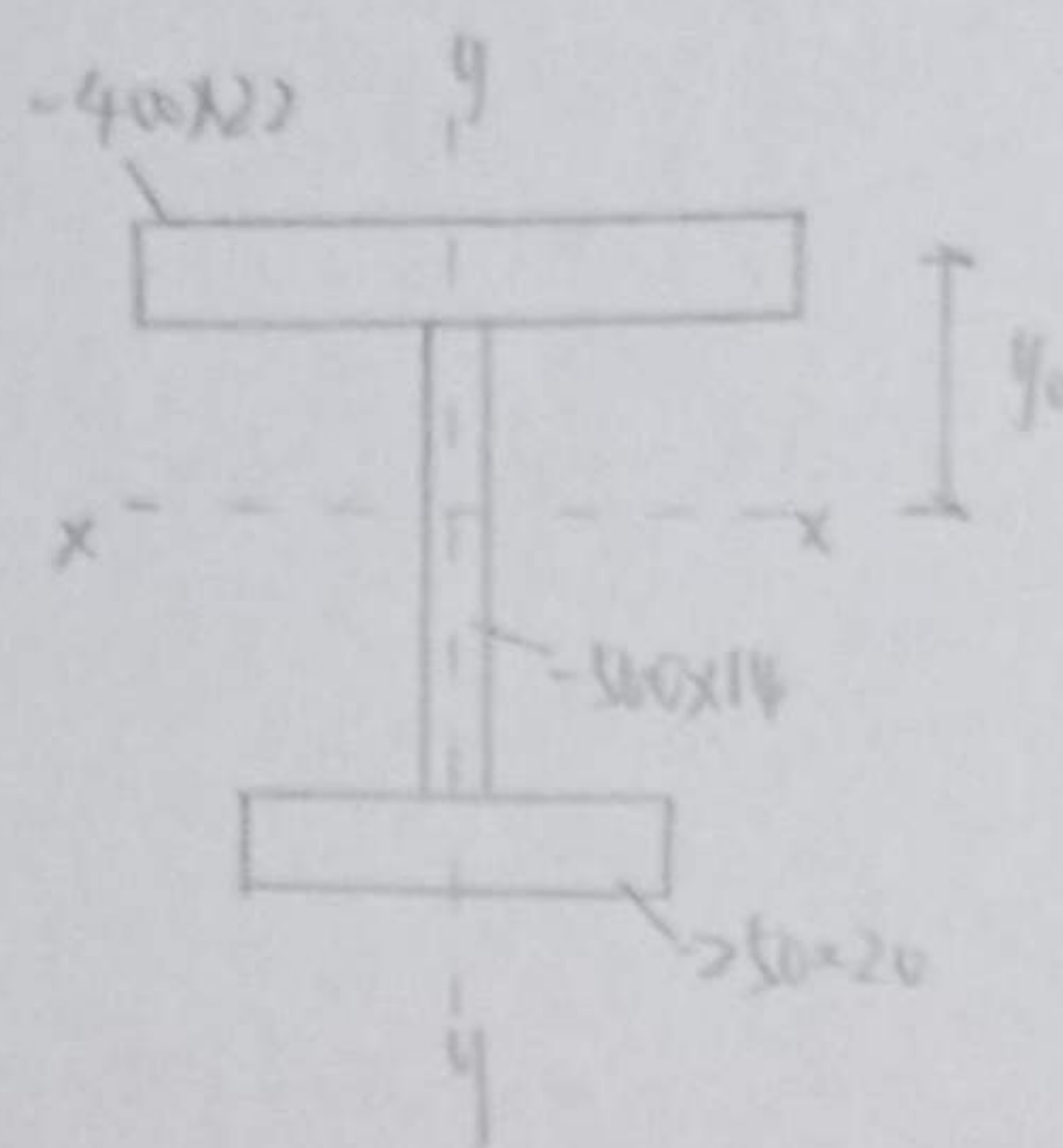
$$\Rightarrow \alpha_f \sigma = 0.8 \times 83.31 = 66.648 \text{ N/mm}^2 \leq [\sigma]_{2 \times 10^6} = 100 \text{ N/mm}^2, \text{ 满足要求.}$$

综上, 该吊杆截面强度和刚度强度均满足要求。

2-3. 吊杆设计-2

解: ① 截面特征计算

$$y_0 = \frac{500 \times 14 \times 261 + 250 \times 20 \times 521}{250 \times 20 + 500 \times 14 + 400 \times 22} = 213 \text{ mm}$$



$$\Rightarrow I_x = 400 \times 22 \times 213^2 + \frac{1}{12} \times 14 \times 500^3 + 14 \times 500 \times (261 - 213)^2 + 250 \times 20 \times (521 - 213)^2 = 1.036 \times 10^9 \text{ mm}^4$$

上翼缘外侧边缘

$$W_{nx1} = \frac{I_x}{y_0 + \frac{t}{2}} = \frac{1.036 \times 10^9}{213 + 11} = 4.625 \times 10^6 \text{ mm}^3$$

下翼缘外侧边缘

$$W_{nx2} = \frac{I_x}{521 - y_0 + \frac{t}{2}} = \frac{1.036 \times 10^9}{521 - 213 + 10} = 3.26 \times 10^6 \text{ mm}^3$$

横杆轴荷仅由翼缘承担, 故只计算上翼缘绕杆的惯性矩:

$$W_{ny} = \frac{1}{6} b h^2 = \frac{1}{6} \times 22 \times 400^2 = 5.87 \times 10^5 \text{ mm}^3$$

② 强度验算

对上翼缘外侧边缘:

$$\sigma = \frac{M_x}{W_{nx1}} + \frac{M_y}{W_{ny}} = \frac{300 \times 10^6}{4.625 \times 10^6} + \frac{100 \times 10^6}{5.87 \times 10^5} = 235.2 \text{ N/mm}^2 > f = 205 \text{ N/mm}^2 \quad (t = 22 \text{ mm} > 16 \text{ mm})$$

对下翼缘:

$$\sigma = \frac{M_x}{W_{nx2}} = \frac{300 \times 10^6}{3.26 \times 10^6} = 92.02 \text{ N/mm}^2 \leq f = 205 \text{ N/mm}^2 \quad (t = 20 \text{ mm} > 16 \text{ mm})$$

强度验算不满足要求。

② 整体稳定性验算

$$\frac{M_x}{\varphi_b W_x} + \frac{M_y}{W_y} \leq f. \quad \text{其中 } \varphi_b = \beta_b \cdot \frac{4320}{\lambda_y^2} \cdot \frac{A h}{W_x} \cdot [\sqrt{1 + (\frac{\lambda_y t_1}{4.4 h})^2} + \eta_b] \varepsilon_k^2,$$

查表得 β_b . 得 $\lambda_y = \frac{l_{y1}}{b_1 h} = \frac{6000 \times 22}{400 \times 542} = 0.609. \Rightarrow \beta_b = 0.73 + 0.18 \lambda_y = 0.73 + 0.18 \times 0.609 = 0.84.$

截面惯性矩和面积矩:

$$I_{y1} = \frac{1}{12} \times 22 \times 400^3 + \frac{1}{12} \times 20 \times 250^3 = 1.43 \times 10^8 \text{ mm}^4. \quad A = 400 \times 22 + 500 \times 14 + 250 \times 20 = 20800 \text{ mm}^2.$$

$$i_y = \sqrt{\frac{I_{y1}}{A}} = \sqrt{\frac{1.43 \times 10^8}{20800}} = 82.92 \text{ mm}. \quad \lambda_y = \frac{l_{y1}}{i_y} = \frac{6000}{82.92} = 72.36.$$

$$\eta_b = 0.8(2\alpha_b - 1). \quad \alpha_b = \frac{I_1}{I_1 + I_2}, \quad I_1 = \frac{1}{12} \times 22 \times 400^3 = 1.17 \times 10^8 \text{ mm}^4. \quad I_2 = \frac{1}{12} \times 20 \times 250^3 = 2.6 \times 10^7 \text{ mm}^4.$$

$$\Rightarrow \alpha_b = 0.818, \quad \eta_b = 0.8(2 \times 0.818 - 1) = 0.8 \times (2 \times 0.818 - 1) = 0.5088.$$

$$\Rightarrow \varphi_b = 0.84 \times \frac{4320}{72.36^2} \times \frac{20800 \times 542}{4.625 \times 10^6} \times [\sqrt{1 + (\frac{72.36 \times 22}{4.4 \times 542})^2} + 0.5088] \times 1^2 = 2.91 > 0.6$$

$$\Rightarrow \varphi_b' = 1.07 - \frac{0.282}{\varphi_b} = 1.07 - \frac{0.282}{2.91} = 0.973$$

$$\Rightarrow \frac{M_x}{\varphi_b W_x} + \frac{M_y}{W_y} = \frac{300 \times 10^6}{0.973 \times 4.625 \times 10^6} + \frac{100 \times 10^6}{587 \times 10^3} = 237 \text{ N/mm}^2 > f = 205 \text{ N/mm}^2.$$

整体稳定不满足。

③ 刚度验算

$$M_{Kx} = \frac{M}{\alpha \beta} = \frac{300}{1.05 \times 1.5} = 190.5 \text{ kNm}. \quad E = 206 \text{ GPa}.$$

$$\Rightarrow v = \frac{M_{Kx} l^2}{10 E I_x} = \frac{190.5 \times 10^6 \times 6000^3}{10 \times 206 \times 10^3 \times 1.036 \times 10^9} = 3.21 \text{ mm} \leq [v] = \frac{l}{900} = \frac{6000}{900} = 6.67 \text{ mm}.$$

满足要求。

④疲劳验算

受拉翼缘与平接焊缝处:

$$\Delta\sigma = \frac{M_{Fx}}{I_x} y = \frac{190.5 \times 10^6}{1.036 \times 10^9} \times (511 - 213) = 54.79 \text{ N/mm}^2.$$

查表-12, 得 $\sigma_f = 0.5$. 查附录表 8-1, 吊车不利, 取为 5 类. 即 $[\sigma]_{2 \times 10^6} = 100 \text{ N/mm}^2$.

$$\Rightarrow \sigma_{\Delta\sigma} = 27.395 \text{ N/mm}^2 \leq [\sigma]_{2 \times 10^6} = 100 \text{ N/mm}^2.$$

疲劳强度满足要求。

综上, 此吊杆强度和整体稳定性均满足要求。