

# Supplementary Material for: “A Stackelberg Game for Robust Cyber-Security Investment of Network Control Systems”

**\*For Review Purpose Only**

## INTRODUCTION

### I. DATA SET FOR NEW ENGLAND POWER GRID MODEL

In section IV of the paper, we validate the results on the IEEE 39-bus New England Power Grid system. The nonlinear model MATLAB file for the system can be found in [1]. The model consists of 10 synchronous generators and 19 loads. Generator 1 is modeled by 7 states and generators 2 through 9 are modeled by 8 states each while generator 10 is an equivalent aggregated model for the part of the network that we do not have control over, modeled by 4 states. The design matrices  $\mathbf{Q}$ ,  $\mathbf{R}$  in (5) along with the linearized system matrices for the nominal system as used in Section IV.B of the paper can be found in Section A of this site. For the application of the proposed SGs in Section III.E of the paper, 19 load buses are perturbed one at a time in the nonlinear model to generate 19 uncertain models. The linearized systems along with design matrices can be found in Section B of this site.

### II. UNCERTAIN MODELS

To validate the SGs proposed in Section III.D of the paper, uncertain models are generated by varying the load bus values of the nonlinear model. There are 19 load buses in the 39-bus nonlinear model. For each load bus  $i$ , the active and reactive setpoints are increased gradually to reach critical values beyond which the load flow fails to converge. Let  $P_i^L$  be the active power at a load bus  $i$  and  $\beta_{f_i}^P$  be the bias value that causes the load flow to fail, leading to instability. We then choose  $\beta_i^P = \beta_{f_i}^P - \epsilon$  for small  $\epsilon > 0$  and redefine the load bus value as

$$P_i^L \leftarrow P_i^L + \beta_i^P, \quad Q_i^L \leftarrow Q_i^L + \beta_i^Q \quad (\text{S1})$$

where  $Q_i^L$  is the reactive power at lead bus  $i$ , and the choice of  $\beta_i^Q$  is similar to that for  $\beta_i^P$ . The bias values  $\beta_i^P, \beta_i^Q$  for all  $i = 1, \dots, \ell$  of the nonlinear model can be found in Section A of this site. The resulting *uncertain* nonlinear system is converted to a state-space model by linearizing about its equilibrium point as described in Section IV.A of the paper. The resulting small-signal model can be written as (1) for each of the uncertain models generated using (S1). The objective function (5) is then formulated by choosing the design matrices as stated in the paper. The system, input, disturbance and design matrices for each uncertain system in  $\mathcal{M}$  can also be found in Section B of this site.

To assess the degree of uncertainty among the models in set  $\mathcal{M}$  we applied the optimal controller designed for the nominal model,  $\mathbf{K}_{nom, \mathcal{H}_2}^*$  to each model  $M_i \in \mathcal{M}$ . Fig. 1 shows the mismatch of the  $\mathcal{H}_2$ —performance of for

models in  $\mathcal{M}$  when the optimal  $\mathcal{H}_2$  controller for the nominal model,  $\mathbf{K}_{nom, \mathcal{H}_2}^*$ , is applied to the model  $M_i$ , i.e.

$$J_{i,nom} \% = \left| \frac{J_{M_i}(\mathbf{K}_{nom, \mathcal{H}_2}^*) - J_{M_i}(\mathbf{K}_{i, \mathcal{H}_2}^*)}{J_{M_i}(\mathbf{K}_{i, \mathcal{H}_2}^*)} \right| \times 100 \quad (\text{S2})$$

where  $J_{M_i}(\mathbf{K})$  is the  $\mathcal{H}_2$ -performance for a feedback matrix  $\mathbf{K}$  and  $\mathbf{K}_{i, \mathcal{H}_2}^*$  is the optimal controller designed for the model  $M_i \in \mathcal{M}$ . We found that the mismatched controller can result in up to 13.71% higher  $\mathcal{H}_2$  value than the

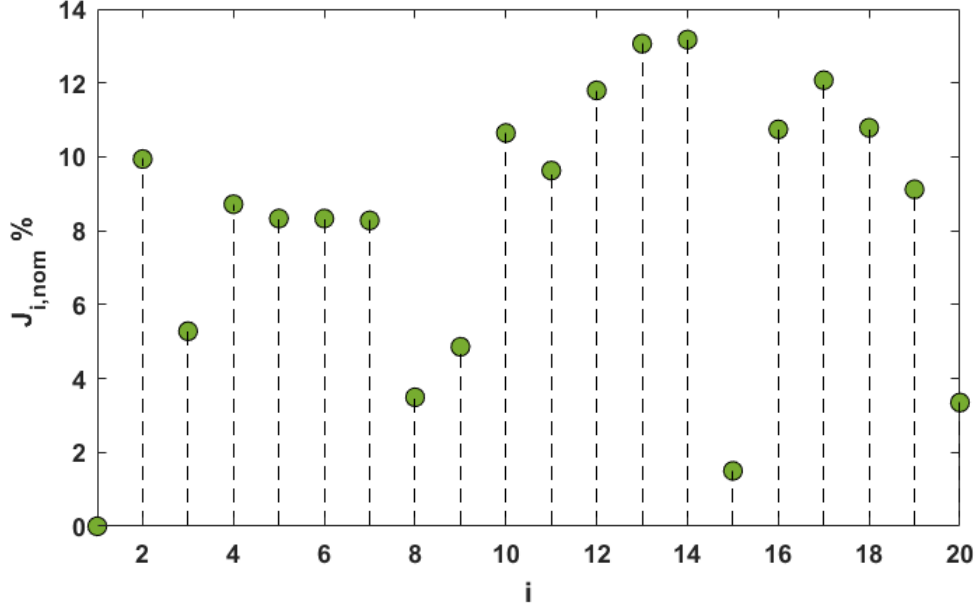


Fig. 1: Mismatch (S2) for model  $M_i$ ,  $i = 1, \dots, 20$

optimal controller  $\mathbf{K}_{i, \mathcal{H}_2}^*$ , thus confirming large diversity of the model set  $\mathcal{M}$ . Similar experiments were performed when the controller for model  $M_i \in \mathcal{M}$ , i.e  $\mathbf{K}_{i, \mathcal{H}_2}^*$ , is applied to the systems  $M_j$ ,  $i \neq j$ , and similar performance was observed in this case.

## REFERENCES

- [1] F. Dorfler and M. R. Jovanovic, "Wide Area Control of IEEE 39 New England Power Grid Model," 2014. [Online]. Available: <http://people.ece.umn.edu/users/mihailo/software/lqrsp/wac.html>