Supplementary Material for: "Stackelberg Security Investment Game for Networked Control Systems with Model Uncertainty"

*Written by the authors of "Stackelberg Security Investment Game for Networked Control Systems with Model Uncertainty"

***For Review Purpose Only**

I. Dataset for New England Power Grid Model

In section IV of the paper, we validate the results on the IEEE 39-bus New England Power Grid system. The nonlinear model MATLAB file for the system can be found in [1]. The model consists of 10 synchronous generators and 19 loads, to evaluate the performance of the proposed SGs. Generator 1 is modeled by 7 states, and generators 2 through 9 are modeled by 8 states each while generator 10 is an equivalent aggregated model for the part of the network that we do not have control over, modeled by 4 states. The design matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ in (5) along with the linearized system matrices for the nominal system as used in Section IV.B can be found in [2]. For the application of the proposed SGs in Section III.E of the paper, 19 load buses are perturbed one at a time in the nonlinear model to generate 19 uncertain models. The linearized systems along with design matrices can be found in [2].

II. Uncertain models

The models generated for validation of robust games by varying the load bus values of the nonlinear model. There are 19 load buses in the 39 bus non-linear model. For each load bus i, we increase the active and reactive setpoints are increased gradually to reach critical values, beyond which the load flow fails to converge. Let P_i^L be the active power at a load bus i and $\beta_{f_i}^P$ be the bias value that causes load flow to fail i.e., leading to instability. We then choose $\beta_i^P = \beta_{f_i}^P - \epsilon$, for small $\epsilon > 0$, and redefine the load bus value as

$$P_i^L \leftarrow P_i^L + \beta_i^P,\ Q_i^L \leftarrow Q_i^L + \beta_i^Q \tag{S1}$$

where Q_i^L is the reactive power at lead bus i, $\beta_{f_i}^P$ be the bias value that causes load flow to fail, β_i^Q is similar to β_i^P. The bias values β_i^P, β_i^P for all $i = 1, ..., \ell$ of the nonlinear model, can be found in [2]. The resulting *uncertain* nonlinear system is converted to a state-space model by linearized about its equilibrium point as described in Section IV of the paper.as described above. The resulting small-signal model can be written as (1) for each of the uncertain models generated using (S1). The objective function (5) is then formulated by choosing the design matrices as stated in the paper. the system, input, disturbance and design matrices for each uncertain system in $\mathcal{M}$ can also be found in [2].

The system generated are "critical" for each of the load bus, i.e a slight increase in load setpoint would result in instability and thus, these systems lead to be protected. However, the generated systems are varied with different controllers and control performance. We discuss the differences in the generated uncertain "critical" models with respect to the LQR objective function of the systems. For the long-term robust game based on the nominal-model, we test the performance of the nominal model's LQR controller when applied on occurring uncertain models. Fig. 1 shows the $\%$ differences between the energies of models in $\mathcal{M}$ when the optimal $\mathcal{H}_2$-norm controller for the nominal model $\boldsymbol{K}^*_{nom,\mathcal{H}_2}$ is applied on system M_i, i.e.

$$J_{i,nom}\ \% = |\frac{J_{M_j}(\boldsymbol{K}^*_{nom,\mathcal{H}_2}) - J_{M_j}(\boldsymbol{K}^*_{j,\mathcal{H}_2})}{J_{M_j}(\boldsymbol{K}^*_{j,\mathcal{H}_2})}| \times 100 \tag{S2}$$

We observe that the differences in $\%$ lie in the range of about 15% with the application of nominal model LQR

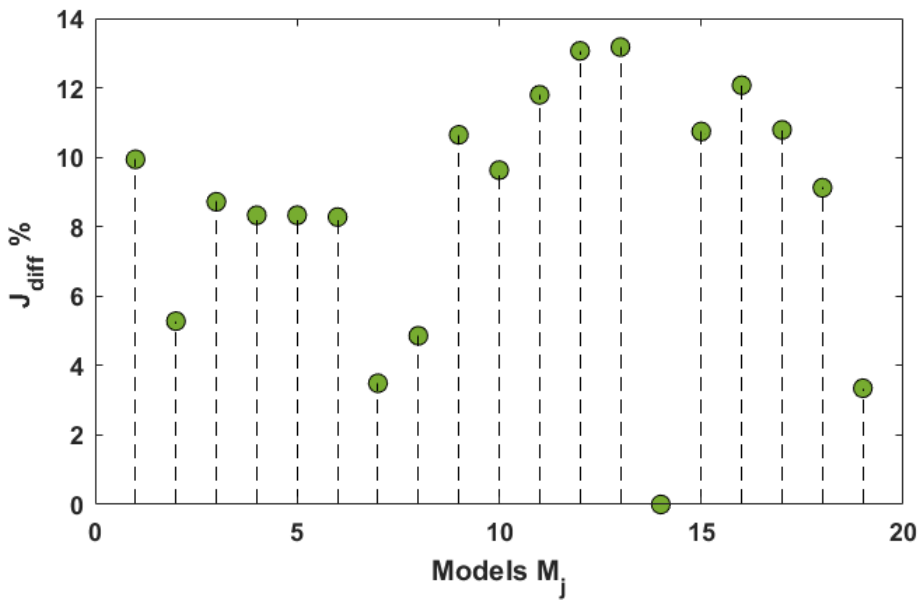

Fig. 1: Differences between the J with respect to their respective controllers and the J for the nominal system

controller on other models. Similar experiments were performed when the controller for model $M_i \in \mathcal{M}$, i.e $\boldsymbol{K}^*_{j,\mathcal{H}_2}$ is used on all systems M_j, $i \neq j$ and similar performance was observed in this case.

References

[1] F. Dorfler and M. R. Jovanovic, "Wide Area Control of IEEE 39 New England Power Grid Model," 2014. [Online]. Available: http://people.ece.umn.edu/users/mihailo/software/lqrsp/wac.html

[2] "Dataset for: *Stackelberg Security Investment Game for Networked Control Systems with Model Uncertainty*," 2020. [Online]. Available: https://github.com/Husky-hp/Stackelberg-Game-New-England-Power-model/tree/master