Supplementary Material for: “A Stackelberg Game for Robust Cyber-Security Investment of Network Control Systems”

***For Review Purpose Only**

INTRODUCTION

I. Data set for New England Power Grid Model

In section IV of the paper, we validate the results on the IEEE 39-bus New England Power Grid system. The nonlinear model MATLAB file for the system can be found in [1]. The model consists of 10 synchronous generators and 19 loads. Generator 1 is modeled by 7 states and generators 2 through 9 are modeled by 8 states each while generator 10 is an equivalent aggregated model for the part of the network that we do not have control over, modeled by 4 states. The design matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ in (5) along with the linearized system matrices for the nominal system as used in Section IV.B of the paper can be found in Section A of this site. For the application of the proposed SGs in Section III.E of the paper, 19 load buses are perturbed one at a time in the nonlinear model to generate 19 uncertain models. The linearized systems along with design matrices can be found in Section B of this site.

II. Uncertain models

To validate the SGs proposed in Section III.D of the paper, uncertain models are generated by varying the load bus values of the nonlinear model. There are 19 load buses in the 39-bus nonlinear model. For each load bus i, the active and reactive setpoints are increased gradually to reach critical values beyond which the load flow fails to converge. Let P_i^L be the active power at a load bus i and $\beta_{f_i}^P$ be the bias value that causes the load flow to fail, leading to instability. We then choose $\beta_i^P = \beta_{f_i}^P - \epsilon$ for small $\epsilon > 0$ and redefine the load bus value as

$$P_i^L \leftarrow P_i^L + \beta_i^P,\ Q_i^L \leftarrow Q_i^L + \beta_i^Q \tag{S1}$$

where Q_i^L is the reactive power at lead bus i, and the choice of β_i^Q is similar to that for β_i^P. The bias values β_i^P, β_i^Q for all $i = 1, ..., \ell$ of the nonlinear model can be found in Section A of this site. The resulting *uncertain* nonlinear system is converted to a state-space model by linearizing about its equilibrium point as described in Section IV.A of the paper. The resulting small-signal model can be written as (1) for each of the uncertain models generated using (S1). The objective function (5) is then formulated by choosing the design matrices as stated in the paper. The system, input, disturbance and design matrices for each uncertain system in $\mathcal{M}$ can also be found in Section B of this site.

To illustrate the degree of uncertainty among the models in set $\mathcal{M}$, we show the mismatch of the $\mathcal{H}_2-$performance of models in $\mathcal{M}$ when the optimal $\mathcal{H}_2$ controller for the nominal model, $\boldsymbol{K}^*_{nom,\mathcal{H}_2}$, is applied to each model M_i,

$i = 1, ..., M$ in Fig. S1. This mismatch is given by:

$$J_{i,nom}\ \% = |\frac{J_{M_i}(\boldsymbol{K}^*_{nom,\mathcal{H}_2}) - J_{M_i}(\boldsymbol{K}^*_{i,\mathcal{H}_2})}{J_{M_i}(\boldsymbol{K}^*_{i,\mathcal{H}_2})}| \times 100 \tag{S2}$$

where $J_{M_i}(\boldsymbol{K})$ is the $\mathcal{H}_2$-performance of the model M_i for a feedback matrix $\boldsymbol{K}$ and $\boldsymbol{K}^*_{i,\mathcal{H}_2}$ is the optimal controller designed for the model $M_i \in \mathcal{M}$. We found that the mismatched controller can result in up to 13.71% higher $\mathcal{H}_2$

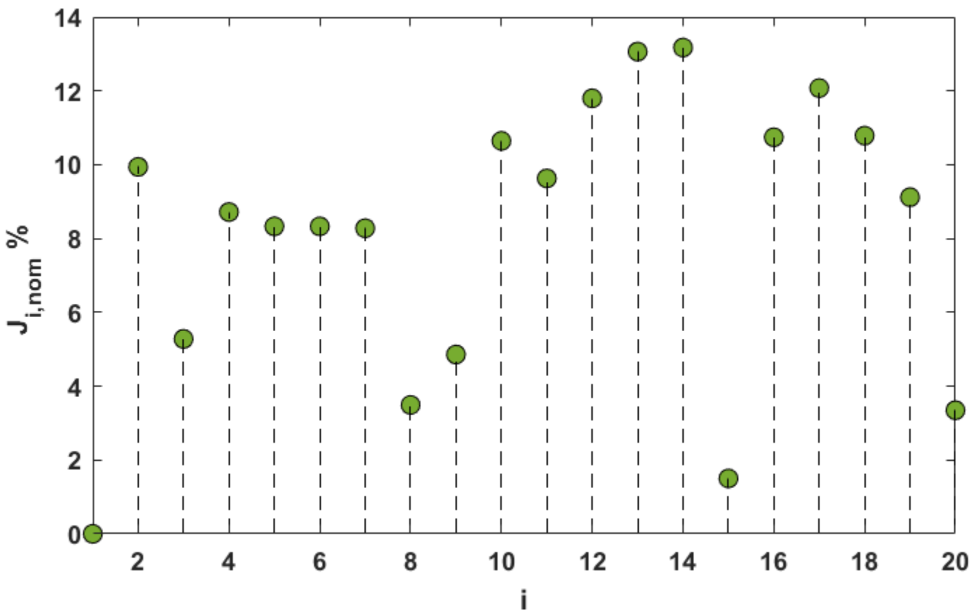

Fig. S1. Mismatch (S2) for model M_i, $i = 1, ..., 20$

value than the optimal controller $\boldsymbol{K}^*_{i,\mathcal{H}_2}$, thus confirming large diversity of the model set $\mathcal{M}$. We observed similar performance trends when the optimal controller for model $M_i \in \mathcal{M}$, $\boldsymbol{K}^*_{i,\mathcal{H}_2}$, was applied to the systems M_j, $i \neq j$.

References

[1] F. Dorfler and M. R. Jovanovic, "Wide Area Control of IEEE 39 New England Power Grid Model," 2014. [Online]. Available: http://people.ece.umn.edu/users/mihailo/software/lqrsp/wac.html