

1)

```
def cutting(wire_len):  
    if(wire_len < 2):  
        return 0  
  
    if(wire_len == 2):  
        return 1  
  
    return cutting(math.ceil(wire_len/2))+1
```

Recurrence relation is $T(n) = T(n/2) + 1$

$a = 1, b=2, d=0$

$T(n) \in \Theta(\log(n))$

2)

```
def worst_best(arr):  
    if len(arr) == 0:  
        return None, None  
  
    if len(arr) == 1:  
        return arr[0], arr[0]  
  
    if len(arr) == 2:  
        e_min = arr[0] if arr[0] < arr[1] else arr[1]  
        e_max = arr[0] if arr[0] > arr[1] else arr[1]  
  
        return e_min, e_max  
  
    e_min1, e_max1 = worst_best(arr[0 : math.floor(len(arr)/2)])  
    e_min2, e_max2 = worst_best(arr[math.ceil(len(arr)/2) : len(arr)])  
  
    e_min = e_min1 if e_min1 < e_min2 else e_min2  
    e_max = e_max1 if e_max1 > e_max2 else e_max2  
  
    return e_min, e_max
```

Recurrence relation is $T(n) = 2T(n/2)+1$

$a = 2, b=2, d=0$

$T(n) \in \Theta(n)$

3)

```
def lamuto_partition(arr):  
    pivot = arr[0]  
    small = 0  
    for i in range(len(arr)):  
        if arr[i] < pivot:  
            small = small + 1  
            swap(arr, small, i)  
    swap(arr, 0, small)  
    return small  
    O(n)  
  
def swap(arr, a, b):  
    temp = arr[a]  
    arr[a] = arr[b]  
    arr[b] = temp  
    O(1)  
  
def meaningful(arr, k):  
    s = lamuto_partition(arr) O(n)  
    if s == k-1:  
        return arr[s]  
    elif s > k-1:  
        return meaningful(arr[:s], k)  
    else:  
        return meaningful(arr[s+1:], k-s-1)  
    T(n/2)
```

Recurrence relation, $T(n) = T(n/2) + n$

$a = 1, b = 2, d = 1$

Complexity = $n + \log(n) = n$

$T(n) \in \Theta(n)$

4)

```
def merge_sort_and_count(arr):

    if len(arr) == 1:
        return 0, arr
    else:
        mid = len(arr)//2
        r_a, a = merge_sort_and_count(arr[:mid])
        r_b, b = merge_sort_and_count(arr[mid:])
        r_m, m = merge_and_count(a, b)
        return r_m+r_a+r_b, m

def merge_and_count(arr1, arr2):

    reverse_order = 0
    i = 0
    j = 0
    res = []
    while i < len(arr1) and j < len(arr2):

        if arr1[i] <= arr2[j] :
            res.append(arr1[i])
            i = i+1
        else :
            res.append(arr2[j])
            reverse_order = reverse_order+len(arr1)-i
            j = j+1

    while i < len(arr1):
        res.append(arr1[i])
        i = i+1

    while j < len(arr2):
        res.append(arr2[j])
        j = j+1

    return reverse_order, res
```

$O(n \log(n))$

$O(\log n)$

$O(n)$

$O(n)$

Recurrence relation is the same with merge sort, the only difference is we are also counting in constant time complexity.

$$T(n) = 2T(n/2) + n$$

$$a = 2, b = 2, d = 1$$

$$T(n) \in \Theta(n \log(n))$$

5)

```
def exp_bf(a, n):  
    if a <= 0:  
        return 0  
    elif n == 0:  
        return 1  
    res = 1  
    for i in range(n):  
        res = res*a  
    return res
```

$O(1)$

$O(n)$

```
def exp_dq(a, n):  
    if a == 0:  
        return 0  
    if n == 0:  
        return 1  
    if n == 1:  
        return a  
    return exp_dq(a, n//2) * exp_dq(a, n-(n//2))
```

$O(1)$

$2T(n/2)$

For exp_bf is $O(n)$

For exp_dq = $T(n) = 2T(n/2)$

$a = 1, b=2, d=0$

$T(n) \in \Theta(\log(n))$