

**MIDDLE EAST TECHNICAL UNIVERSITY**

**DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING**

**EE568 – Selected Topics on Electrical Machines**

**Project #1**

*Torque in a Variable Reluctance Machine*

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## Introduction

In this project, a variable reluctance machine is investigated. Knowing its dimensions, permeability, and the number of turns and current of the winding, the machine is first analyzed analytically to find reluctance, inductance and torque and plot them as a function of rotor angle under DC excitation of stator windings. Then, the machine is modelled in a 2D FEA software to examine flux density vectors, inductance, stored energy, and torque of the system having a linear material core. These phenomena are analyzed also for a system having non-linear effects. Moreover, a control method to make the rotor turn continuously is proposed.

Q1)

a)

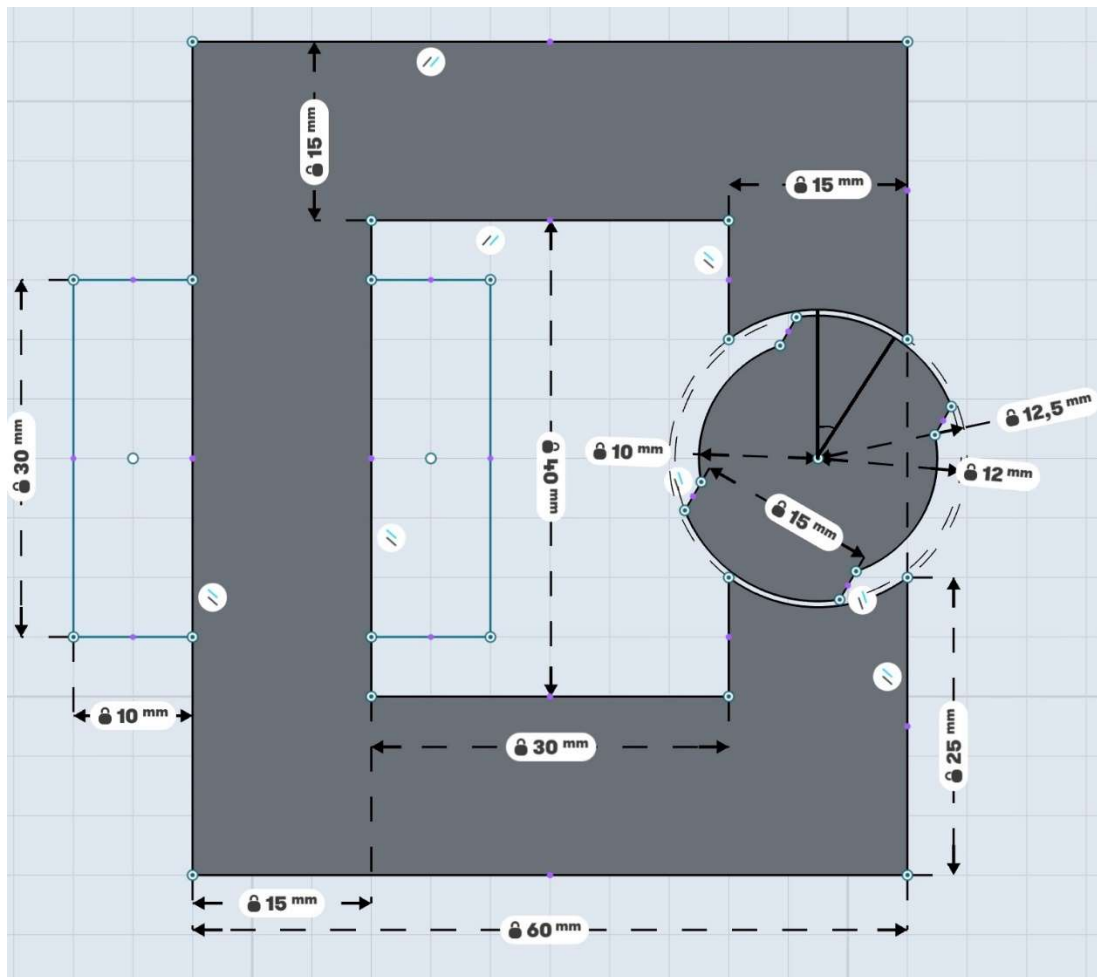


Fig-1: Variable Reluctance Machine

As seen in Fig-1, the minimum airgap length is 0.5mm and the maximum airgap length is 2.5mm. Assuming core is infinitely permeable, and area of the airgap is rectangular, maximum and minimum reluctance and inductance values can be found as follows:

$$\mathfrak{R}_{g,min} = \frac{2 * l_{g,min}}{\mu_0 A_g} \cong 2.65 \times 10^6 (1/H) \quad \mathfrak{R}_{g,max} = \frac{l_{g,max}}{\mu_0 A_g} \cong 13.26 \times 10^6 (1/H)$$

$$L_{min} = \frac{N^2}{\mathfrak{R}_{g,max}} \cong 4.71 \text{ mH} \quad L_{max} = \frac{N^2}{\mathfrak{R}_{g,min}} \cong 23.5 \text{ mH}$$

where  $N = 250, l_{g,min} = 0.5\text{mm}, l_{g,max} = 2.5\text{mm}, \mu_0 = 4\pi \times 10^{-7}, A_g = 300\text{mm}^2$

Also, it is assumed that the inductance varies with a sinusoidal expression shown below where  $\theta$  is the angle between the stator and rotor as shown in Fig-1:

$$L(\theta) = \frac{L_{max} - L_{min}}{2} \cos(2\theta) + \frac{L_{max} + L_{min}}{2}$$

Hence, the resulting graph is as seen in Fig-2.

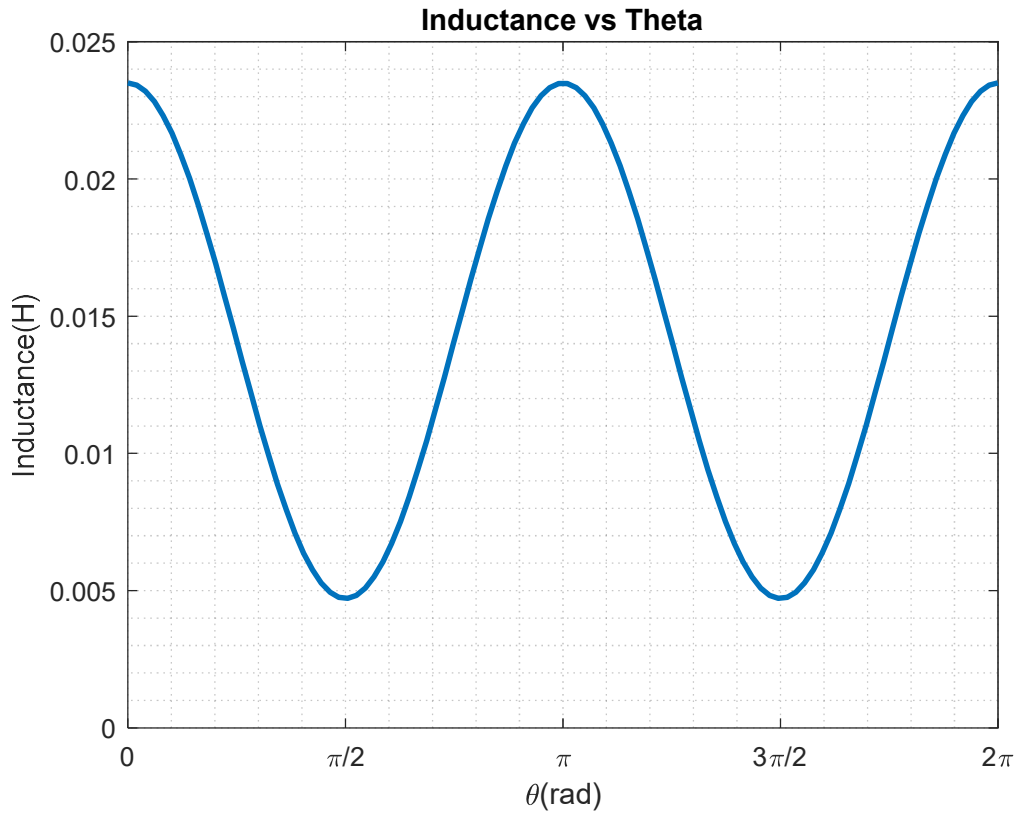


Fig-2: Inductance vs Rotation Angle graph

**b)**

$$T(\theta) = -\frac{\partial W}{\partial \theta} = \frac{1}{2} I^2 \frac{dL(\theta)}{d\theta} = -I^2 \frac{L_{max} - L_{min}}{2} \sin(2\theta) = 0.0845 * \sin(2\theta) \text{ with } I = 3A$$

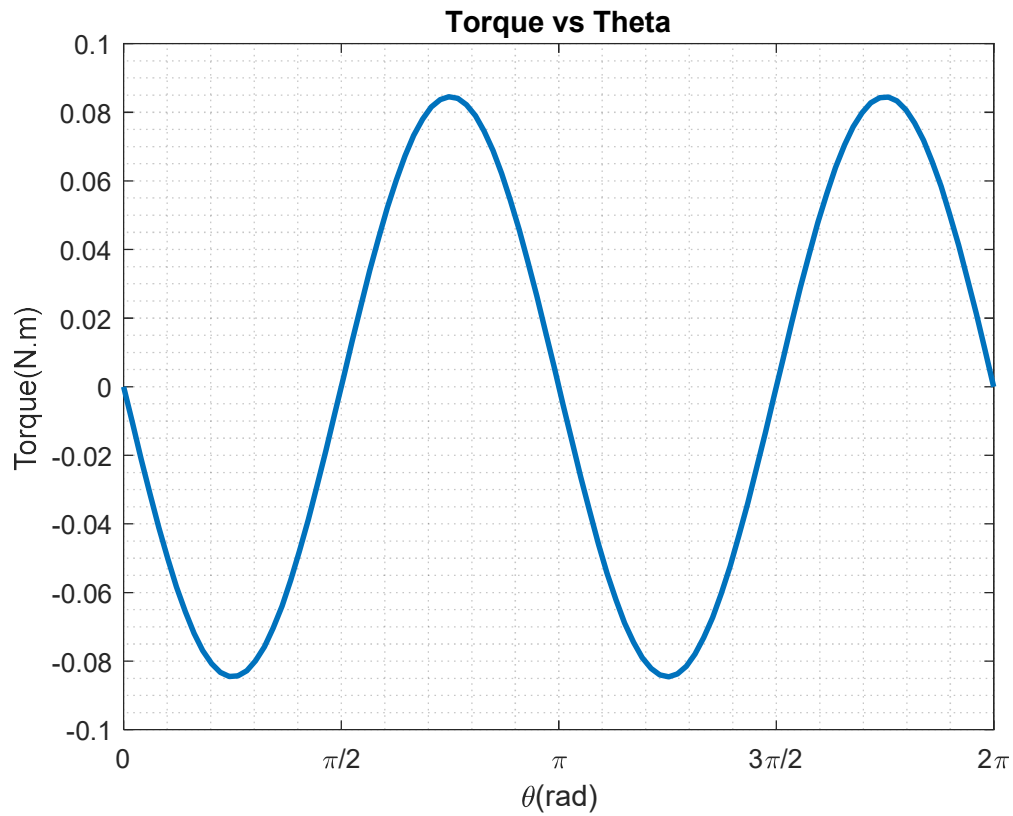


Fig-3: Torque vs Rotation Angle graph

c)

It is assumed that the flux density is uniform, and core has an infinite permeability, but that is not the case for real world. To show these effects, permeability of the core should be a non-zero number, and integral should be taken over the area of the core to calculate the reluctance.

Q2)

a)

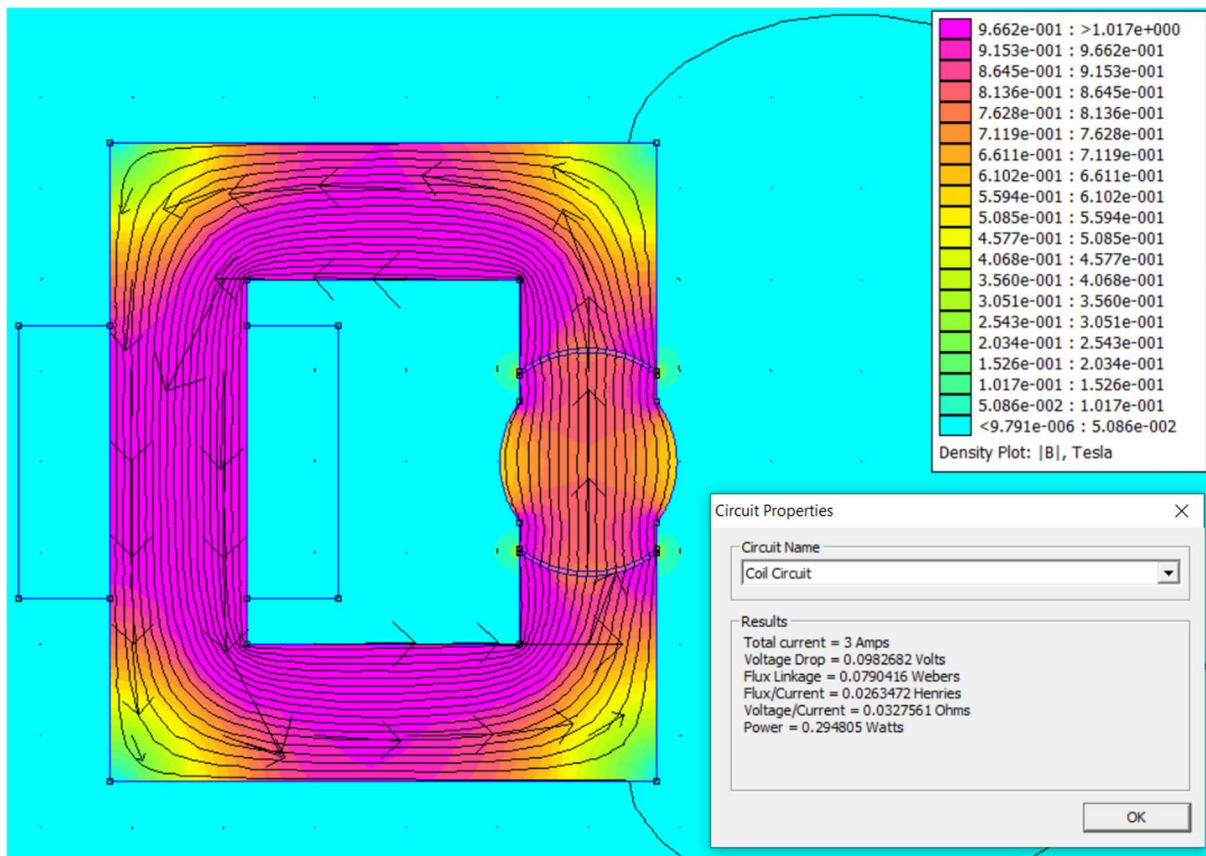


Fig-4: Flux Density Vectors for 0°

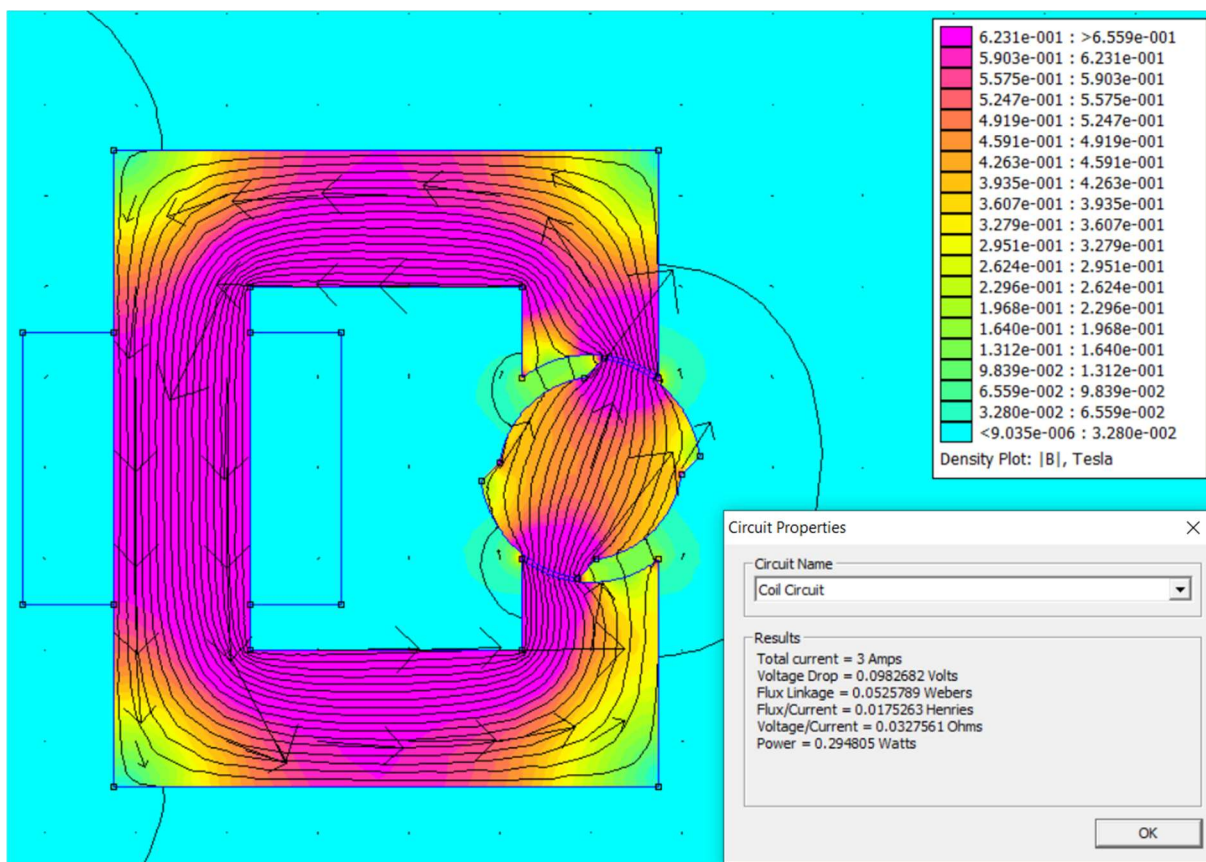


Fig-5: Flux Density Vectors for 45°

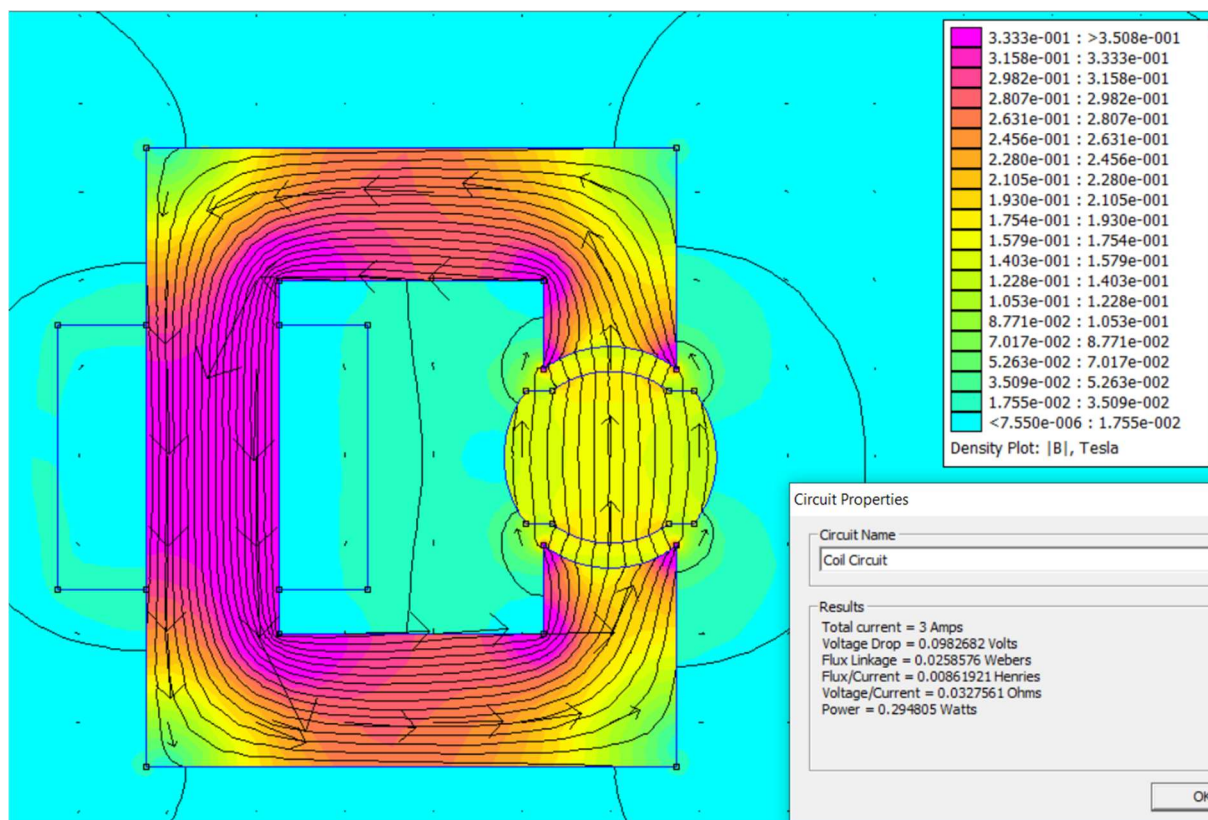


Fig-6: Flux Density Vectors for 90°

b)

In this part, linear material is assumed, and relative permeability of 1000H/m is used in the core. As seen in Fig-4-5-6, flux linkages are found as 0.079 Webers, 0.05257 Webers, 0.02585 Webers for 0°, 45° and 90°, respectively. Since  $\lambda = L \cdot I$  and  $W = 0.5 \cdot L \cdot I^2$  where  $I = 3A$ , inductances are calculated as 26.3mH, 17.5mH and 8.61mH, and the stored energy in the systems are calculated as 0.1176 J, 0.0778 J and 0.0377 J for 0°, 45° and 90°, respectively.

c)

Degree	Torque using simulation (Nm)	Torque using analytical model (Nm)
0	0.00016	0
15	0.05381	0.04225
30	0.05443	0.07317
45	0.05736	0.0845
60	0.06465	0.07317
75	0.07179	0.04225
90	0.0000285	0
105	-0.07174	-0.04225
120	-0.06470	-0.07317
135	-0.05760	-0.0845
150	-0.05475	-0.07317
165	-0.05410	-0.04225

Table-1: Simulated and analytical torque values for different rotor positions



As seen in Table-1, the simulated and analytical torque values are different for all positions since there is an assumption of sinusoidal variation in inductance of the machine, while this not true. The simulation takes into account of different parameters and integrates through materials instead of using a single formula. That is why the values are not matching. Moreover, the maximum torque angles are different for both approaches,  $45^\circ$  for analytical model and around  $75^\circ$  for simulation model. It is known that the salient pole machines have their maximum torque around  $75^\circ$ , and  $\sin(90)$  is the maximum value for analytical formula, hence this difference is expected.

**Q3)**

**a)**

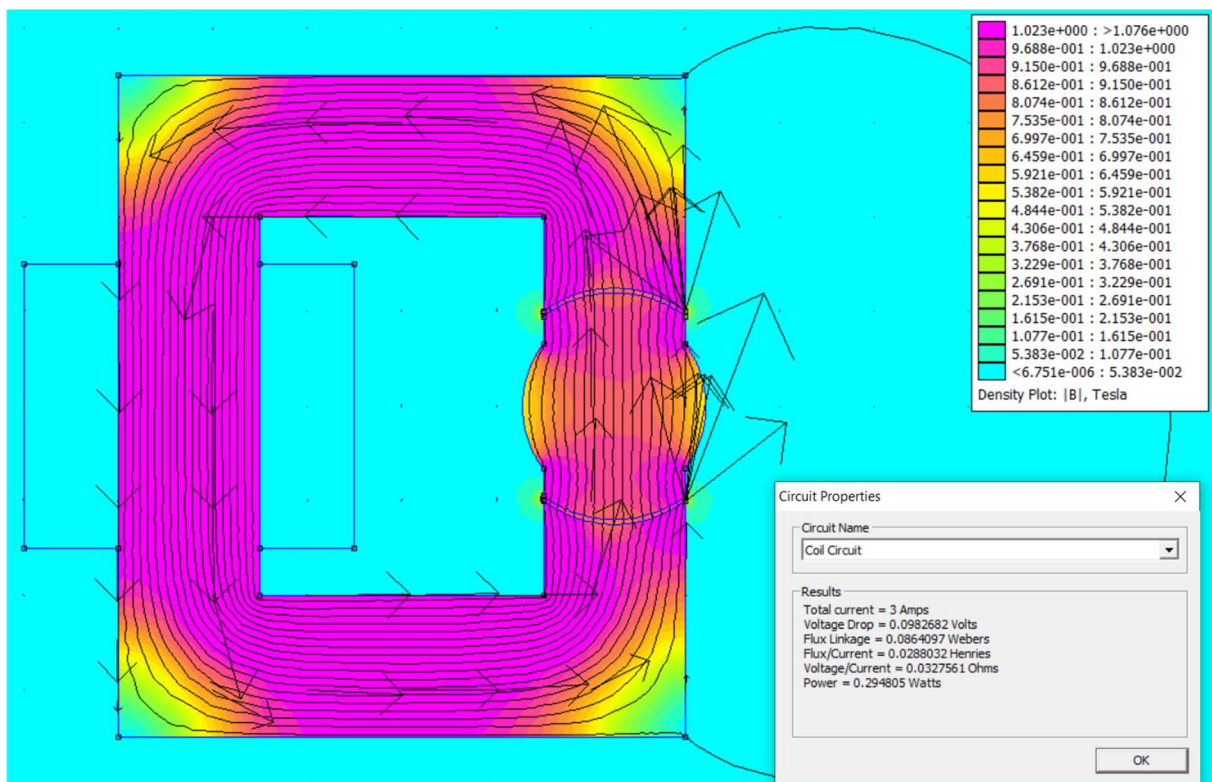


Fig-7: Flux Density Vectors for  $0^\circ$

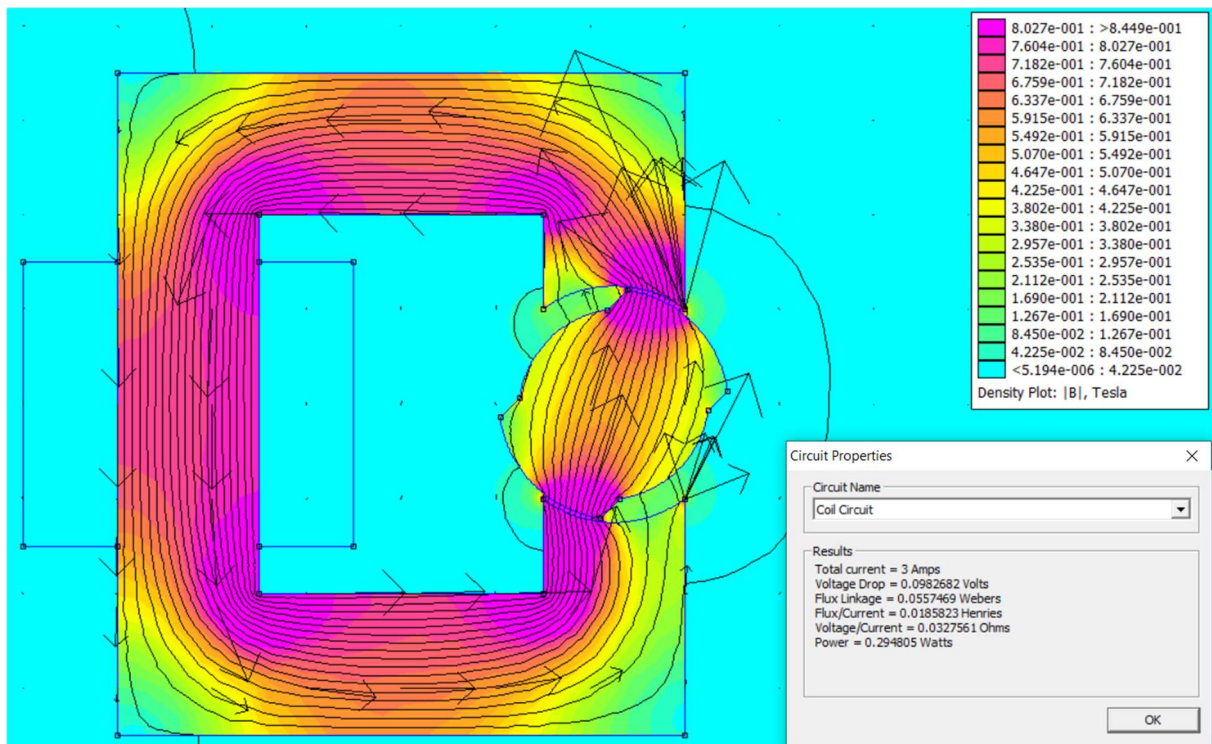


Fig-8: Flux Density Vectors for 45°

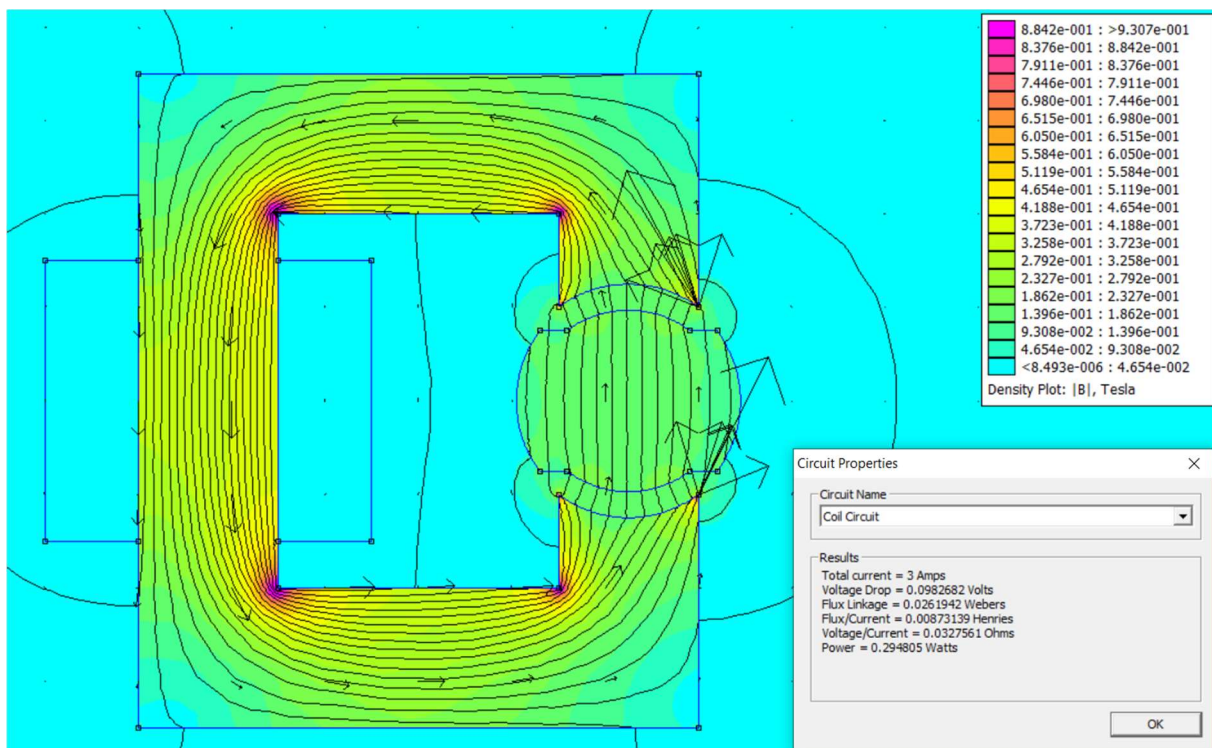


Fig-9: Flux Density Vectors for 90°



**b)**

In this part, non-linear material M-50 steel is used in the core and it has a relative permeability higher than 500H/m according to the datasheets. As seen in Fig-7-8-9, flux linkages are found as 0.08640 Webers, 0.05574 Webers, 0.02619 Webers for 0°, 45° and 90°, respectively. Since  $\lambda = L \cdot I$  and  $W = 0.5 \cdot L \cdot I^2$  where  $I = 3A$ , inductances are calculated as 28.8mH, 18.5mH and 8.73mH, and the stored energy in the systems are calculated as 0.12922 J, 0.08378 J and 0.0386 J for 0°, 45° and 90°, respectively.

**c)**

Degree	Torque using simulation (Nm)	Torque using analytical model (Nm)
0	0.000215	0
15	0.06428	0.04225
30	0.06333	0.07317
45	0.0649	0.0845
60	0.0712	0.07317
75	0.07318	0.04225
90	0.000043	0
105	-0.07312	-0.04225
120	-0.07115	-0.07317
135	-0.06518	-0.0845
150	-0.06367	-0.07317
165	-0.06451	-0.04225

Table-2: Simulated and analytical torque values for different rotor positions

As seen in Table-2, the simulated and analytical torque values are different for all positions since there is an assumption of sinusoidal variation in inductance of the machine, while this not true. This behavior is the same as Q2-c).

**d)**

It can be seen from Fig-6 and Fig-9 that the fringing flux ratio increases in the non-linear case, also non-linear material saturates.

**Q4)**

In order to satisfy continuous rotation, using 0-180° interval for explanation due to symmetry, and assuming we are starting at a proper rotor angle such as 135°, one can cut the DC current at certain point right after passing 0° rotor angle provided that there is enough torque to rotate the rotor until 90°, and give DC current again right after 90°. Another approach can be using AC excitation at a proper frequency.

**Q5)**

A GIF for the flux density vectors of non-linear material case is uploaded to the Github repository.

## **Appendix**

[1] <https://www.techsteel.net/alloy/steel/m-50>  
M50 Steel Material Properties