1 TP1 : propagation of a sound wave and acquisition with a microphone array

> ***Note that the first section has to be prepared before the session. Please answer those questions carefully directly in your report, which will be evaluated.***

1.1 In preparation

In this first tutorial, space is represented as a plane with an orthonormal basis $[x,z]$ and a reference point O. A microphone array is placed on the x-axis and centred at O. It is equipped with N microphones (N even), placed at the coordinates $(x = 0,\ z = z_n)$,$n \in [1, N]$, and equally spaced by the distance d.

1. Express z_n as a function of the spacing d between the microphones.

A sound source arrives at the origin O at an angle α_s to the axis $[O, x)$. We consider a situation where the wavefront is planar, see Fig1, and another where the wavefront is spherical, see Fig2. In both situations :

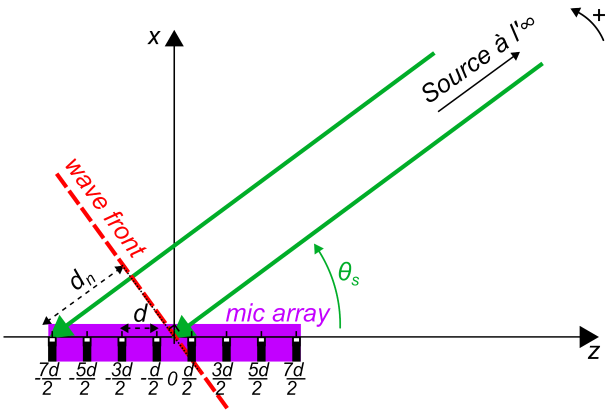

Fig1: plane wave

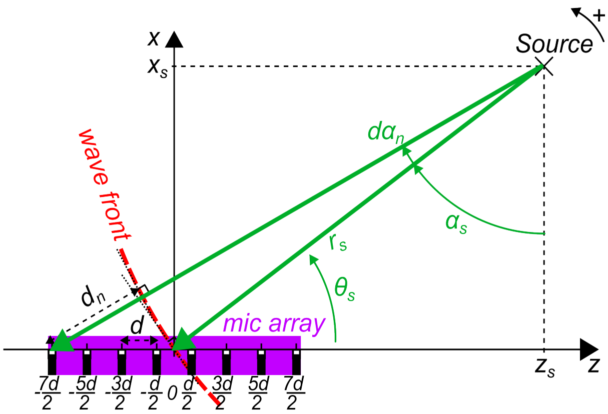

Fig2: spherical wave

- d_n represents the extra distance the wave has to travel to reach the nth microphone, compared to the distance it has to travel to reach the origin of the reference frame ;
- δ_{τ_n} represents the corresponding delay.

1.1.1 Wave fronts

Assumption 1 : the wave front is assumed to be a plane, cf. Fig1.

2. Express d_n as a function of z_n and the angle θ_s, which is the angle of incidence defined with respect to the axis $[O, z]$ of the microphone array.

NB : In this expression, pay attention to the sign of d_n, noting that the angle θ_s is counted positively, cf Fig1.

3. Deduce the angle of incidence θ_s, as a function of the delay δ_{τ_n}.

In practice, it will therefore be sufficient to measure this delay δ_{τ_n} to know the angle of incidence θ_s.

Assumption 2 : the wave front is assumed to be spherical, cf. Fig2.

4. Express the (Cartesian) coordinates of the source (x_s, z_s) as a function of the distance r_s between the source and the centre of the microphone array, the position of the n^{th} microphone z_n, the distance d_n and the angles α_s and $d\alpha_n$, the angle between the wave arriving at the n^{th} microphone and that arriving at O.

5. Deduce the equation verified by d_n :

$$d_n^2 + 2d_n r_s = z_n^2 - 2r_s z_n \cos\theta_s \tag{1}$$

6. Propose a method to derive the position of the source, given by the angle of incidence θ_s with respect to the axis $[O, z)$ of the microphone array, and its distance from the origin r_s.
7. What happens to equation (1) when $r_s \to +\infty$? Conclude.

1.1.2 Sound source

In this tutorial, the sound source will be the speaker integrated in your smartphone. To control this source, download and install the application `ATG lite` for `IOS` or `Frequency Sound Generator` for `Android`.

1.2 In session

For all the experimental part of this tutorial, define an orthonormal reference frame $[0, x, z)$ on your workbench and place the microphone array on the axis $[0, z)$ centred at O.

1.2.1 Getting started

. To begin, start the acquisition of the audio system, and capture one audio buffer. Plot the resulting signals as a function of time.

1.2.2 Speed of sound

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1. Place the source on the $[0, z)$ axis towards the origin O, at a distance greater than the length of the microphone array.
2. Generate a pure tone at a fixed frequency chosen between 100 Hz and 1 kHz using the app previously installed on your smart phone.
3. Record the sound wave with the microphones of the array, over a period of 1 s, and then display the recorded waveforms on the same graph with different colours.
4. Calculate their discrete Fourier transforms on a judiciously chosen number of points. For each Fourier transform, plot the modulus and the unwrapped phase.
5. Measure the time needed by the wave to travel the distance between the microphones, using :
   — (a) the observation of the waveform
   — (b) the phase of the spectra.
6. Deduce the speed of sound propagation, also known as the celerity. Compare with the theoretical value at room temperature.
7. Repeat the previous steps at a frequency of 3 kHz.

Caution ! As the human ear is very sensitive to this frequency, make sure to reduce the sound level of the source.

8. Use the two previous methods - in the time domain (a) and in the frequency domain (b) - to estimate the speed of sound. Interpret the results obtained.
9. Deduce the limit frequency above which it is no longer possible to determine the delay between the acoustic waves measured, only with the observation of the waveforms. In the following, this limit frequency is noted f_{lim}.

1.2.3 Radiation from the source

10. Place the source in the axis of one microphone of the array, as close as possible (1 cm) to the microphone capsule and directed towards the microphone under consideration.
11. Generate a pure tone at 1 kHz.
12. Display the recorded waveform, calculate its RMS value.
13. Repeat the above steps for different values of the source frequency, evenly distributed over the audible spectrum.

For each frequency, make sure that the same "sound volume" is maintained at the source interface.

14. Plot the RMS values as a function of frequency.
15. Interpret the shape of this curve, assuming that the sensitivity curve of the microphone array is flat over the entire audible spectrum (see the specifications of the microphone array, if available).

1.2.4 Wave fronts

16. Place the source in the direction of the centre of the microphone array, at a distance well above the length of the array.
17. Read the coordinates (x_s, y_s) of the source and deduce the angle of incidence θ_s of the source, relative to the $[O, z)$ axis of the microphone array.
18. Generate a pure sound at a frequency below the limit frequency previously measured.
19. Record and display the waveforms measured by each microphone of the array.
20. Estimate the time delay between each of these waveforms.
21. Deduce for each microphone, the delay δ_{τ_n} needed by the wave to travel the distance d_n defined in preparation (see Fig.1).
22. Interpret the shape of the curve δ_{τ_n} as a function of z_n and conclude on the approximation to adopt on the shape of the wavefront.
23. Locate the source using these measurements and the equations obtained in preparation. Compare with the real position of the source.
24. Place the source towards the origin O, at a distance of the same order of magnitude as the length of the microphone array. Then, repeat all the previous steps.
25. At what distance from the source to the microphone array can the sound wave be considered as a plane wave ?

1.2.5 Directivity of the microphone array

The microphone array is designed to locate a sound source in space, especially its angle of incidence. For this purpose, the waveforms recorded by the microphones are phase shifted and then summed. The diagram of directivity shows the RMS value of the sensitivity of the microphone array on a polar coordinate diagram, typically in V/Pa, for each angle of incidence θ_s between the position of the source and the $[O, z)$ axis of the microphone array, typically between $0°$ and $+180°$.

In the absence of phase-shifting filters. As a first step, we propose to plot the directivity pattern of the microphone array in the absence of phase-shifting filters.

26. Place the source on the axis $[0, x)$ perpendicular to the microphone array, towards the middle of the array, at a distance of about 1 m. Read the coordinates of the source and the angle of incidence on the microphone array. Generate a sine wave with a frequency below the limit frequency. Calculate and plot the output signal of the microphone array. Calculate its RMS value.

27. Repeat the previous steps by varying the angle of incidence θ_s from $0°$ to $+180°$, in steps of about $5°$ to $10°$. To do this, in order not to change the source-array distance, the source can be moved to a circle centred on the centre of the microphone array, or the microphone array can be rotated around an axis passing through its centre.
28. Plot the results on a diagram in polar coordinates : each point (r, θ) having as angle : θ_s, the angle of incidence of the source (with the axis $[O, z)$ of the microphone array), and having as radius : r, the RMS value of the microphone array output.

In the presence of phase-shifting filters. In order to assess the influence of the phase-shifting filters on the directivity pattern of the microphone array, the source is placed in the direction of the origin O, always at the same distance from the centre of the microphone array but with a non-zero angle of incidence.

29. Read the coordinates of the source and the angle of incidence on the microphone array. Generate a sine wave with the same frequency as in the previous section (below the limit frequency).
30. Display the waveforms recorded by each microphone and determine the phase-shifting filters that maximise the amplitude of the microphone array output.
31. Apply these filters to the measured signals, calculate and display the microphone array output.
32. Calculate its RMS value.
33. Repeat the above steps, varying the angle of incidence θ_s from $0°$ to $+180°$, in steps of about $5°$ to $10°$. Make sure that the source-array distance and the phase-shifting filters are not changed.
34. Plot the results on a polar diagram.
35. Compare this directivity diagram with the previous one, measured without phase shifters.

1.2.6 BONUS :

36. Plot the directivity pattern of the microphone array without phase shifters for a higher frequency than the value used in the previous section (but below the limit frequency) and for a lower frequency than the value used in the previous section.
37. Conclude on the influence of the frequency on the directivity of the microphone array.