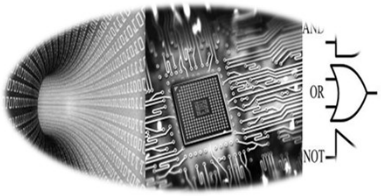
DIGITAL ELECTRONICS

Lecture 05: Karnaugh Map (K-Map)



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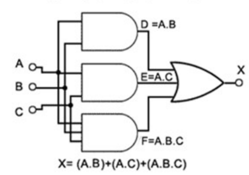
Why Minterm and Maxterm?

- Developing a digital logic circuit using logic gates is very straight forward when a Boolean Expression is given.

X= (A.B)+(A.C)+(A.B.C)



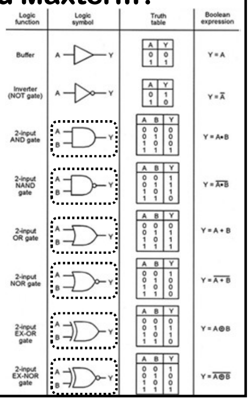
- Even for a large or complex BE the design is possible

$$f(A,B,C) = \left(\overline{AB + AB\overline{C} + BCD + \overline{E}(A+\overline{B}) + A\overline{B}\overline{E}}\right)F$$

IS IT POSSIBLE TO DESIGN A DIGITAL CIRCUIT FROM A TRUTH TABLE?

Why Minterm and Maxterm?

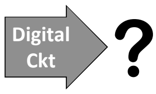
- The function of each logic gates could be possible to represent through its Truth Table.
- Hence:
  Designing a digital circuit from a Truth Table could be possible for a known or standard digital logic.
- Thus, if a Truth table is provided the gate could be designed.



Circuit Design from Truth Table

- But, if a unknown or non-standard or large truth table is provided then designing a digital circuit need Minterm and Maxterm based Boolean representation

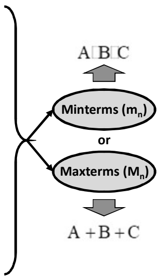
A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



Minterms or Maxterms from Truth Table

- From the truth table Minterms or Maxterms are found and accordingly the SSOP or SPOS are formed.
- Using the SSOP or SPOS, the digital circuits are formed.

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



Minterm and SSOP from Truth Table

- From the truth table Minterms or Maxterms are found and accordingly the SSOP or SPOS are formed.
- Using the SSOP or SPOS, the digital circuits are formed.

A	B	C	X	Minterms	Product Terms
0	0	0	1	m_0	$\bar{A}\bar{B}\bar{C}$
0	0	1	1	m_1	$\bar{A}\bar{B}C$
0	1	0	1	m_2	$\bar{A}B\bar{C}$
0	1	1	0	m_3	$\bar{A}BC$
1	0	0	0	m_4	$A\bar{B}\bar{C}$
1	0	1	1	m_5	$A\bar{B}C$
1	1	0	0	m_6	$AB\bar{C}$
1	1	1	1	m_7	ABC

Minterm and SSOP from Truth Table

- Check the 1s in the output (X) variable.
- Corresponding product terms will be there in SSOP.

A	B	C	X	Minterms	Product Terms
0	0	0	1	m_0	$\overline{A}\,\overline{B}\,\overline{C}$ ✓
0	0	1	1	m_1	$\overline{A}\,\overline{B}\,C$ ✓
0	1	0	1	m_2	$\overline{A}\,B\,\overline{C}$ ✓
0	1	1	0	m_3	$\overline{A}\,B\,C$ X
1	0	0	0	m_4	$A\,\overline{B}\,\overline{C}$ X
1	0	1	1	m_5	$A\,\overline{B}\,C$ ✓
1	1	0	0	m_6	$A\,B\,\overline{C}$ X
1	1	1	1	m_7	$A\,B\,C$ ✓

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C + A\,B\,C$$

Maxterm and SPOS from Truth Table

- From the truth table Maxterms are found and accordingly the SPOS are formed.
- Using the SSOP or SPOS, the digital circuits are formed.

A	B	C	X	Maxterms	Sum Terms
0	0	0	1	m_0	$A+B+C$
0	0	1	1	m_1	$A+B+\overline{C}$
0	1	0	1	m_2	$A+\overline{B}+C$
0	1	1	0	m_3	$A+\overline{B}+\overline{C}$
1	0	0	0	m_4	$\overline{A}+B+C$
1	0	1	1	m_5	$\overline{A}+B+\overline{C}$
1	1	0	0	m_6	$\overline{A}+\overline{B}+C$
1	1	1	1	m_7	$\overline{A}+\overline{B}+\overline{C}$

Maxterm and SPOS from Truth Table

- Check the 1s in the output (X) variable.
- Corresponding product terms will be there in SPOS.

A	B	C	X	Minterms	Product Terms
0	0	0	1	M_0	$A+B+C$ X
0	0	1	1	M_1	$A+B+\overline{C}$ X
0	1	0	1	M_2	$A+\overline{B}+C$ X
0	1	1	0	M_3	$A+\overline{B}+\overline{C}$ ✓
1	0	0	0	M_4	$\overline{A}+B+C$ ✓
1	0	1	1	M_5	$\overline{A}+B+\overline{C}$ X
1	1	0	0	M_6	$\overline{A}+\overline{B}+C$ ✓
1	1	1	1	M_7	$\overline{A}+\overline{B}+\overline{C}$ X

$$f(A,B,C) = (A+\overline{B}+\overline{C})\cdot(\overline{A}+B+C)\cdot(\overline{A}+\overline{B}+C)$$

Maxterm and SPOS from Truth Table

- From the truth table Minterms or Maxterms are found and accordingly the SSOP or SPOS are formed.
- Using the SSOP or SPOS, the digital circuits are formed.

A	B	C	X	Maxterms	Sum Terms
0	0	0	1	m_0	$A+B+C$
0	0	1	1	m_1	$A+B+\overline{C}$
0	1	0	1	m_2	$A+\overline{B}+C$
0	1	1	0	m_3	$A+\overline{B}+\overline{C}$
1	0	0	0	m_4	$\overline{A}+B+C$
1	0	1	1	m_5	$\overline{A}+B+\overline{C}$
1	1	0	0	m_6	$\overline{A}+\overline{B}+C$
1	1	1	1	m_7	$\overline{A}+\overline{B}+\overline{C}$

K-Map

Karnaugh Map (K-Map)

- Karnaugh Mapping is used to minimize the number of logic gates that are required in a digital circuit.
- This will replace Boolean reduction when the circuit is large.
- Write the Boolean equation in a SOP form first and then place each term on a map.

LARGE BOOLEAN EXPRESSION → K-Map → SIMPLEST BOOLEAN EXPRESSION

- **Karnaugh Mapping**
  1. SOP-Based (BE should be in SSOP form)
  2. POS-Based (BE should be in SPOS form)

Karnaugh Map (K-Map)

- The map is made up of a table of every possible SOP using the number of variables that are being used.
  - **2-variable function: 2×2 map**
  - **3-variable function: 4×2 map**
  - **4-variable function: 4×4 map**
  - **5-variable function: 2×(4×4) map**
  - **6-variable function: 4×(4×4) map**
  - **7-variable function: K-Map is not used**

K-Map SOP Minimization

Logic Simplification With Karnaugh Maps

- The logic simplification examples that we have done so far could have been performed with Boolean algebra about as quickly.
- Real world logic simplification problems call for larger Karnaugh maps so that we may do serious work.
- We will work some contrived examples in this section, leaving most of the real world applications for the Combinatorial Logic chapter.
- By contrived, we mean examples which illustrate techniques.
- This approach will develop the tools we need to transition to the more complex applications in the Combinatorial Logic chapter.

Karnaugh Map Representation of BE

- The K-Map could be represented either with SSOP i.e. either with Minterms (m_n).

$$f(A,B,C) = A\overline{B}C + \overline{A}BC + A\overline{B}\,\overline{C} + ABC$$

$$f(A,B,C) = m_3 + m_4 + m_5 + m_7 = \sum m(3,4,5,7)$$

- The K-Map could be represented either with SPOS i.e. with Maxterms (M_n).

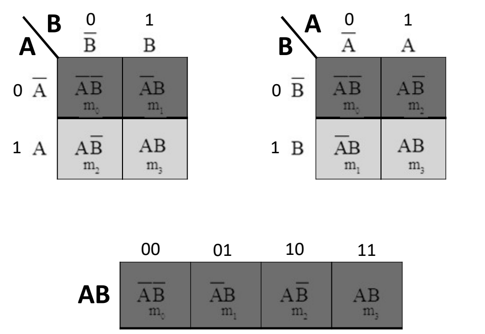
$$f(A,B,C) = (A+\overline{B}+C)\cdot(\overline{A}+B+C)\cdot(A+\overline{B}+\overline{C})\cdot(A+B+C)$$

$$f(A,B,C) = M_2 + M_4 + M_3 + M_0 = \sum m(0,2,3,4)$$

Karnaugh Map Representation

- The K-Map is represented by cells which represent each product term or min term in a SSOP.
- Total cells is given by
- The 2-variable K-Map has: $N_{Cell} = 2^2 = 4$
- The 3-variable K-Map has: $N_{Cell} = 2^3 = 8$
- The 4-variable K-Map has: $N_{Cell} = 2^4 = 16$
- The 5-variable K-Map has: $N_{Cell} = 2^5 = 32$
- The 6-variable K-Map has: $N_{Cell} = 2^6 = 64$

K-Map: 2-Variable Representation



K-Map: 2-Variable Representation

A \ B	0 $\bar{B}$	1 B
0 $\bar{A}$	m_0	m_1
1 A	m_2	m_3

B \ A	0 $\bar{A}$	1 A
0 $\bar{B}$	m_0	m_2
1 B	m_1	m_3

	00	01	10	11
AB	m_0	m_1	m_2	m_3

K-Map: 3-Variable Representation

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1 A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

BC \ A	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1 A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

K-Map: 3-Variable Representation

AB \ C	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1 A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

C \ AB	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1 A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

K-Map: 4-Variable Representation

AB \ CD	00 $\bar{C}\bar{D}$	01 $\bar{C}D$	11 CD	10 $C\bar{D}$
00 $\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0000	$\bar{A}\bar{B}\bar{C}D$ 0001	$\bar{A}\bar{B}CD$ 0011	$\bar{A}\bar{B}C\bar{D}$ 0010
01 $\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 0100	$\bar{A}B\bar{C}D$ 0101	$\bar{A}BCD$ 0111	$\bar{A}BC\bar{D}$ 0110
11 AB	$AB\bar{C}\bar{D}$ 1100	$AB\bar{C}D$ 1100	$ABCD$ 1100	$ABC\bar{D}$ 1100
10 $A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 1000	$A\bar{B}\bar{C}D$ 1000	$A\bar{B}CD$ 1000	$A\bar{B}C\bar{D}$ 1000

K-Map: 4-Variable Representation

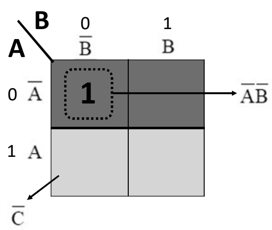
AB CD

AB \ CD	00 $\bar{C}\bar{D}$	01 $\bar{C}D$	11 CD	10 $C\bar{D}$
00 $\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0000_0	$\bar{A}\bar{B}\bar{C}D$ 0001_1	$\bar{A}\bar{B}CD$ 0011_3	$\bar{A}\bar{B}C\bar{D}$ 0010_2
01 $\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 0100_4	$\bar{A}B\bar{C}D$ 0101_5	$\bar{A}BCD$ 0111_7	$\bar{A}BC\bar{D}$ 0110_6
11 AB	$AB\bar{C}\bar{D}$ 1100_{12}	$AB\bar{C}D$ 1101_{13}	$ABCD$ 1111_{15}	$ABC\bar{D}$ 1110_{14}
10 $A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 1000_8	$A\bar{B}\bar{C}D$ 1001_9	$A\bar{B}CD$ 1011_{11}	$A\bar{B}C\bar{D}$ 1010_{10}

$N_{Cell} = 2^4 = 16$

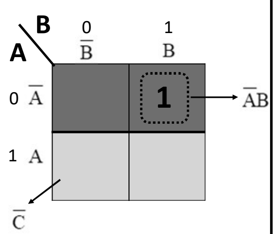
K-Map: SSOP Representation

$f(A,B,C) = \bar{A}\bar{B}$



$f(A,B,C) = \bar{A}\bar{B}$

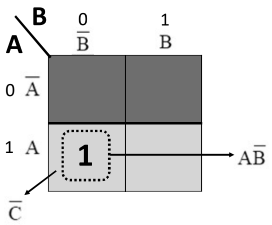
$f(A,B,C) = \bar{A}B$

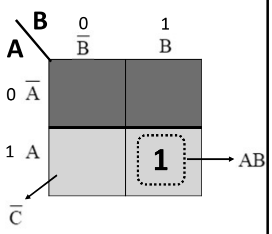


$f(A,B,C) = \bar{A}B$

K-Map: SSOP Representation

$f(A,B,C) = A\bar{B}$ $f(A,B,C) = AB$

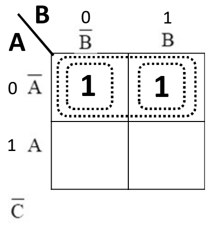


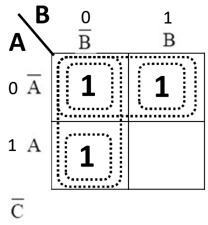


$f(A,B,C) = A\bar{B}$ $f(A,B,C) = AB$

K-Map: Grouping and Implicant

$f(A,B,C) = \bar{A}\bar{B} + \bar{A}B$



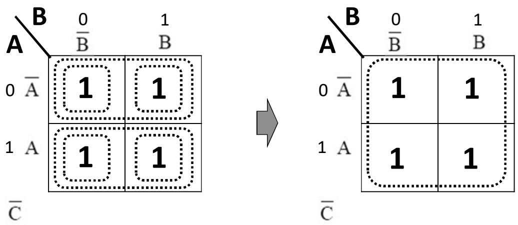


- The simplified Boolean expression is given by:

$f(A,B,C) = \bar{A}$

K-Map: Grouping and Implicant

$f(A,B,C) = \bar{A}\bar{B} + \bar{A}B$



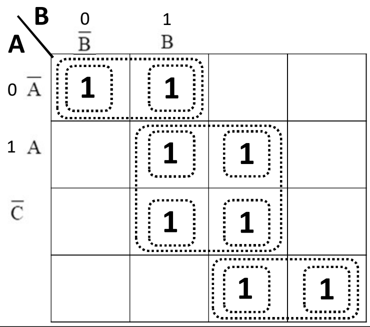
- The simplified Boolean expression is given by:

$f(A,B,C) = \bar{A}$

K-Map: Product Term Representation

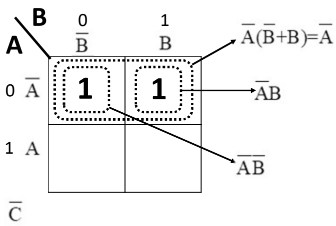
$f(A,B,C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC$

- **Implicants:** 1s
- **Prime Implicants (PI)**
  - **Largest possible group of 1s**
- **Essential Prime Implicants (EPI)**
  - **PI that contains at least one Minterm which could not be further combined**



K-Map: Grouping and Prime Implicant

$f(A,B,C) = \bar{A}\bar{B} + \bar{A}B$

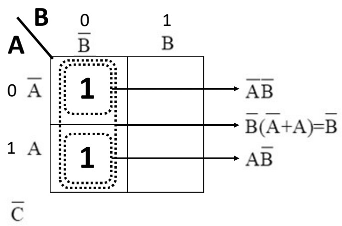


- The simplified Boolean expression is given by:

$f(A,B,C) = \bar{A}$

K-Map: Grouping and Prime Implicant

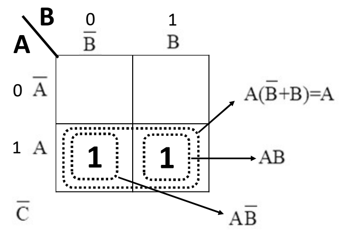
$f(A,B,C) = \bar{A}\bar{B} + A\bar{B}$



$f(A,B,C) = \bar{B}$

K-Map: Grouping and Prime Implicant

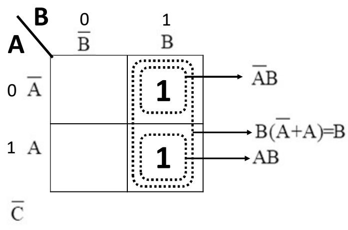
$f(A,B,C) = A\overline{B} + AB$



$f(A,B,C) = A$

K-Map: Grouping and Prime Implicant

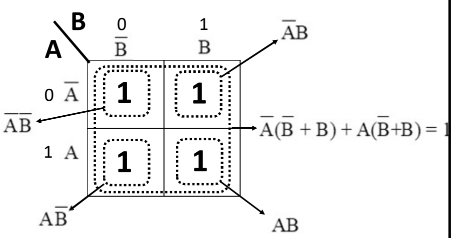
$f(A,B,C) = \overline{A}B + AB$



$f(A,B,C) = B$

K-Map: Grouping and Prime Implicant

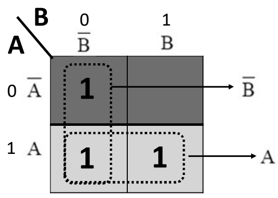
$f(A,B,C) = \overline{A}\,\overline{B} + \overline{A}B + A\overline{B} + AB$



$f(A,B,C) = 1$

K-Map Simplification: Example 01

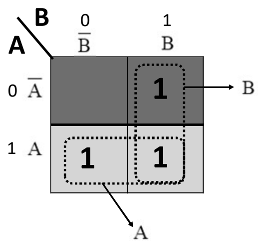
$f(A,B,C) = \overline{A}\,\overline{B} + A\overline{B} + AB$



$f(A,B,C) = A+\overline{B}$

K-Map Simplification: Example 02

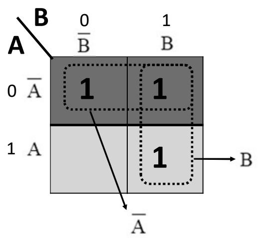
$f(A,B,C) = \overline{A}B + A\overline{B} + AB$



$f(A,B,C) = A+B$

K-Map Simplification: Example 02

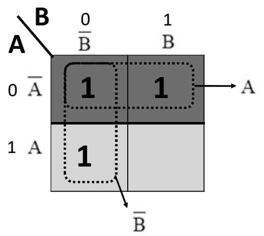
$f(A,B,C) = \overline{A}\,\overline{B} + \overline{A}B + AB$



$f(A,B,C) = \overline{A}+B$

K-Map Simplification: Example 02

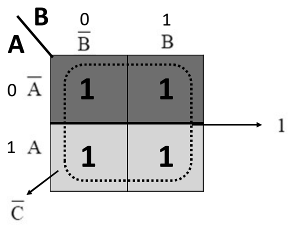
$f(A,B,C) = \bar{A}\bar{B} + \bar{A}B + A\bar{B}$



$f(A,B,C) = A + \bar{B}$

K-Map Simplification: Example 03

$f(A,B,C) = \bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$



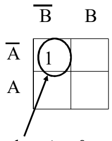
$f(A,B,C) = 1$

2 Variables Karnaugh Map

	$\bar{B}$	B
$\bar{A}$	0	1
A	2	3

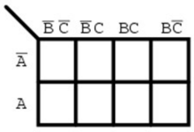
Notice that the map is going false to true, left to right and top to bottom

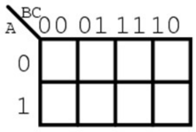
The upper right hand cell is $\bar{A}\,\bar{B}$ if X= $\bar{A}\,\bar{B}$ then put an X in that cell



This show the expression true when A = 0 and B = 0

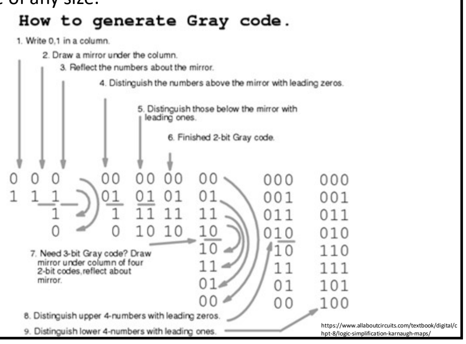
3 Variables Karnaugh Map





Generating Gray Code

- If we sketch our own Karnaugh maps, we need to generate Gray code for any size map that we may use. This is how we generate Gray code of any size.



3 Variables Karnaugh Map

Gray Code		0 $\bar{C}$	1 C
00	$\bar{A}\bar{B}$	0	1
01	$\bar{A}B$	2	3
11	AB	6	7
10	$A\bar{B}$	4	5

2 Variables Karnaugh Map: Grouping

If X=$\bar{A}\bar{B} + A\bar{B}$ then put an X in both of these cells

	$\bar{B}$	B
$\bar{A}$	1	
A	1	

From Boolean reduction we know that $\bar{A}\,\bar{B} + A\,\bar{B} = \bar{B}$

From the Karnaugh map we can circle adjacent cell and find that X = $\bar{B}$

	$\bar{B}$	B
$\bar{A}$	1	
A	1	

3 Variables Karnaugh Map (cont'd)

$f(A,B,C) = \bar{A}\bar{B}C+\bar{A}\bar{B}C+A\bar{B}\bar{C}+A\bar{B}C$

Gray Code		0 $\bar{C}$	1 C
00	$\bar{A}\,\bar{B}$	1	1
01	$\bar{A}\,B$		
11	A B		
10	A $\bar{B}$	1	1

One simplification could be

X = $\bar{A}\,\bar{B} + A\,\bar{B}$

3 Variables Karnaugh Map (cont'd)

$f(A,B,C) = \bar{A}\bar{B}C+\bar{A}\bar{B}C+A\bar{B}\bar{C}+A\bar{B}C$

Gray Code		0 $\bar{C}$	1 C
00	$\bar{A}\,\bar{B}$	1	1
01	$\bar{A}\,B$		
11	A B		
10	A $\bar{B}$	1	1

Another simplification could be

X = $\bar{B}\,\bar{C} + \bar{B}\,C$

A Karnaugh Map does wrap around

3 Variables Karnaugh Map (cont'd)

$f(A,B,C) = \bar{A}\bar{B}C+\bar{A}\bar{B}C+A\bar{B}\bar{C}+A\bar{B}C$

Gray Code		0 $\bar{C}$	1 C
00	$\bar{A}\,\bar{B}$	1	1
01	$\bar{A}\,B$		
11	A B		
10	A $\bar{B}$	1	1

The Best simplification would be

X = $\bar{B}$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}\bar{C}+\bar{A}\bar{B}C$

Out= $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$

A \ BC	00	01	11	10
0	1	1		
1				

Out= $\bar{A}\bar{B}$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$

Out= $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$

A \ BC	00	01	11	10
0	1	1	1	1
1				

Out= $\bar{A}$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$Out = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

A \ BC	00	01	11	10
0		1	1	
1		1	1	

$Out = C$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

$Out = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

A \ BC	00	01	11	10
0	1	1	1	1
1			1	1

$Out = \bar{A} + B$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

$Out = \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

A \ BC	00	01	11	10
0			1	1
1			1	1

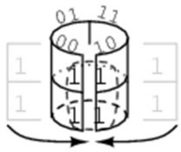
$Out = B$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C}$

$Out = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + AB\bar{C}$

A \ BC	00	01	11	10
0	1			1
1	1			1



$Out = \bar{C}$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$Out = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

A \ BC	00	01	11	10
0	1	1	1	1
1	1			1

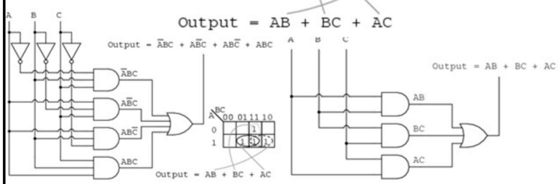
$Out = \bar{A} + \bar{C}$

3 Variables Karnaugh Map

$f(A,B,C) = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

A \ BC	00	01	11	10
0			1	
1		1	1	1

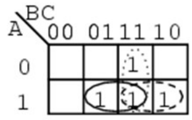
Output = AB + BC + AC



3 Variables Karnaugh Map

$f(A,B,C) =$

$\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$



Output = AB + BC + AC

$\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

↓ Factoring BC out of 1st and 4th terms

$BC(\bar{A} + A) + A\bar{B}C + AB\bar{C}$

↓ Applying identity $A + \bar{A} = 1$

$BC(1) + A\bar{B}C + AB\bar{C}$

↓ Applying identity $1A = A$

$BC + A\bar{B}C + AB\bar{C}$

↓ Factoring B out of 1st and 3rd terms

$B(C + A\bar{C}) + A\bar{B}C$

↓ Applying rule $A + \bar{A}B = A + B$ to the $C + A\bar{C}$ term

$B(C + A) + A\bar{B}C$

↓ Distributing terms

$BC + AB + A\bar{B}C$

↓ Factoring A out of 2nd and 3rd terms

$BC + A(B + \bar{B}C)$

↓ Applying rule $A + \bar{A}B = A + B$ to the $B + \bar{B}C$ term

$BC + A(B + C)$

↓ Distributing terms

$BC + AB + AC$

or Simplified result

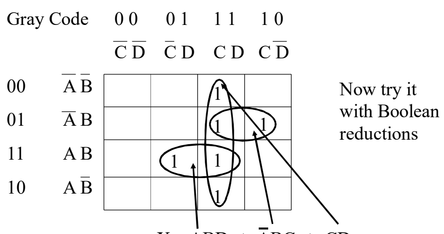
$AB + BC + AC$

On a 3 Variables Karnaugh Map

- One cell requires 3 Variables
- Two adjacent cells require 2 variables
- Four adjacent cells require 1 variable
- Eight adjacent cells is a 1

4 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$

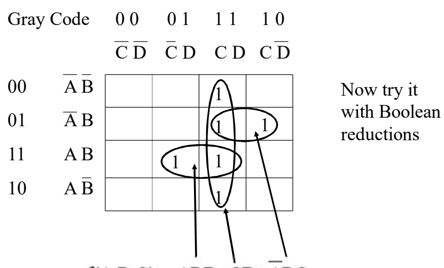


Now try it with Boolean reductions

$X = ABD + \bar{A}BC + CD$

4 Variables Karnaugh Map

$f(A,B,C) = \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$

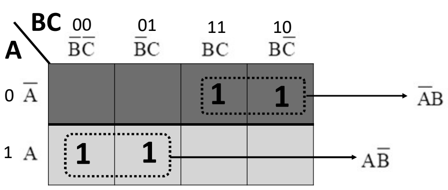


Now try it with Boolean reductions

$f(A,B,C) = ABD + CD + \bar{A}BC$

3 Variables K-Map: Example 01

$f(A,B,C) = \bar{A}BC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C}$



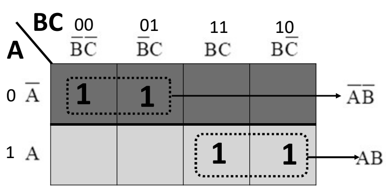
$f(A,B,C) = \bar{A}B + A\bar{B}$

$f(A,B,C) = A \oplus B$

Ref: TKB

3 Variables K-Map: Example 01

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + AB\bar{C}$



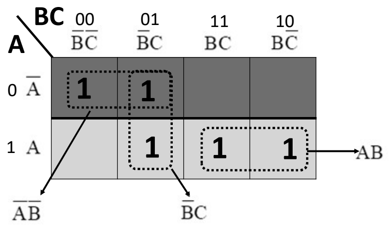
$f(A,B,C) = \bar{A}\bar{B} + AB$

f(A,B,C) = A □ B

Ref: TKB

3 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + ABC + AB\bar{C}$

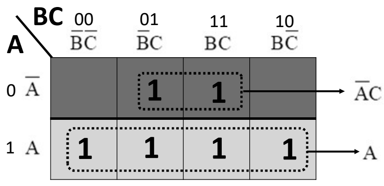


f(A,B,C) = $\bar{A}\bar{B} + AB + \bar{B}C$

Ref: TKB

3 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC + AB\bar{C}$

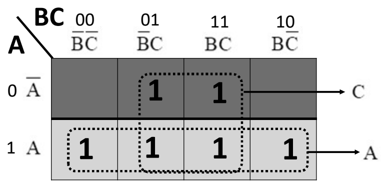


f(A,B,C) = $A + \bar{A}C$

Ref: TKB

3 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC + AB\bar{C}$

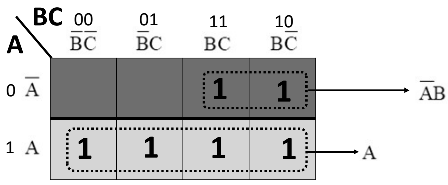


f(A,B,C) = $A + \bar{A}C$

Ref: TKB

3 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC + AB\bar{C}$

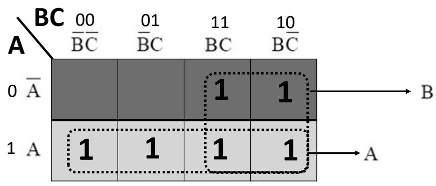


f(A,B,C) = $A + \bar{A}C$

Ref: TKB

3 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC + AB\bar{C}$

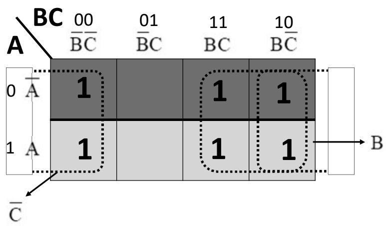


f(A,B,C) = A + B

Ref: TKB

2 Variables K-Map: Example 01

f(A,B,C) = $\bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC + AB\bar{C}$

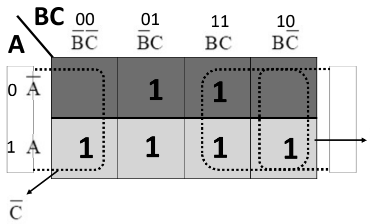


f(A,B,C) = $B + \bar{C}$

Ref: Digital Technology: Principles and Practice, Virendra Kumar, New Age International Pub.

3 Variables K-Map: Example 01

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC + AB\bar{C}$

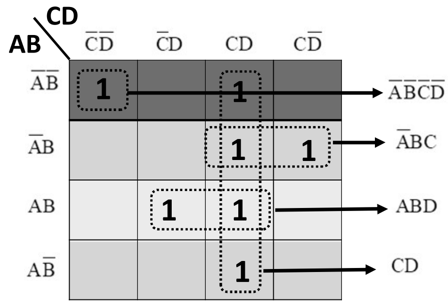


$f(A,B,C) = B + \bar{C}$

Ref: Digital Technology: Principles and Practice, Virendra Kumar, New Age International Pub.

4 Variables K-Map: Example 01

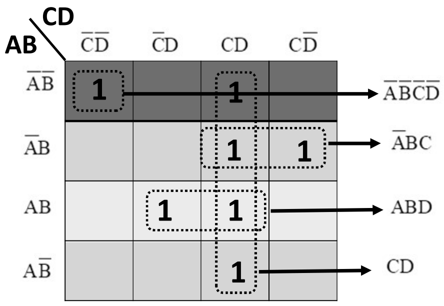
$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$



$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D}\ \bar{A}BC + ABD + CD$

4 Variables K-Map: Example 02

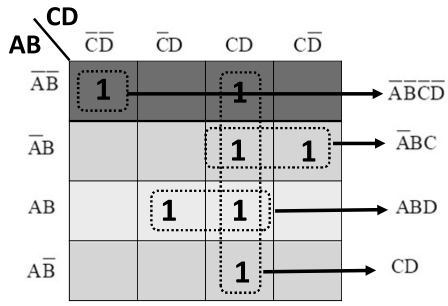
$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$



$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D}\ \bar{A}BC + ABD + CD$

4 Variables K-Map: Example 03

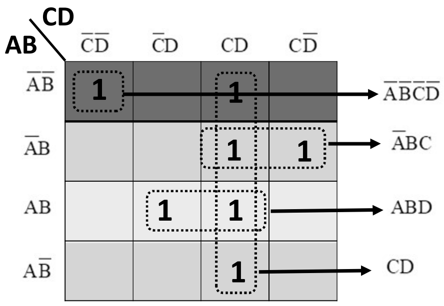
$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$



$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D}\ \bar{A}BC + ABD + CD$

4 Variables K-Map: Example 04

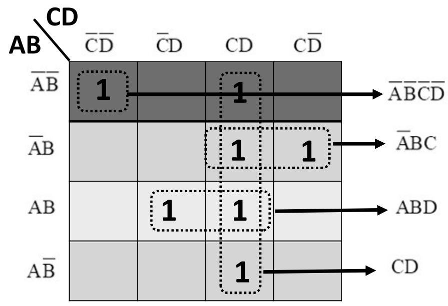
$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$



$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D}\ \bar{A}BC + ABD + CD$

4 Variables K-Map: Example 05

$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + AB\bar{C}D + A\bar{B}CD$



$f(A,B,C) = \bar{A}\bar{B}\bar{C}\bar{D}\ \bar{A}BC + ABD + CD$

4 Variables K-Map: Example 06

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$$

AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$	
$\overline{A}\,\overline{B}$	1		1		→ $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$\overline{A}B$			1	1	→ $\overline{A}BC$
AB		1	1		→ ABD
$A\overline{B}$			1		→ CD

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\ \overline{A}BC + ABD + CD$$

4 Variables K-Map: Example 07

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$$

AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$	
$\overline{A}\,\overline{B}$	1		1		→ $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$\overline{A}B$			1	1	→ $\overline{A}BC$
AB		1	1		→ ABD
$A\overline{B}$			1		→ CD

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\ \overline{A}BC + ABD + CD$$

4 Variables K-Map: Example 08

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$$

AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$	
$\overline{A}\,\overline{B}$	1		1		→ $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$\overline{A}B$			1	1	→ $\overline{A}BC$
AB		1	1		→ ABD
$A\overline{B}$			1		→ CD

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\ \overline{A}BC + ABD + CD$$

4 Variables K-Map: Example 09

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$$

AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$	
$\overline{A}\,\overline{B}$	1		1		→ $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$\overline{A}B$			1	1	→ $\overline{A}BC$
AB		1	1		→ ABD
$A\overline{B}$			1		→ CD

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\ \overline{A}BC + ABD + CD$$

4 Variables K-Map: Example 10

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$$

AB \ CD	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$	
$\overline{A}\,\overline{B}$	1		1		→ $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$
$\overline{A}B$			1	1	→ $\overline{A}BC$
AB		1	1		→ ABD
$A\overline{B}$			1		→ CD

$$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\ \overline{A}BC + ABD + CD$$

K-Map: Don’t Care Conditions

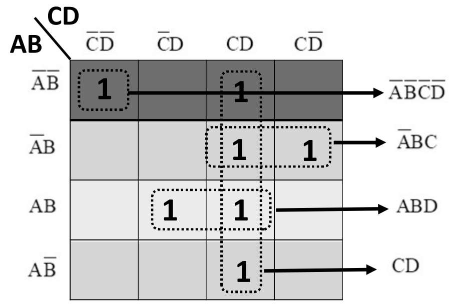
- In some logic circuits certain input conditions never occur.
- Therefore, corresponding outputs never appear.
- In such cases outputs are not defined.
- Outputs either be HIGH or LOW.
- Such outputs conditions are represented as **“Don’t-Care”** conditions in the Boolean Algebra.
- Output for the **“Don’t-Care”** conditions in the Truth table and K-Map is represented as “x”.
- In the minterms based BE the function with the **“Don’t-Care”** conditions could be represented as follows:

$$f(A,B,C) = \sum m(0,1,2,4,8,10) + d(5,7)$$

- The **“Don’t-Care”** conditions are represented as d(5,7)

K-Map: Don't Care Conditions

$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}BC\overline{D} + ABCD + AB\overline{C}D + A\overline{B}CD$

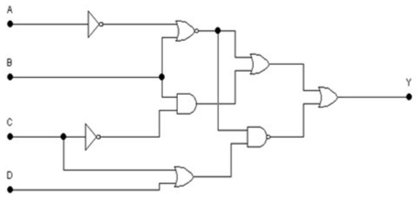


$f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\;\overline{A}BC+ABD+CD$

On a 4 Variables Karnaugh map

- One Cell requires 4 variables
- Two adjacent cells require 3 variables
- Four adjacent cells require 2 variables
- Eight adjacent cells require 1 variable
- Sixteen adjacent cells give a 1 or true

Simplify using Karnaugh map



First, we need to change the circuit to an SOP expression

Simplify using Karnaugh Map

$$Y = \overline{\overline{A}+B} + B\overline{C} + \overline{(\overline{\overline{\overline{A}+B}})(C+D)}$$

$$Y = \overline{\overline{A}}\,\overline{B} + B\overline{C} + \overline{\overline{\overline{A}}\,\overline{B}(C+D)}$$

$$Y = A\overline{B} + B\overline{C} + \overline{A\overline{B}C + A\overline{B}D}$$

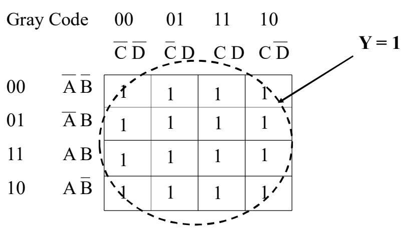
$$Y = A\overline{B} + B\overline{C} + \overline{A\overline{B}C}\;\overline{A\overline{B}D}$$

$$Y = A\overline{B} + B\overline{C} + (\overline{A} + B + \overline{C})(\overline{A} + B + \overline{D})$$

$$Y = A\overline{B} + B\overline{C} + \overline{A} + \overline{A}B + \overline{A}\,\overline{D} + B + B\overline{D} + \overline{A}\,\overline{C} + \overline{C}\,\overline{D}$$

SOP expression

Simplify using Karnaugh map (cont'd)



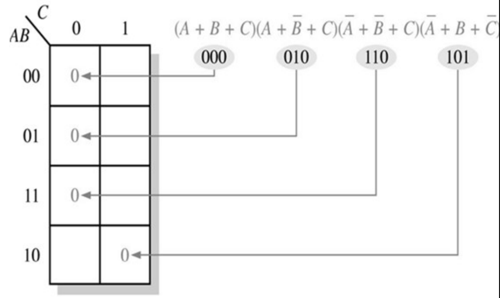
K-Map POS Minimization

3 Variables Karnaugh Map

Gray Code

AB \ C	0	1
0 0	0	1
0 1	2	3
1 1	6	7
1 0	4	5

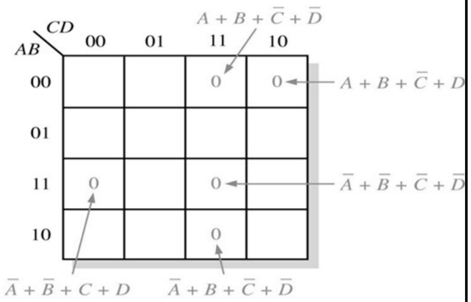
3 Variables Karnaugh Map (cont'd)



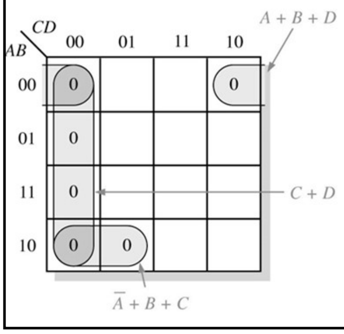
4 Variables Karnaugh Map

AB \ CD	0 0	0 1	1 1	1 0
0 0	0	1	3	2
0 1	4	5	7	6
1 1	12	13	15	14
1 0	8	9	11	10

4 Variables Karnaugh Map (cont'd)



4 Variables Karnaugh Map (cont'd)



Karnaugh Map - Example

- **Mapping a Standard SOP expression**
  - Example:

$$Y = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}BCD + \overline{A}\,\overline{B}C\overline{D} + A\overline{B}C\overline{D}$$

Answer:

$$Y = \overline{B}\,\overline{D} + \overline{A}CD$$

- **Mapping a Standard POS expression**
  - Example:
    Using K-Map, convert the following standard POS expression into a minimum SOP expression

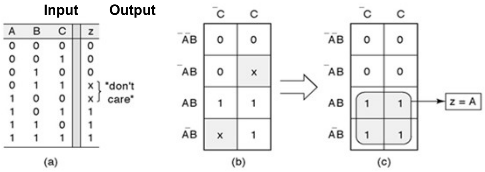
$$Y = A(\overline{B} + C)$$

Answer:

$Y = \overline{A}B + AC$ or standard SOP : $Y = A\overline{B}\,\overline{C} + A\overline{B}C + ABC$

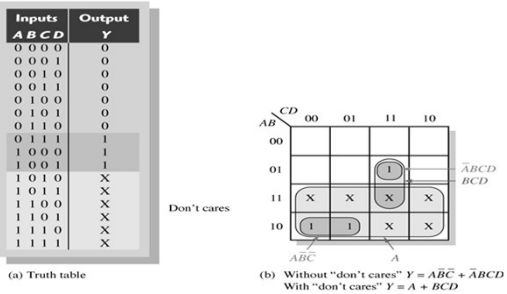
K-Map with “Don’t Care” Conditions

Example :



3 variables with output “don’t care (X)”

K-Map with “Don’t Care” Conditions (cont’d)



4 variables with output “don’t care (X)”

K-Map with “Don’t Care” Conditions (cont’d)

- **“Don’t Care” Conditions**
  - Example:
    Determine the minimal SOP using K-Map:

$$F(A, B, C, D) = \pi M(0,2,6,8,9,10)\ D(5,12,13,14,15)$$

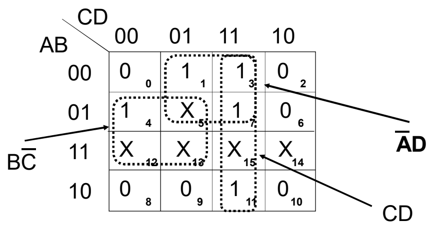
Answer:

$$F(A,B,C,D) = CD + B\overline{C} + \overline{A}D$$

K-Map with “Don’t Care” Conditions (cont’d)

Solution :

$$F(A, B, C, D) = \pi M(0,2,6,8,9,10)\ D(5,12,13,14,15)$$



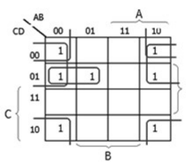
Minimum SOP expression is

$$F(A,B,C,D) = CD + B\overline{C} + \overline{A}D$$

K-map Product of Sums simplification

Example: Simplify the Boolean function $F_{(ABCD)}=\Sigma(0,1,2,5,8,9,10)$ in

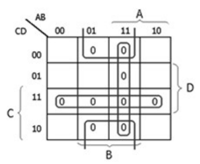
(a) S-of-p



Using the minterms (1’s)

$F_{(ABCD)}= B'D'+B'C'+A'C'D$

(b) P-of-s



Using the maxterms (0’s) and complimenting F
Grouping as if they were minterms, then using De Morgen’s theorem to get F.

$F'_{(ABCD)}= BD'+CD+AB$

$F_{(ABCD)}= (B'+D)(C'+D')(A'+B')$

Quine-McCluskey Method

- A **method** used for minimization of Boolean functions.
- Developed by Willard V. **Quine** and extended by Edward J. **McCluskey.**
- McCluskey developed the first algorithm for designing combinational circuits - the Quine-McCluskey logic minimization procedure as a doctoral student at MIT.
- Notable Publications:

1. *Quine, Willard Van Orman (October 1952). "The Problem of Simplifying Truth Functions". The American Mathematical Monthly. **59** (8): 521–531. doi:10.2307/2308219. JSTOR 2308219.*
2. *Quine, Willard Van Orman (November 1955). "A Way to Simplify Truth Functions". The American Mathematical Monthly. **62** (9): 627–631. doi:10.2307/2307285. JSTOR 2307285.*
3. *McCluskey, Jr., Edward J. (November 1956). "Minimization of Boolean Functions". Bell System Technical Journal. **35** (6): 1417–1444. doi:10.1002/j.1538-7305.1956.tb03835.x. Retrieved 2014-08-24.*

Willard Van Orman Quine

Edward J. McCluskey

Quine-McCluskey Method: Important Terms

- **Implicants:** 1s $f(A,B,C) = \overline{A}\overline{B}C + \overline{A}B\overline{C} + ABC$
- **Prime Implicants (PI)**
  - Largest possible group of 1s
- **Essential Prime Implicants (EPI)**
  - PI that contains at least one Minterm which could not be further combined

AB \ CD	00 $\overline{C}\overline{D}$	01 $\overline{C}D$	11 CD	10 $C\overline{D}$
00 $\overline{A}\overline{B}$	1	1		
01 $\overline{A}B$		1	1	
11 AB	1	1	1	1
10 $A\overline{B}$			1	1

Quine-McCluskey Method

$$f(A,B,C) = \sum m(0, 1, 3, 7, 8, 9, 11, 15)$$

- **Minterms**

Minterms	Designations	Binary	A B C D
m_0	0	0 0 0 0	$\overline{A}\overline{B}\overline{C}\overline{D}$
m_1	1	0 0 0 1	$\overline{A}\overline{B}\overline{C}D$
m_0	3	0 0 1 1	$\overline{A}\overline{B}CD$
m_1	7	0 1 1 1	$\overline{A}BCD$
m_0	8	1 0 0 0	$A\overline{B}\overline{C}\overline{D}$
m_1	9	1 0 0 1	$A\overline{B}\overline{C}D$
m_0	11	1 0 1 1	$A\overline{B}CD$
m_1	15	1 1 1 1	$ABCD$

Quine-McCluskey Method

- **Step 2: Table 2 [To make matched pairs between nth and (n+1)th]**

Group of 1s	Matched Pairs	A B C D
0 (No of 1s is 0)	0	0 0 0 0
1 (No of 1s is 1)	1 8	0 0 0 1 1 0 0 0
2 (No of 1s is 2)	3 9	0 0 1 1 1 0 0 1
3 (No of 1s is 3)	7 11	0 1 1 1 1 0 1 1
4 (No of 1s is 4)	15	1 1 1 1

Quine-McCluskey Method

- **Step 1: Table 1**

Group	Minterms	A B C D	Matched Pairs	Variable
0	0	0 0 0 0	0, 1 0, 8	0 0 0 X X 0 0 0
1	1 8	0 0 0 1 1 0 0 0	1, 3 1, 9 8, 9	0 0 X 1 X 0 0 1 1 0 0 X
2	3 9	0 0 1 1 1 0 0 1	3, 7 3, 11 9, 11	0 X 1 1 X 0 1 1 1 0 X 1
3	7 11	0 1 1 1 1 0 1 1	7, 15 11, 15	X 1 1 1 1 X 1 1
4	15	1 1 1 1		

- Step 2:
- Table 2

Quine-McCluskey Method

Groups	Matched Minterms	A B C D	Matched Pairs	
0	0, 1 0, 8	0 0 0 X X 0 0 0	0,1; 8,9 0,8; 1,9	X 0 0 X X 0 0 X $\overline{B}\overline{C}$
1	1, 3 1, 9 8, 9	0 0 X 1 X 0 0 1 1 0 0 X	1,3; 9,11 1,9; 3,11	X 0 X 1 X 0 X 1 $\overline{B}D$
2	3, 7 3, 11 9, 11	0 X 1 1 X 0 1 1 1 0 X 1	3,7; 11,15 3,11; 7,15	X X 1 1 X X 1 1 CD
3		X 1 1 1 1 X 1 1		
4				

- Step 3:
- Table 3

Quine-McCluskey Method

PI	Minterms Involved	0	1	3	7	8	9	11	15
$\overline{B}\overline{C}$	0, 1, 8, 9	X	X			X	X		
$\overline{B}D$	1, 3, 9, 11		X	X			X	X	
CD	3, 7, 11, 15			X	X			X	X

$$f = \overline{B}\overline{C} + CD$$

Solve it with K-Map and Check