Condition for maximum efficiency :-

$$\frac{d\eta}{dI_2} = \frac{\left[V_2 I_2 \cos\theta_2 + (P_i + I_2^2 R_2)\right] V_2 \cos\theta_2 - V_2 I_2 \cos\theta_2 \left[V_2 \cos\theta_2 + 2 I_2 R_2\right]}{\left[V_2 I_2 \cos\theta_2 + (P_i + I^2 R_2)\right]^2}$$

Equating numerator to zero,

$$\left[V_2 I_2 \cos\theta_2 + (P_i + I_2^2 R_2)\right] V_2 \cos\theta_2 = V_2 I_2 \cos\theta_2 \left[V_2 \cos\theta_2 + 2 I_2 R_2\right]$$

$$\Rightarrow V_2 I_2 \cos\theta_2 + P_i + I_2^2 R_2 = V_2 I_2 \cos\theta_2 + 2 I_2^2 R_2$$

$$\Rightarrow P_i = I_2^2 R_2 .$$

$\therefore$ Iron loss = Copper loss.

So when iron loss is equal to variable copper loss, the efficiency attains the maximum value.

$I_2 = \sqrt{\frac{P_i}{R_2}}$ = output current corresponding to maximum efficiency.

Ex:- In a transformer if the load current is kept constant, find the power factor at which the maximum efficiency occurs.

Here, Copper loss $= I_2^2 R_2 =$ constant.

Let, $P_i + I_2^2 R_2 = C$.

$$\therefore \eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + C}$$

$$\therefore \frac{d\eta}{d\theta_2} = \frac{(V_2 I_2 \cos\theta_2 + C).(-V_2 I_2 \sin\theta_2) - V_2 I_2 \cos\theta_2 (-V_2 I_2 \sin\theta_2)}{(V_2 I_2 \cos\theta_2 + C)^2}$$

$$= 0.$$

$$\therefore (V_2 I_2 \cos\theta_2 + C) V_2 I_2 \sin\theta_2 = V_2^2 I_2^2 \cos\theta_2 \sin\theta_2$$

$$\Rightarrow V_2^2 I_2^2 \sin\theta_2 \cos\theta_2 + C V_2 I_2 \sin\theta_2 = V_2^2 I_2^2 \sin\theta_2 \cos\theta_2$$

$$\Rightarrow \sin\theta_2 = 0. \quad \therefore \cos\theta_2 = 1 = \text{Power factor.}$$

All-day efficiency :- Distribution transformers have their primaries energised all the twenty four hours, although their secondaries supply little or no-load much of the time during the day, except during the house lighting period. It means that core loss occurs during the day and copper loss occurs only when the transformer is loaded. Therefore, a better method of assessing the efficiency of a transformer working on variable load is on the energy basis.

The all-day efficiency is defined as the ratio of the total energy output to that of the total energy input over a given period (generally 24-hours).

$$\therefore \quad \eta_{all\text{-}day} = \frac{\text{Output in KWh.}}{\text{Input in KWh.}} \text{ (for 24-hours).}$$

It is also called energy efficiency.

(3) A 5 KVA., 400/200 V., 50 c/s., 1-phase transformer gave the following results —

No-load : 400 V., 1 A., 50 W. (H.V. side)

Short-Circuit : 12 V., 10 A., 40 W. (H.V. side)

Calculate — (i) the components of the no-load current
(ii) the efficiency and regulations at full load and power factor of 0.8 lagging.

Soln. :— (i) $I_c = I_0 \cos\phi_0 = \frac{W_0}{V_0} = \frac{50}{400} = 0.125\ A.$

$I_0 = 1\ A.$

$\therefore$ Magnetising component, $I_m = \sqrt{I_0^2 - I_c^2}$

$= \sqrt{1^2 - (0.125)^2} = 0.992\ A.$

(ii) Measurements are made on the primary side again, during the short circuit test:

$Z_1 = \frac{12}{10} = 1.2\ \Omega.$, $R_1 = \frac{40}{(10)^2} = 0.4\ \Omega$

$\therefore X_1 = \sqrt{(1.2)^2 - (0.4)^2} = 1.13\ \Omega.$

Full-load current on the primary side,

$I_1 = \frac{5000}{400} = 12.5\ A.$

$\therefore$ Full-load copper loss, $P_c = I_1^2 R_1$

$= (12.5)^2 \times 0.4 = \cancel{40\ W.}\ 62.5\ W$

$\therefore \eta_{full\text{-}load} = \frac{5000 \times 0.8}{5000 \times 0.8 + 50 + 62.5} \times 100\%$

$= 97.26\%.$

Regulation at full-load and 0.8 lagging power factor,

$\%\ Reg. = \frac{I_1(R_1 \cos\phi + X_1 \sin\phi)}{V_1} \times 100\%$

$= \frac{12.5(0.4 \times 0.8 + 1.13 \times 0.6)}{400} \times 100\% = 3.13\%.$

(4) A 50 kVA, transformer has 5:1 ratio of turns. The secondary full-load current is 200 A. The primary and secondary resistances are respectively 0.55 Ω and 0.023 Ω. If the transformer is designed for maximum efficiency at $\frac{2}{3}$ of full-load, find its efficiency when delivering full load at 0.8 power factor.

Soln.:- Turns ratio, $K = 1/5$.

$\therefore R_2 = r_1' + r_2 = K^2 r_1 + r_2 = \left(\frac{1}{5}\right)^2 \times 0.55 + 0.023$

$= 0.045\ \Omega$.

Full-load copper loss $= I_2^2 R_2 = (200)^2 \times 0.045$

$= 1800$ W.

$\therefore$ Copper loss at $\frac{2}{3}$ of full-load

$= \left(\frac{2}{3}\right)^2 \times 1800 = 800$ W.

$\therefore$ Iron loss $= 800$ W.

Full-load output at 0.8 p.f. $= 50 \times 0.8 = 40$ kW.

$= 40,000$ W.

Total losses at full-load $= 1800 + 800 = 2,600$ W

$\therefore$ Full-load efficiency, $= \frac{40,000}{40,000 + 2,600} = 93.8\%$.