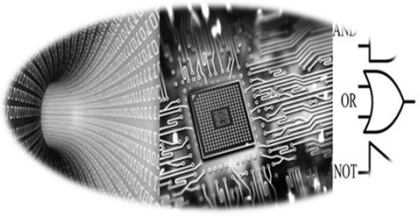
DIGITAL ELECTRONICS

Lecture Note 03: Minterms and Maxterms



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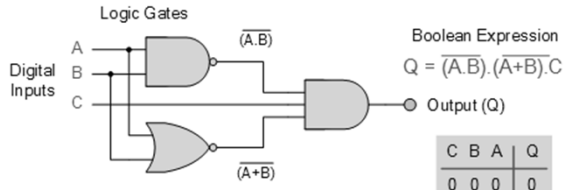
Gate Combination: Boolean Expressions

Any logic circuit could be represented with a Boolean expression and a Truth Table associated with the expression.



Logic Diagram

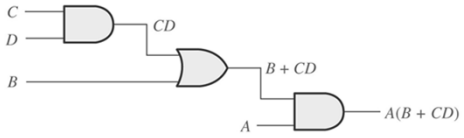
Typical Truth Table

C	B	A	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

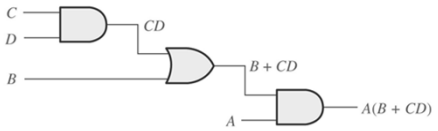
Gate Combination: Boolean Expressions

A logic circuit showing the development of the Boolean expression for the output.

Example 1

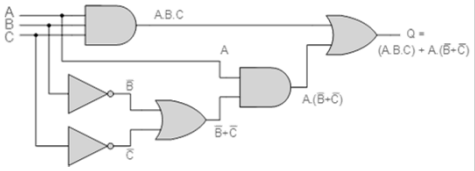


Example 2



Gate Combination: Boolean Expressions

Example 3

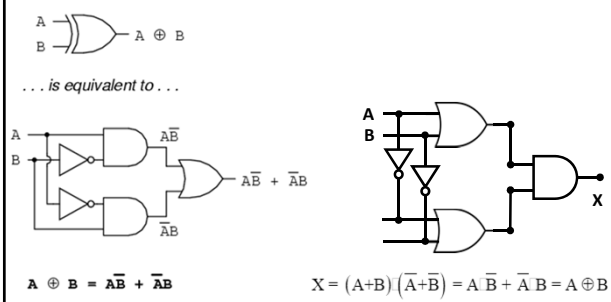


Truth Table

A	B	C	$\overline{B}$	$\overline{C}$	A.B.C	$\overline{B}+\overline{C}$	$A.(\overline{B}+\overline{C})$	$(A.B.C) + A.(\overline{B}+\overline{C})$
0	0	0						
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

Boolean Expressions: SOP and POS

- **Boolean Expressions could be represented with two Standard Forms of Expressions.**



$A \oplus B = A\overline{B} + \overline{A}B$

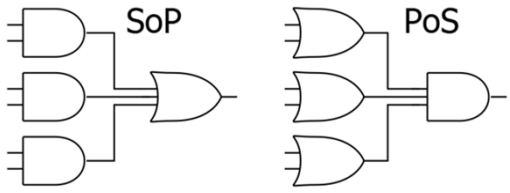
$X = (A+B).(\overline{A}+\overline{B}) = A.\overline{B} + \overline{A}.B = A \oplus B$

Boolean Expressions: SOP and POS

- **Boolean Expressions could be represented with two Standard Forms of Expressions**
- **The sum-of-product (SOP) form**

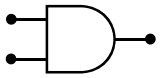
  Example: X = AB + CD + EF
- **The product of sum (POS) form**

  Example: X = (A + B)(C + D)(E + F)

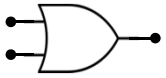


Boolean Expressions: SOP and POS

- The terms "product" and "sum" have been borrowed from mathematics to describe AND and OR logic operations.
- **Any logic system can be represented in one of these two logic ways.**
- As it will be explained in this section, the two forms are equivalent ways of expressing a logic system;
- However, some logic systems lend themselves to one rather than the other.



PRODUCT ≡ AND

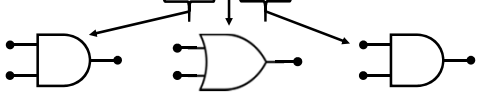


SUM ≡ OR

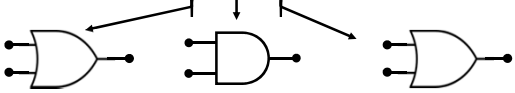
Boolean Expressions: SOP and POS

- *Sum of Products (SOP):* OR'ing of AND'ed terms

$$Y_{SOP} = \overline{A} \cdot B + A \cdot \overline{B}$$

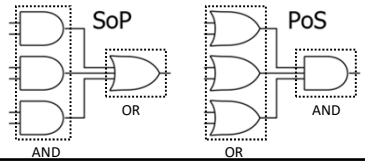


$$Y_{POS} = (A + B) \cdot (\overline{A} + \overline{B})$$



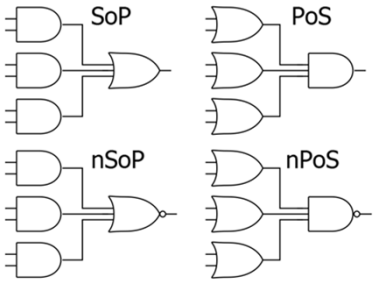
Boolean Expressions: SOP and POS

- **SOP**
  - Product → AND
  - Sum → OR
  - Expression: **OR-ed combination of AND-ed variables**
- **POS**
  - Product → AND
  - Sum → OR
  - Expression: **OR-ed combination of AND-ed variables**



nSOP and nPOS

- **nSOP and nPOS**
  - If output of the SOP is inverted (with a NOT operation) then it is called nSOP.
  - If output of the POS is inverted (with a NOT operation) then it is called nPOS.



Boolean Expressions: SOP

- SSOP
  - In some cases, all the AND-ed functions may not have all the variables
  - **SSOP: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - SOP could be converted into a SSOP form

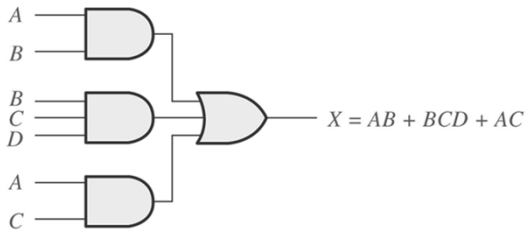


Figure 4–18 Implementation of the SOP expression *AB + BCD + AC*.

SOP without Inverted Variables

- SOP may or may not contain inverted variables

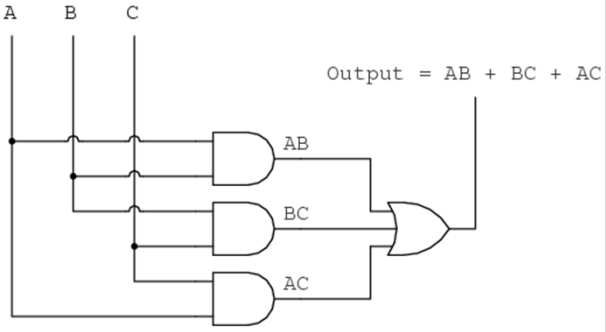
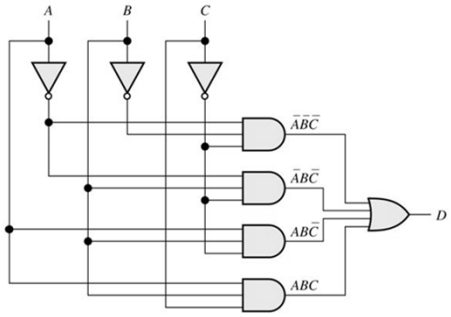


Figure 4–18 Implementation of the SOP expression *AB + BCD + AC*.

SOP with Inverted Variables

- SOP may or may not contain inverted variables



POS with Inverted Variables

- POS may or may not contain inverted variables

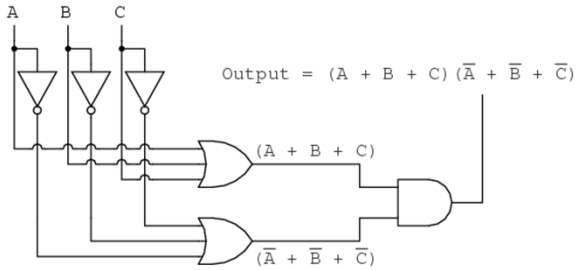


Figure 4–18 Implementation of the SOP expression *AB + BCD + AC*.

Standard SOP (SSOP)

- SSOP
  - In some cases, all the AND-ed functions may not have all the variables
  - **SSOP: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - SOP could be converted into a SSOP form

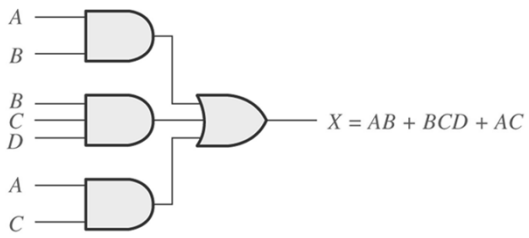
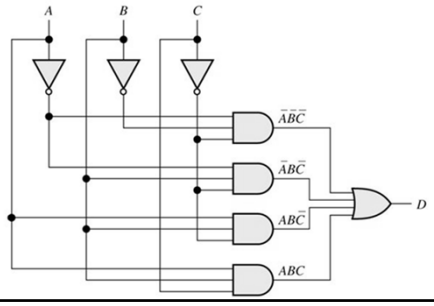


Figure 4–18 Implementation of the SOP expression *AB + BCD + AC*.

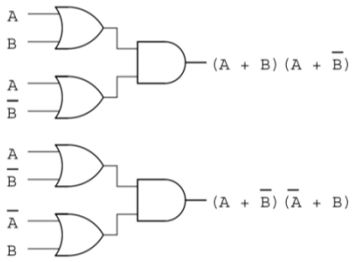
Standard SOP (SSOP)

- SSOP
  - **SSOP: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - SOP could be converted into a SSOP form



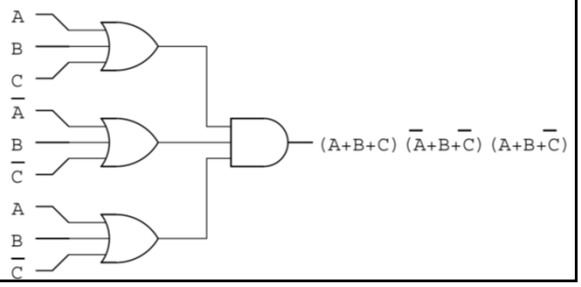
Standard POS (SPOS): Two Variables

- SPOS
  - In some cases, all the OR-ed functions may not have all the variables
  - **SPOS: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - POS could be converted into a SPOS form



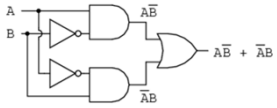
Standard POS (SPOS): Three Variables

- SPOS
  - In some cases, all the OR-ed functions may not have all the variables
  - **SPOS: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - POS could be converted into a SPOS form

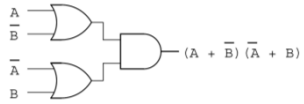


Minterm and Maxterm

- **Minterm: SOP**
- **Maxterm: POS**
- **Minterm: Each individual AND-ed (product) term in SSOP**



- **Maxterm: Each individual OR-ed (sum) term in SPOS form**



- **Maximum possible number of Minterm or Maxterm (NMT_{Max}):**

$$NMT_{Max} = 2^n$$

2 Variable Minterms: SSOP

- 2 Variable Minterm

$$NMT_{Max} = 2^n = 2^2 = 4$$

- **For Minterms:**
  - **Main variable (A) is considered as 1**
  - **Complemented variable ($\overline{A}$) is considered as 0**

A	B	Minterm	Symbols
0	0	$\overline{A}\cdot\overline{B}$	m_0
0	1	$\overline{A}\cdot B$	m_1
1	0	$A\cdot\overline{B}$	m_2
1	1	$A\cdot B$	m_3

2 Variable Minterms: Representation

- 2 Variable Minterm

$$NMT_{Max} = 2^n = 2^2 = 4$$

- **For Minterms:**
  - **Main variable (A) is considered as 1**
  - **Complemented variable ($\overline{A}$) is considered as 0**
- **The Boolean expression can be represented in the following generalized form:**

$$f(A_1,A_2,A_3,...A_n) = \sum m\left(0,1,2,3,...2^n\right)$$

- **Hence for 2-variable systems, the Boolean expression can be represented in the following generalized form:**

$$f(A,B) = \sum m\left(0,1,2,3\right)$$

2 Variable Minterms: Representation

- **Example**

$$f(A,B) = AB + A\overline{B}$$

- **Minterms representation:**
  - **AB stands for $11_2 = 3_{10}$ →**
  - **$A\overline{B}$ stands for $10_2 = 2_{10}$**
- **Therefore**
  - **AB stands for m_3**
  - **$A\overline{B}$ stands for m_2**
- **Hence the above Boolean expression can be represented in Minterm for as follows:**

$$f(A,B) = \sum m\left(2,3\right)$$

3 Variable Minterms: SSOP

$$NMT_{Max} = 2^n = 2^3 = 8$$

- **Minterms:**
  - **Main variable (A) is considered as 1**
  - **Complemented variable ($\overline{A}$) is considered as 0**

A	B	C	Minterm	Symbols	Decimal Suffix
0	0	0	$\overline{A}\cdot\overline{B}\cdot\overline{C}$	m_0	
0	0	1	$\overline{A}\cdot\overline{B}\cdot C$	m_1	
0	1	0	$\overline{A}\cdot B\cdot\overline{C}$	m_2	
0	1	1	$\overline{A}\cdot B\cdot C$	m_3	
1	0	0	$A\cdot\overline{B}\cdot\overline{C}$	m_4	
1	0	1	$A\cdot\overline{B}\cdot C$	m_5	
1	1	0	$A\cdot B\cdot\overline{C}$	m_6	
1	1	1	$A\cdot B\cdot C$	m_7	

3 Variable Minterms: Representation

- 2 Variable Minterm

$$NMT_{Max} = 2^n = 2^3 = 8$$

- **For Minterms:**
  - **Main variable (A) is considered as 1**
  - **Complemented variable ($\overline{A}$) is considered as 0**
- **The Boolean expression can be represented in the following generalized form:**

$$f(A_1,A_2,A_3,...A_n) = \sum m\left(0,1,2,3,...2^n\right)$$

- **Hence for 2-variable systems, the Boolean expression can be represented in the following generalized form:**

$$f(A,B,C) = \sum m\left(0,1,2,3,4,5,6,7\right)$$

3 Variable Minterms: Representation

- **Example**

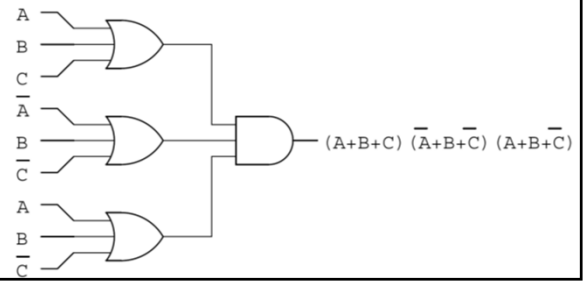
$$f(A,B,C) = A\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC + A\bar{B}\bar{C}$$

- **Minterms representation:**
  - $A\bar{B}C$ **stands for** $101_2 = 5_{10}$
  - $\bar{A}\bar{B}\bar{C}$ **stands for** $000_2 = 0_{10}$
  - ABC **stands for** $111_2 = 7_{10}$
  - $A\bar{B}\bar{C}$ **stands for** $100_2 = 4_{10}$
- **Minterms representation:**
  - $A\bar{B}C$ **represents** m_5
  - $\bar{A}\bar{B}\bar{C}$ **represents** m_0
  - ABC **represents** m_7
  - $A\bar{B}\bar{C}$ **represents** m_4
- **Hence the Minterm expression can be represented as:**

$$f(A,B,C) = \sum m(0,4,5,15)$$

Maxterm

- SPOS
  - In some cases, all the OR-ed functions may not have all the variables
  - **SPOS: All the variable (either in actual form or in inverted form) should be there in the expression.**
  - POS could be converted into a SPOS form



2 Variable Maxterms: SPOS

- 2 Variable Maxterm

$$NMT_{Max} = 2^n = 2^2 = 4$$

- **For Maxterms:**
  - **Main variable (A) is considered as 0**
  - **Complemented variable ($\bar{A}$) is considered as 1**

A	B	Maxterm	Symbols
0	0	$A+B$	M_0
0	1	$A+\bar{B}$	M_1
1	0	$\bar{A}+B$	M_2
1	1	$\bar{A}+\bar{B}$	M_3

2 Variable Maxterms: Representation

- 2 Variable Maxterm

$$NMT_{Max} = 2^n = 2^2 = 4$$

- **For Maxterms:**
  - **Main variable (A) is considered as 0**
  - **Complemented variable ($\bar{A}$) is considered as 1**
- **The Boolean expression can be represented in the following generalized form:**

$$f(A_1,A_2,A_3,\ldots A_n) = \prod M(0,1,2,3,\ldots 2^n)$$

- **Hence for 2-variable systems, the Boolean expression can be represented in the following generalized form:**

$$f(A,B) = \prod M(0,1,2,3)$$

3 Variable Maxterms: SPOS

$$NMT_{Max} = 2^n = 2^3 = 8$$

- **Minterms:**
  - **Main variable (A) is considered as 0**
  - **Complemented variable ($\bar{A}$) is considered as 1**

A	B	C	Maxterm	Symbols	Decimal Suffix
0	0	0	$A+B+C$	M_0	
0	0	1	$A+B+\bar{C}$	M_1	
0	1	0	$A+\bar{B}+C$	M_2	
0	1	1	$A+\bar{B}+\bar{C}$	M_3	
1	0	0	$\bar{A}+B+C$	M_4	
1	0	1	$\bar{A}+B+\bar{C}$	M_5	
1	1	0	$\bar{A}+\bar{B}+C$	M_6	
1	1	1	$\bar{A}+\bar{B}+\bar{C}$	M_7	

3 Variable Maxterms: Representation

- 2 Variable Maxterm

$$NMT_{Max} = 2^n = 2^3 = 8$$

- **For Maxterms:**
  - **Main variable (A) is considered as 0**
  - **Complemented variable ($\bar{A}$) is considered as 1**
- **The Boolean expression can be represented in the following generalized form:**

$$f(A_1,A_2,A_3,\ldots A_n) = \prod M(0,1,2,3,\ldots 2^n)$$

- **Hence for 2-variable systems, the Boolean expression can be represented in the following generalized form:**

$$f(A,B,C) = \prod M(0,1,2,3,4,5,6,7)$$

Minterm to Boolean Expression

- **Example 01:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C)= \sum m(0,2,4)$$

- **That means:**

$$f(A,B,C)= m_0 + m_2 + m_4$$

m_0 → 000 → $\bar{A}\bar{B}\bar{C}$; m_2 → 010 → $\bar{A}B\bar{C}$; m_4 → 100 → $A\bar{B}\bar{C}$

For Minterms:
- **Main variable (A) is considered as 1**
- **Complemented variable ($\bar{A}$) is considered as 0**

- **Hence for the above Minterm equation, the SSOP Boolean expression can be represented as:**

$$f(A,B,C)= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

Minterm to Boolean Expression

- **Example 02:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C)= \sum m(0,1,3,4,6)$$

- **That means:**

$$f(A,B,C)= m_0 + m_1 + m_3 + m_4 + m_6$$

m_0 → 000 → $\bar{A}\bar{B}\bar{C}$; m_1 → 001 → $\bar{A}\bar{B}C$; m_3 → 011 → $\bar{A}BC$; m_4 → 100 → $A\bar{B}\bar{C}$; m_6 → 110 → $AB\bar{C}$

For Minterms:
- **Main variable (A) is considered as 1**
- **Complemented variable ($\bar{A}$) is considered as 0**

- **Hence for the above Minterm equation, the SSOP Boolean expression can be represented as:**

$$f(A,B,C)= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$$

Maxterm to Boolean Expression

- **Example 01:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C)= \prod M(0,2,4)$$

- **That means:**

$$f(A,B,C)= M_0 \cdot M_2 \cdot M_4$$

M_0 → 000 → $(A+B+C)$; M_2 → 010 → $(A+\bar{B}+C)$; M_4 → 100 → $(\bar{A}+B+C)$

For Minterms:
- **Main variable (A) is considered as 1**
- **Complemented variable ($\bar{A}$) is considered as 0**

- **Hence for the above Minterm equation, the SSOP Boolean expression can be represented as:**

$$f(A,B,C)= (A+B+C)\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+C)$$

Maxterm to Boolean Expression

- **Example 02:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C)= \prod M(0,1,3,4,6)$$

- **That means:**

$$f(A,B,C)= M_0 \cdot M_1 \cdot M_3 \cdot M_4 \cdot M_6$$

M_0 → 000 → $A+B+C$; M_1 → 001 → $A+B+\bar{C}$; M_3 → 011 → $A+\bar{B}+\bar{C}$; M_4 → 100 → $\bar{A}+B+C$; M_6 → 110 → $\bar{A}+\bar{B}+C$

For Maxterms:
- **Main variable (A) is considered as 1**
- **Complemented variable ($\bar{A}$) is considered as 0**

- **Hence for the above Minterm equation, the SSOP Boolean expression can be represented as:**

$$f(A,B,C)= (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+C)$$

SOP to SSOP

- **Let us assume the following SSOP Boolean expression**

$$f(A,B,C) = f_1(A,B,C) + f_2(A,B,C) + f_3(A,B,C) + \ldots + f_n$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = AB + AB\bar{C} + BC$$

C is missing; A is missing

- **Missed parameter is required to be represent as**

$$\text{Missed term} = 1 = (X+\bar{X})$$

- **Missed parameter is required to be added as a multiplier with the product term**
- **Common terms will be neglected after simplification.**

SOP to SSOP

- **Example 01:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = AB + AB\bar{C} + BC$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = AB\cdot 1 + AB\bar{C} + 1\cdot BC$$

$$f(A,B,C) = AB\cdot 1 + AB\bar{C} + 1\cdot BC$$

$$f(A,B,C) = AB\cdot(C+\bar{C}) + AB\bar{C} + (A+\bar{A})\cdot BC$$

$$f(A,B,C) = ABC + AB\bar{C} + AB\bar{C} + ABC + \bar{A}BC$$

Common; Common

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = ABC + AB\bar{C} + \bar{A}BC$$

SOP to Minterm

- **Example 01:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = AB + AB\overline{C} + BC$$

- **Now for getting the min term based expression we must need SSOP**

$$f(A,B,C) = AB.1 + AB\overline{C} + 1.BC$$

$$f(A,B,C) = AB.(C+\overline{C}) + AB\overline{C} + (A+\overline{A}).BC$$

$$f(A,B,C) = ABC+AB\overline{C}+AB\overline{C}+ABC+\overline{A}BC$$

$$f(A,B,C) = ABC+AB\overline{C}+\overline{A}BC$$

- **Hence the Minterm based expression can be found as:**

$$f(A,B,C) = \sum m(3,6,7)$$

SOP to SSOP

- **Example 02:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = AC + AB\overline{C} + BC + ABC$$

- **Now for getting the min term based expression we must need SSOP**

$$f(A,B,C) = AC.1 + AB\overline{C} + 1.BC+ABC$$

$$f(A,B,C) = AC.(B+\overline{B}) + AB\overline{C} + (A+\overline{A}).BC + ABC$$

$$f(A,B,C) = ABC+A\overline{B}C+AB\overline{C}+ABC+\overline{A}BC + ABC$$

$$f(A,B,C) = ABC+A\overline{B}C+AB\overline{C}+\overline{A}BC$$

- **Hence the Minterm based expression can be found as:**

$$f(A,B,C) = \sum m(3,5,6,7)$$

SOP to SSOP

- **Example 03:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = C\overline{B} + \overline{A}B + ABC + AB\overline{C} + AC$$

- **Now for getting the min term based expression we must need SSOP**

$$f(A,B,C) = ?$$

$$f(A,B,C) = ?$$

- **Hence the Minterm based expression can be found as:**

$$f(A,B,C) = \sum m(?)$$

SOP to SSOP

- **Example 04:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = AC\overline{B} + \overline{A}B + \overline{B}C + AB\overline{C} + AB$$

- **Now for getting the min term based expression we must need SSOP**

$$f(A,B,C) = ?$$

$$f(A,B,C) = AC\overline{B} + \overline{A}B.1 + 1.\overline{B}C + AB\overline{C} + AB.1$$

$$f(A,B,C) = AC\overline{B} + \overline{A}B.(C+\overline{C}) + (A+\overline{A}).\overline{B}C + AB\overline{C} + AB.(C+\overline{C})$$

$$f(A,B,C) = AC\overline{B} + \overline{A}BC+\overline{A}B\overline{C} + A\overline{B}C+\overline{A}\overline{B}C + AB\overline{C} + ABC+ AB\overline{C}$$

$$f(A,B,C) = A\overline{B}C + \overline{A}BC + \overline{A}B\overline{C} + \overline{A}\overline{B}C + ABC + AB\overline{C}$$

- **Hence the Minterm based expression can be found as:**

$$f(A,B,C) = \sum m(1,2,5,6,7)$$

SOP to SSOP

- **Example 05:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = AC\overline{B} + AC + B\overline{C} + \overline{A}BC + AB\overline{C} + \overline{A}B$$

- **Now for getting the min term based expression we must need SSOP**

$$f(A,B,C) = ?$$

$$f(A,B,C) = AC\overline{B} + A.1.C + 1.B\overline{C} + \overline{A}BC + AB\overline{C}+ \overline{A}B.1$$

$$f(A,B,C) = AC\overline{B} + A(B+\overline{B}).C + (A+\overline{A}).B\overline{C} + \overline{A}BC + AB\overline{C}+ \overline{A}B.(C+\overline{C})$$

$$f(A,B,C) = AC\overline{B} + ABC+A\overline{B}C + AB\overline{C}+\overline{A}B\overline{C} + \overline{A}BC+ AB\overline{C} + \overline{A}BC+ \overline{A}B\overline{C}$$

$$f(A,B,C) = A\overline{B}C + ABC + \overline{A}BC+ AB\overline{C} + \overline{A}B\overline{C}$$

- **Hence the Minterm based expression can be found as:**

$$f(A,B,C) = \sum m(2,3,5,6,7)$$

POS to SPOS

- **Let us assume the following SSOP Boolean expression**

$$f(A,B,C) = f_1(A,B,C) \cdot f_2(A,B,C) \cdot f_3(A,B,C) \cdot \ldots \cdot f_n$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (B + C)\ (A + C)$$

A is missing — B is missing

- **Missed parameter is required to be represent as**

$$\text{Missed term} = 0 = (X.\overline{X})$$

- **Missed parameter is required to be added as a multiplier with the product term**
- **Common terms will be neglected after simplification.**

POS to SPOS

- **Example 01:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = (B + C)\,(A + C)$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (B + C + 0)\,(A + C + 0)$$

$$f(A,B,C) = (B + C + A\overline{A})\,(A + C + B\overline{B})$$

$$f(A,B,C) = (B + C + A\overline{A})\,(A + C + B\overline{B})$$

$$f(A,B,C) = (B + C + A)\,(B + C + \overline{A})(A + C + B)(A + C + B)$$

$$f(A,B,C) = (A + B + C)\,(\overline{A} + B + C)(A + B + C\,)(A + B + C)$$

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = (A + B + C)\,(\overline{A} + B + C)$$

POS to SPOS

- **Example 02:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = (A + B)(B + \overline{C})\,(A + C)$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (A + B + 0)\,(B + \overline{C} + 0)(A + C + 0)$$

$$f(A,B,C) = (A + B + C\overline{C})\,(B + \overline{C} + A\overline{A})(A + C + B\overline{B})$$

$$f(A,B,C) = (A + B + C)\,(A + B + \overline{C})(B + C + A)(B + C + \overline{A})$$

$$(A + C + B)(A + C + \overline{B})$$

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(A + \overline{B} + C)$$

$$f(A,B,C) = \prod M(0,1,2,4)$$

POS to SPOS

- **Example 03:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = (A + B)(B + \overline{C})\,(A + C)$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (A + B + 0)\,(B + \overline{C} + 0)(A + C + 0)$$

$$f(A,B,C) = (A + B + C\overline{C})\,(B + \overline{C} + A\overline{A})(A + C + B\overline{B})$$

$$f(A,B,C) = (A + B + C)\,(A + B + \overline{C})(B + C + A)(B + C + \overline{A})$$

$$(A + C + B)(A + C + \overline{B})$$

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(A + \overline{B} + C)$$

$$f(A,B,C) = \prod M(0,1,2,4)$$

POS to SPOS

- **Example 04:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = (A + B)(B + \overline{C})\,(A + C)$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (A + B + 0)\,(B + \overline{C} + 0)(A + C + 0)$$

$$f(A,B,C) = (A + B + C\overline{C})\,(B + \overline{C} + A\overline{A})(A + C + B\overline{B})$$

$$f(A,B,C) = (A + B + C)\,(A + B + \overline{C})(B + C + A)(B + C + \overline{A})$$

$$(A + C + B)(A + C + \overline{B})$$

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(A + \overline{B} + C)$$

$$f(A,B,C) = \prod M(0,1,2,4)$$

POS to SPOS

- **Example 05:**
  - **Let us assume the following Boolean expression**

$$f(A,B,C) = (A + B)(B + \overline{C})\,(A + C)$$

- **Now find the terms are missing in all the product terms:**

$$f(A,B,C) = (A + B + 0)\,(B + \overline{C} + 0)(A + C + 0)$$

$$f(A,B,C) = (A + B + C\overline{C})\,(B + \overline{C} + A\overline{A})(A + C + B\overline{B})$$

$$f(A,B,C) = (A + B + C)\,(A + B + \overline{C})(B + C + A)(B + C + \overline{A})$$

$$(A + C + B)(A + C + \overline{B})$$

- **Hence for the above SOP, the SSOP can be represented as:**

$$f(A,B,C) = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C)(A + \overline{B} + C)$$

$$f(A,B,C) = \prod M(0,1,2,4)$$

Boolean Expression Conversion

Different Representation Techniques

We have learnt

1. **SOP = Sum of product**
2. **POS = product of Sum**
3. **SSOP = Standard SOP or Canonical SOP**
4. **SPOS = Standard POS or Canonical POS**
5. **Minterm**
6. **Maxterm**
7. **SOP to SSOP**
8. **POS to SPOS**

We will learn

1. **Minterm to Mxterm**
2. **Maxterm o Minterm**
3. **POS to SOP**
4. **SOP to POS**
5. **SSOP to SPOS**
6. **SPOS to SSOP**

Minterms to Maxterms

- **RULES: Let us assume the following Boolean expression**

$$f(A,B,C) = \sum m(N_1,N_2,N_3,...N_5,N_n)$$

1. **Check the number of the variables in the function f.**
2. **Find the maximum number of possible Maxterms.**

$$NMinT_{Max} = 2^n$$

3. **Check the Minterms available in the Minterm based Boolean Expression (Min-BE).**

Such as: m_1, m_3, m_4, m_6, m_7

4. **Find the absent Minterms in Min-BE.** Such as: m_0, m_2, m_5
5. **Write down the Maxterms based Boolean Expression (Min-BE) with the Maxterms found corresponding to the absent Minterm numbers (0,2,5):** $f(A,B,C) = \prod M(0,2,5)$

- **Note: If the Maxterm based Boolean Expression (Max-BE) is not available find the Max-BE either from POS or SPOS.**

Maxterms to Minterms

- **RULES: Let us assume the following Boolean expression**

$$F(A,B,C) = \prod M(N_1,N_2,N_3,...N_5,N_n)$$

1. **Check the number of the variables in the function f.**
2. **Find the maximum number of possible Maxterms.**

$$NMaxT_{Max} = 2^n$$

3. **Check the Maxterms available in the Maxterm based Boolean Expression (Max-BE).**

Such as: M_1, M_3, M_4, M_6, M_7

4. **Find the absent Maxterms in Min-BE.** Such as: M_0, M_2, M_5
5. **Write down the Minterms based Boolean Expression (Min-BE) with the Minterms found corresponding to the absent Maxterm numbers (0,2,5):** $f(A,B,C) = \sum m(0,2,5)$

- **Note: If the Maxterm based Boolean Expression (Max-BE) is not available find the Max-BE either from POS or SPOS.**

Minterms to Maxterms

- **Example 01: Let us assume the following Boolean expression** $f(A,B,C) = \sum m(1,3,5)$

1. **Check the number of the variables in the function f: 3**
2. **Find the maximum number of possible Minterms: $2^n = 8$**

$$NMinT_{Max} = 2^3 = 8$$

3. **Check the Minterms available in the Minterm based Boolean Expression (Min-BE): m_1, m_3, m_5**
4. **The absent Minterms in the Min-BE: m_0, m_2, m_4, m_6, m_7**
5. **Write down the Maxterms based Boolean Expression (Max-BE) with the Maxterms corresponding to the absent Minterm numbers (0,2,4,6,7).**

$$Maxterms: M_0, M_2, M_4, M_6, M_7$$

$$f(A,B,C) = \prod M(0, 2, 4, 6, 7)$$

Maxterms to Minterms

- **Example 01: Let us assume the following Boolean expression** $F(A,B,C) = \prod M(1,3,5)$

1. **Check the number of the variables in the function f: 3**
2. **Find the maximum number of possible Maxterms: $2^n = 8$**

$$NMaxT_{Max} = 2^3 = 8$$

3. **Check the Maxterms available in the Maxterm based Boolean Expression (Max-BE): M_1, M_3, M_5**
4. **The absent Maxterms in the Min-BE: M_0, M_2, M_4, M_6, M_7**
5. **Write down the Minterms based Boolean Expression (Min-BE) with the Minterms corresponding to the absent Maxterm numbers (0,2,4,6,7).**

$$Minterms: m_0, m_2, m_4, m_6, m_7$$

$$f(A,B,C) = \sum m(0, 2, 4, 6, 7)$$

SSOP to SPOS

- **RULES: Let us assume the following Boolean expression**

$$f(A,B,C) = f_1(A,B,C) + f_2(A,B,C) + f_3(A,B,C)$$

- **Find the Minterms from the SSOP**

$$f(A,B,C) = f_1(A,B,C) + f_2(A,B,C) + f_3(A,B,C)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

Binary $1(N_1)_2$ Binary $2(N_2)_2$ Binary $3(N_3)_2$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

Minterm $1(m_1)$ Minterm $2(m_2)$ Minterm $3(m_3)$

- **Find the Maxterms corresponding to the absent Minterms and write down the Max-BE**

$$f(A,B,C) = \prod M(N_1,N_2,N_3)$$

- **Write down the SPOS Boolean Expression utilizing the Maxterms found:**

$$F(A,B,C) = F_1(A,B,C) + F_2(A,B,C) + F_3(A,B,C)$$

SPOS to SSOP

- **RULES: Let us assume the following Boolean expression**

$$F(A,B,C) = F_1(A,B,C) + F_2(A,B,C) + F_3(A,B,C)$$

- **Find the Maxterms from the SPOS**

$$F(A,B,C) = F_1(A,B,C) + F_2(A,B,C) + F_3(A,B,C)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

Binary $1(N_1)_2$ Binary $2(N_2)_2$ Binary $3(N_3)_2$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

Maxterm $1(M_1)$ Maxterm $2(M_2)$ Maxterm $3(M_3)$

- **Find the Minterms corresponding to the absent Maxterms and write down the Max-BE**

$$f(A,B,C) = \sum m(N_1,N_2,N_3)$$

- **Write down the SSOP Boolean Expression utilizing the Maxterms found:**

$$f(A,B,C) = f_1(A,B,C) + f_2(A,B,C) + f_3(A,B,C)$$