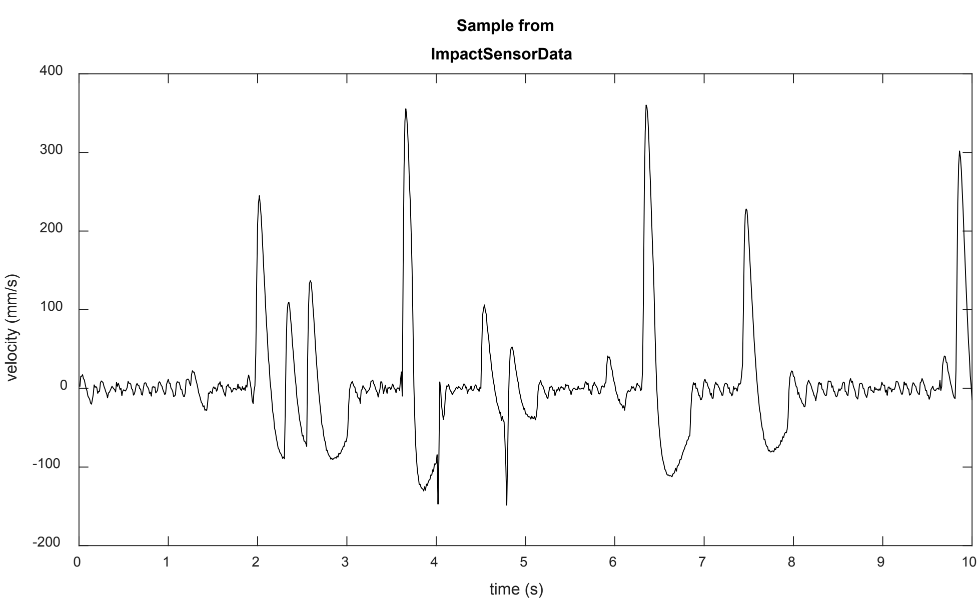
Analysis of impact sensor data

Background

The class demonstration had a shaker plate with a number of ¼" steel balls moving randomly around, colliding with each other and colliding with the walls. Suspended just above the plate there was a light dial, which deflected when impacted by a ball. This deflection generated the voltage, displayed on the oscilloscope screen. The aim of this homework problem set is to analyze the data taken by this impact sensor.

> *As an aside for the aficionados: The impact sensor was made from an old moving coil galvanometer. You may remember from E&M labs that if you send a current through a coil which is suspended in a magnetic field, the coil will feel a torque. This is the principle behind a moving coil galvanometer. However, this can be turned around as well: If you rotate a coil in a magnetic field, then you generate a current which can be measured. This is of course how our electric power is generated. Here we use this same principle to measure the speed at which the dial is deflected. In principle, the current generated by the coil is proportional to its rotational velocity.*

In two separate experiments I ran the shaker plate for 5000 seconds each, and recorded the data. Recording was done with an A/D converter, which takes analog voltages and converts them into binary numbers that can be stored in the computer. The data were sampled at 100 Hz, that is at 10ms intervals. The figure below shows a short segment of the full trace. In the time series you download, the measurements are converted to velocity units. Also, there has been some preprocessing of the data, primarily to remove the undershoots that you can see in the figure, and to filter out high frequency components in the signals.



On impact with a ball, the sensor produces a voltage deflection of a characteristic shape, as shown. Due to some inertia and elasticity in the sensor, when a ball hits the sensor, the current will not change instantaneously but rise to a peak in a few tens of milliseconds. The sensor can respond only in one direction, so you will analyze velocity components of one sign only. Because of the finite time resolution of the system each impact leads to a characteristic time trace: A peak followed by an undershoot, together lasting about 500 milliseconds (the pulse in the figure peaking at about 7.5 s is a good clean example). These time traces may overlap if collisions happen to be close together in time, and that makes the overall trace look a bit messy (see e.g. the episode between 2 and 3 seconds in the figure). As you see, the impact events in the figure have different peak heights, depending on the speed and angle of the impact.

We want to analyze the statistics of the timing and the amplitude of those events. To do that we need the raw data, as well as some Matlab code to analyze the data. You can find the data file and a Matlab script on Canvas: Files/Problemsets/PS2. From there you can download the file "`PS2DataFile_Sept2016.mat`" which contains the raw data for both experiments. The "LowTemp" and "HighTemp" experiments use a low and high setting of the shaker plate motion. You should also download "`PS2_ImpactSensorAnalysis_Sept2016.m`" which is a Matlab script that reads the data file, identifies the peaks in the raw data and plots some statistics (figures 1 and 2 are for the Low temp case, figures 11 and 12 for the high temperature.

The script file is extensively commented. You should read through the statements and try to understand what happens in each line. If things are unclear, use the Matlab HELP command to understand what the different statements mean. You can select portions of the code, and run them, using the F9 key. The script calculates and plots histograms of the time interval between impact events, the velocity distribution of those events, and the distribution of the squared velocities (since that is proportional to the kinetic energy).

For this problem set you should answer the following:

All students:

a) Based on the lectures on statistics, what do you expect for the time interval statistics in these experiments? The time interval histograms are plotted in Fig 2 and 12 (as a count histogram in the top left panel, and as a normalized probability distribution on a logarithmic vertical scale in the bottom left panel). Do the data support this expectation? What deviations do you see from the ideal behavior? Explain your observations. Compare the mean interval for the "Low Temp" and "High Temp" experiments.

b) In an ideal gas, what do you expect for the velocity distribution of the molecules? Assuming that the shaker experiment is a reasonable model of an ideal gas, what do you expect for the distribution of the peak amplitudes in the data? As mentioned earlier, you should assume that the impact sensor produces a signal proportional to one component, say the X-direction, of the velocity of the ball on impact. Also, remember that the sensor is sensitive only in one direction (only positive X, not negative). Within these restrictions, do the data support this expectation? Explain your conclusions. Estimate the standard deviation of the velocity values (note that we measure only positive velocity values, so you cannot just use the Matlab command "`std`"), and try to draw the resulting Gaussian in the histogram. You can assume that the negative half of the velocity distribution is the mirror image of the positive half; the question about the standard deviation of velocity pertains to the full symmetrized version of the distribution. If you choose to find the standard deviation by fitting a Gaussian to the data, then look at the help for the Matlab "`nlinfit`" command.

Compare the widths of the distributions for the "LowTemp" and "HighTemp" experiments. Based on your measurements of time intervals for these two experiments what would you expect for the ratio of velocity standard deviations for the two cases? Is that close to correct here? Given that the mass of the balls used in the experiments is 1 gram, compute the effective temperature of the simulated gas in the two experiments.

c) In the lectures we talked about time series, taking the Poisson process as perhaps the simplest example of a random time series. Convert the data from this experiment into

- A time series of unit height pulses, corresponding to the times of impact (to do this you create an array of zeros (type "help zeros"), and replace only the appropriate bins with ones.
- The same time series as before, except that you replace the ones with the values of the peak amplitudes.

For both time series do the following: chop them up into time windows of 0.1, 0.3, 1 , 3, 10, 30, 100 seconds respectively (use the command RESHAPE to do this). For each window size determine the mean and the variance. Plot the variance as a function the mean. Is the variance proportional to the mean in both cases? Comment on what you find, and explain, qualitatively, the differences if there are any.

Extra, for the Biophysics PhD students:

The probability density of squared velocities, $P(V^2)$, seems to deviate from exponential behavior at low values of V^2: this is most clearly seen in the bottom-right panel of figures 2 and 12.

Since $V^2$is proportional to the kinetic energy K, one might naively expect that, since velocities are distributed as a Gaussian, $P(V) \propto \exp\left(-\frac{V^2}{2\sigma^2}\right)$, that we would have $P(K) \propto$

$\exp\left(-\frac{K}{2\sigma^2}\right)$. However, this does not take account of the fact that the transformation from V to K is not linear. What needs to be true is that:

$$P(K)dK = P(V)dV$$

And this means that:

$$P(K)dK = P(V)\left|\frac{dV}{dK}\right| dK$$

Show that this leads to:

$$P(K)dK \propto \frac{\exp\left(-\frac{K}{\langle K\rangle}\right)}{\sqrt{K}} dK \qquad (1)$$

Where $\langle K\rangle$ is the mean kinetic energy. Are the data shown in the bottom-right panel of figures 2 and 12 consistent with this?

Of course, Eq. (1) is not the Boltzmann distribution. However, if you calculate the total energy of the sum of two distributions of the form of Eq. (1), then you *will* get a Boltzmann distribution (you can calculate this by convolving two distributions of the shape of Eq. (1)). Can you *briefly* comment on what this might mean?