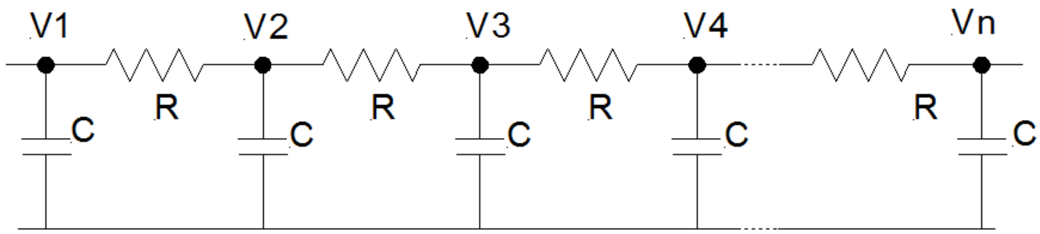
Diffusion – RC network and the Cable Equation



Short description:
As mentioned during the lectures, there is a large variety of systems that exhibit diffusion. One such system is the discrete resistor-capacitor network of the form shown in the figure. Using Ohm's law, and the fact that the voltage across a capacitor is: V=Q/C (C is capacitance, and Q the charge, show that the set of voltages across the resistors obeys a difference equation that can be seen as a discrete approximation to the diffusion equation (compare to par. 4.4.2 in Nelson). You will also need the relation between charge, Q, and current, I: I=dQ/dt. Of course, in a network of discrete elements there is no notion of geometrical space, which means that the effective diffusion constant in this problem does not exhibit space dependence (i.e. its units are not m^2/s).
There is a continuous electronic analog in which there is a diffusion of charge in real space (i.e. with the dimension of meters). This requires a continuous distribution of resistance and capacitance along the length of the model. Such a continuum model is known as the cable model, which is described by the cable equation. Historically this equation has had great practical value: It was first analyzed by Thomson (the later Lord Kelvin) in connection with the early transatlantic telegraph cables. If you are interested you can read about this, and also about how to get rich by doing good mathematics, by downloading "TransatlanticCable.pdf" from the Canvas site. These two chapters from Körner's book on Fourier analysis also give a good picture of some of the (expensive!) misconceptions that people had at the time about the propagation of signals in cables in a conducting medium. The cable equation has great biological relevance, as it offers a good description of the propagation of signals along the axons of nerve cells.

The Matlab file for this problemset contains data taken from a real resistor-capacitor network. An experiment with this network will be demonstrated in class.

References

T.W. Körner, Fourier Analysis, Cambridge University Press, 1989

Problemset

The Folder PS3 for this problemset contains the data file **RCCableModel.mat.** The first capacitor in an RC network of the form illustrated in the figure was connected to a pulse generator, which gave a single pulse lasting 10 ms, and with amplitude 16 Volt. This means that V1 in the figure above had the following time sequence:
V1=0 Volt (t<0)
V1=16 Volt (0 < t < 10ms)
V1=0 Volt (t>10 ms)
(If you want to draw the parallel with concentration of ink in water, this amounts to putting in some ink at one volume element very briefly (i.e. 10 ms), and then clearing all the ink from that volume element after that.) The network used in the experiment had 16 resistors and 17 capacitors (n=1,…,17). The first capacitor was driven directly by the pulse generator, so that its voltage waveform, V1(t), was determined externally. In this particular experiment the last capacitor was shunted, so by definition V17(t)=0 for all t.

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Explanation of the variables in the datafile
%
% Time: 10000 sample times (in seconds) at which the voltages across the
% capacitors were measured.
%
% VC: 10000 sample values of the first ten capacitor voltages, measured at the
% time points defined by Time. VC(:,1) is the time dependent voltage across C1,
% VC(:,2) across C2, etc.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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Assignments for HW3:

1. For the discrete RC model, define the analog of the diffusion constant. What are its physical units, and how do you express that constant in terms of R and C?
2. Verify that the measurements obey the discretized diffusion equation. You may do this either analytically, or by simulating the network in the figure.
3. Given that the capacitors in the network are all equal to 0.1μF, compute the value of the resistors (these are all equal).

Extra for Biophysics PhD students:

4. Set up a model in which resistance and capacitance are distributed continuously so as to lead to the regular 1-dimensional diffusion equation, with conventional space dependence.
5. Work out Nelson, problem 4.8

A Matlab script for reading this file and plotting the data, called RCCableModelRead.m, is saved in the folder as well. Hand in plots and a write up of your analysis.

Good luck!