Nonmobius solution

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June 2021

1 Anstaz

Inspired by the Mobius strip line element, we use the Anstaz,

$$ds^2 = dt^2 - f(u,v)du^2 - dv^2 - dw^2$$

with $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$. The only nonzero entries of the Einstein tensor are

$$G_{tt} = -G_{ww} = \frac{-0.5f(u,v)\frac{\partial^2}{\partial v^2}f(u,v) + 0.25\left(\frac{\partial}{\partial v}f(u,v)\right)^2}{f^2(u,v)}.$$

Setting $G_{tt} = 0$ we obtain the following nonlinear PDE,

$$(\partial_v f)^2 = 2f\partial^2_{vv}f.$$

The solution of which is

$$f(u,v) = \frac{1}{4}(vc_1(u) + c_2(u))^2.$$

2 potenial equivalent solutions in the literature

In [?]

- p102, eq 8.29
- p364, eq 23.13a