Math 525, Spring 2018. First Midterm Practice

Choose three out of the following four problems.

(1) Show that there are no retractions $r : X \longrightarrow A$ in the following cases:
a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to $\mathbb{S}^1$.
b) $X = \mathbb{D}^2 \vee \mathbb{D}^2$ with A equal to its boundary $\mathbb{S}^1 \vee \mathbb{S}^1$.
c) X a Möbius band and A its boundary circle.

(2) Let X be a space obtained from the two-dimensional sphere and the unit interval by identifying both end points of the interval with a point q on the sphere. What is the fundamental group of X? What are the path-connected covering spaces of X, up to equivalence?

(3) Let E_1, E_2, B be path connected and locally path-connected. Suppose $E_1 \xrightarrow{p_1} B$ and $E_2 \xrightarrow{p_2} B$ are coverings and that E_1 is simply connected. Show that E_1 is a covering space of E_2.

(4) a) Show that the free group on three generators is a subgroup of the free group on two generators.
b) Show that any subgroup of the free group on two generators is itself a free group.