Math 525, Spring 2018. Homework 10
Due: Thursday, April 19, 2018

(1) a) Show that a homeomorphism $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ extends to a homeomorphism $\widetilde{f}$ of the one-point compactification $\mathbb{S}^n$. Thus we may define $\deg(f)$ as $\deg(\widetilde{f})$.
b) Using this notion, fill in the details of the following argument of R. Fokkink which shows that $\mathbb{R}^n$ is not homeomorphic to a product $X \times X$ if n is odd:
Assume $\mathbb{R}^n = X \times X$ consider the homeomorphism f of $\mathbb{R}^n \times \mathbb{R}^n$ that cyclically permutes the factors, $f(x_1, x_2, x_3, x_4) = (x_2, x_3, x_4, x_5)$. On the one hand, since f is a homeomorphism, show that $\deg(f^2) = 1$, on the other hand, since f^2 switches the two factors of $\mathbb{R}^n \times \mathbb{R}^n$ and n is odd, show that f^2 has degree -1.

(2) Let $\mathbb{S}^n \longrightarrow \mathbb{RP}^n$ be the quotient map, and consider the cell complex structure on $\mathbb{S}^n$ obtained by lifting the cell complex structure on $\mathbb{RP}^n$ (so, e.g., there are two i-cells for each $i \leq n$). Compute the resulting cellular chain complex and verify that it has the correct homology groups.

(3) Hatcher §2.2 exercise 29

(4) ~~Hatcher §2.2 exercise 34~~ (removed by Hatcher)

(5) Hatcher §2.2 exercise 35