# Math 525, Spring 2018. Second Midterm Practice

*Choose three out of the following four problems.*

(1) Find a $\Delta$-complex structure for, and compute the associated homology groups of, the space $X$ obtained from the annulus $A = \mathbb{S}^1 \times [0,1]$ by gluing $\mathbb{S}^1 \times \{1\}$ to $\mathbb{S}^1 \times \{0\}$ by a map representing 2 times the generator of $\pi_1(\mathbb{S}^1)$.

(2) Show that $X = \mathbb{S}^1 \times \mathbb{S}^1$ and $Y = \mathbb{S}^1 \vee \mathbb{S}^1 \vee \mathbb{S}^2$ have isomorphic homology groups in all degrees, but their universal covering spaces do not.

(3) Suppose that $A$, $B$, and $C$ are Abelian groups and we are given homomorphisms $f : A \longrightarrow B$, $g : B \longrightarrow C$. Show that there are induced homomorphisms making an exact sequence

$$0 \longrightarrow \ker f \xrightarrow{\alpha} \ker(gf) \xrightarrow{\beta} \ker g \xrightarrow{\gamma} \operatorname{coker}(f) \xrightarrow{\zeta} \operatorname{coker}(gf) \xrightarrow{\eta} \operatorname{coker}(g) \longrightarrow 0\ .$$

(Recall that the cokernel of a homomorphism between Abelian groups $h : X \longrightarrow Y$ is the group $Y/h(X)$.)
Hint: $\alpha$ is an inclusion, $\gamma$ and $\eta$ are projections onto quotient groups.

(4) Let $X$ be a cell complex and $p : Y \longrightarrow X$ a two-fold covering. We know from earlier in the semester that $p_* : \pi_1(Y) \longrightarrow \pi_1(X)$ is injective. Is it true that $p_* : H_n(Y) \longrightarrow H_n(X)$ is injective for all $n$? Justify your answer.