Math 525, Spring 2018. Homework 8
Due: Thursday, April 5, 2018

(1) Fix a space Z. Given any pair of topological spaces (X, A) with $A \subseteq X$, define a sequence of Abelian groups
$$H_k^Z(X, A) = H_k(X \times Z, A \times Z).$$
Show that this sequence satisfies many of the same properties as the singular homology of a pair, namely:

a) A map $f : (X, A) \longrightarrow (Y, B)$ induces a map $f_* : H_k^Z(X, A) \longrightarrow H_k^Z(Y, B)$ in such a way that $(gf)_* = g_* f_*$.

b) If $f, g : (X, A) \longrightarrow (Y, B)$ are homotopic through maps $X \longrightarrow Y$ that send A to B, then $f_* = g_*$.

c) If $V \subseteq A$ is such that $\overline{V} \subseteq A^\circ$ then the map
$$H_n^Z(X \setminus V, A \setminus V) \longrightarrow H_n^Z(X, A)$$
is an isomorphism.

d) If we define $H_k^Z(M) = H_k(M \times Z)$, there are connecting maps $\delta : H_k^Z(X, A) \longrightarrow H_{k-1}^Z(A)$ such that the sequence of maps
$$\ldots \longrightarrow H_n^Z(A) \longrightarrow H_n^Z(X) \longrightarrow H_n^Z(X, A) \longrightarrow H_{n-1}^Z(A) \longrightarrow \ldots$$
is exact. Moreover, for any $f : (X, A) \longrightarrow (Y, B)$ we have $\delta f_* = f_* \delta$.

e) If X is a disjoint union of connected spaces X_α then $H_n^Z(X) = \oplus_\alpha H_n^Z(X_\alpha)$.

(These properties show that H_*^Z is a 'generalized homology theory'.)

(2) Hatcher §2.2 exercise 1

(3) Hatcher §2.2 exercise 2

(4) Hatcher §2.2 exercise 7

(5) Hatcher §2.2 exercise 8