Math 525, Spring 2018. Homework 4
Due: Thursday, February 15, 2018

(1) For a covering space $p : \widetilde{X} \longrightarrow X$ and a subspace $A \subseteq X$, let $\widetilde{A} = p^{-1}(A)$. Show that the restriction $p : \widetilde{A} \longrightarrow A$ is a covering space.

(2) Let $p : \widetilde{X} \longrightarrow X$ be a covering space with $p^{-1}(x)$ finite and non-empty for all $x \in X$. Show that $\widetilde{X}$ is compact Hausdorff if and only if X is compact Hausdorff.

(3) Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be such that gf and g are covering maps. If Z is locally path-connected, show that f is also a covering map.

(4) Let $X = \mathbb{S}^1 \vee \mathbb{S}^1$ and let $Y \subseteq \mathbb{R}^2$ be the union of all horizontal and vertical lines which pass through integer lattice points (i.e., $Y = \bigcup_{n \in \mathbb{Z}} (\mathbb{R} \times \{n\}) \cup (\{n\} \times \mathbb{R})$). Define a covering map $p : Y \longrightarrow X$ and describe the subgroup $p_*(\pi_1(Y; (0,0)))$ of $\pi_1(X)$.

(5) Construct a simply-connected covering space for each of the following spaces:
(a) $\mathbb{S}^1 \vee \mathbb{S}^2$,
(b) The union of $\mathbb{S}^2$ with an arc joining two distinct points of $\mathbb{S}^2$,
(c) $\mathbb{S}^2$ with two points identified,
(d) $\mathbb{RP}^2 \vee \mathbb{RP}^2$,
(e) $\mathbb{S}^1 \vee \mathbb{T}^2$.