Math 525, Spring 2018. Homework 11
Due: Thursday, April 26, 2018

(1) Hatcher §3.3 exercise 3

(2) Hatcher §3.3 exercise 5
You may use the following description of the orientation cover of $M \times N$: If $\widetilde{M} \longrightarrow M$ is the orientation cover of M, so that $\widetilde{M} = \{(x, \mu_x)\}_{x \in M}$, there is a natural $\mathbb{Z}_2$-action induced by taking (x, μ_x) to $(x, -\mu_x)$, i.e., the same point with the opposite orientation. If $\widetilde{M}$ and $\widetilde{N}$ are the orientation covers of M and N respectively, let Op : $\widetilde{M} \times \widetilde{N} \longrightarrow \widetilde{M} \times \widetilde{N}$ be the map that changes the orientation in both factors (i.e., $Op(x, \mu_x, y, \mu_y) = (x, -\mu_x, y, -\mu_y)$). The orientation cover of $M \times N$ is obtained from the product of the orientation covers of M and N by identifying each point with its image under Op, that is

$$\widetilde{M \times N} = \widetilde{M} \times \widetilde{N}/(z \sim Op(z)).$$

(3) Hatcher §3.3 exercise 7

(4) By a morphism between two generalized homology theories, $h.$ and $k.$, we mean a collection of natural transformations $\Phi_n : h_n \Longrightarrow k_n$ such that $\partial^k \circ \Phi_n = \Phi_{n-1} \circ \partial^h$. Show that generalized homology theories form a category.

(5) Recall that in HW8 we showed that, for each topological space Z, the functors $(X, A) \mapsto H_n^Z(X, A) = H_n(X \times Z, A \times Z)$, together with the connecting map from the l.e.s. of $(X \times Z, A \times Z)$ form a generalized homology theory, $H_.^Z$. Given a map of spaces $f : Z \longrightarrow Z'$, explain why we get a map of generalized homology theories $H^f : H^Z \longrightarrow H^{Z'}$. Show that this is a functor from spaces to generalized homology theories.