Introduction

- Introduction to portfolio management
- Expected returns and risk for a portfolio
- Portfolio construction with two-security case
- Portfolio construction with *N*-security case
- Risk diversification with portfolios

Portfolio Construction with Two Securities: Expected Returns

Portfolio Construction with Two Securities

What is a portfolio and why to invest in it?

- What happens to the (1) expected return and (2) risk when you combine two securities (or multiple securities)?
- What is diversification?
- Investing in mutual funds and index investing
- What is the difference in risk of investing in Nifty-50 vs. HDFC?

Expected Returns for Two-Security Case

Consider a portfolio constructed from two-security case with actual return distributions as R_1 and R_2

- The proportionate amounts invested in these assets are w_1 and w_2, where $w_1 + w_2 = 1$
- Please also remember that expected returns E(R_1) = $\overline{R_1}$ and E(R_2) = $\overline{R_2}$
- Now, let us try to understand the return for the portfolio
- The actual return from the portfolio R_p
- $R_p = w_1 * R_1 + w_2 * R_2$ (1)

Expected Returns for Two-Security Case

What about expected returns?

- $\mathrm{E}(R_p) = E(w_1 * R_1 + w_2 * R_2)$ (2)
- $\mathrm{E}(R_p) = E(w_1 * R_1) + E(w2 * R_2)$ (3)
- $\mathrm{E}(R_p) = w_1 * E(R_1) + w2 * E(R_2)$ (4)

  where w_1 and w_2 are constants. Therefore, $\mathrm{E}(R_1 w_1) = w_1 \mathrm{E}(R_1)$.

- However, R_1 and R_2 are probabilistic variables with finite distributions.

Expected Returns for Two-Security Case

What about expected returns?

- For these variables, the expectation operator returns the probability weightage average. That is, $E(R_1) = \overline{R_1}$; therefore,

$$\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2} \tag{5}$$

- Expected returns from the portfolio are simply the weighted average of expected returns of individual securities in the portfolio.

Expected Returns for Two-Security Case

What about expected returns?

- This can be generalized into three securities and multi-security as well

$$\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2} + w3 * \overline{R_3}, \text{ where } w_1 + w_2 + w_3 = 1$$

- For "N" securities
- $\overline{R_p} = \sum_{i=1}^{N} w_i * \overline{R_i}$, where $\sum_{i=1}^{N} w_i = 1$ (6)

Expected Returns from Portfolio: A Simple Example

Expected Returns: Case 1 (Different Probabilities)

Pt	Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb	Pt*R_p
0.20	9.00%	6.00%	3.60%	3.60%	7.20%	1.44%
0.15	8.00%	5.00%	3.20%	3.00%	6.20%	0.93%
0.10	7.00%	8.00%	2.80%	4.80%	7.60%	0.76%
0.15	11.00%	9.00%	4.40%	5.40%	9.80%	1.47%
0.25	12.00%	10.00%	4.80%	6.00%	10.80%	2.70%
0.15	6.00%	11.00%	2.40%	6.60%	9.00%	1.35%
	Wa	**Wb**			**Total**	**8.65%**
	0.40	0.60	E(R_p)=P1*R_{p1}+P2*R_{p2} … … .+P6*R_{p6}			

Expected Returns: Case 2 (Equal Probabilities)

Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb
9.00%	6.00%	3.60%	3.60%	7.20%
8.00%	5.00%	3.20%	3.00%	6.20%
7.00%	8.00%	2.80%	4.80%	7.60%
11.00%	9.00%	4.40%	5.40%	9.80%
12.00%	10.00%	4.80%	6.00%	10.80%
6.00%	11.00%	2.40%	6.60%	9.00%
Wa	Wb		Average	8.43%
0.40	0.60	E(R_p)=(1/ N)* (R_{p1}+R_{p2} … … .+R_{p6})		

Portfolio Construction with Two Securities: Risk

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- Variance (σ_i^2)= $\sum_{t=1}^{T} P_t (R_{i,t} - \bar{R_i})^2$
- Again, for past observations that are equally likely
- That is, $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T$. Since $\sum_{i=1}^{T} P_i$ =1, we have $P_1 = P_2 = P_3 = P_4 \ldots \ldots . = P_T = \frac{1}{T}$
- Variance (σ_i^2)= $\frac{1}{T} \sum_{t=1}^{T} (R_{i,t} - \bar{R_i})^2$

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- Think of $(A+B)^2 = A^2 + B^2 + 2AB$
- $\boldsymbol{\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * (w_1 * \sigma_1)(w_2 * \sigma_2)\rho_{12}}$ (7)
- where σ_p is the portfolio standard deviation (SD). σ_1 and $\boldsymbol{\sigma_2}$ are SD of the individual securities. w_1 and w_2 are the investment proportions in each of the securities. ρ_{12} is the correlation between the two securities, and varies from −1.0 to 1.0
- What if $\boldsymbol{\rho_{12}}$=1?

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ (7)
- $\boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ is called the covariance between securities 1 and 2, also $\boldsymbol{\rho_{12}} = \boldsymbol{\rho_{21}}$
- This variance (or SD) is less or more than the value given by Eq. (8)?
- For $\boldsymbol{\rho_{12}}$=1, $\sigma_p^2 = (w_1 * \sigma_1 + w_2 * \sigma_2)^2$
- $\boldsymbol{\sigma_p = w_1 * \sigma_1 + w_2 * \sigma_2}$ (8)
- For all the values of ρ_{12} (except ρ_{12} =1), the value of Eq. (7) will be less than that of Eq. (8); What are the implications?

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ (7)
- For $\boldsymbol{\rho_{12}} = -1$, $\sigma_p^2 = (w_1 * \sigma_1 - w_2 * \sigma_2)^2$
- $\boldsymbol{\sigma_p = w_1 * \sigma_1 - w_2 * \sigma_2}$ (9)
- For all the values of ρ_{12} (except $\rho_{12} = -1$), the value of Eq. (7) will be more than Eq. (9); What are the implications?

Risk: Standard Deviation for Two Securities

$$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$$

	$1\ (w_1, \sigma_1)$	$2\ (w_2, \sigma_2)$
$1\ (\mathrm{w}_1, \sigma_1)$	$w_1^2 * \sigma_1^2$	$\rho_{12} * w_1 * \sigma_1 * w_2 * \sigma_2$
$2\ (\mathrm{w}_2, \sigma_2)$	$\rho_{12} * w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$

Portfolio Construction with Multiple Securities: Risk

Risk: Standard Deviation for Multiple Securities

$$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$$

	1 (w_1, σ_1)	2 (w_2, σ_2)	3 (w_3, σ_3)
1 (w_1, σ_1)	$w_1^2 * \sigma_1^2$	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$
2 (w_2, σ_2)	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$
3 (w_3, σ_3)	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$	$w_3^2 * \sigma_3^2$

Risk: Standard Deviation for *N*-Security

	1 (w_1, σ_1)	2 (w_2, σ_2)	…..	……	N (w_N, σ_N)
1 (w_1, σ_1)					
2 (w_2, σ_2)					
…..					
…..					
N (w_N, σ_N)					

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- There will be "*N*" such boxes with entries of $w_i^2\sigma_i^2$
- Variance terms = $\sum_{i=1}^{N} w_i^2\sigma_i^2$
- Also, let us assume that all these stocks we have amounts invested in equal proportion (1/*N*).
- $\sum_{i=1}^{N} w_i^2\sigma_i^2 = \sum_{i=1}^{N} \frac{1}{N^2}\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{\mathbf{N}}\boldsymbol{\sigma_i^2}$ because $w_i = \frac{1}{N}$
- Define $\sigma_{\text{avg}}^2 = \sum_{i=1}^{N} \frac{1}{N}\sigma_i^2$, Variance terms= $(\frac{1}{N}) * \sigma_{\text{avg}}^2$

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- There will also be "$N^2 - N$" boxes with covariance terms and cross products of weights invested in both the securities with the following entries: $w_i w_j \sigma_i \sigma_j \rho_{ij}$
- Covariance terms = $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$, also $w_i = w_j = \frac{1}{N}$
- Covariance terms = $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \left(\frac{1}{N^2}\right) \sigma_i \sigma_j \rho_{ij} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$
- $\sigma_{\text{avg-cov}} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- Covariance terms = $\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}(\frac{1}{N^2})\sigma_i\sigma_j\rho_{ij} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}\sigma_i\sigma_j\rho_{ij}$
- $\sigma_{\text{avg-cov}} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}\sigma_i\sigma_j\rho_{ij}$
- $\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i\neq j}}^{N}\sigma_i\sigma_j\rho_{ij}$ = Covariance terms$*N^2 = \sigma_{\text{avg-cov}}*N(N-1)$
- Covariance terms=$(N^2-N)*\left(\frac{1}{N}\right)^2*\sigma_{\text{avg-cov}} = (\frac{N-1}{N})*\sigma_{\text{avg-cov}}$

Risk: Standard Deviation for *N*-Security

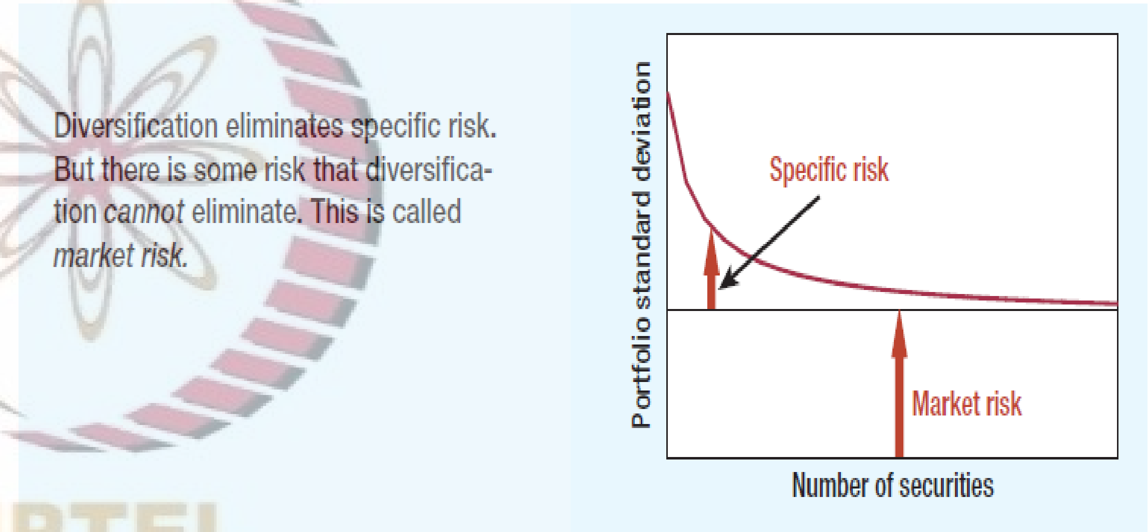
The variance of *N*-security portfolio

- Variance terms= $(\frac{1}{N}) * \sigma_{\text{avg}}^2$; Covariance terms=$(\frac{N-1}{N}) * \sigma_{\text{avg-cov}}$
- $\sigma_P^2 = (\frac{1}{N}) * \sigma_{\text{avg}}^2 + (\frac{N-1}{N}) * \sigma_{\text{avg-cov}}$
- Now, if *N* is very large ($N \rightarrow \infty$), then variance term will be close to zero
- Covariance term will be close to the average covariance
- The portfolio variance will be close to the average covariance
- $\sigma_P^2 = \sigma_{\text{avg-cov}}$
- What are the implications?

Risk Diversification with Portfolios

Risk Diversification with Portfolios

- For a well-diversified portfolio with a large number of securities, the variance terms will be close to zero
- Only the average covariances across the stocks will contribute to the portfolio risk

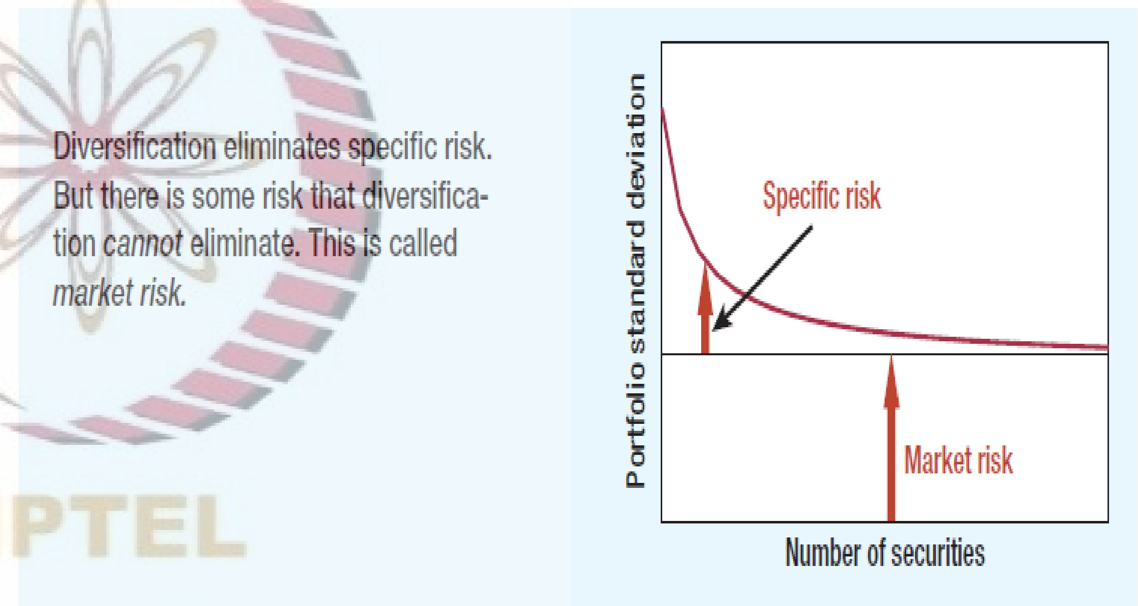


- These covariances arise due to the correlations between the security returns
- For a portfolio with low correlations across securities, the portfolio risk can be lower

Brealey, Myers, and Allen, *Principles of Corporate Finance*, 10th, 11th, or 12th edition (Chapter 7)

Risk Diversification with Portfolios

- The component associated with variances is called diversifiable risk or specific risk
- Later, we will see that market does not reward this risk
- The risk that is associated with covariances is often called market risk or non-diversifiable risk



- Market only rewards for bearing this non-diversifiable risk (market risk)

Brealey, Myers and Allen; Principles of Corporate Finance. 10th, 11th, or 12th editions. Chapter 7

Example: Computation of Expected Portfolio Returns

- For example, if we invest 60% of the money in security 1 and 40% of the money in security 2, and the expected returns from security 1 and security 2 are, respectively, 8% and 18.8%. Then, the expected returns from the portfolio are computed as follows:

  $\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2}$

- $R_p = 0.60 * 8.0\% + 0.40 * 18.8\% = 12.30\%$

Example: Computation of Expected Portfolio SD

- Consider the same previous example (w_1 = 60%, w_2 = 40%). Now, some additional information is given to compute the portfolio variance: σ_1 = 13.2% and σ_2 = 31.0%. Consider five cases of correlation coefficients: ρ_{12} = −1.0, −0.5, 0, 0.5, and 1. Now, let us compute the SD of the portfolio for all the five scenarios
- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$

Example: Computation of Expected Portfolio SD

Case	Variance (σ_P^2)	Standard Deviation (σ_P)
ρ_{12}=1	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 1 * 0.132 * 0.31 = 0.0413$	20.32%, which is same as = 0.6*13.2%+0.4*31.0%
ρ_{12}=0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.50 * 0.132 * 0.31 = 0.0315$	17.74%
ρ_{12}=0.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.00 * 0.132 * 0.31 = 0.0217$	14.71%
ρ_{12}=-0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0118$	10.88%
ρ_{12}=-1.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0020$	4.48%

Summary and Concluding Remarks

Summary and Concluding Remarks

- Adding more securities that are less correlated (have lower covariance) in the portfolio leads to diversification
- Diversification here means the reduction of stock-specific risk
- The part of the risk that is non-diversifiable is on account of the covariances across securities
- Often this risk is called market risk or systematic risk

Summary and Concluding Remarks

- Markets do not reward for bearing stock-specific diversifiable risks
- Since these risks can be easily mitigated, when we say that we expect certain return for bearing risk, that risk is systematic/non-diversifiable/market risk

Thanks!

Introduction

- Portfolio construction: expected returns, risk, correlation, and covariance
- Portfolio optimization and mean-variance framework: two-security case and N-security case
- Portfolio possibilities curve and feasible region
- Feasible region with short sales
- Minimum variance portfolio
- Introduction to risk-free lending and borrowing
- Market risk and beta

Portfolio Construction Recap I

Expected Returns on a Portfolio

Actual returns on the portfolio can be represented by the following model:

- $R_{Pt} = \sum_{i=1}^{N} X_i R_{it}$ (1)
- Where 'i' depicts one of the 'N' securities, and 'Xi' is the weight invested in the security 'i'
- Now, the expected returns of the portfolio can also be written as:
- $\bar{R}_P = E(R_{Pt}) = E(\sum_{i=1}^{N} X_i R_{it})$
- This can also be written as follows: $\sum_{i=1}^{N} E(X_i R_{it})\ or\ \sum_{i=1}^{N} X_i E(R_{it})$
- $\bar{R}_P = \sum_{i=1}^{N} X_i \bar{R}_i$ (2)

Risk of a Two-Security Portfolio

Risk of a two-security portfolio can be shown as

- $\sigma_p^2 = E\left(R_{pt} - \bar{R}_p\right)^2 = E[X_1 R_{1t} + X_2 R_{2t} - (X_1 \bar{R}_1 + X_2 \bar{R}_2)]^2$
- $= E[\, X_1 (R_{1t} - \bar{R}_1) + X_2 (R_{2t} - \bar{R}_2)]^2$
- $= E[\, X_1^2 (R_{1t} - \bar{R}_1)^2 + X_2^2 (R_{2t} - \bar{R}_2)^2 + 2X_1 X_2 (R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]$
- $= X_1^2 E[\, (R_{1t} - \bar{R}_1)]^2 + X_2^2 E[(R_{2t} - \bar{R}_2)^2] + 2X_1 X_2 E[(R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]$
- The third term, "$E[(R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]$", is called covariance and can be depicted as σ_{12} (here $\sigma_{12} = \sigma_{21}$)

Risk of a Two-Security Portfolio

The resulting final expression can be shown as

- $\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_{12}$ (3)
- This expression can be extended for a three-security portfolio, as shown below
- $\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_1X_3\sigma_{23}$ (4)

Few Words on Covariance

Please note that this covariance is the product of two deviations
$E[(R_{1t} - \bar{R}_1)(R_{2t} - \bar{R}_2)]$

- If both the securities move together, i.e., positive deviations and negative deviations are observed for both securities together, then covariance is expected to be positive
- Conversely, if positive deviations of one security occur together with negative deviations of the other security, then the covariance is expected to be negative

Few Words on Covariance

If the securities do not move together, then the covariance is expected to be low

- This covariance is standardized in the following manner to obtain the correlation coefficient, as follows
- $$\rho_{ik} = \frac{\sigma_{ik}}{\sigma_i \sigma_k} \qquad (5)$$
- The standardized measure is known as the correlation coefficient
- It varies between +1 and -1

Portfolio Construction Recap II

N-Security Case

Let us start with the variance and covariance expression for a three-security case.

- $\boldsymbol{\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_1X_3\sigma_{23}}$
- These terms can be segregated into two segments
  - Terms like $X_1^2\sigma_1^2$, called variance terms
  - Terms like $2X_1X_2\sigma_{12}$, called covariance terms
- For 'N' securities variance, the generalized term can be simply written as $\sum_{i=1}^{N} X_i^2\sigma_i^2$.
- The covariance [N*(N-1)] term looks like this: $\sum_{j=1}^{N}\sum_{\substack{k=1 \\ j\neq k}}^{N}(X_jX_k\sigma_{jk})$

- $\boldsymbol{\sigma_p^2 = \sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{j=1}^{N}\sum_{\substack{k=1 \\ j\neq k}}^{N}(X_jX_k\sigma_{jk})}$

N-Security Case: Variance Terms

Assume that we are investing equal amounts in each of these securities

- Then, $X_1 = X_2 \ldots\ldots = X_N = \frac{1}{N}$
- This means that the variance term will become $\frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2$ or $\frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_i^2}{N}$
- Assuming the average variance of $\bar{\sigma}_i^2$, the variance term can also be written as $\frac{1}{N}\bar{\sigma}_i^2$
- For a portfolio with a large number of securities, this variance term will be closer to zero or very small

N-Security Case: Covariance Terms

What about the covariance term? $\sum_{j=1}^{N}\sum_{\substack{k=1 \\ j\neq k}}^{N}(X_j X_k \sigma_{jk}) = \sum_{j=1}^{N}\sum_{k=1}^{N}(\frac{1}{N^2}\sigma_{jk})$;

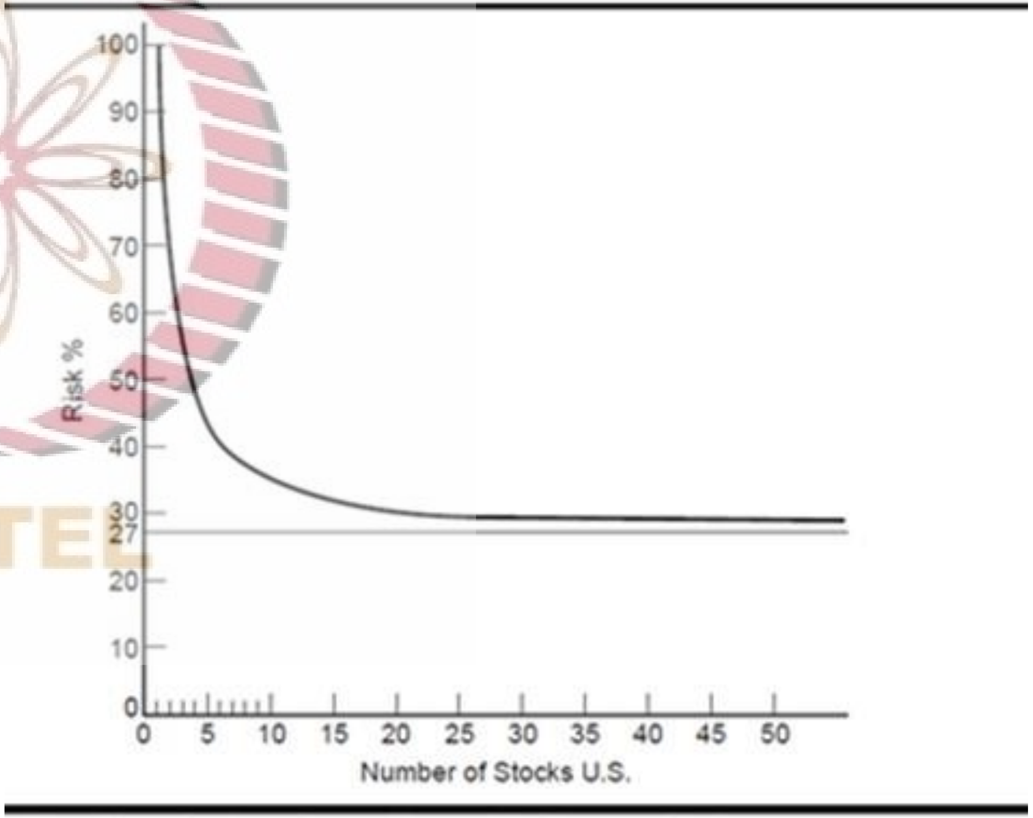
assuming equal investment in each security

- $= \frac{N-1}{N}\sum_{j=1}^{N}\sum_{k=1}^{N}(\frac{1}{N(N-1)}\sigma_{jk})$
- The term $\sum_{j=1}^{N}\sum_{k=1}^{N}(\frac{1}{N(N-1)}\sigma_{jk})$, is the summation of covariances divided by the number of covariances: average covariance $(\bar{\sigma}_{jk})$
- Resulting covariance term will become: $\frac{N-1}{N}\bar{\sigma}_{jk}$
- As we increase N, this term approaches $\bar{\sigma}_{jk}$

N-Security Case

Total standard deviation of the N-security portfolio converges to

- $\sigma_p^2 = \frac{1}{N}\overline{\sigma}_i^2 + \frac{N-1}{N}\overline{\sigma}_{jk}$
- For a large number of securities, this formula simplifies to
- $\sigma_p^2 \approx \overline{\sigma}_{jk}$

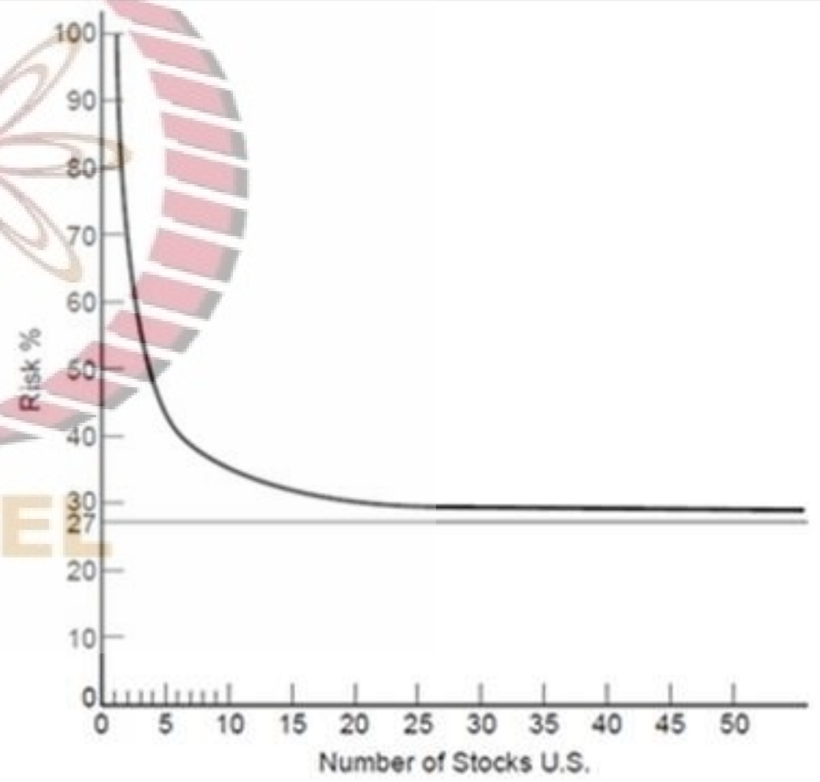


The effect of number of securities on risk of the portfolio

Elton, Gruber, Brown, and Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 9th edition (Chapter 4)

N-Security Case

- This gives us the intuition that as the number of securities is increased, the variance terms that represent the risk of individual securities are offset
- What is left is that the covariance terms can not be diversified away



The effect of number of securities on risk of the portfolio

Elton, Gruber, Brown, and Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 9th edition (Chapter 4)

Mean Variance Framework

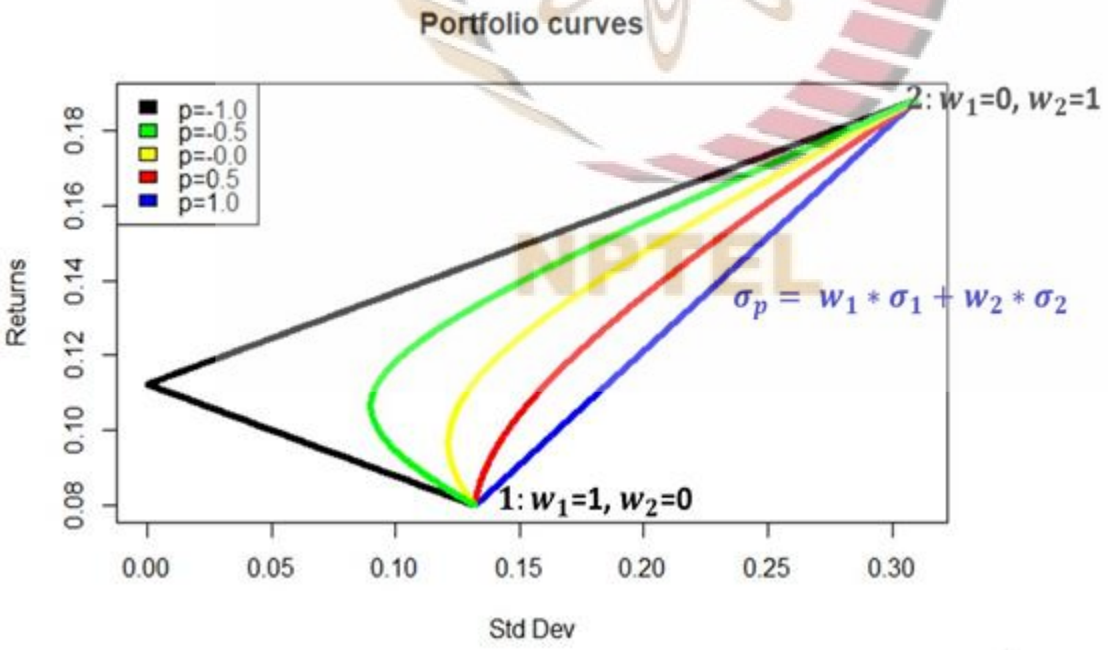
Portfolio Risk and Return Profile

Consider the following equations describing expected returns and risk from a two-stock portfolio.

- $\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2}$ (1)
- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$ (2)
- Consider two securities 1 and 2. Security 1 offers 8% expected return, and 2 offers 18.8% return. SD of 1 is 13.2% and that of 2 is 31%.

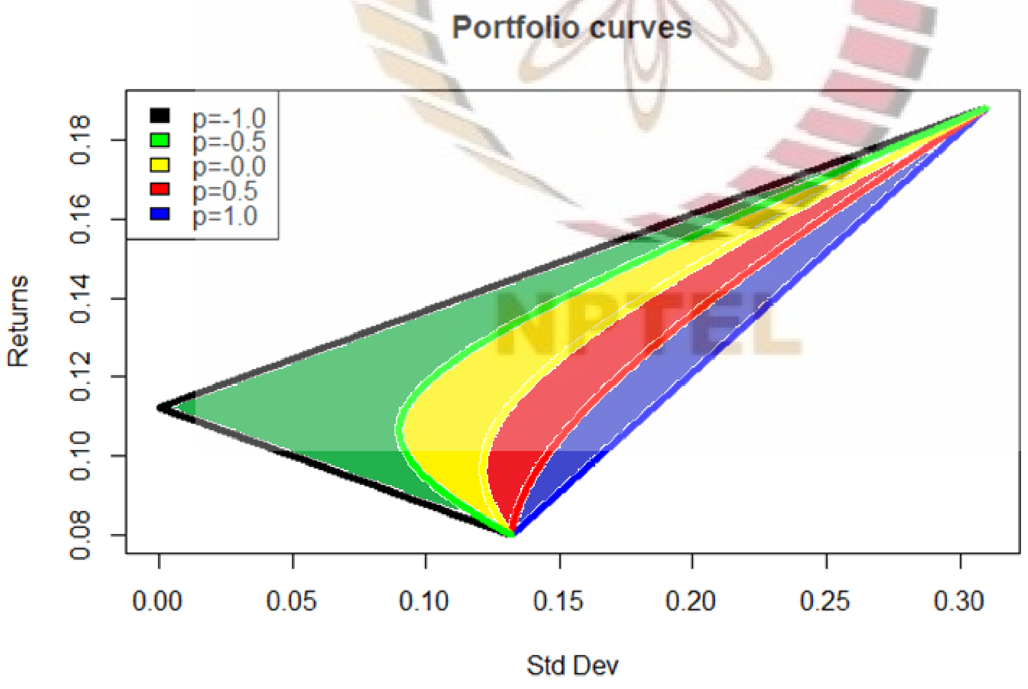
Portfolio Risk and Return Profile

We will examine how the risk-return profile looks for ρ_{12}= 1.0 (blue), ρ_{12}=0.5 (red), ρ_{12}=0 (yellow), ρ_{12}=-0.5 (green), and ρ_{12}=-1.0 (black).



Portfolio Risk and Return Profile

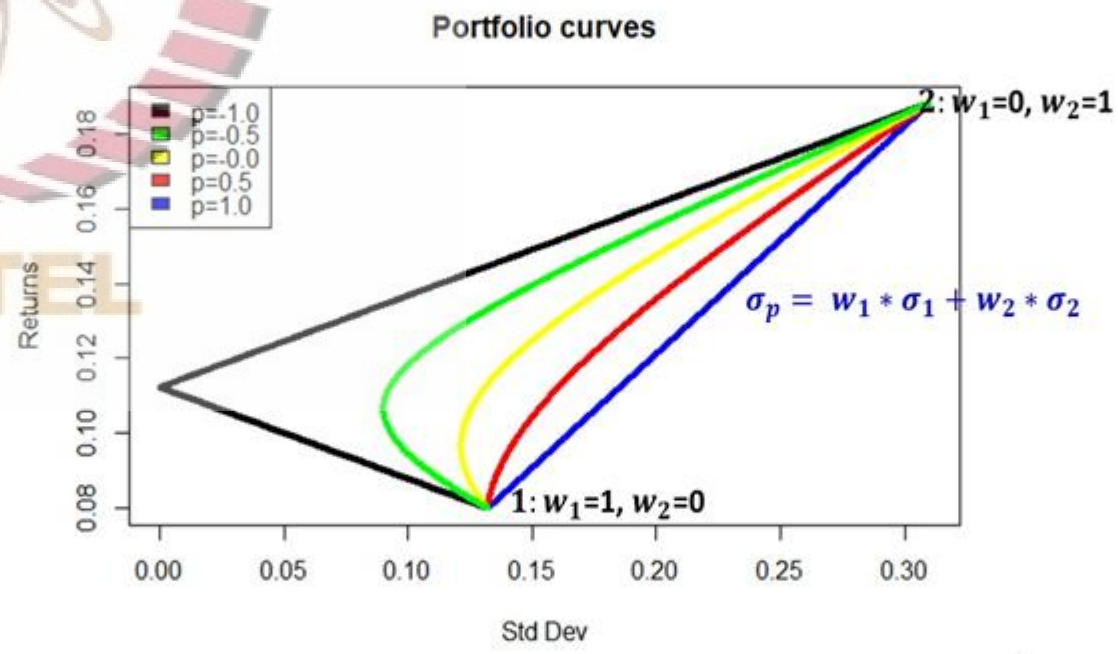
We will vary the proportionate amounts, that is, w_1 and w_2, between 0 and 1 where $w_1 + w_2 = 1$



Portfolio Risk and Return Profile

Consider the blue line with ρ_{12}=1 correlation. In this special case, the equation becomes a straight line: $\sigma_p = w_1 * \sigma_1 + w_2 * \sigma_2$ (blue line)

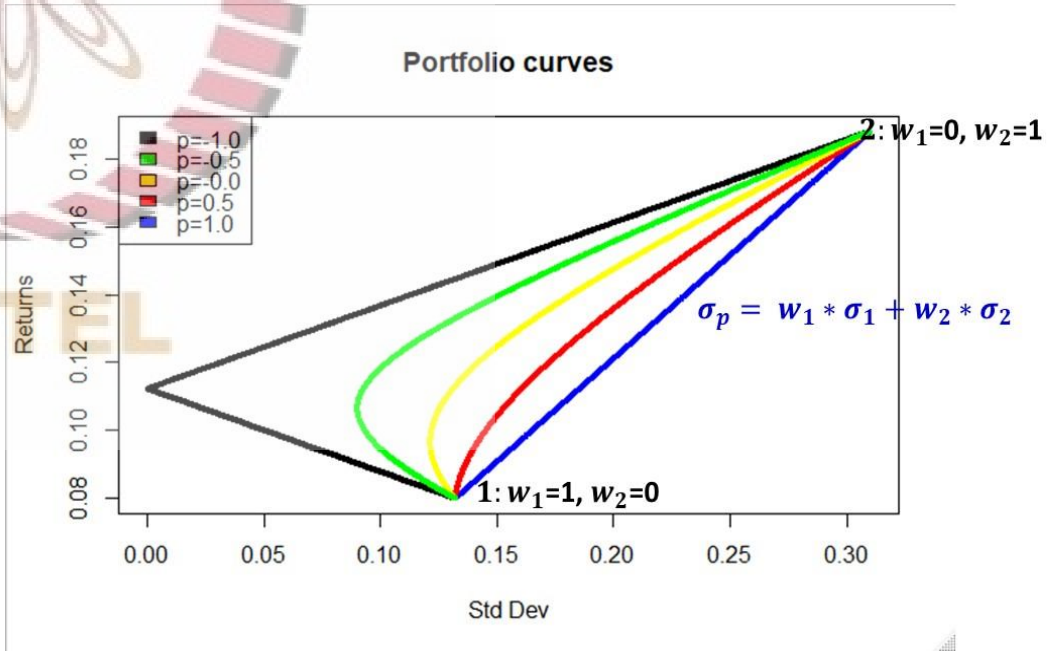
- Across all the graphs, the lowest amount of diversification (highest portfolio risk, σ_p^2) for a given level of return is associated with the blue line (ρ_{12}=1)



Portfolio Risk and Return Profile

Next, we examine the other extreme case corresponding to ρ_{12} = -1 correlation shown in black

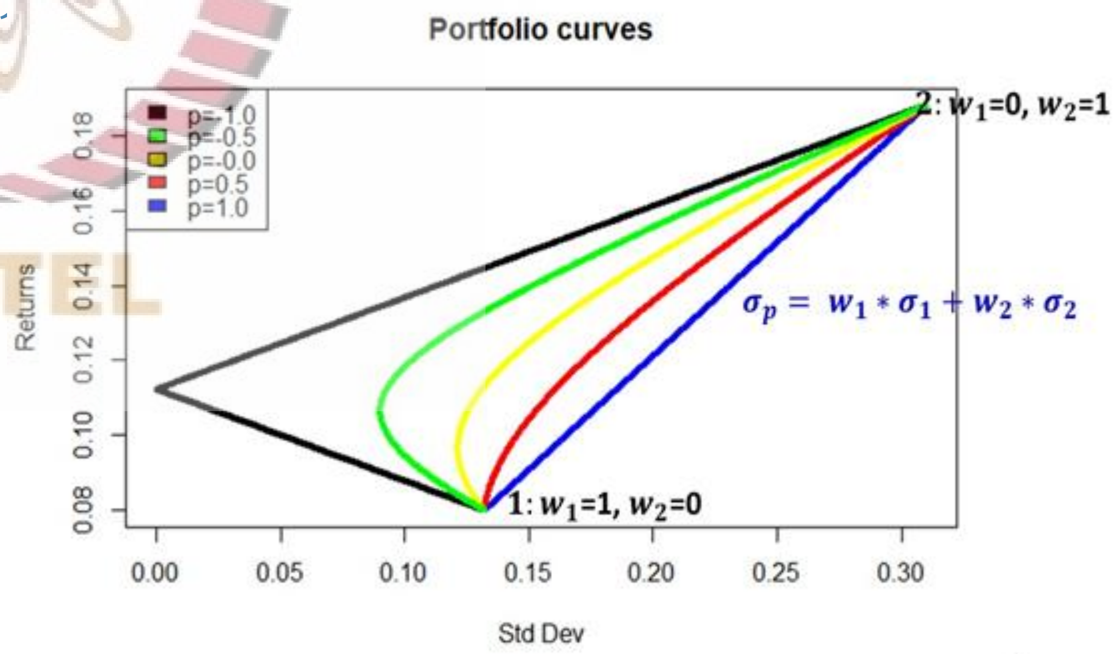
- This case (black line) offers the highest diversification, as it carries the lowest levels of risk for a given level of returns. In this case, the equation for risk: $\sigma_p^2 = (w_1 * \sigma_1 - w_2 * \sigma_2)^2$



Portfolio Risk and Return Profile

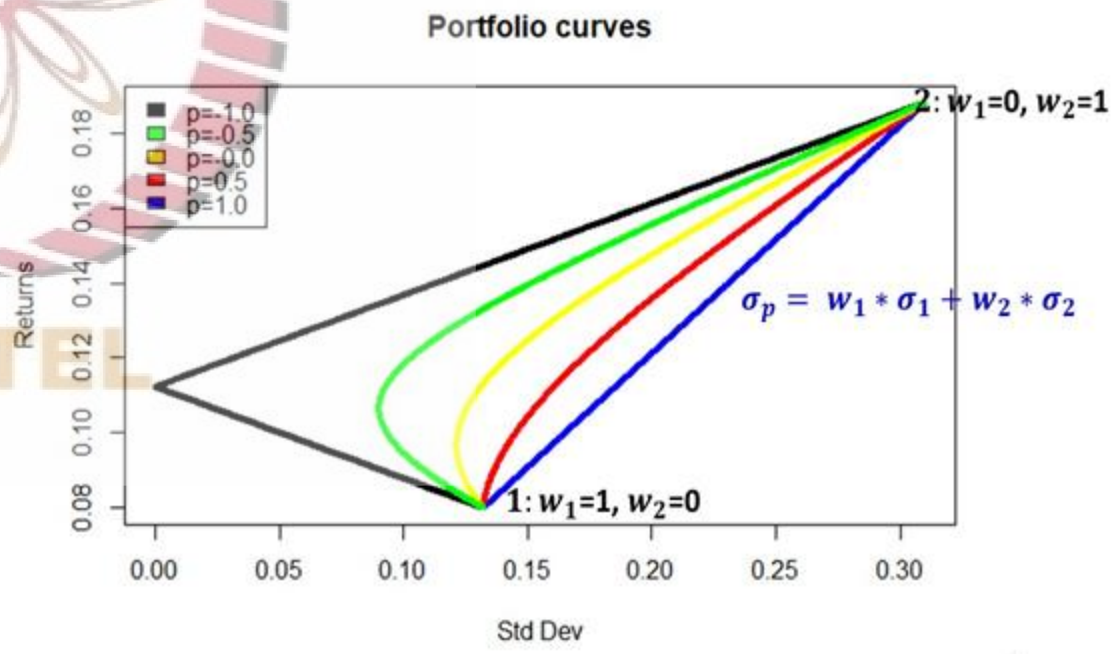
This equation has two solutions, each representing a straight line: (a) $\sigma_p = (w_1 * \sigma_1 - w_2 * \sigma_2\ )$ when $(w_1 * \sigma_1 - w_2 * \sigma_2\ )$>=0; and $\sigma_p = -(w_1 * \sigma_1 - w_2 * \sigma_2\ )$ when $(w_1 * \sigma_1 - w_2 * \sigma_2\ )$<0

- These two lines intersect at $\sigma_p = 0, where\ (w_1 * \sigma_1 = w_2 * \sigma_2\ )$. This is a special though impractical case where we attained complete diversification with zero risks



Portfolio Risk and Return Profile

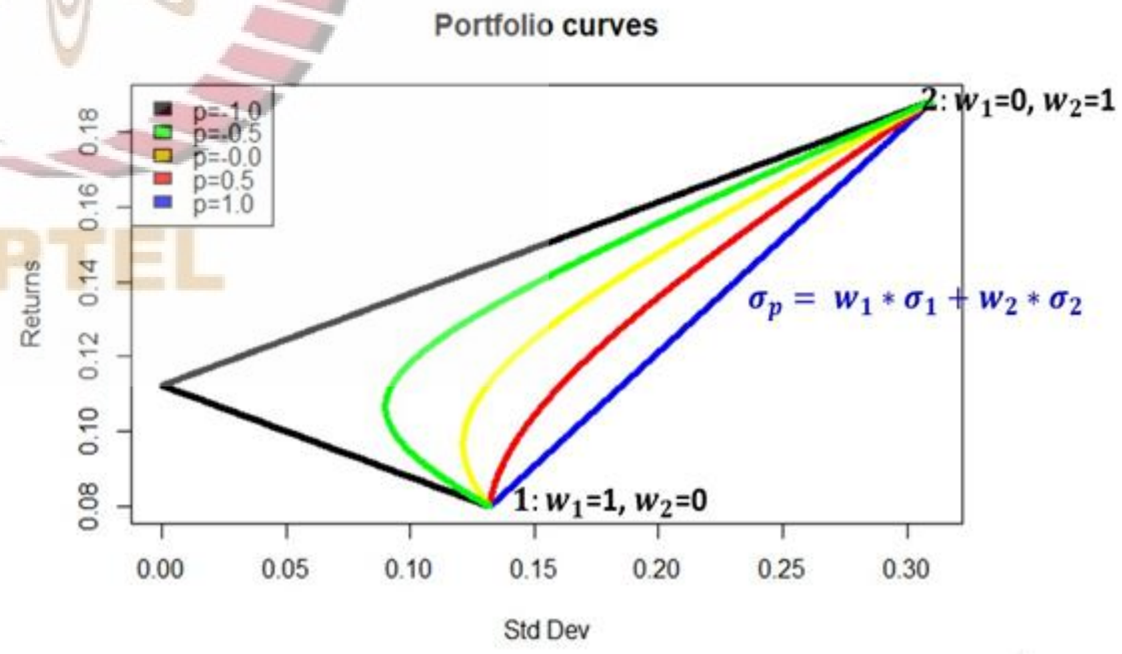
- The cases where ρ_{12} lies between -1 and +1 are concave kinds of curves in-between the two extreme cases
- An important observation here is that the risk of the portfolio, for a given level of returns, is sometimes even less than the least risky security in the portfolio, even more so when the correlation between the securities is low



Portfolio Risk and Return Profile

Adding more securities to the portfolio surely lowers the specific risk of the portfolio. Even say 15-20 stocks can offer a considerable amount of diversification

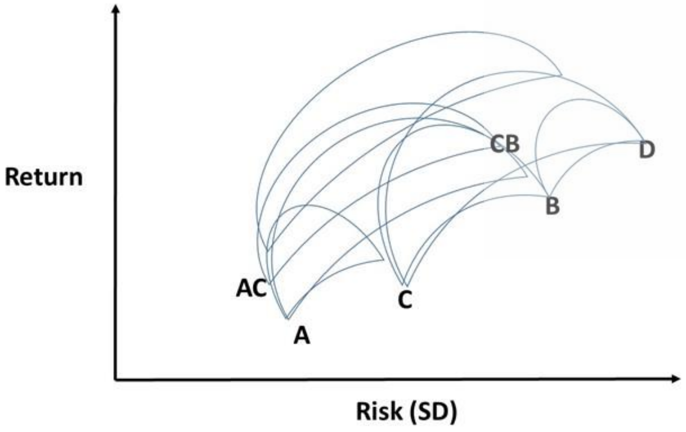
- What happens when we add more and more securities? How does the feasible region of the area of possibilities changes

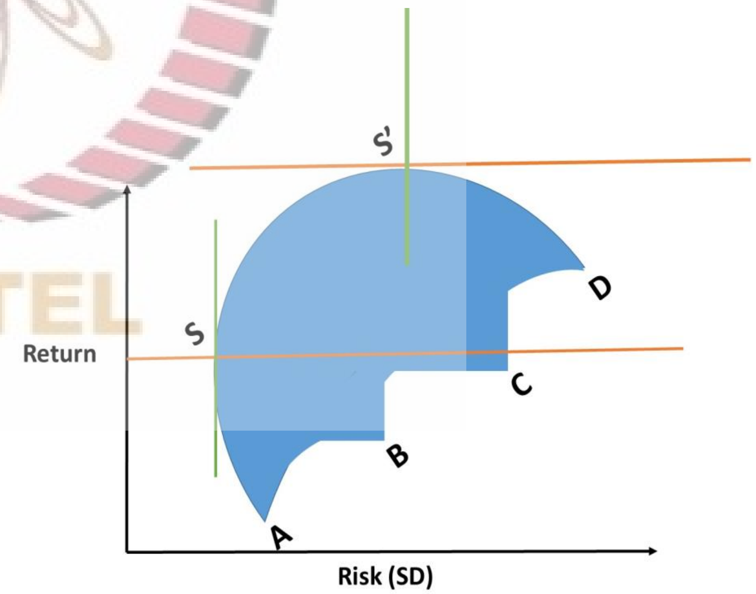


Portfolio Possibilities Curve

Portfolio Risk and Return Profile

As we keep on forming these combinations infinitely, we will get the following convex egg-cut shape.

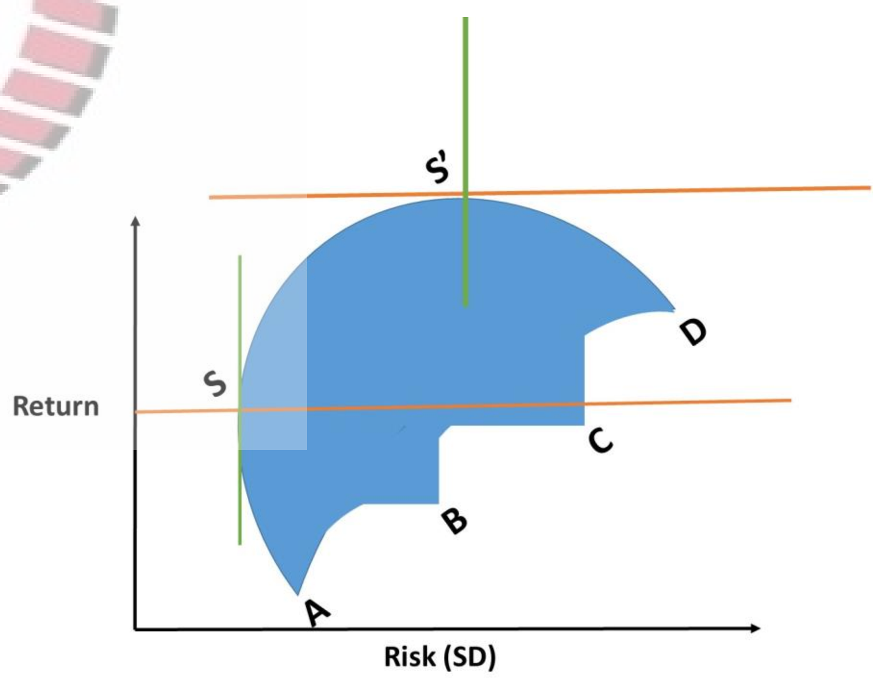




Portfolio Risk and Return Profile

The region of possibilities is shown in blue

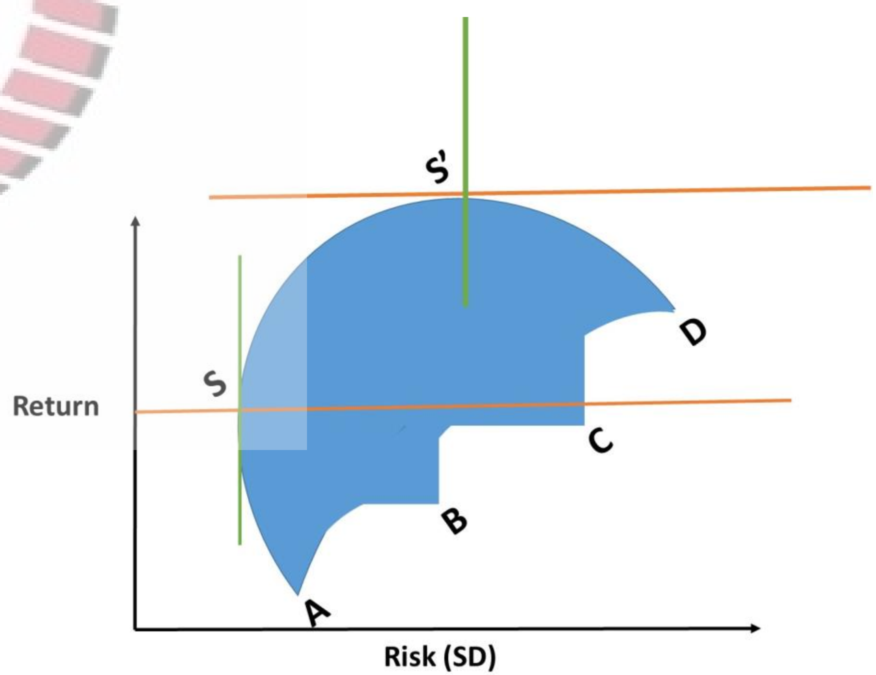
- The blue area is effectively the region of expected return and risk possibilities that an investor can attain
- Each point represents the combination of risk and returns that is available to investors in the form of investment in portfolios
- Together, all these points (portfolios) comprise the region of possibilities (or the feasible region)



How to Improve Our Position in This Region?

We want to move up (increase returns) and move to the left (reduce risk)

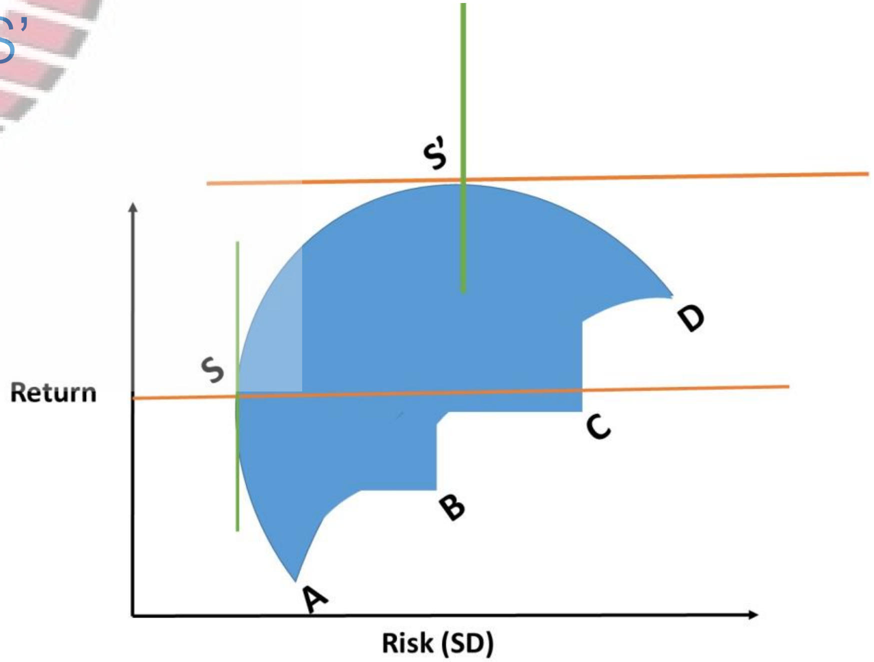
- As we do that, we reach the top surface of the region of possibilities, that is, the surface SS'
- There are no more points where we can move further left or up on this curve (SS')
- This region would be called the efficient frontier. And all the points on this region offer the highest return for the given level of risk (or the lowest risk for a given level of returns)



How to Improve Our Position in This Region?

Also, each investor depending upon his risk preference may choose a specific risk level

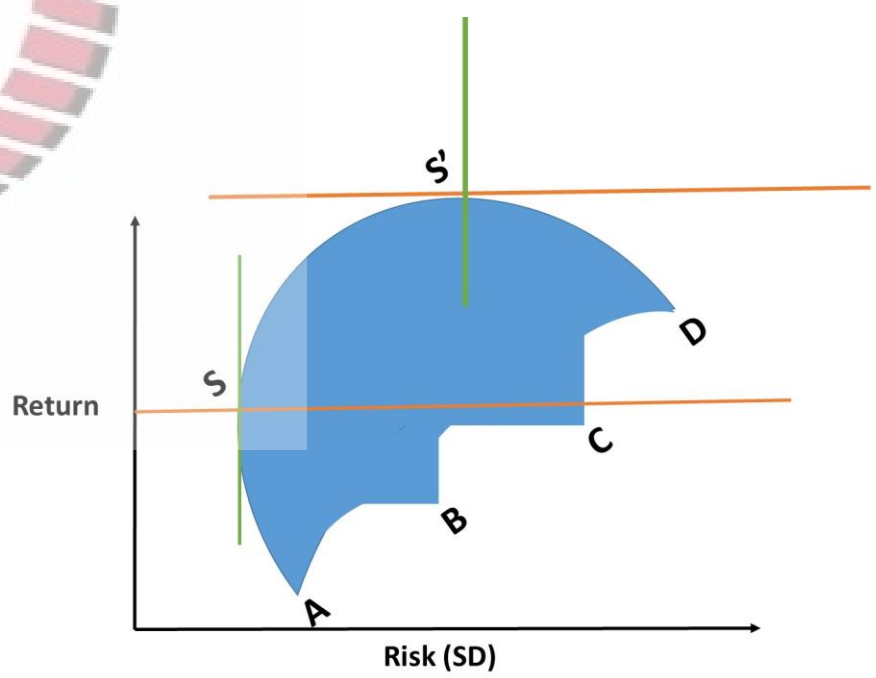
- Once he decides on a specific risk level, he will have a given certain expected return level on the surface SS'
- Once he decides on a specific risk level, he will have a given certain expected return level on the surface SS'
- Two points in this region are particularly important for us



How to Improve Our Position in This Region?

Two points in this region are particularly important for us

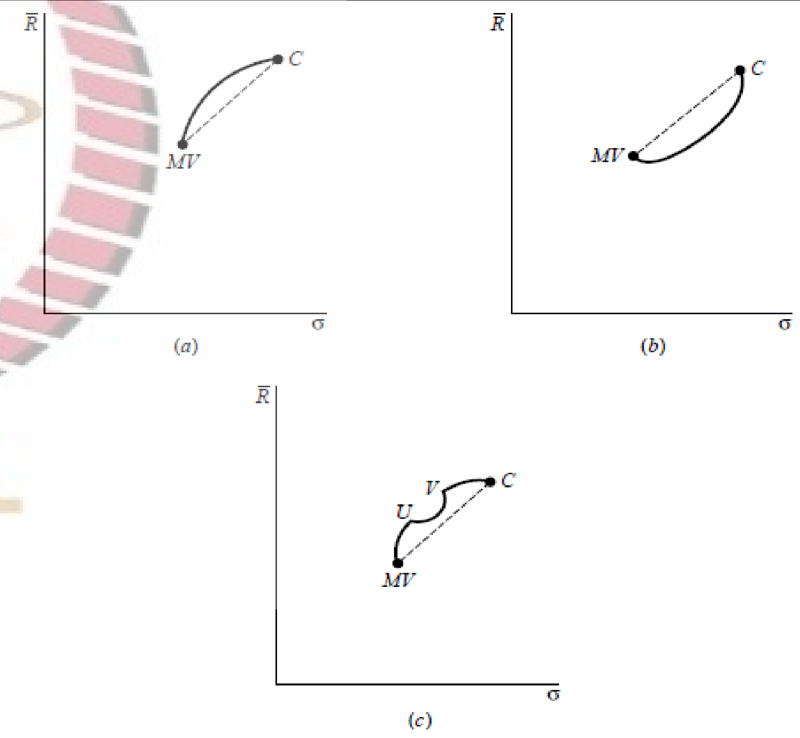
- Point S has minimum risk as compared to any other point in the feasible region
- Point S' that has maximum return as compared to any other point on the feasible region
- All the points between SS' presents the unique and best combinations of risk and return on the feasible region



Feasible Frontiers

Feasible Frontiers

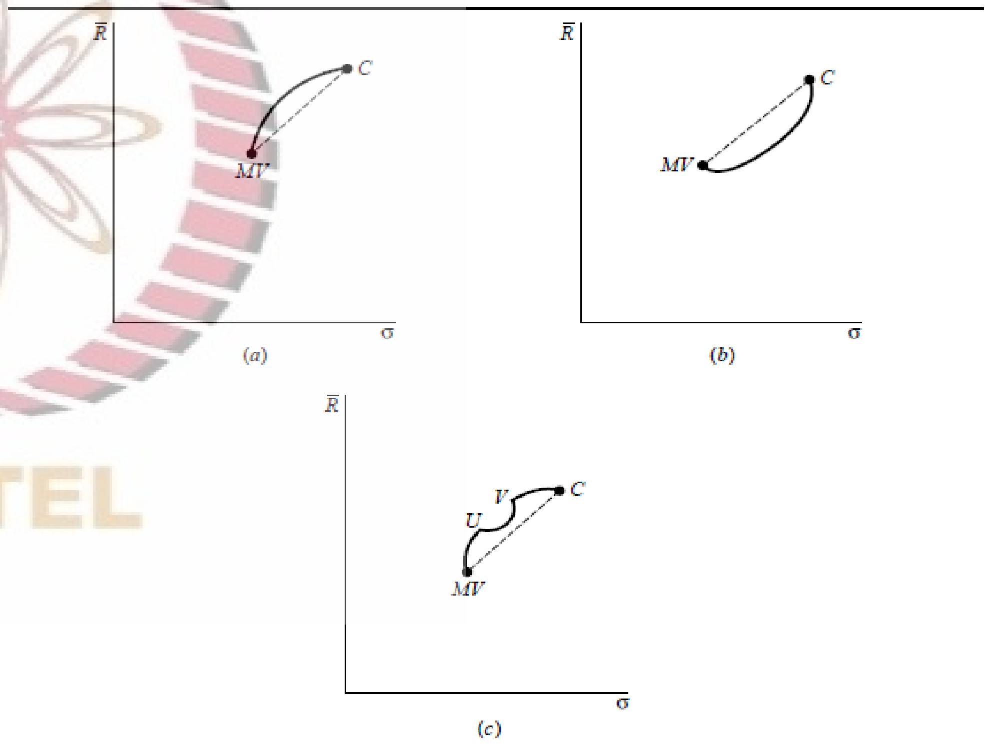
- The portfolio possibility curve that lies above the minimum variance portfolio is concave, whereas that which lies below the minimum variance portfolio is convex
- (b) is not possible because the combination of assets can not have more risk than that found on a straight line connecting two assets, and that is only the case where perfect correlation exists



Elton, Gruber, Brown, and Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 9th edition, Chapter 5

Feasible Frontiers

- In (c), all the combinations of U and V must lie on the line joining U and V or above such line hence the given shape is not possible
- Here, U and V themselves are combinations of MV and C
- Thus, only proper shape is (a), which is a concave curve



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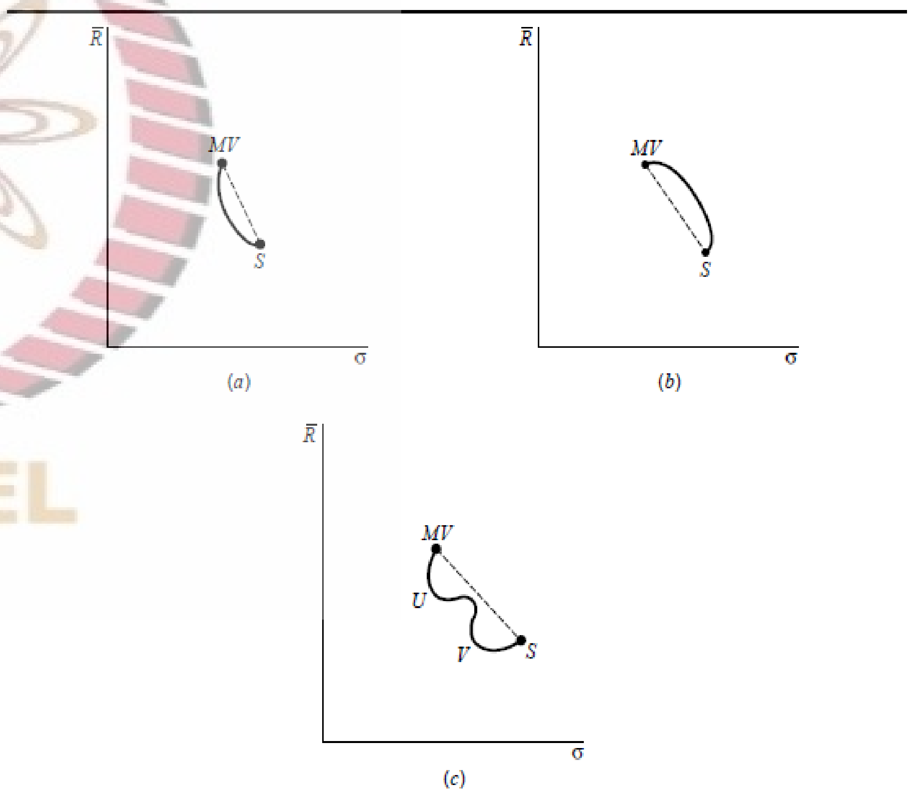
Feasible Frontiers

- With the same logic as discussed, MV and any portfolio below MV (higher variance and lower return), the resulting curve is convex
- Thus, both (b) and (c) are not feasible, only (a) is possible
- Now that we understand the risk-return properties of combinations of two assets, we are in a position to study the attributes of combinations of all risky assets



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Efficient Frontier Scenarios: Multi-Security Case I

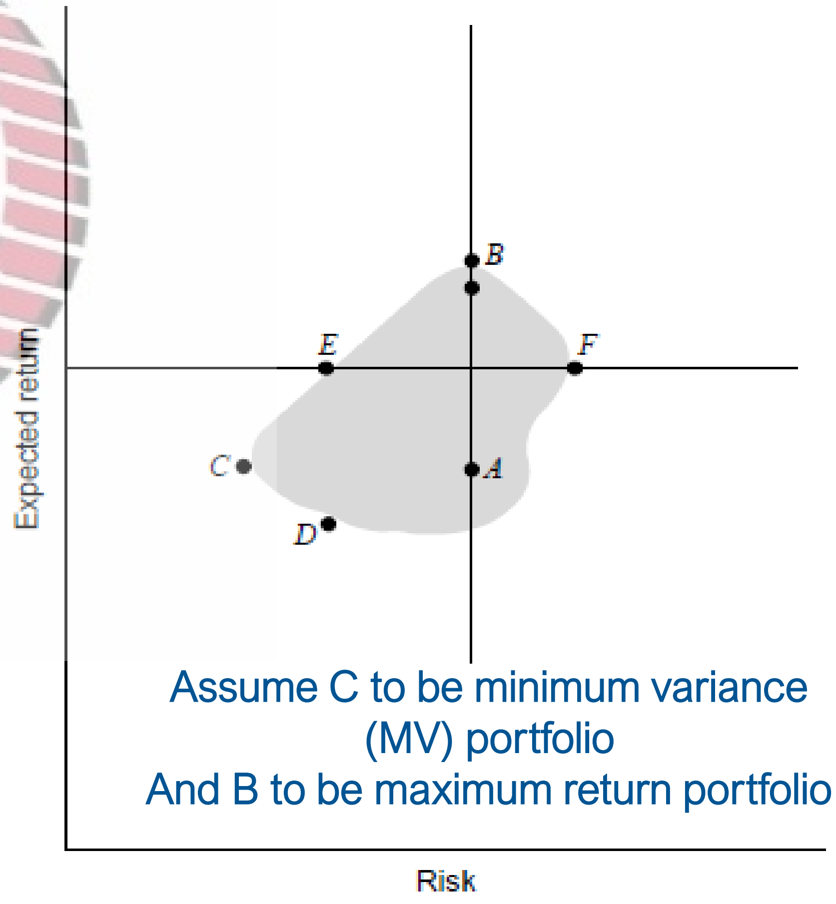
Efficient Frontier Scenarios: Multi-Security Case

- Efficient frontier with no-short sales
- Efficient frontier with short sales (no risk-free lending and borrowing)

Efficient Frontier with No-Short Sales: Multi-Security Case

In this diagram, we try to find portfolios that offer a higher returns for a given level of risk or

- Offered a lower risk for the same return
- Here, portfolio B would be preferred over portfolio A, and portfolio C would be preferred over portfolio A
- No portfolio dominates a portfolio such as B or C



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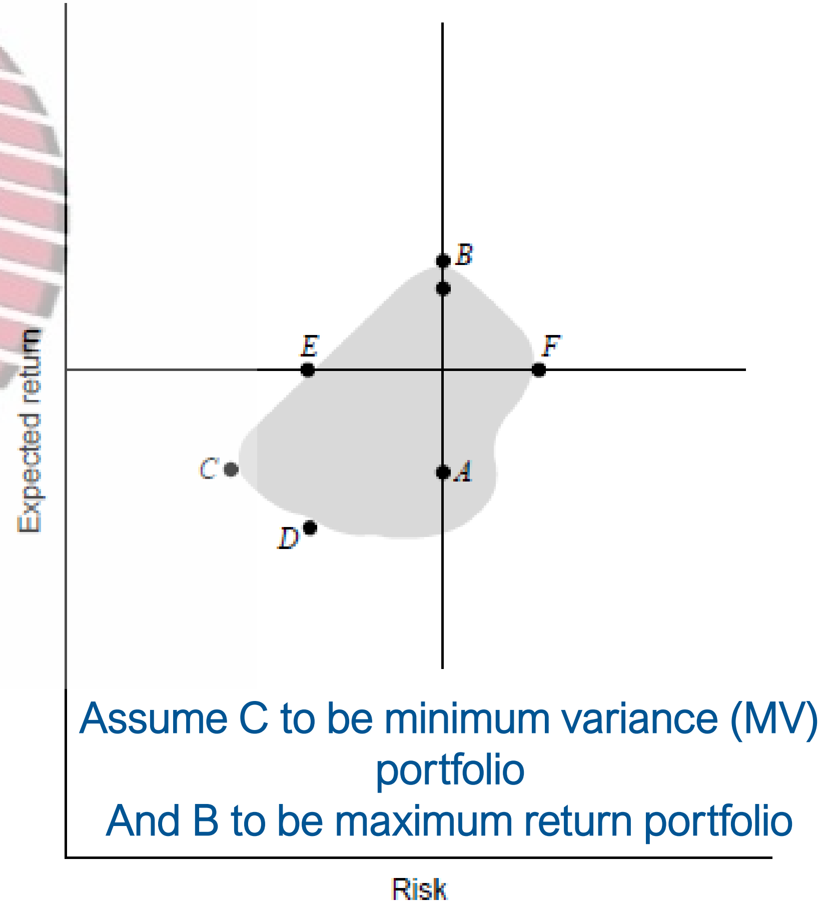
Efficient Frontier with No-Short Sales: Multi-Security Case

- C here is the global minimum variance portfolio
- Portfolio E is superior to portfolio F
- Thus, efficient sets of portfolios are those that lie between the global minimum variance portfolio and the maximum return portfolio
- This is referred to as the efficient frontier



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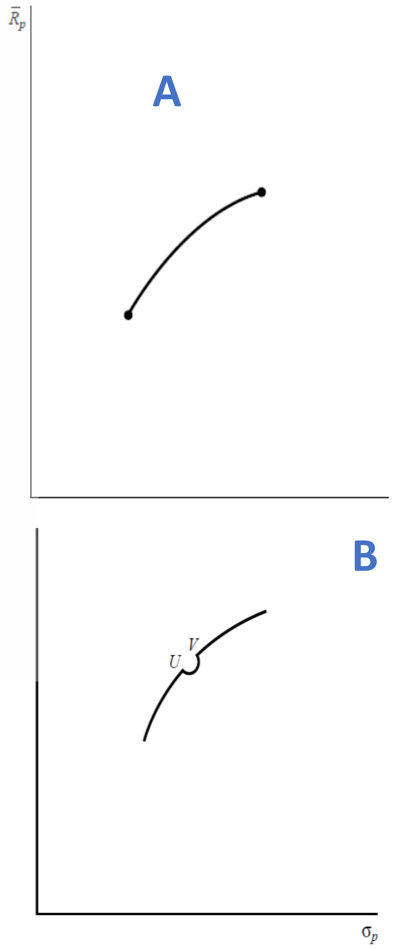
Efficient Frontier with No-Short Sales: Multi-Security Case

- The efficient frontier here is a concave curve (A)
- Why should it be a concave curve (not convex like the segment between U and V on B)?
- In this case (A), the efficient frontier (EF) is a concave function; EF extends from minimum variance portfolio to maximum return portfolio



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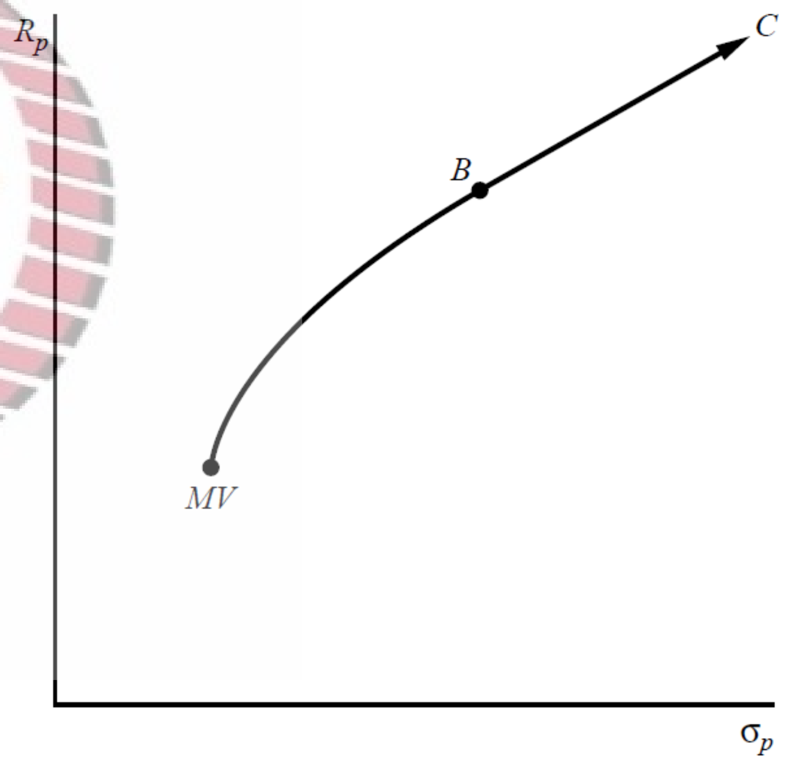
Efficient Frontier with Short Sales



- With short sales, one can sell securities with low expected returns and use the proceeds to buy securities with high expected returns
- Theoretically, this leads to infinite expected rates of return but extremely high standard deviations as well
- MVBC becomes the efficient frontier which is concave
- The efficient set still starts with the minimum variance portfolio, but when short sales are allowed, it has no finite upper bound

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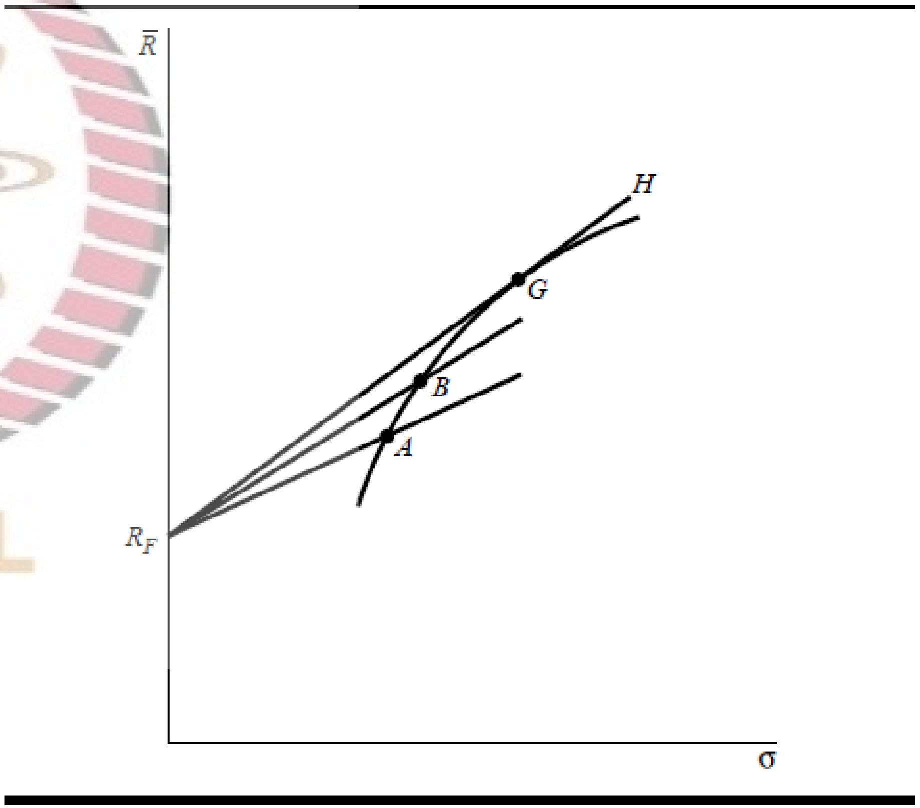
Efficient Frontier Scenarios: Multi-Security Case: II

Efficient Frontier Scenarios: Multi-Security Case

- Efficient frontier with riskless lending and borrowing
- Only riskless lending is allowed; not borrowing
- Riskless lending and borrowing at different rates

Efficient Frontier with Riskless Lending and Borrowing

- Introduction of riskless assets considerably simplifies the analysis
- Tangent line from R_F to G offers a new set of the efficient portfolios with a maximum expected return premium for a given level of risk $\frac{(R_G - R_F)}{\sigma_G}$; where G is the tangent portfolio

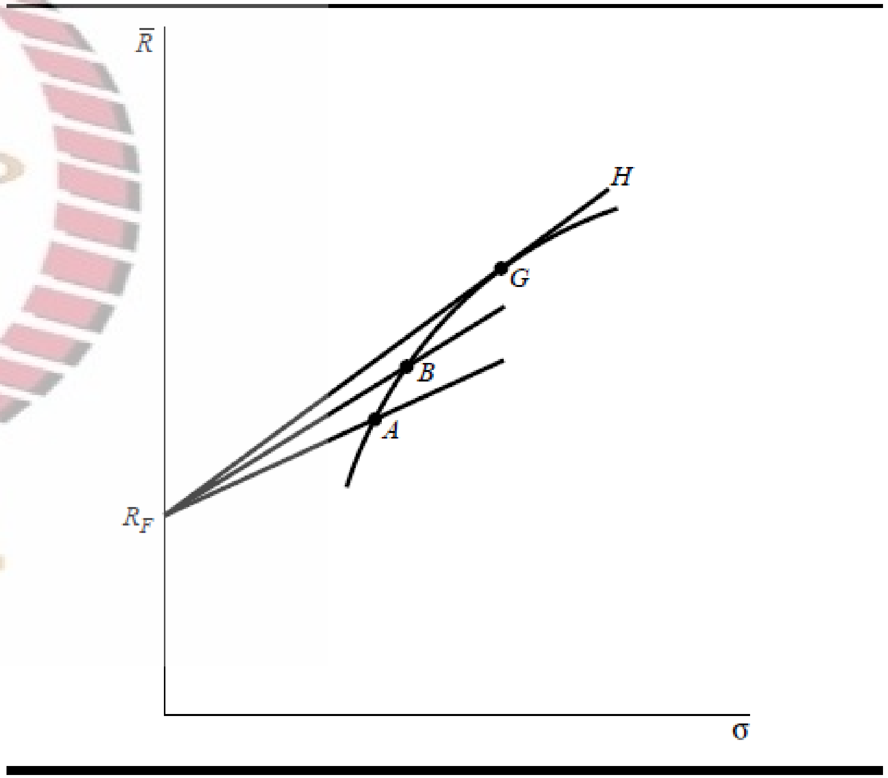


Combinations of the riskless asset and various risky portfolios.

Elton, Gruber, Brown, and Goetzmann, *Modern Portfolio Theory and Investment Analysis*, 9th edition, Chapter 5

Efficient Frontier with Riskless Lending and Borrowing

- Very risk-averse investors would hold portfolio G along with some investment in risk-free assets: R_F–G (lending portion)
- Those who are more risk-tolerant would borrow some amount at R_F and invest the entire money in the tangent portfolio (G): $G - H$ (borrowing portion)
- Separation theorem: identification of optimum portfolio does not require knowledge of investor preference



Combinations of the riskless asset and various risky portfolios.

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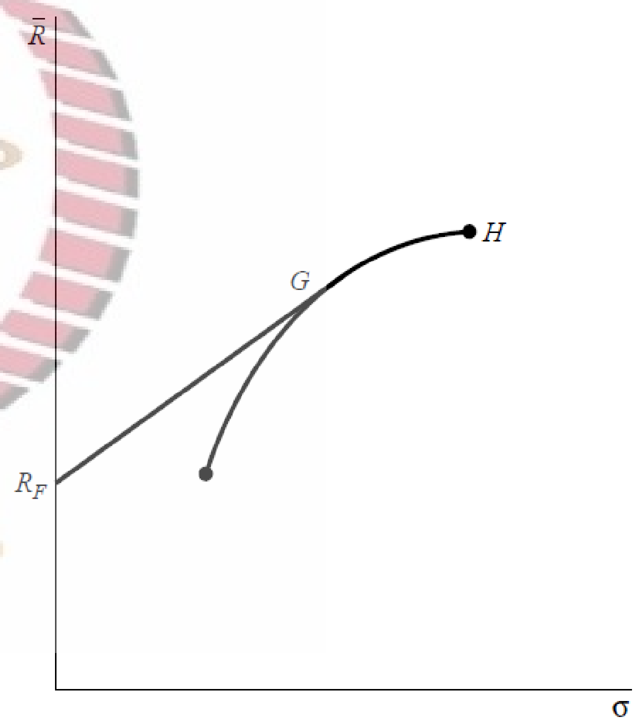
Only Riskless Lending Is Allowed; Not Borrowing

- If investors can lend but not borrow at the risk-free rate, then the efficient frontier becomes $R_F - G - H$
- Some investors will hold R_F and G (positioned on the line $R_F - G$), and others will hold a risky portfolio between G and H



The efficient frontier with lending but not borrowing at the riskless rate.

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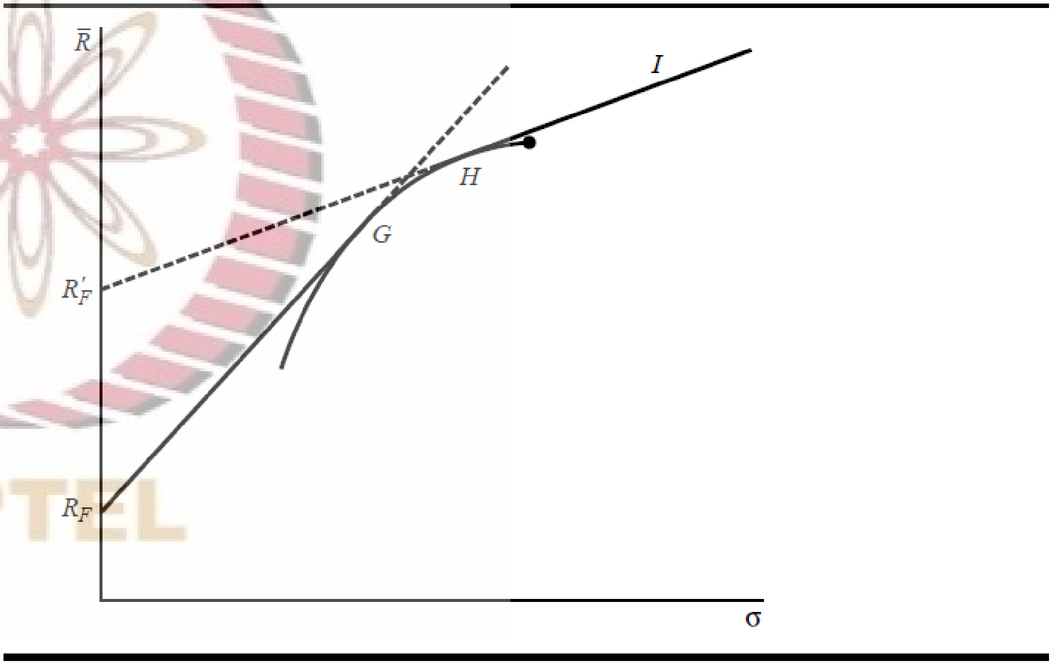
Riskless Lending and Borrowing at Different Rates

- Another possibility is that investors can lend at one rate but must pay a different and presumably higher rate to borrow (R_F and R'_F)
- The efficient frontier

$$R_F - G - H - I$$



The efficient frontier with riskless lending and borrowing at different rates.

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Minimum Variance Portfolio

Minimum Variance Portfolio

In the absence of short sales, two points become extremely important on the efficient frontier

- First, the portfolio with maximum return, and second the minimum variance portfolio
- In the absence of short sales, these portfolios define the two extreme ends of the efficient frontier
- While it is easy to understand that a maximum return portfolio will be the security in the portfolio that offers the maximum return
- The same is not the case for minimum variance portfolio

Minimum Variance Portfolio

This portfolio is often expected to be different from the security with minimum risk (SD) in the portfolio. How do we compute this portfolio?

- $\sigma_P = [X_A^2\sigma_A^2 + X_B^2\sigma_B^2 + 2X_AX_B\rho_{AB}\sigma_A\sigma_B]^{\frac{1}{2}}$ (1)
- What exactly do we want to compute here?
- $\sigma_P = [X_A^2\sigma_A^2 + (1 - X_A)^2\sigma_B^2 + 2X_A(1 - X_A)\rho_{AB}\sigma_A\sigma_B]^{\frac{1}{2}}$ (2)
- To obtain the minima, we need to set the derivative = 0 in Eq. (2), and solving this for X_A, we get
- $X_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{(\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B)}$ (3)

Minimum Variance Portfolio

Consider the example below

Stock	Expected Returns	SD
A	14%	6%
B	8%	3%

- Assume a correlation of 0, try to find the amount invested in MV portfolio
- $X_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{(\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B)} = \frac{\sigma_B^2}{(\sigma_A^2 + \sigma_B^2)} = \frac{3^2}{6^2 + 3^2} = \frac{1}{5} = 0.2$, $X_B = 0.8$
- $\sigma_P = (0.2^2 * 6^2 + 0.8^2 3^2)^{\frac{1}{2}} = 2.68\%$

Minimum Variance Portfolio

Consider the example below

Stock	Expected Returns	SD
A	14%	6%
B	8%	3%

- Assume a correlation of 0.5, try to find the amount invested
- $X_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{(\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B)} = \frac{3^2 - 0.5*6*3}{6^2 + 3^2 - 2*0.5*6*3} = 0$
- What is the implication? No combination of securities A and B has less risk than security B itself. So, the minimum variance portfolio is security B itself. That also means for any correlation higher than 0.5, security B will itself B the minimum variance portfolio ($X_A = 0$)

Minimum Variance Portfolio

Consider the example below

Stock	Expected Returns	SD
A	14%	6%
B	8%	3%

- Assume a correlation of 1, try to find the amount invested
- $X_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{(\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B)} = \frac{\sigma_B(\sigma_B - \sigma_A)}{(\sigma_A - \sigma_B)^2} = \frac{\sigma_B}{\sigma_B - \sigma_A} < 0$
- With limiting constraint that any weight cannot be equal to zero, this gives us $X_A = 0$

Minimum Variance Portfolio

Consider the example below

Stock	Expected Returns	SD
A	14%	6%
B	8%	3%

- Assume a correlation of -1, try to find the amount invested
- $X_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{(\sigma_A^2+\sigma_B^2-2\rho_{AB}\sigma_A\sigma_B)} = \frac{\sigma_B(\sigma_B+\sigma_A)}{(\sigma_A+\sigma_B)^2} = \frac{\sigma_B}{\sigma_B+\sigma_A} = 1/3$, $X_B = \frac{2}{3}$
- $\sigma_P = \left(\frac{1}{3} * 6 - \frac{2}{3} * 3\right) = 0$
- $W_A\sigma_A - \mathrm{W_B}\sigma_B$=0

Introduction to Risk-Free Lending and Borrowing I

Introduction to Risk-Free Lending and Borrowing

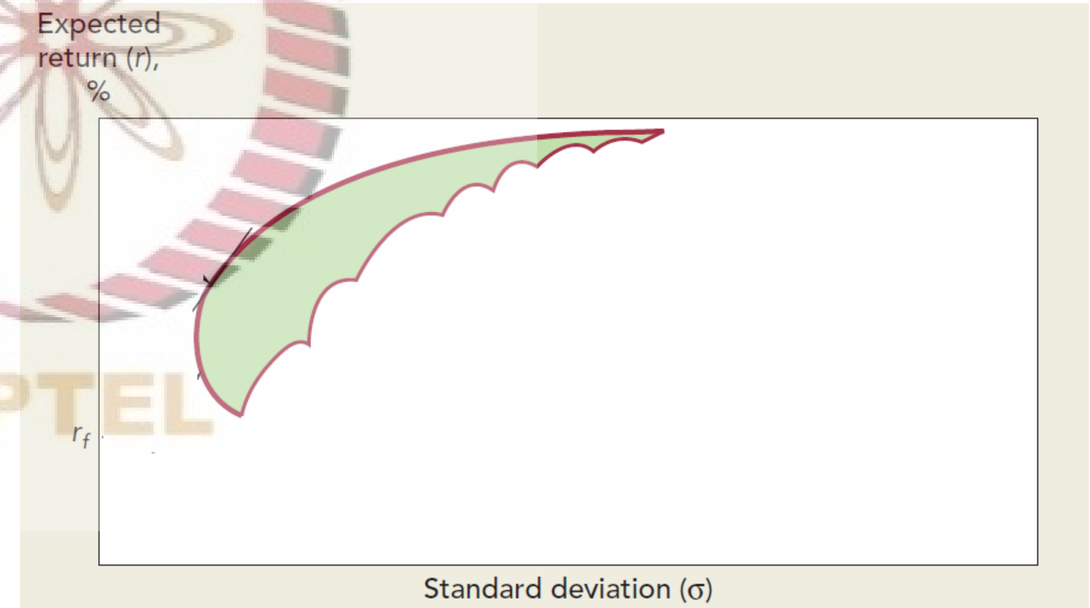
Let us introduce risk-free lending and borrowing at the risk-free rate of interest r_f

- What are the practical challenges with this assumption
- Can we borrow at the same rate from the State Bank of India (SBI) at which we make fixed deposits with SBI
- However, this assumption has several important implications for portfolio construction
- Consider that a large number of stocks are employed to construct a feasible region of possibilities

Introduction to Risk-Free Lending and Borrowing

In practice, you invest in a portfolio of number of stocks

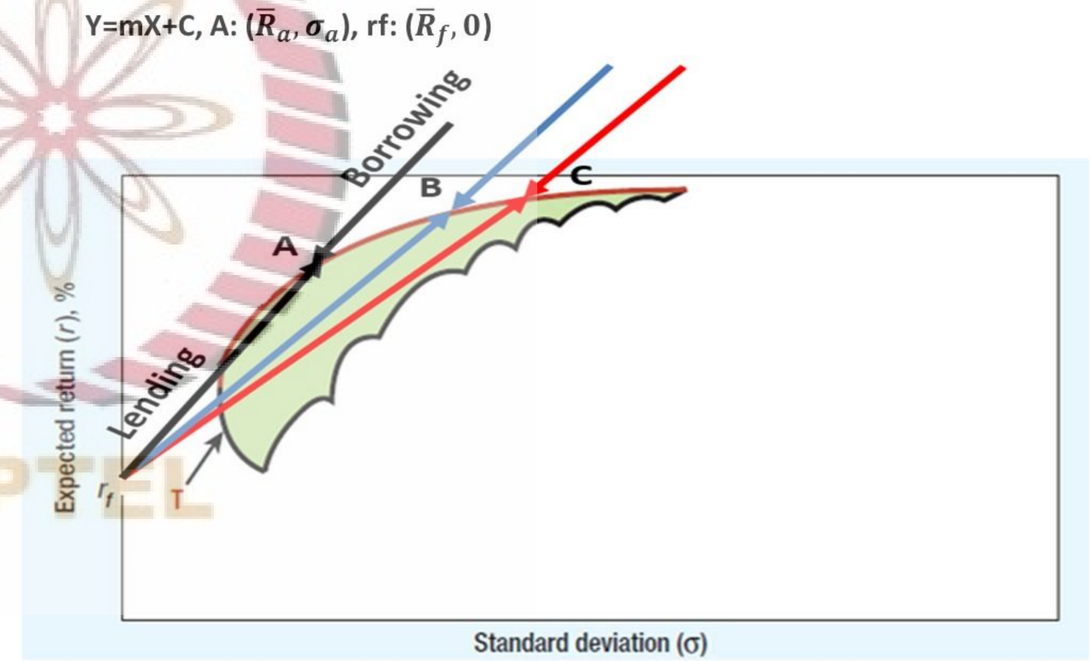
- Thus, you obtain a wider selection of risks and return
- You also obtain the efficient frontier by going up (increase expected return) and to the left (reduce risk)
- This becomes a capital rationing problem, which can be solved with quadratic programming



Brealey, Myers, and Allen, *Principles of Corporate Finance,* 10th, 11th, or 12th editions, Chapter 8

The Efficient Frontier with Riskless Lending and Borrowing

- The addition of riskless securities considerably simplifies the analysis and opens new possibilities for investment
- Consider two investments (1) a portfolio of assets A that lies on the efficient frontier; and (2) one risk-free asset



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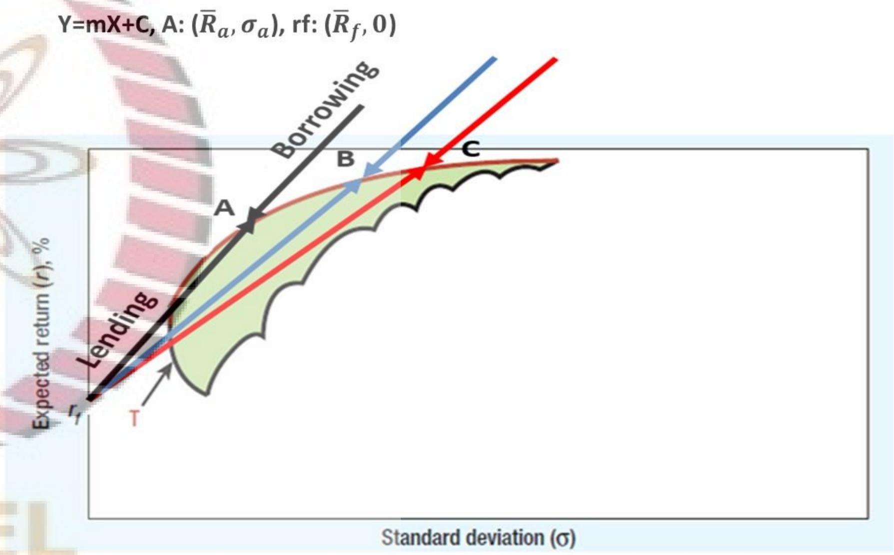
The Efficient Frontier with Riskless Lending and Borrowing

If X fraction of the amount is placed in the portfolio, then 1 − X fraction will be placed in the riskless asset

- The expected return on this portfolio can be expressed by the following equation:



- $\bar{R}_p = X\bar{R}_A + (1 - X)R_f$ (1)
- $\sigma_p^2 = X^2\sigma_A^2 + (1 - X)^2\sigma_f^2 + 2X(1 - X)\rho_{Af}\sigma_A\sigma_f$ (2)

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The Efficient Frontier with Riskless Lending and Borrowing

The equation for risk can be simplified with the introduction of risk-free instrument

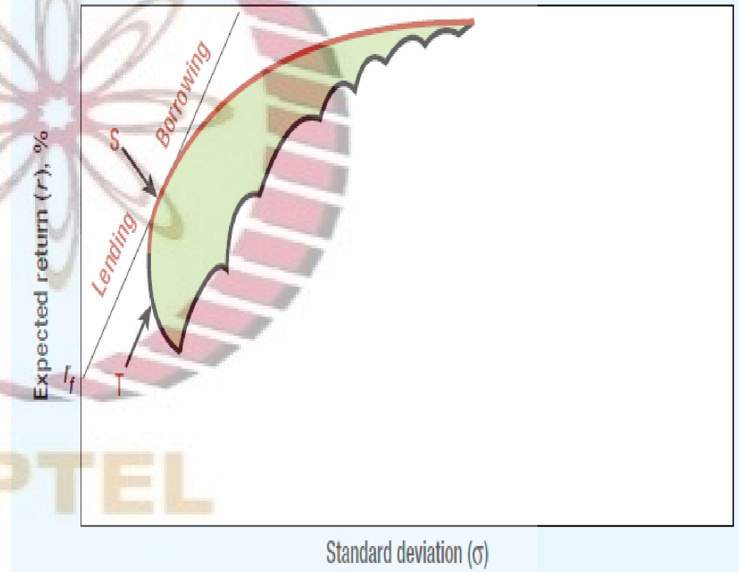
- $\sigma_p^2 = X^2\sigma_A^2 + (1-X)^2\sigma_f^2 + 2X(1-X)\rho_{Af}\sigma_A\sigma_f$
- Because $\sigma_f = 0$, the following expression of the portfolio risk is obtained
- $\sigma_p = X\sigma_A$ (1)
- $\bar{R}_p = X\bar{R}_A + (1-X)R_f$ (2)
- $\bar{R}_p = R_f + (\frac{\bar{R}_A - \bar{R}_f}{\sigma_A})\sigma_p$ (3)
- This (Eq. 3) is the equation of a straight line that passes through all the combinations of riskless lending or borrowing with portfolio A

Introduction to Risk-Free Lending and Borrowing II

Introduction to Risk-Free Lending and Borrowing

The brown line represents the most efficient portfolios or the efficient frontier

- Now that you have risk-free asset, you can invest a certain amount in the risk-free investment at r_f and the remaining amount on any portfolio available on the surface "S" corresponding to the efficient frontier



Lending and borrowing extend the range of investment possibilities. If you invest in portfolio S and lend or borrow at the risk-free interest rate, r_f, you can achieve any point along the straight line from r_f through S. This gives you the highest expected return for any level of risk. There is no point in investing in a portfolio like T.

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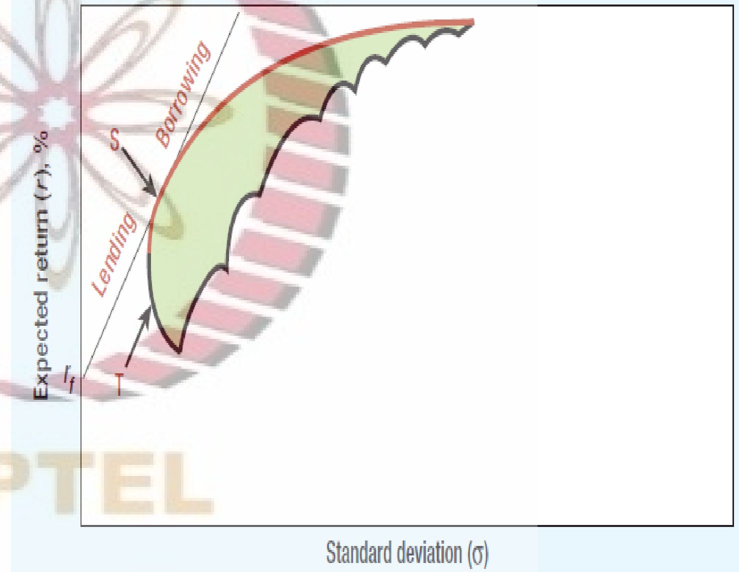
Introduction to Risk-Free Lending and Borrowing

Let us draw a line tangent from the point r_f to the red line curve

- The line that is the steepest among all is the tangent line
- The slope of this line is the amount of return per unit of risk. That is, $\frac{r_S - r_f}{\sigma_p}$
- This means that per unit of risk, this portfolio offers the highest return



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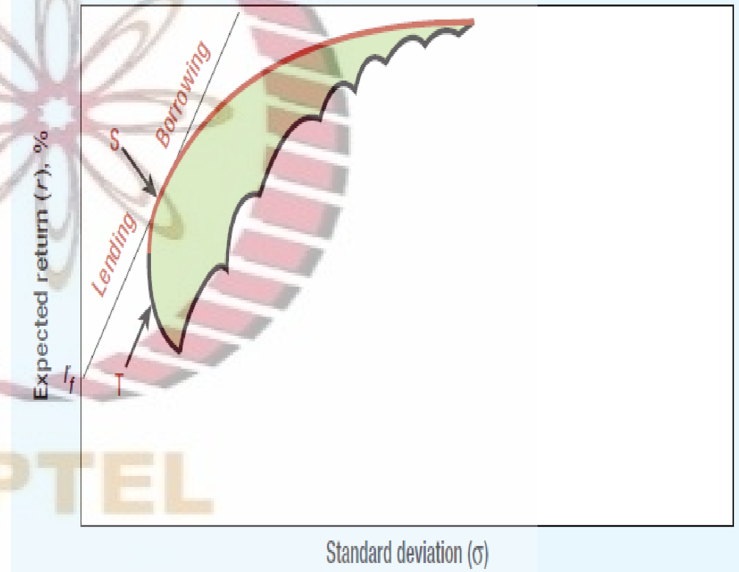
Introduction to Risk-Free Lending and Borrowing

Now, we have an even better position, which is shown by the line going through rf and rs

- It has two segments borrowing and lending for investors with high and low-risk preference
- This strategy of borrowing at r_f and investing at r_S is depicted by the line segment called borrowing



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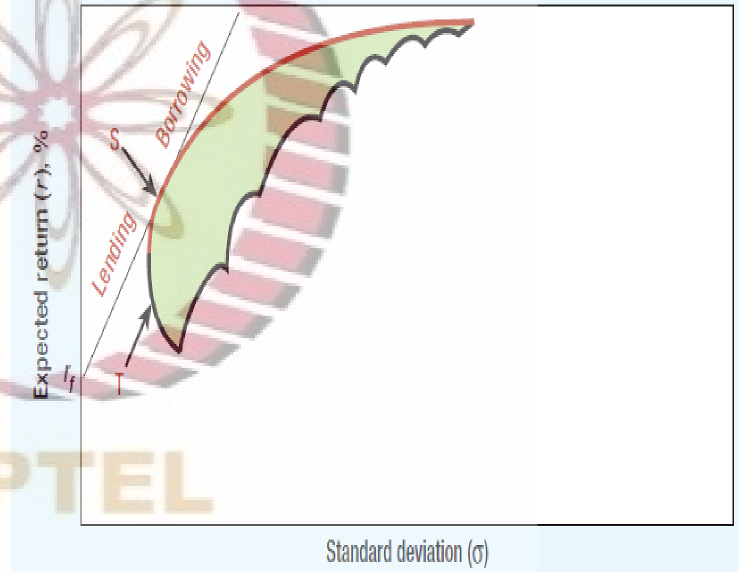
Introduction to Risk-Free Lending and Borrowing

I can invest partially at r_f and partially at r_S, and hold a portfolio on the line segment called lending

- If the portfolio S is known with reasonable certainty, everybody should hold this portfolio, and this will be called market portfolio



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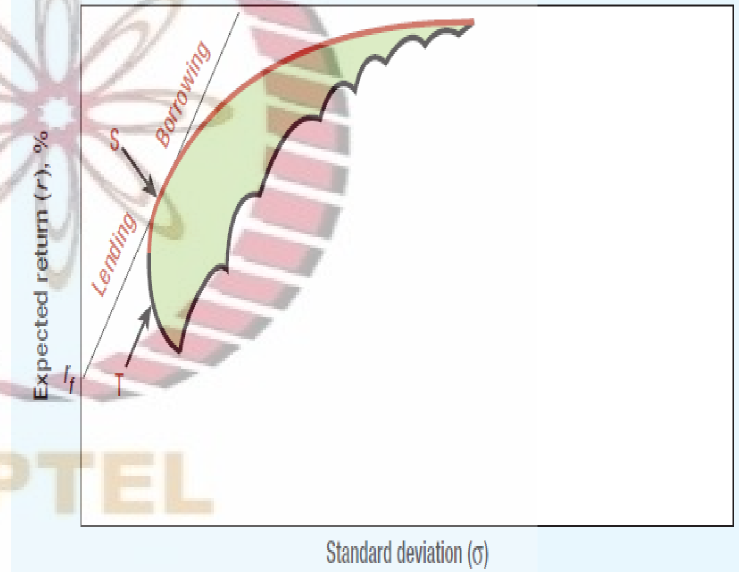
Introduction to Risk-Free Lending and Borrowing

In a competitive market, everybody is expected to hold this market portfolio, and the job of the investment manager is expected to be fairly easy

- One must identify the market portfolio of common stocks
- Then mix this portfolio with risk-free lending or borrowing to create a product that suits the taste and risk preference of investors



Lending and borrowing extend the range of investment possibilities. If you invest in portfolio S and lend or borrow at the risk-free interest rate, r_f, you can achieve any point along the straight line from r_f through S. This gives you the highest expected return for any level of risk. There is no point in investing in a portfolio like T.

Brealey, Myers, and Allen, *Principles of Corporate Finance,* 10th, 11th, or 12th editions, Chapter 8

Introduction to Risk-Free Lending and Borrowing III

Introduction to Risk-Free Lending and Borrowing: Simple Example

Suppose market portfolio S here offers 15% expected returns and SD of 16%. The risk-free instrument offers a 5% uniform rate of lending and borrowing, with an SD=0.

You are a risk-averse investor; therefore, you would like to invest 50% into rf and balance into S. What does your portfolio look like. The corresponding equations for the risk and expected returns on the portfolio are provided below

$$\sigma_p = X\sigma_A$$

$$\bar{R}_p = X\bar{R}_A + (1 - X)R_f$$

Introduction to Risk-Free Lending and Borrowing: Simple Example

You are a risk-averse investor; therefore, you would like to invest 50% into rf and balance into S.

$$\boldsymbol{\sigma_p = X\sigma_A}$$

$$\bar{R}_p = X\bar{R}_A + (1 - X)R_f$$

The expected returns on your portfolio are
$r_f * 0.5 + r_S * 0.5 = 5\% * .5 + 15\% * 0.5 = 10\%$.
The standard deviation of the portfolio will be $\sigma_p = 0.5 * 16\% = 8\%$.

You are standing on the lending segment of the line of investment at a point, that is, midway between rf and rs.

Introduction to Risk-Free Lending and Borrowing: Simple Example

- Another investor who is more risk-taking in his approach will borrow at rf almost 100% and invest 200% in the market portfolio. The risk-return profile of this investor is shown below. His return will be $r_f * (-1.0) + r_S * 2.0 = 5\% * -1.0 + 15\% * 2.0 = 25\%$. At the same time, his risk will be $\sigma_p = 2 * 16\% = 32\%$.
- This investor has extended his possibilities and operates on the borrowing segment of the line.

Introduction to Risk-Free Lending and Borrowing: Simple Example

So, whether it is fearful chickens or risky lions, both will prefer this market portfolio as compared to any of the portfolios on the efficient frontier

- Therefore, this market portfolio is the best efficient portfolio for the entire set of investors
- And we also know how to identify this portfolio by drawing a tangent line from rf to on the surface of efficient portfolios
- This portfolio, as we discussed earlier, offers the highest risk premium to the standard deviation: Sharpe ratio: $\frac{Risk\ Premium}{Standard\ Deviation} = \frac{r_S - r_f}{\sigma_p}$

Market Risk and Beta

Market Risk and Beta

Market risk is the risk associated with a well-diversified portfolio, often called a market portfolio (Nifty 50)

- If a sufficiently large number of securities are added to a portfolio, the only risk that remains is the non-diversifiable/systematic/market risk
- What is this market risk?
- The contribution of a security to the portfolio is determined by the correlation of a security (or the covariance) with the market portfolio

Market Risk and Beta

This correlation or the sensitivity of the security (i) with the market portfolio is represented through beta (β_i)

- For example, if security moves by 1.5% for a 1% movement in the market portfolio, then the beta of a security is said to be 1.5
- If the beta of a security is 1.0, then security is said to be having same risk as that of the market
- If beta is 0, then the security doesn't have any market risk: government securities
- In summary, this beta represents the sensitivity of the security to market movements

Market Risk and Beta

Beta of a portfolio is weightage average betas of the individual securities

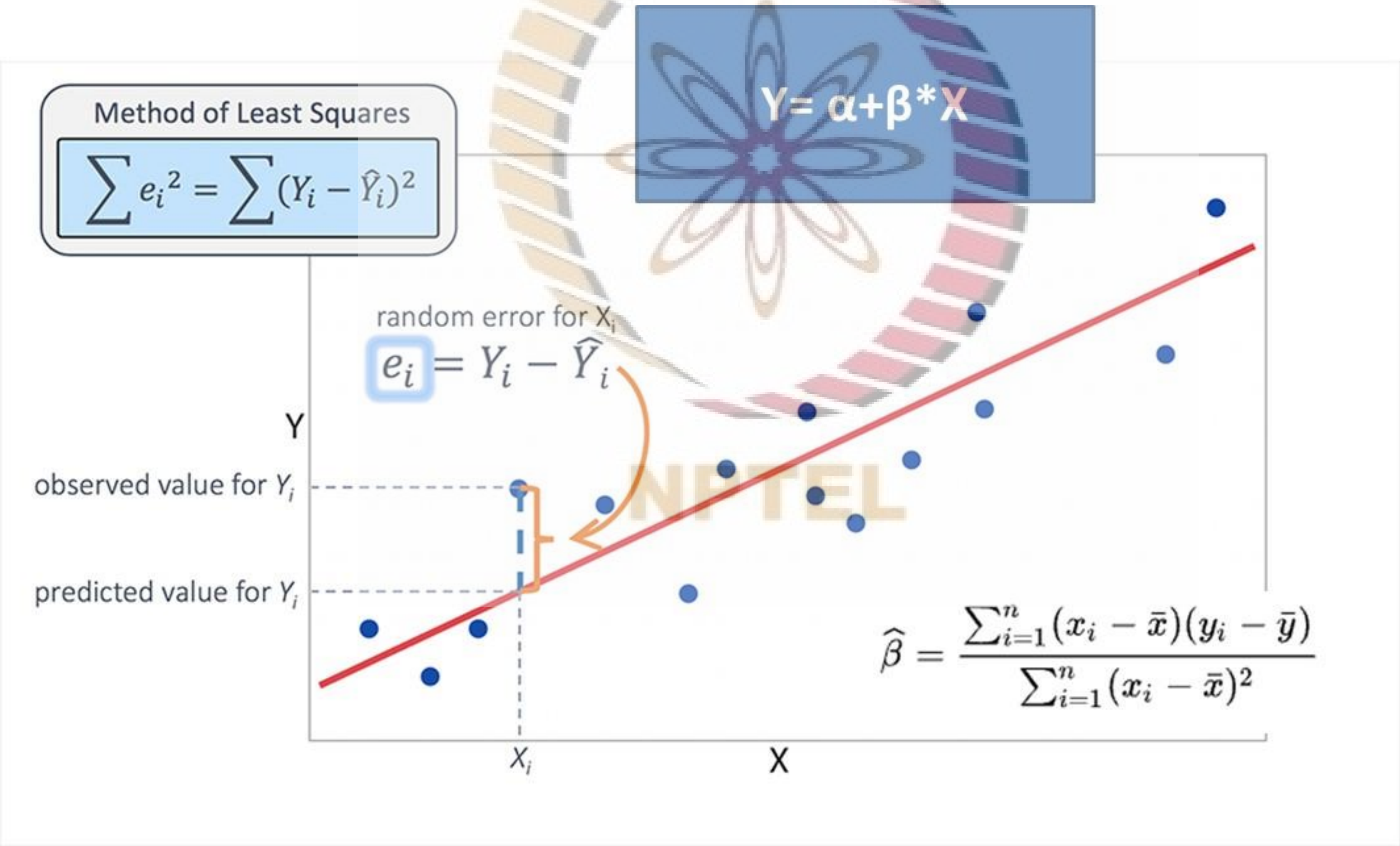
- For example, if we have N securities with individual betas $(\beta_1, \beta_2, \beta_3, \beta_4 \ldots\ldots \beta_N)$ and proportionate amounts invested in these securities are $w_1, w_2 \ldots\ldots w_N$. Then, the beta of the portfolio can be written as below
- $\beta_P = w_1 * \beta_1 + w_2 * \beta_2 \ldots w_N * \beta_N \quad = \sum_{i=1}^{N} w_i * \beta_i$
- If the observed standard deviation of the market is 20%. Now we construct a portfolio from a large number of securities with an average beta of 1.5
- The standard deviation of this portfolio will be 30% (1.5*20%)

Market Risk and Beta

Beta of individual security (β_i) is defined and computed as follows.

- $\beta_i = \sigma_{im}/\sigma_m^2$; here, σ_{im} is the covariance between the security and the market returns (expected). σ_m is the standard deviation of the expected market returns
- How to compute betas in real life
- Returns of the security are regressed on the market returns. Market returns can be proxied using broad indices such as Nifty, NYSE

Market Risk and Beta: Regression Analysis



Example: Beta Computation

	A	B	C	D	E	F
Period	R_m	R_1	$R_m - \bar{R}_m$	$R_1 - \bar{R}_1$	$\sigma_m^2 = (R_m - \bar{R}_m)^2$	$\sigma_{im} = (R_1 - \bar{R}_1) * (R_m - \bar{R}_m)$
1	-1.00	3.60	-1.30	-0.10	1.69	0.14
2	-6.00	3.20	-6.30	-0.50	39.69	3.18
3	10.00	4.48	9.70	0.78	94.09	7.53
4	10.00	4.48	9.70	0.78	94.09	7.53
5	-3.00	3.44	-3.30	-0.26	10.89	0.87
6	-11.00	2.80	-11.30	-0.90	127.69	10.22
7	8.00	4.32	7.70	0.62	59.29	4.74
8	-6.00	3.20	-6.30	-0.50	39.69	3.18
9	10.00	4.48	9.70	0.78	94.09	7.53
10	-8.00	3.04	-8.30	-0.66	68.89	5.51
Avg.	0.30	3.70			63.01	5.04

$$\sigma_{im} = 5.04, \sigma_m^2 = 63.01, and\ \beta_i = \frac{\sigma_{im}}{\sigma_m^2} = 0.08$$

Summary and Concluding Remarks

Summary and Concluding Remarks

- For a portfolio with a large number of securities, only systematic (or market) risk is relevant
- Idiosyncratic stock-specific risk is eliminated due to diversification
- When two securities are perfectly correlated ρ_{12} = 1, no diversification is achieved
- When two securities are perfectly negatively correlated ρ_{12} = -1, maximum diversification is achieved
- As we keep on adding more and more securities, the region of all possible risk-return scenarios is obtained (feasible region)

Summary and Concluding Remarks

- On this feasible region, we would like to go up (increase expected returns) and go to the left (decrease the risk)
- When short-selling is not allowed, a set of best efficient portfolios from minimum variance portfolio to maximum return portfolio are obtained that dominate all other risk-return profiles: efficient frontier (EF)
- When short-selling is allowed, an extended feasible region is obtained, the efficient frontier is also extended on the top-right

Summary and Concluding Remarks

- In the presence of risk-free security, a new efficient frontier is obtained, which is a tangent line joining risk-free security to the tangency point
- On this new efficient frontier, the line segment toward the left of the tangency point is called the lending segment: a mix of investment into risk-free security and tangency portfolio
- The line segment towards the right of the tangency point is called the borrowing segment: borrowing at the risk-free rate and investing the complete amount into the tangency portfolio

Thanks!