



B.Tech(honours) in Data Science Department
University Teaching Department, CSVTU, Bhilai
Subject –EM-1
Class Test 1-February 2022

Time: 1:30 hrs.

Max. Marks: 40

Attempt all questions from each section.

Section A: Multiple Choice Questions:

(1×10)

1. Which of the following statement is/are correct.

- a. Every polynomial function is differentiable function.
- b. The modulus function $f(x)=|x-1|$, is differentiable at $x=1$.
- c. A function $f(x) = x \sin(1/x)$, is differentiable at $x=0$.
- d. All of above.

2. If $x = \cos \theta$, $y = \sin \theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$

- a. r
- b. 1
- c. 0
- d. -1

3. The value of $\int_0^{\pi} \cos x \, dx =$

- a. 0
- b. 1
- c. 2
- d. -1

4. If $y = \sin^{-1}(x)$ then value of $(y_2)_{x=0}$, where y_2 is second derivative of y .

- a. 0
- b. 1
- c. 2
- d. -1

5. n^{th} derivative of $\sin x$

- a. $\sin(n\pi + x)$
- b. $\cos(n\pi + x)$
- c. $\sec(n\pi + x)$
- d. $\sin(n\frac{\pi}{2} + x)$

6. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, $(n-1)(I_n + I_{n-2}) =$

- a. 0
- b. 1
- c. 2
- d. -1

7. $\int_0^{\pi/2} \sin^4 x \, dx =$

- a. $\frac{\pi}{4}$
- b. $\frac{\pi}{8}$
- c. $\frac{\pi}{32}$
- d. $\frac{3\pi}{16}$

8. The value of the series $\sum_{r=0}^{n-1} \frac{1}{n^2 + r^2}$

- a. $\frac{\pi}{2}$
- b. $\frac{\pi}{4}$
- c. $\frac{\pi}{3}$
- d. $\frac{\pi}{6}$

9. If $y = \log(\sin x)$ then value of $(y_2)_{x=\pi/2}$, where y_2 is second derivative of y .

- a. $\cot(2)$
- b. $\operatorname{cosec}(2)$
- c. $-\operatorname{cosec}^2(2)$
- d. $\operatorname{cosec}^2(2)$

10. In which statement is incorrect

- a. Every proper integral is convergent.
- b. Every improper integral is convergent .
- c. Gamma function is convergent for $n > 0$.
- d. $I = \int_0^1 \frac{dx}{x}$ is improper integral of second kind

Section B:

Descriptive Type Questions:

(6×5)

1. Prove that $\beta(n, m) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)}$

2. If $y = \{x + \sqrt{x^2 - 1}\}^m$, show that

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

3. Prove that $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx = \frac{5\pi a^4}{8}$.

4. Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$.

And hence evaluate.

5. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

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