Solving the Dead Mileage Problem for the Bus Network of the Greater Cairo Region

1. Research Question

This research attempts to enhance the operation of the Cairo Transport Authority (CTA); the government agency in charge of operating the public bus network in the Greater Cairo Region (GCR). *It proposes bus route allocation to depots in a way that minimizes the total dead mileage of the bus network.*

The removal of long-standing fuel subsidies as part of the conditions of a $12 billion IMF loan (IMF 2019) means that fuel costs have become increasingly significant to operators. The GCR is also characterized by high pollution levels, with $PM_{2.5}$ concentrations in 2016 being over 6 times the limits set by the World Health Organization (Larsen 2019). Minimizing dead mileage would help cut both fuel costs and emissions.

2. Literature Review

2.1 Bus Operations in Cairo

Dead mileage is the distance travelled by a bus between the depot it is parked in overnight and its starting/end point. A study done in 2016 found that the allocation of CTA buses to depots was very inefficient (ACE Consulting Engineers and COWI 2016). The study was only able to use data for 98 CTA bus routes (out of 400) but found that the average dead mileage for these routes was 13.4km. It calculated that almost 70% of these routes could benefit from better allocation to depots.

2.2 Framing the Dead Mileage Problem

The dead mileage problem is modelled as a 0-1 assignment problem with constraints on depot capacities (Sharma and Prakash 1986). Hsu (1988) added an all or nothing constraint to the problem; all buses on the same route must be assigned to one depot. This was done to resemble real-life bus operations where depots are used for fleet management as well as vehicle storage. Prakash et. al (1999) formulate the problem as a multi-objective problem, with the objectives being 1) minimizing dead mileage, and 2) minimizing the maximum distance covered by any bus from its depot to its starting terminal. Nasibov et al. (2013) solve the dead mileage problem for the bus transport system of Izmir by removing depot capacity constraints to show which depots could be expanded to reduce dead mileage.

3. Data

I attempt to solve the dead mileage problem using data on 179 unique CTA routes. The data is made available by Transport for Cairo, a Cairo-based transport consultancy. It includes:

- Shapefile with CTA routes, including metadata on beginning and end terminals as well as estimated duration and headway[1] for each route
- Shapefile with location of all 20 CTA bus depots

Data is also obtained from tables in a 2016 study on the CTA (ACE Consulting Engineers and COWI 2016), showing:

- Capacity of each CTA depot (20 in total)
- Total number of operational buses as of 2016 (2650)

A straight-line distance matrix between the depots and bus terminals is calculated on QGIS, and the distance matrix between routes and depots is obtained from it based on the starting terminal of each route.

Figure 1: Depot Locations in Cairo

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[1] Headway is the time between consecutive buses on the same route

4. Methodology

Data on actual bus depot allocation is not available, and so this study cannot calculate the current dead mileage on the network. The problem is solved to show ideal allocation and the corresponding dead mileage.

4.1. Preparing the Data

The problem is solved using the all or nothing constraint proposed by Hsu (1988). One alteration is that this constraint is applied on a trip level, not a route level.

$$Route_{AB} = Trip_A + Trip_B$$

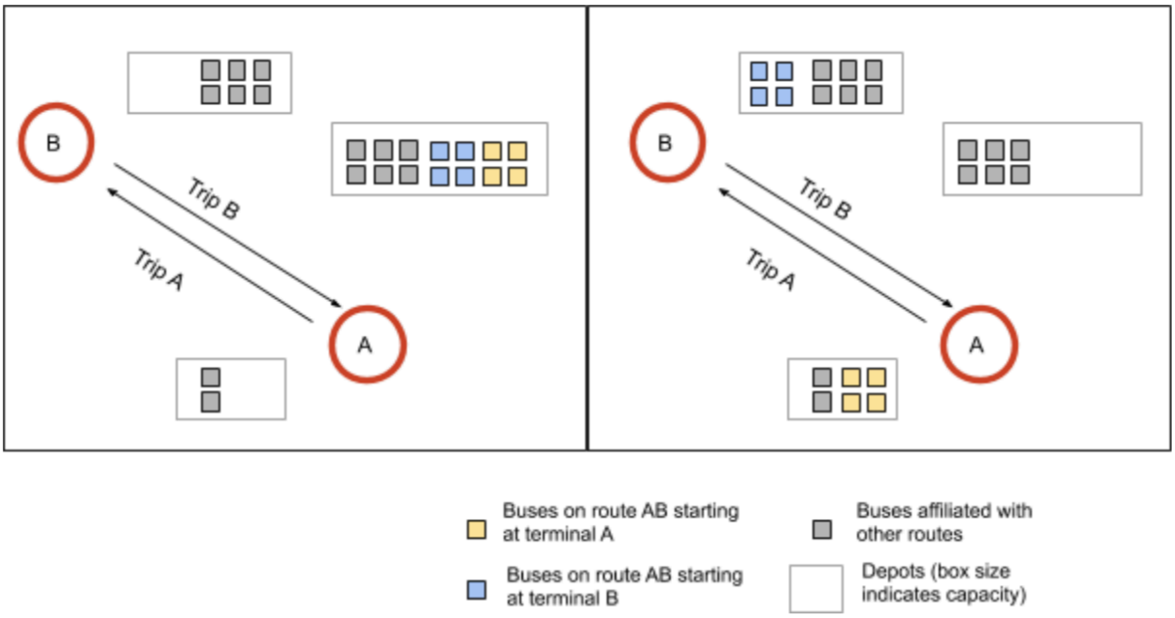

Figure 2: Potential benefit of trip level constraint (right) over route-level constraint (left)

If the constraint was applied on the level of the route, the bus allocation would be a compromise between the distance of the depot to terminal A and the distance to terminal B (Mahadikar et al. 2015). I assume that buses on $Route_{AB}$ beginning their shift at A also end it at A, while buses that begin at B end at B. As buses on $Trip_A$ are half of the total buses on $Route_{AB}$, an additional benefit is that the constraint applies to smaller groups and vacant depot spaces can be better utilized (see Figure 2).

The number of buses on each individual route is not available. I therefore estimate this number from the headway and trip time figures provided using the following equations:

$$No.of\ buses\ on\ Route = \frac{Route\ Duration}{Route\ Headway}$$

The number of buses for each trip is half the number of buses for its parent route. The results are scaled up by multiplying them by the ratio of their sum to their reported sum in the literature. Without this last step, the cumulative number of buses would not be an accurate representation of reality.

4.2. Problem Formulation

Following Hsu (1988), the problem can be laid out as follows:

Let:

$i \;\; : depots\ (i = 1, \dots, m)$
$j \;\; : trips\ (j = 1, \dots, n)$
$e_{ij} : dead\ mileage\ associated\ with\ the\ pull\ in\ and\ pull\ out\ journeys\ between\ trip\ j\ and\ depot\ i$
$c_i \;\; : capacity\ of\ depot\ i$
$b_j \;\; : number\ of\ buses\ on\ trip\ j$
$x_{ij} : binary\ variable\ (1\ if\ buses\ on\ trip\ j\ are\ assigned\ to\ depot\ i; 0\ otherwise)$

Objective Function

$$Minimize \sum_{j=1}^{n}\sum_{i=1}^{m} e_{ij} * b_j * x_{ij} \qquad (1)$$

Constraints

$$x_{ij} = 0\ or\ 1 \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad for\ (j = 1, \dots, n) \qquad (3)$$

$$\sum_{j=1}^{n} b_j * x_{ij} \le c_i \quad for\ (i = 1, \dots, m) \qquad (4)$$

Constraint (2) prevents buses on the same trip from being split up between different depots. Constraint (3) ensures that each trip is only assigned to one depot, and Constraint (4) is added so that the buses allocated to each depot do not exceed its capacity.

The problem is solved twice:

- Problem 1: Applying all constraints (2, 3 and 4)
- Problem 2: Removing the capacity constraint (4)

Solving the problem without depot capacity constraints gives an indication of the dead mileage reduction obtained if the depot capacities could be increased (Nasibov et al 2013).

4.3. The Dead Mileage Problem and Complexity

Pure binary assignment problems (variables must be integers; 0 or 1) are known to be NP-Complete (Floudas 1995). If n variables are each assigned one of 2 integer values, there are 2^n solutions. Finding the exact solution by enumerating all possible solutions is prohibitively time consuming and so approximation algorithms are used (Vazirani 2001). This problem is solved using the Gurobi Solver (Gurobi Optimization, LLC 2019) which utilizes the branch and bound algorithm. This algorithm is able to cover all solutions even though it evaluates only a fraction of them. A search tree is generated for the problem, and at each node on the tree one of the x_{ij} variables branches out as 0 and 1, and constraint (2) is 'relaxed' to $0 \le x_{ij} \le 1$ for the remaining variables. As we go down the node, constraint (2) is added incrementally to variables. For a minimization problem, nodes seize to be explored further if any of the following conditions apply:

- All constraints of original problem are satisfied, including constraint (2)
- Current constraints make solution infeasible
- Optimal solution to relaxed problem has an objective value higher than the best current integer solution

There are different methods of exploring the search tree, but the main idea is that nodes are expanded only if they have potential to produce a better integer solution (Floudas 1995).

5. Results

Solving the problem with all constraints yields a total dead mileage of 33,324 km. Removing the depot capacity constraint (4) reduces the dead mileage by almost 2,500 km.

Table 1: Dead Mileage Results

	Problem 1	Problem 2
Total Dead Mileage (km)	33,324	30,886
Average Dead Mileage Per Bus (km)	12.2	11.3

These values show the dead mileage for the pull in (shift beginning) and pull out (shift end) trips. A dead mileage of 12.2km per bus means that buses are, on average, parked 6.1km from their starting points.

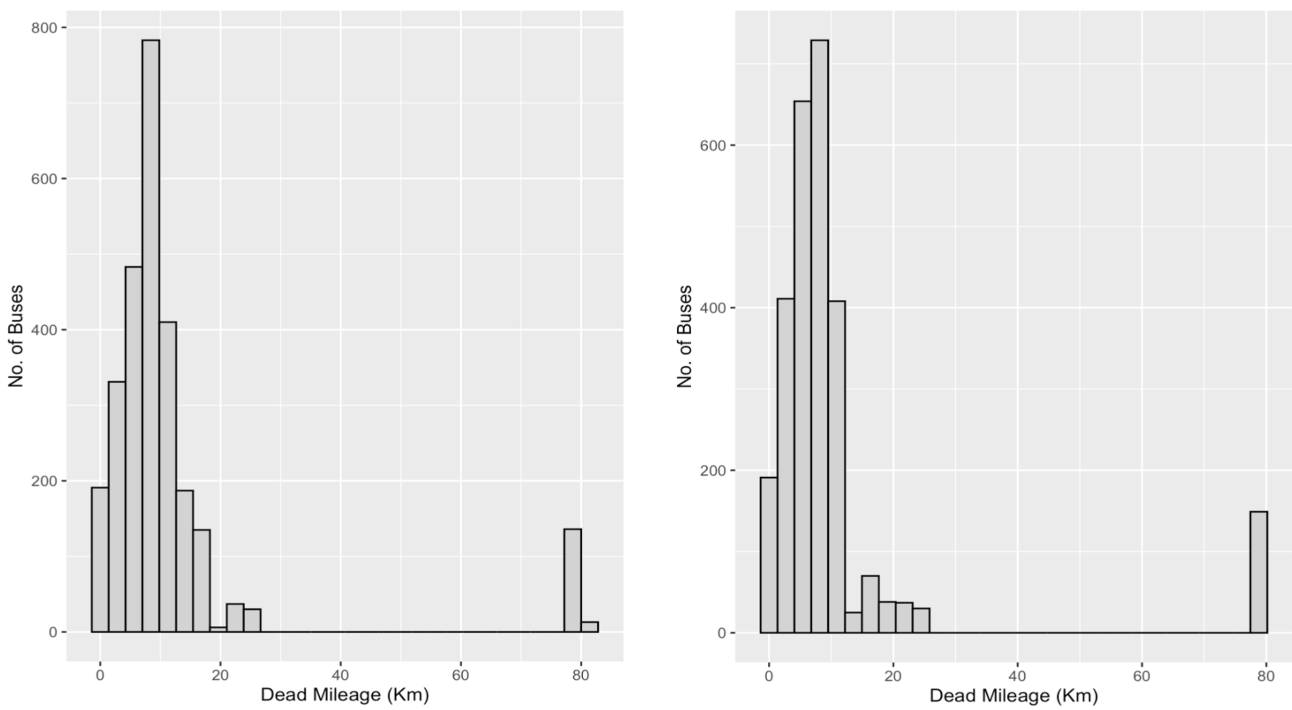

Figure 3: Distribution of Dead Mileage for Problem 1 (left) and Problem 2 (Right)

The results of problem 1 (Figure 3 – left) show that the dead mileage is less than 30km for most buses, but there are a few with over 80km. Even after removing the depot capacity constraints (Figure 3 – right), some buses still have a dead mileage of 80km.

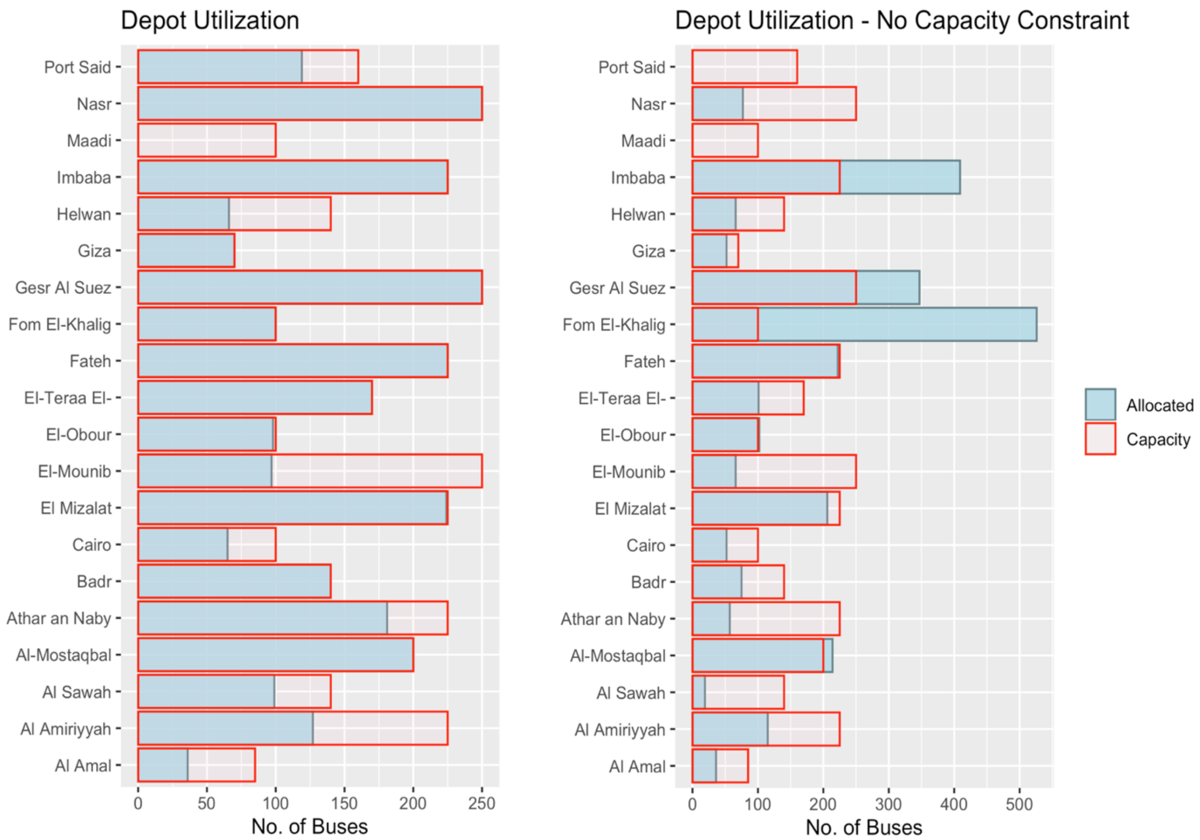

Figure 4: Depot Utilization for Problem 1 (left) and Problem 2 (right)

No buses are allocated to the Maadi depot, indicating that the land could be freed up for a different purpose. Removing the capacity constraints is shown to increase the allocation of buses well beyond capacity at Imbaba, Gesr Al Suez, and Fom El-Khalig.

6. Discussion

The results for problem 1 show that depot capacity exceeds the number of buses in operation (Figure 4 – left). If buses were to be allocated according to the results of problem 1, a few depots would be underutilized (the Maadi depot would be completely empty). Under such conditions, the feasibility of reducing the size of these depots and using the land for a different purpose should be considered.

Removing the depot capacity constraint reduces the dead mileage as expected, but there are still around 150 buses with a dead mileage of 80km (Figure 3 – right). A closer look reveals that these buses belong to trips originating in the city of Badr on the far East (see Figure 1). The closest depot from this starting point is 40km away. Attempting to add another constraint on the maximum distance covered by any bus provides infeasible solutions if the maximum threshold is set below 79km. Setting up the problem as a multi-objective problem, as laid out by Prakash et al. (1999) would not have reduced the maximum dead mileage covered by any bus. This is a strong indication that the depots are not distributed optimally across the city. Figure 1 shows that many of these depots are within close proximity of one another.

The bus allocation results for problem 2 (Figure 4 – right) highlight the significance of three depots in particular: Imbaba, Gesr Al Suez, and Fom El-Khalig. These three depots are also at full capacity in Problem 1. Expanding their capacity would help in reducing dead mileage. Given information on the potential for expansion of each depot in the study area, the model could be rerun to show which expansions would benefit the network.

6.1 Limitations

There is no available public data on the actual parking locations of the different buses. If such data were available, then the reduction in dead mileage could be calculated. The study also calculates dead mileage based on the straight-line distance between depots and bus starting terminals. A more accurate approach would be to use the actual distance using the road network. The number of buses on each trip is also estimated (as explained in section 4.1.). Given the actual number of buses, more accurate results would have been provided.

7. Conclusion

Solving the dead mileage problem provides valuable insight on how to streamline bus operations. Not only does it give an indication of where to allocate buses, but it also shows where empty spaces would exist given this reallocation. Further research could be done to determine a) where to allocate new depots to reduce dead mileage for buses operating on the fringes of the city and b) which depots to repurpose to make use of unutilized space. The availability of bus schedules would allow the problem to be formulated with an additional scheduling constraint to ensure that depot capacity is not exceeded at any time period, as done by Djiba et. al (2012). This could provide better dead mileage results as different buses would utilize the same depot at different times of the day.

Word Count: 1811

8. Bibliography

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