```
> #Libraries
> library(devtools)
> devtools::install github("jlmelville/snedata", force=TRUE)
Downloading GitHub repo jlmelville/snedata@HEAD
- R CMD build ·

✓ checking for file

'C:\Users\husse\AppData\Local\Temp\Rtmp4ECQIM\remotes3ffc6b8e769\jlmelvill
e-snedata-c8cfdbf/DESCRIPTION' ...
- preparing 'snedata': (578ms)

✓ checking DESCRIPTION meta-information ...
- checking for LF line-endings in source and make files and shell scripts
- checking for empty or unneeded directories
- building 'snedata 0.0.0.9000.tar.gz'
Warning: package 'snedata' is in use and will not be installed
> library(snedata)
> library(plotly)
> library(ISLR)
> library(e1071)
> library(ISLR2)
> library(MASS)
> library(glmnet)
> library(randomForest)
> library(caret)
> library(keras)
> library(reticulate)
> library(tensorflow)
> library(class)
> library(datasets)
> library(nnet)
> #Splitting data
> dat<- snedata::download fashion mnist()</pre>
> set.seed(1)
> train<-sample(60000,1200)</pre>
> x.train<-dat[train,1:784]</pre>
> y.train<-dat[train,785]</pre>
> table(y.train)
v.train
          2 3 4 5 6 7 8
      1
123 107 136 122 101 158 115 109 112 117
> types<-dat[train,786]</pre>
> table(types)
types
T-shirt/top
                Trouser
                          Pullover
                                           Dress
                                                        Coat
        123
                 107
                                136
                                            122
                                                         101
     Sandal
                  Shirt
                            Sneaker
                                            Bag Ankle boot
                    115
                                 109
                                             112
                                                         117
        158
> types[326]
[1] Dress
10 Levels: T-shirt/top Trouser Pullover Dress Coat ... Ankle boot
> x<-matrix(as.numeric(x.train[326,]),nrow=28)</pre>
> image(x[,28:1])
> types[900]
```

```
[1] Coat
10 Levels: T-shirt/top Trouser Pullover Dress Coat ... Ankle boot
> x<-matrix(as.numeric(x.train[900,]),nrow=28)</pre>
> image(x[,28:1])
> ind<-c(60001:70000)
> test<-sample(ind,600)
> x.test<-dat[test,1:784]
> y.test<-dat[test,785]</pre>
> table(y.test)
v.test
0 1 2 3 4 5 6 7 8 9
47 65 60 60 72 52 54 65 68 57
> #Part I: PCA
> pca.train<-prcomp(x.train,center=TRUE,scale.=FALSE)</pre>
> var.explained<-cumsum(pca.train$sdev^2)/sum(pca.train$sdev^2)</pre>
> var.explained
  [1] 0.2939141 0.4667171 0.5261355 0.5757745 0.6152747 0.6503085
  [7] 0.6738004 0.6930777 0.7071935 0.7201411 0.7305922 0.7391539
 [13] 0.7467871 0.7538116 0.7603720 0.7663954 0.7719123 0.7771586
 [19] 0.7823608 0.7871666 0.7917093 0.7959012 0.8000147 0.8039821
 [25] 0.8077562 0.8114544 0.8148207 0.8180929 0.8212354 0.8243591
 [31] 0.8274291 0.8303315 0.8331312 0.8359010 0.8386366 0.8411421
 [37] 0.8436193 0.8460434 0.8483398 0.8505417 0.8527035 0.8547643
 [43] 0.8567217 0.8586735 0.8605693 0.8624287 0.8642389 0.8660099
 [49] 0.8677462 0.8694367 0.8710730 0.8726873 0.8742418 0.8757725
 [55] 0.8772810 0.8787540 0.8801883 0.8815841 0.8829687 0.8843215
 [61] 0.8856271 0.8869275 0.8882067 0.8894631 0.8906818 0.8918903
 [67] 0.8930939 0.8942717 0.8954084 0.8965313 0.8976280 0.8987059
 [73] 0.8997777 0.9008332 0.9018649 0.9028817 0.9038931 0.9048687
 [79] 0.9058401 0.9067869 0.9077194 0.9086487 0.9095699 0.9104677
 [85] 0.9113584 0.9122284 0.9130872 0.9139395 0.9147733 0.9156030
 [91] 0.9164162 0.9172089 0.9179996 0.9187850 0.9195508 0.9203074
 [97] 0.9210542 0.9217913 0.9225235 0.9232409 0.9239476 0.9246498
[103] 0.9253439 0.9260331 0.9267108 0.9273826 0.9280496 0.9287058
[109] 0.9293538 0.9299897 0.9306149 0.9312342 0.9318513 0.9324611
[115] 0.9330691 0.9336743 0.9342706 0.9348585 0.9354380 0.9360017
[121] 0.9365606 0.9371183 0.9376682 0.9382121 0.9387475 0.9392770
[127] 0.9398007 0.9403137 0.9408191 0.9413198 0.9418158 0.9423075
[133] 0.9427892 0.9432671 0.9437388 0.9442076 0.9446736 0.9451348
[139] 0.9455890 0.9460419 0.9464908 0.9469376 0.9473802 0.9478174
[145] 0.9482525 0.9486854 0.9491081 0.9495300 0.9499452 0.9503568
[151] 0.9507647 0.9511702 0.9515699 0.9519653 0.9523585 0.9527477
[157] 0.9531295 0.9535070 0.9538840 0.9542582 0.9546289 0.9549987
[163] 0.9553607 0.9557180 0.9560726 0.9564257 0.9567759 0.9571248
[169] 0.9574685 0.9578081 0.9581435 0.9584762 0.9588069 0.9591354
[175] 0.9594602 0.9597849 0.9601080 0.9604259 0.9607398 0.9610528
[181] 0.9613643 0.9616695 0.9619715 0.9622713 0.9625683 0.9628636
[187] 0.9631569 0.9634491 0.9637381 0.9640254 0.9643090 0.9645905
[193] 0.9648703 0.9651461 0.9654213 0.9656936 0.9659635 0.9662310
[199] 0.9664977 0.9667605 0.9670213 0.9672789 0.9675361 0.9677917
[205] 0.9680448 0.9682974 0.9685465 0.9687950 0.9690402 0.9692827
[211] 0.9695235 0.9697631 0.9700005 0.9702351 0.9704691 0.9707003
[217] 0.9709294 0.9711575 0.9713834 0.9716066 0.9718290 0.9720484
[223] 0.9722673 0.9724839 0.9726971 0.9729086 0.9731185 0.9733272
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[229] 0.9735353 0.9737413 0.9739466 0.9741492 0.9743501 0.9745501
[235] 0.9747488 0.9749460 0.9751423 0.9753358 0.9755279 0.9757184
[241] 0.9759079 0.9760968 0.9762843 0.9764698 0.9766538 0.9768365
[247] 0.9770185 0.9771985 0.9773770 0.9775538 0.9777292 0.9779041
[253] 0.9780768 0.9782487 0.9784190 0.9785876 0.9787558 0.9789229
[259] 0.9790890 0.9792534 0.9794150 0.9795748 0.9797337 0.9798909
[265] 0.9800470 0.9802020 0.9803563 0.9805099 0.9806623 0.9808141
[271] 0.9809642 0.9811135 0.9812622 0.9814091 0.9815536 0.9816972
[277] 0.9818401 0.9819822 0.9821234 0.9822634 0.9824029 0.9825415
[283] 0.9826782 0.9828141 0.9829495 0.9830840 0.9832179 0.9833513
[289] 0.9834829 0.9836144 0.9837443 0.9838735 0.9840008 0.9841279
[295] 0.9842546 0.9843795 0.9845040 0.9846282 0.9847515 0.9848737
[301] 0.9849954 0.9851146 0.9852323 0.9853494 0.9854662 0.9855825
[307] 0.9856984 0.9858135 0.9859278 0.9860417 0.9861551 0.9862674
[313] 0.9863776 0.9864872 0.9865962 0.9867044 0.9868120 0.9869192
[319] 0.9870251 0.9871309 0.9872359 0.9873406 0.9874441 0.9875471
[325] 0.9876494 0.9877505 0.9878509 0.9879505 0.9880492 0.9881475
[331] 0.9882451 0.9883422 0.9884386 0.9885340 0.9886287 0.9887229
[337] 0.9888165 0.9889093 0.9890015 0.9890925 0.9891830 0.9892729
[343] 0.9893616 0.9894497 0.9895363 0.9896228 0.9897092 0.9897951
[349] 0.9898796 0.9899636 0.9900470 0.9901300 0.9902124 0.9902947
[355] 0.9903756 0.9904562 0.9905356 0.9906148 0.9906934 0.9907714
[361] 0.9908489 0.9909259 0.9910025 0.9910790 0.9911550 0.9912297
[367] 0.9913042 0.9913785 0.9914525 0.9915260 0.9915985 0.9916704
[373] 0.9917421 0.9918128 0.9918831 0.9919528 0.9920221 0.9920912
[379] 0.9921598 0.9922281 0.9922955 0.9923628 0.9924294 0.9924958
[385] 0.9925614 0.9926267 0.9926913 0.9927553 0.9928189 0.9928821
[391] 0.9929451 0.9930077 0.9930696 0.9931313 0.9931920 0.9932525
[397] 0.9933123 0.9933722 0.9934315 0.9934904 0.9935485 0.9936064
[403] 0.9936634 0.9937201 0.9937766 0.9938322 0.9938873 0.9939423
[409] 0.9939969 0.9940511 0.9941048 0.9941583 0.9942114 0.9942644
[415] 0.9943168 0.9943688 0.9944205 0.9944719 0.9945229 0.9945738
[421] 0.9946241 0.9946737 0.9947230 0.9947717 0.9948197 0.9948676
[427] 0.9949153 0.9949625 0.9950094 0.9950560 0.9951023 0.9951483
[433] 0.9951941 0.9952396 0.9952847 0.9953294 0.9953738 0.9954179
[439] 0.9954615 0.9955046 0.9955474 0.9955900 0.9956321 0.9956742
[445] 0.9957157 0.9957569 0.9957977 0.9958385 0.9958791 0.9959194
[451] 0.9959591 0.9959987 0.9960380 0.9960771 0.9961159 0.9961542
[457] 0.9961920 0.9962298 0.9962669 0.9963038 0.9963405 0.9963768
[463] 0.9964127 0.9964483 0.9964837 0.9965187 0.9965536 0.9965883
[469] 0.9966228 0.9966569 0.9966907 0.9967244 0.9967576 0.9967907
[475] 0.9968235 0.9968560 0.9968885 0.9969208 0.9969527 0.9969843
[481] 0.9970154 0.9970463 0.9970770 0.9971077 0.9971378 0.9971677
[487] 0.9971973 0.9972267 0.9972558 0.9972849 0.9973135 0.9973420
[493] 0.9973704 0.9973985 0.9974262 0.9974537 0.9974811 0.9975082
[499] 0.9975350 0.9975616 0.9975881 0.9976143 0.9976404 0.9976664
[505] 0.9976919 0.9977172 0.9977424 0.9977675 0.9977925 0.9978171
[511] 0.9978415 0.9978657 0.9978898 0.9979135 0.9979370 0.9979603
[517] 0.9979835 0.9980065 0.9980294 0.9980519 0.9980744 0.9980967
[523] 0.9981186 0.9981404 0.9981620 0.9981834 0.9982045 0.9982256
[529] 0.9982466 0.9982673 0.9982879 0.9983083 0.9983284 0.9983485
[535] 0.9983682 0.9983876 0.9984068 0.9984260 0.9984449 0.9984637
[541] 0.9984822 0.9985007 0.9985191 0.9985372 0.9985550 0.9985726
[547] 0.9985903 0.9986078 0.9986251 0.9986424 0.9986595 0.9986763
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[553] 0.9986930 0.9987095 0.9987259 0.9987422 0.9987583 0.9987745
[559] 0.9987903 0.9988058 0.9988213 0.9988366 0.9988517 0.9988668
[565] 0.9988818 0.9988966 0.9989114 0.9989260 0.9989403 0.9989545
[571] 0.9989685 0.9989824 0.9989962 0.9990098 0.9990233 0.9990366
[577] 0.9990499 0.9990630 0.9990760 0.9990889 0.9991017 0.9991143
[583] 0.9991270 0.9991394 0.9991515 0.9991636 0.9991755 0.9991874
[589] 0.9991993 0.9992111 0.9992227 0.9992342 0.9992456 0.9992569
[595] 0.9992681 0.9992791 0.9992900 0.9993007 0.9993114 0.9993219
[601] 0.9993324 0.9993427 0.9993529 0.9993630 0.9993731 0.9993830
[607] 0.9993928 0.9994024 0.9994119 0.9994214 0.9994309 0.9994401
[613] 0.9994492 0.9994582 0.9994671 0.9994759 0.9994847 0.9994934
[619] 0.9995021 0.9995106 0.9995191 0.9995274 0.9995356 0.9995437
[625] 0.9995517 0.9995596 0.9995674 0.9995752 0.9995829 0.9995904
[631] 0.9995979 0.9996053 0.9996127 0.9996198 0.9996269 0.9996338
[637] 0.9996407 0.9996475 0.9996542 0.9996609 0.9996674 0.9996738
[643] 0.9996801 0.9996864 0.9996926 0.9996987 0.9997047 0.9997107
[649] 0.9997166 0.9997224 0.9997282 0.9997339 0.9997395 0.9997451
[655] 0.9997506 0.9997561 0.9997614 0.9997668 0.9997720 0.9997771
[661] 0.9997822 0.9997872 0.9997922 0.9997970 0.9998018 0.9998064
[667] 0.9998110 0.9998156 0.9998201 0.9998245 0.9998288 0.9998331
[673] 0.9998374 0.9998415 0.9998455 0.9998495 0.9998535 0.9998573
[679] 0.9998612 0.9998649 0.9998686 0.9998722 0.9998757 0.9998792
[685] 0.9998827 0.9998860 0.9998894 0.9998926 0.9998959 0.9998990
[691] 0.9999021 0.9999052 0.9999082 0.9999112 0.9999141 0.9999169
[697] 0.9999197 0.9999225 0.9999251 0.9999278 0.9999304 0.9999330
[703] 0.9999355 0.9999379 0.99999403 0.99999426 0.99999449 0.99999471
[709] 0.9999492 0.9999513 0.9999534 0.9999555 0.9999574 0.9999593
[715] 0.9999612 0.9999630 0.9999647 0.9999664 0.9999681 0.9999697
[721] 0.9999713 0.9999728 0.9999742 0.9999757 0.9999770 0.9999783
[727] 0.9999796 0.9999809 0.99999821 0.9999833 0.9999845 0.9999856
[733] 0.9999866 0.9999876 0.9999886 0.9999895 0.9999904 0.9999912
[739] 0.9999920 0.9999927 0.9999934 0.9999941 0.9999948 0.9999953
[745] 0.9999958 0.9999963 0.9999967 0.9999971 0.9999975 0.9999978
[751] 0.9999981 0.9999984 0.99999987 0.9999989 0.99999991 0.99999992
[757] 0.9999993 0.99999995 0.99999996 0.99999997 0.99999997 0.99999998
[763] 0.9999999 0.9999999 0.9999999 0.9999999 1.0000000 1.0000000
[769] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
[775] 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
[781] 1.0000000 1.0000000 1.0000000 1.0000000
> pca.dim = 74
> x.pca.train<-pca.train$x[,1:pca.dim]</pre>
> dim(x.pca.train)
[1] 1200
          74
> pairs(x.pca.train[,1:5],col=y.train)
> x.pca.test<-predict(pca.train,newdata=x.test)</pre>
> x.pca.test<-x.pca.test[,1:pca.dim]</pre>
> dim(x.pca.test)
[1] 600 74
> #Embedded into 74 dimensions
> #KNN
> set.seed(12345)
> K folds <- trainControl(method = "repeatedcv",
                              number=10, repeats =10)
> #KNN Using PCA
```

```
> dat.pca.train=data.frame(x=x.pca.train, y=as.factor(y.train))
> dat.pca.test=data.frame(x=x.pca.test, y=as.factor(y.test))
> tuned K <- caret::train(y~., data = dat.pca.train,
                         method = "knn", trControl = K folds)
> tuned K
k-Nearest Neighbors
1200 samples
  74 predictor
  10 classes: '0', '1', '2', '3', '4', '5', '6', '7', '8', '9'
No pre-processing
Resampling: Cross-Validated (10 fold, repeated 10 times)
Summary of sample sizes: 1079, 1077, 1080, 1083, 1081, 1081, ...
Resampling results across tuning parameters:
  k Accuracy
               Kappa
  5 0.7885100 0.7646598
  7 0.7921245 0.7686598
  9 0.7922035 0.7686752
Accuracy was used to select the optimal model using the
largest value.
The final value used for the model was k = 9.
> cl <- dat.pca.train$y</pre>
> system.time(
  k<-knn(dat.pca.train, dat.pca.test, cl, k = 7, prob=TRUE))
  user system elapsed
         0.00
   0.26
                 0.36
> #Error rate:
> 1-sum(k[1:600]==c1)/600
[1] 0.7866667
> #KNN using full dimensions
> dat.train=data.frame(x=x.train, y=as.factor(y.train))
> dat.test=data.frame(x=x.test, y=as.factor(y.test))
> tuned K og <- caret::train(y~., data = dat.train,
                         method = "knn")
> tuned K og
k-Nearest Neighbors
1200 samples
784 predictor
  10 classes: '0', '1', '2', '3', '4', '5', '6', '7', '8', '9'
No pre-processing
Resampling: Bootstrapped (25 reps)
Summary of sample sizes: 1200, 1200, 1200, 1200, 1200, 1200, ...
Resampling results across tuning parameters:
  k Accuracy Kappa
  5 0.7388591 0.7094660
  7 0.7385178 0.7090982
  9 0.7383398 0.7088904
```

```
Accuracy was used to select the optimal model using the
largest value.
The final value used for the model was k = 5.
> cl og <- dat.train$y</pre>
> system.time(
   k og<-knn(dat.train, dat.test, cl = cl og, k = 7, prob=TRUE))</pre>
   user system elapsed
          0.00
                 2.42
> #Error rate:
> 1-sum(k og[1:600]==c1)/600
[1] 0.7883333
> #Part II: Comparing Across Classifiers
> Linear SVM using PCA
> system.time(tuned L svm <- tune(svm, y~., dat=dat.pca.train,
                      kernel = "linear",
                      ranges=list(cost=seq(0.05, 2, length = 20))))
         system elapsed
   user
  57.07
          0.40
                 57.99
> summary(tuned L svm)
Parameter tuning of 'svm':
   sampling method: 10-fold cross validation
- best parameters:
cost
0.05
  best performance: 0.2158333
  Detailed performance results:
                 error dispersion
        cost
1 0.0500000 0.2158333 0.03273773
2 0.1526316 0.2316667 0.02827336
3 0.2552632 0.2416667 0.02832789
4 0.3578947 0.2425000 0.02845019
5 0.4605263 0.2450000 0.03406602
6 0.5631579 0.2441667 0.03216710
7 0.6657895 0.2475000 0.04121047
  0.7684211 0.2475000 0.03909667
9 0.8710526 0.2483333 0.03963569
10 0.9736842 0.2475000 0.03889881
11 1.0763158 0.2475000 0.04195272
12 1.1789474 0.2475000 0.03929353
13 1.2815789 0.2483333 0.04040688
14 1.3842105 0.2483333 0.03924441
15 1.4868421 0.2508333 0.03837960
16 1.5894737 0.2491667 0.03937200
17 1.6921053 0.2466667 0.04179610
18 1.7947368 0.2450000 0.04216370
19 1.8973684 0.2433333 0.04097575
20 2.0000000 0.2450000 0.04029214
```

```
> best L svm <- tuned L svm$best.model
> summary(best L svm)
Call:
best.tune(METHOD = svm, train.x = y \sim ., data = dat.pca.train,
    ranges = list(cost = seq(0.05, 2, length = 20)),
    kernel = "linear")
Parameters:
   SVM-Type: C-classification
 SVM-Kernel: linear
       cost: 0.05
Number of Support Vectors: 774
 ( 100 83 79 78 39 54 57 61 122 101 )
Number of Classes: 10
Levels:
0 1 2 3 4 5 6 7 8 9
> L svm pred <- predict(best L svm, dat.pca.test,
                        type="response")
> L_svm_error =
+ 1-(sum(diag(table
                (predict = L svm pred, truth=dat.pca.test$y))))/600
> #Error rate:
> L svm error
[1] 0.23
> #Linear SVM using full dimensions
> system.time(tuned_full_L_svm <- tune(svm, y~., dat=dat.train,
                                  kernel = "linear",
                                  ranges=list(cost=seq(0.05, 2, length =
20))))
   user system elapsed
 600.73
           3.70 606.78
There were 50 or more warnings (use warnings() to see the first 50)
> summary(tuned full L svm)
Parameter tuning of 'svm':
- sampling method: 10-fold cross validation
- best parameters:
cost
0.05
- best performance: 0.195
```

```
- Detailed performance results:
        cost error dispersion
1 0.0500000 0.195 0.0304797
2 0.1526316 0.195 0.0304797
3 0.2552632 0.195 0.0304797
4 0.3578947 0.195 0.0304797
5 0.4605263 0.195 0.0304797
6 0.5631579 0.195 0.0304797
7 0.6657895 0.195 0.0304797
8 0.7684211 0.195 0.0304797
9 0.8710526 0.195 0.0304797
10 0.9736842 0.195 0.0304797
11 1.0763158 0.195 0.0304797
12 1.1789474 0.195 0.0304797
13 1.2815789 0.195 0.0304797
14 1.3842105 0.195 0.0304797
15 1.4868421 0.195 0.0304797
16 1.5894737 0.195 0.0304797
17 1.6921053 0.195 0.0304797
18 1.7947368 0.195 0.0304797
19 1.8973684 0.195 0.0304797
20 2.0000000 0.195 0.0304797
> best full L svm <- tuned full L svm$best.model
> L full svm pred <- predict(best full L svm, dat.test,
                        type="response")
> summary(best full L svm)
Call:
best.tune(METHOD = svm, train.x = y ~ ., data = dat.train,
    ranges = list(cost = seq(0.05, 2, length = 20)),
    kernel = "linear")
Parameters:
  SVM-Type: C-classification
 SVM-Kernel: linear
      cost: 0.05
Number of Support Vectors: 672
 ( 93 76 70 71 26 51 52 31 103 99 )
Number of Classes: 10
Levels:
0 1 2 3 4 5 6 7 8 9
> L full svm error =
```

```
+ 1-(sum(diag(table
                 (predict = L full svm pred, truth=dat.test$y))))/600
> #Error rate:
> L full svm error
[1] 0.21
> #Radial SVM using PCA
> system.time(radial svm <- tune(svm, y~., dat=dat.pca.train,
                                   kernel = "radial",
                                   ranges=list(cost=seq(0.05, 2, length =
10),
                                   sigma=C(0.01, 5, length = 5)
                                   )))
Error in `contrasts<-`(`*tmp*`, how.many, value = contr) :</pre>
 contrasts can be applied only to factors with 2 or more levels
Timing stopped at: 0 0 0
> #Radial SVM using full dimensions
> system.time(radial fsvm <- tune(svm, y~., dat=dat.train,
                                  kernel = "radial",
                                  ranges=list(cost=seq(0.05, 2, length = \frac{1}{2}
10),
                                              sigma=C(0.01, 5, length = 5)
                                  )))
Error in `contrasts<-`(`*tmp*`, how.many, value = contr) :</pre>
  contrasts can be applied only to factors with 2 or more levels
Timing stopped at: 0 0 0
> #Random Forest using PCA
> system.time(rf <- randomForest(y~., data= dat.pca.train,
                     mtry=8, importance=TRUE))
   user system elapsed
   4.33
           0.00
                   4.40
> rf pred <- predict(rf, newdata= dat.pca.test)</pre>
> #Error rate:
> mean((rf pred!=dat.pca.test$y))
[1] 0.205
> #Random Forest using full dimensions
> system.time(rf full <- randomForest(y~., data= dat.train,
                     mtry=28, importance=TRUE))
  user system elapsed
  23.26 0.02
                 23.44
> rf pred full <- predict(rf full, newdata= dat.test)</pre>
> #Error rate:
> mean((rf pred full!=dat.test$y))
[1] 0.1966667
> #LDA using PCA
> system.time(lda.fit<-lda(y~., data=dat.pca.train))
   user system elapsed
   0.07
          0.00 0.05
> lda.fit
Call:
lda(y ~ ., data = dat.pca.train)
```

```
3
                                                             5
                  1
0.10250000 0.08916667 0.113333333 0.10166667 0.08416667 0.13166667
0.09583333 0.09083333 0.09333333 0.09750000
Group means:
       x.PC1 x.PC2 x.PC3 x.PC4
                                                  x.PC5
                                                            x.PC6
           x.PC8 x.PC9 x.PC10 x.PC11
x.PC7
                                                   x.PC12
0 - 764.05710 \qquad 693.36939 - 155.55580 - 148.46180 \qquad -7.337393 \qquad 707.22224 -
105.75837 11.7634545 -16.2946515 -34.74666 77.72095 -2.327815 -
64.72897
1 95.66339 1398.06749 -182.66459 327.68519 288.971748 -364.26354
282.88772 159.3187987 340.9250460 54.77037 -7.98132 21.388956 -
2 -904.56984 -259.88791 490.04439 -203.44607 235.953419 -98.61471
26.40454 -46.7625018 -29.2220470 -99.17127 44.13156 29.387096 -
12.02044
3 -141.18251 1110.99435 -288.24086 139.22419 -189.055011 -127.56917
20.08120 -22.6706425 -281.6182581 -99.88930 -49.95557 2.249283
86.22974
4 -1073.55749 -182.30826 435.43897 -22.48462 -13.713699 -325.21008 -
223.04340 26.6256347 15.9469066 124.76117 -62.22953 15.865455
69.41020
5 1453.49562 -219.73259 104.35957 -200.03629 76.698670
                                                         60.90032 -
47.14059 32.1681772 15.5127439 -17.92852 8.78871 9.292916
95.23817
6 \quad -633.65537 \quad -80.02712 \quad 337.71923 \quad -215.92841 \quad 65.490308 \quad 53.73187 \quad -
22.19901 -0.2329146 -50.6236613 43.79859 -89.31235 -45.590898
32.46572
7 1382.29575 -466.61103 171.43298 559.03541 -209.058056 122.77092
23.68542 -359.3659630 38.2899642 37.34411 69.53084 -29.786297
20.03907
8 - 148.59048 - 826.48541 36.98883 - 44.84459 - 614.841129 - 10.62753
286.58053 277.6409359 -0.8156656 68.00780 22.65358 24.460768 -
119.43522
  355.62452 -1105.24138 -982.36737 -28.39332 293.524886 -121.23112 -
217.53740 -77.2510130 13.1171495 -20.55721 -30.44072 -30.717887 -
     x.PC14 x.PC15 x.PC16 x.PC17 x.PC18 x.PC19
x.PC20 x.PC21 x.PC22 x.PC23 x.PC24 x.PC25 x.PC26
x.PC27
0\ -24.541495\ -7.030338\ -24.97631\ 4.055421\ -79.87306\ -10.862662\ -
16.295756 1.128505 21.709032 -8.093458 -2.072301 -14.928474
25.834634 -15.683478
  6.853842 49.446089 -27.86143 -42.173484 -21.58218 -1.865776 -
21.967576 8.167367 25.382195 6.771001 -10.236557 47.797976 -
4.468072 -0.380031
2 \quad 18.134310 \quad 27.195676 \quad 18.63724 \quad -10.598201 \quad 55.72810 \quad 65.444250
45.446948 31.739627 7.118714 14.176084 -24.022566 9.911452
```

Prior probabilities of groups:

1.637830 21.549056

```
3 1.944539 -23.802504 44.91180 60.376923 61.86947 28.381057
28.817075 -21.801958 -49.427580 23.503856 -23.093350 -33.969003 -
38.710249 19.273580
4 -2.845603 -38.777858 23.73091 -37.842113 -95.27665 -38.510429 -
4.963525 -14.783413 -30.055466 -56.425072 20.777737 -18.333841 13.590875
17.360106
5 -28.488257 -26.835217 -94.69423 -22.945256 61.98943 -25.608461
8.299244 -21.966630 -15.793708 22.789544 29.101908 -7.190621 5.978456
1.159357
6 33.277830 -7.499987 35.82298 16.322987 13.86073 -69.452395
4.742326 -15.387490 38.546798 8.074732 20.960993 6.913245 -16.418731
-62.988210
7 31.597896 85.997623 50.37479 14.102621 -20.39702 -5.083443
13.273376 6.396868 35.725912 -24.358498 -8.912932 13.256769
21.854226 12.769635
8 - 36.257772 - 33.075149 - 32.39927 \quad 39.673921 \quad 10.47836 \quad 67.134337 - 10.47836 \quad 67.13437 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4
91.244295 -3.762691 -15.973205 1.516351 -9.782938 -20.665305
3.022961 18.232222
      9.914896 -15.991490 39.50807 -19.840067 -31.70164 -15.977746
17.740782 32.377022 -11.376797 -7.432347 3.373452 22.054862 -
11.533962 -12.300182
          x.PC28 x.PC29 x.PC30 x.PC31 x.PC32
                                                                                                              x.PC33
                  x.PC35 x.PC36 x.PC37 x.PC38 x.PC39
x.PC34
x.PC40
0 \quad 30.426380 \quad 1.371630 \quad -7.2120726 \quad -20.2841109 \quad -7.0916226 \quad 13.575761
6.786120 - 0.6662605 - 17.930032  11.7555530  0.5663023  1.027203 -
3.3904127
1 21.025941 9.344218 4.4348680 26.6456527 -6.2335716 11.916089
1.276021 31.3592138 -2.712494 -1.8103055 12.5980331 -11.219077
5.8962463
2 - 19.568090 - 18.013744 \ 17.4673176 - 9.1741413 \ 18.3475947 - 10.434934 -
1.259149 - 8.8444783 - 24.733036 - 7.3661594 - 19.5822038 17.004890 -
2.1491633
3 - 23.104582 - 5.052624 - 0.7376129 - 0.1568662 0.5638363 - 29.446862 -
13.480023 - 19.0734018 \quad 15.257512 \quad -4.7280894 \quad -30.5350359 \quad 4.734392 \quad -
7.0493779
4 -9.309736
                       1.601478 -18.9897359 -15.1980617 3.0522619 -9.858398
15.987383 10.3954406 -12.012298 -10.1550535 -12.6831419 -19.138311
0.5085038
                       2.919921 -19.2368900 28.5925585 -26.7310161 10.386937
5 14.092503
18.253155 1.4507386 5.211583 -17.7981942 -5.2533864 8.834056 -
2.1333443
6 - 2.170913 - 13.117568 - 18.9025235 - 1.6558734 1.3028824 21.996550 -
15.407025 -18.9478387 21.764075 14.8021033 47.9012219 -2.331930
1.1468529
7 21.325445 -12.425882 15.5045594 -17.7217700 -21.2685221 -5.253033 -
16.462709 - 8.6451248 - 19.072292 9.9354788 - 4.7936343 - 3.364994
19.9917029
      1.744293 35.399932 18.8124481 10.0430153 18.3201260
                                                                                                          0.243495
11.049195 0.7156720 31.717814 11.3206278 17.5680864 2.827444
4.6883934
9 -34.775322 1.476922 12.4887060 -9.1851081 25.7009081 -4.811144 -
11.328299 17.2506564 3.515687 0.9491449 1.0947153 -8.211064 -
13.7772779
```

```
x.PC41 x.PC42 x.PC43 x.PC44 x.PC45
                                                                                                                  x.PC46
              x.PC48 x.PC49 x.PC50 x.PC51
x.PC47
                                                                                                                   x.PC52
x.PC53
0 - 2.7089264 \quad 12.484422 \quad -4.736564 \quad 6.523969 \quad -1.7100349 \quad 3.732052
6.8421103 - 2.9905119 3.031015 - 15.747692864 0.9959531 0.4132903 -
7.0091869
1 19.0127552 11.732709 3.535813 2.585805 6.7746773 -7.810791 -
4.1568575 -23.4155459 -5.629362 -7.845227214 15.1615604 -16.9268592
4.4173575
2 10.4969225 10.906544 16.997721
                                                                      1.899589 5.0473956 -1.705347
1.5467194 - 8.8472443 - 16.699360 - 16.914821108 - 2.7712593 6.6160867
3 -5.6598022 -17.319495  5.013454 -18.014022 -8.0774039 -5.728943 -
13.6150831 20.5804481 14.637666 19.208634628 -18.2797027 21.4893045
3.5186994
4 - 17.4094789 - 3.463938 - 6.196157 10.015211 - 2.2228522 1.317471 -
0.3150766 20.3760436 -4.249328 1.399131655 6.2177219 13.0478638 -
4.1988622
       2.2245937 -13.564381 27.642408 13.671814 -4.5870910 -6.981977
16.8389091
6 - 8.4665511 - 6.933485 - 40.804437 - 10.575284 - 4.7089046 14.373720 -
11.5682168 -7.0540216 11.545559 14.556208861 8.7444890 -21.4967304
7 -8.8195020 22.713491 -32.336188 -26.954978 -0.8569414 1.404662
1.3101801 0.9129498 11.176423 -9.543881506 -8.4692482 1.3961451
1.6293294
    8.3646045 -6.960586 13.097341 2.613008 4.5005420 6.170967
3.1702289 6.8193134 -14.525323 -1.870620792 -9.9550466 -2.0325047 -
2.4413927
9 \quad -0.2840812 \quad -4.847003 \quad 2.474342 \quad 13.249126 \quad 7.3895701 \quad -1.876719 \quad 
14.8142110 -5.1424872 -1.110130 0.005374994 11.8304281 1.0961149
21.7784140
            x.PC54 x.PC55 x.PC56 x.PC57 x.PC58 x.PC59
x.PC60
                     x.PC61 x.PC62 x.PC63 x.PC64 x.PC65
x.PC66
0 -1.0180187 -3.5882662 5.5239500 -5.172675 -10.0669269 -0.9038855 -
2.56444829 0.58598224 2.1205643 9.566402 -3.937092 -0.7819553
1.581355
      3.7980125 - 6.0192179 - 0.3919504 - 1.207581 5.6551614 7.8150661 -
11.11654503 7.46718380 8.4968384 -2.010329 1.578382 3.1643922 -
3.609001
       7.5289703 - 7.7803211 - 7.0485245 9.419235 0.8612474 - 16.2616888
4.61127752 0.03907355 4.5232888 2.720235 12.235023 3.6803323
8.177804
3 -1.6443968 4.2636283 -10.6418748 -6.674595 -0.3085238
                                                                                                                 2.3285450
6.61797773 -7.01868596 -0.5247082 3.937541 4.914695 5.6979732
10.775997
4 - 8.4138862 - 0.2944623 - 4.1470330 - 1.252446 8.7798335 0.3831331
4.02221096 - 4.46378452 6.2486074 - 3.050565 - 7.907850 - 3.6700269 -
4.503034
5 - 13.8742640 3.3888876 -2.8661476 -2.236215 -4.9295679 10.9801321 -
4.01064045 0.97648370 -13.3624536 -2.606208 -2.385560 0.8814345
11.028453
```

```
9.2761969 0.3597750 13.0141949 -7.044355 1.6961744 0.2385126
1.50862087 8.76929745 -7.7424993 -13.598481 -10.618747 -11.9293890 -
7.896427
        9.2610532 -0.2667769 9.2625108 4.164463
                                                                                                   9.6147010 -9.5311297
                                                                            4.099797 6.901972 -6.6513982 -
2.20117879 -11.86499100 4.2422538
8.808596
                                                                                                  6.7663347 1.9286563 -
8 -1.0766650 15.1701033 14.6957713 7.161339
0.94154579 1.96078322 -1.0426614 6.509262 -5.798756
                                                                                                                            6.0502101
1.533344
9 -0.1554645 -5.0741580 -14.1971788 2.843231 -13.2918240
                                                                                                                            1.9176958
0.08682429 2.92022313 2.5963603 -6.017914 2.954978 1.8168999 -
12.285306
                                   x.PC68
                                                                                     x.PC70
            x.PC67
                                                              x.PC69
                                                                                                            x.PC71
                                                                                                                                   x.PC72
x.PC73 x.PC74
     1.429914 -3.31886266 3.3212827 -3.0210364 -2.532040 0.1932055
2.0005665 5.7539210
1 \quad -2.979899 \quad 7.04768639 \quad -2.5384711 \quad -3.3613944 \quad 4.748624 \quad 0.5213181 \quad -4.748624 \quad 0.5213181 
0.3384956 0.3443229
2 -1.126958 5.49197131 1.9747392 -4.3398997 -11.574341 2.0982575
12.6053342 1.2976511
       6.017167 - 4.46406594 - 0.8790432 - 1.4794235 1.551159 - 3.9144256 -
5.3571188 -4.2244288
4 - 7.013861 - 5.04057031 3.2471341 14.1368063 -8.597618 -1.5113857 -
3.8667446 -5.4831166
        1.139487 0.04989391 -13.9266015 5.0923210 -5.083219 0.8025129
3.9247816 -5.5048930
       4.456981 0.29478398
                                                       0.8352892 -0.5638588 12.724475 1.2544699 -
6.5123723 5.6645875
7 -10.538076 6.21727665
                                                        2.7954719 2.2626387
                                                                                                    9.221793 -5.8887337
7.7557313 6.4780741
      9.315196 -7.33199780
                                                        6.4233113 -8.9464222 -3.489707 3.2344084 -
6.3659546 -8.1738655
                                                        3.8807207 0.7673845 6.684379 2.3406726 -
9 -2.706854 0.53538815
7.5525374 4.9216078
Coefficients of linear discriminants:
                                                                                             LD3
                                                                                                                           LD4
                                   LD1
                                                              LD2
LD5
                             LD6
                                                          LD7
                                                                                       LD8
                                                                                                                    LD9
              1.697174e-03 -6.606717e-04 -5.450389e-04 4.807970e-04 2.571845e-
04 7.500205e-05 3.550419e-04 -3.477358e-05 1.673472e-05
x.PC2 -2.569703e-03 -1.388107e-03 3.489235e-04 8.500758e-04
                                                                                                                                    7.818390e-
05 \quad 1.033108e-04 \quad 1.078327e-04 \quad -1.414518e-04 \quad -6.893833e-05
x.PC3 -2.608617e-03 2.058952e-03 -1.404406e-03 -1.358753e-04
                                                                                                                                    1.833380e-
03 -4.806026e-04 1.427326e-04 6.696579e-05 1.263039e-05
x.PC4 6.075367e-04 -1.748018e-03 -6.753659e-04 5.664506e-04 4.189763e-
04 5.868057e-05 -1.373488e-03 -2.200214e-04 -3.133236e-04
x.PC5
               5.267314e-05 -1.567990e-03 1.921748e-03 -1.147080e-03 1.175945e-
03 -5.981357e-04 9.012315e-04 6.273547e-04 2.656928e-04
              4.419331e-04 1.719297e-03 7.604831e-04 2.427055e-03 -9.840904e-
x.PC6
```

04 -2.071568e-03 4.021078e-04 2.693453e-04 1.040979e-04

04 1.044197e-04 -2.956733e-04 8.296367e-04 1.044374e-03

03 3.352901e-04 1.390969e-03 -3.733094e-04 -5.103532e-05

x.PC7 -1.988356e-03 -1.390980e-03 -2.415953e-03 9.672935e-05 -7.607304e-

x.PC8 -3.011804e-03 -1.544072e-04 -1.131300e-03 -1.265562e-03 -2.195767e-

```
x.PC9 -1.164487e-04 -3.519337e-03 -1.485212e-03 -2.014760e-03 2.841965e-
04 -2.583812e-03 3.601772e-04 -1.102851e-04 -4.358497e-04
x.PC10 6.544472e-05 -1.160785e-04 -1.146092e-03 -9.751891e-04 -1.073182e-
05 -7.389784e-04 -4.272566e-04 -1.774678e-03 -3.078768e-04
x.PC11 5.459812e-04 9.809708e-05 -5.958687e-04 9.175812e-04 -1.830047e-
04 -1.214680e-03 -1.703154e-04 1.446795e-03 -9.130206e-04
x.PC12 -1.154205e-03 -6.281302e-05 -5.192428e-04 -2.628214e-04 -1.461891e-
04 2.124312e-04 3.092345e-04 5.762254e-04 -8.512572e-04
x.PC13 9.337349e-05 9.041131e-04 5.128426e-04 1.366882e-03 2.965369e-
03 1.927663e-03 1.329656e-03 -1.316748e-03 -4.178152e-04
x.PC14 1.165950e-04 -2.550889e-04 4.459209e-04 -2.583051e-04 9.037543e-
   9.788019e-08 -7.878591e-04 7.164901e-05 8.758063e-04
x.PC15 2.832549e-04 -1.495767e-03 -5.052944e-04 3.828731e-04
                                                              1.215281e-
03 -1.018543e-03 -1.194584e-03 9.736743e-04 5.108463e-04
x.PC16 1.758497e-04 4.554157e-04 1.523102e-03 -3.251332e-04 6.792188e-
04 7.814347e-04 -2.865868e-03 -1.394176e-04 9.400567e-04
x.PC17 -3.692070e-04 1.131218e-03 -3.433043e-04 1.509264e-03 -8.862917e-
04 1.088545e-03 -8.917411e-04 9.373733e-05 1.048058e-03
x.PC18 2.458509e-04 2.835193e-04 -1.282488e-03 8.463281e-04 8.834528e-
   2.634347e-03 1.772883e-03 2.220125e-03 2.313607e-03
x.PC19 -1.104625e-03 4.141582e-04 -1.277932e-03 -1.962139e-05 -1.130680e-
   1.027312e-03 -8.526181e-04 2.870290e-03 -7.870127e-04
x.PC20 6.788534e-04 -1.316599e-04 2.121543e-03 3.461407e-04 2.269837e-
03 7.821226e-04 1.206739e-04 1.081556e-03 2.577494e-04
x.PC21 8.419983e-04 -3.614072e-04 5.379002e-04 -9.070692e-04 -1.288898e-
04 -5.612064e-04 -6.734132e-04 1.357413e-03 1.845329e-05
x.PC22 -1.527367e-04 -5.061355e-04 -1.865055e-04 6.755951e-05 7.106245e-
04 -2.409899e-03 -5.462667e-04 2.082995e-04 1.706727e-03
x.PC23 -4.500465e-04 -5.473943e-04 -1.388645e-04 7.089913e-04 -3.235537e-
04 8.071133e-04 1.262313e-03 1.190771e-03 1.493301e-03
x.PC24 1.055229e-03 4.999500e-04 1.881622e-05 -3.308093e-04 4.100872e-
04 -4.400472e-04 1.210360e-03 -1.450183e-03 8.605837e-06
x.PC25 7.903666e-04 -2.284499e-03 4.608123e-05 -1.251013e-03 6.674412e-
04 -1.115729e-03 -2.087296e-04 5.716106e-04 9.598572e-04
x.PC26 5.294355e-04 6.064980e-04 -5.672750e-04 1.337246e-04
                                                             2.558212e-
04 -1.747543e-03 -1.263661e-05 7.878541e-05 -1.288457e-03
x.PC27 -1.355075e-04 -2.003555e-04 -1.225879e-03 9.802347e-05
                                                             1.443164e-
04 1.278607e-03 -6.327963e-04 1.472715e-03 -2.722477e-03
x.PC28 -8.307025e-04 -7.705447e-04 -1.247201e-03 1.383068e-03 1.171319e-
04 -1.960097e-03 4.965731e-04 -5.425816e-04 -3.587779e-04
x.PC29 6.323830e-05 -2.812325e-04 -1.138181e-03 -3.299223e-04 -1.554786e-
03 4.637142e-05 2.500656e-04 -4.502863e-04 -5.683709e-04
x.PC30 8.267866e-04 -3.819244e-04 -5.219567e-04 -3.242170e-04 -5.768787e-
04 8.523883e-05 -1.247154e-03 1.536865e-03 6.028361e-05
x.PC31 -2.124506e-04 -1.521063e-03 -1.326105e-03 -2.358303e-04 -1.516444e-
04 5.811230e-04 1.663711e-03 -9.148218e-05 4.779028e-04
x.PC32 -2.494191e-04 8.067299e-04 7.432142e-04 -1.693374e-03 -1.189656e-
03 5.745420e-04 -9.340580e-04 8.078207e-04 3.826708e-04
x.PC33 -9.863714e-05 -1.848959e-04 -3.085693e-04 -1.962478e-04 -1.698507e-
04 -1.713235e-03 1.009049e-03 -6.633060e-04 1.061184e-03
x.PC34 -6.289398e-04 4.578721e-04 -7.433490e-04 -3.649218e-04 -3.138626e-
04 -3.415199e-04 1.253226e-03 -1.702715e-04 -1.505204e-03
x.PC35 4.684483e-04 -2.071002e-03 -1.042933e-04 -1.324886e-03 -5.521788e-
04 - 6.739553e - 04 \ 2.950254e - 04 - 2.211679e - 04 - 1.147387e - 03
```

```
x.PC36 2.788119e-04 2.221461e-04 -1.052876e-03 -1.214653e-04 -1.657665e-
03 1.474558e-03 5.768163e-04 -1.366857e-03 1.858087e-03
x.PC37 -9.208440e-05 3.913647e-04 -4.892738e-05 3.437295e-04 -8.041161e-
04 -8.650334e-04 -1.109882e-03 -3.305196e-04 1.133808e-03
x.PC38 -6.214445e-05 1.577972e-04 -8.197547e-04 -1.006152e-03 -8.261193e-
04 -1.747575e-03 2.323155e-04 -1.692814e-03 3.121039e-03
x.PC39 -1.640280e-04 9.087589e-04 -2.414832e-04 6.672289e-04 9.756774e-
05 3.108802e-04 6.887620e-04 1.500863e-03 4.957652e-04
x.PC40 -8.663359e-05 -1.979930e-04 -1.275681e-03 3.601094e-04 6.534953e-
04 -6.095824e-04 -6.763882e-04 -1.874548e-04 8.037994e-05
x.PC41 - 6.661973e - 04 - 1.284497e - 03 - 6.929424e - 04 - 5.293991e - 04 - 5.736018e - 04 - 04 - 05.293991e - 04 - 05.736018e - 05.
04 -2.021256e-04 5.296949e-04 1.378520e-03 3.445311e-04
x.PC42 -2.830573e-04 -6.947238e-04 -1.519215e-04 1.145892e-04 6.909250e-
04 -1.947458e-03 -1.337077e-03 1.010434e-03 -3.870187e-04
x.PC43 -3.895114e-05 -5.169612e-04 -7.372477e-04 -6.048314e-04 -1.166757e-
03 1.499281e-03 2.741325e-03 2.403503e-03 -2.548665e-03
x.PC44 4.209528e-04 1.169860e-04 5.289572e-04 -1.606939e-03 -9.262348e-
04 -5.781564e-04 1.896296e-03 1.029335e-04 -1.392088e-03
x.PC45 3.555971e-04 -4.840675e-04 -1.261608e-04 -8.468601e-04 -3.578016e-
04 -3.701955e-04 -2.754607e-04 6.023513e-04 -6.659227e-05
x.PC46 -2.539728e-04 1.284401e-03 -2.545494e-05 -1.387005e-04 -2.439750e-
04 -5.580536e-04 -4.817789e-04 -5.555186e-04 9.215954e-04
x.PC47 4.618785e-04 6.708341e-04 -1.350450e-03 6.988059e-04 4.568668e-
04 -9.117681e-04 1.813606e-03 3.882961e-04 -1.347625e-03
x.PC48 1.002569e-04 1.979840e-03 -2.109743e-05 8.920996e-04 -1.479437e-
04 1.844177e-03 -5.214328e-04 -1.200917e-03 -1.593938e-03
x.PC49 8.095537e-04 -4.607318e-04 8.301469e-04 1.838900e-03 5.966873e-
04 3.998238e-04 -2.095127e-04 -1.444392e-03 9.056382e-04
x.PC50 6.387627e-04 1.214409e-04 1.337875e-05 5.684394e-04
                                                                                                 1.624033e-
04 2.187129e-03 1.224017e-03 -1.891344e-03 1.190382e-03
x.PC51 1.350268e-04 -1.289535e-03 8.900000e-04 -1.781899e-03
                                                                                                 1.696480e-
04 -1.352521e-03 5.484946e-04 -7.405237e-04 4.092818e-04
x.PC52 -1.853522e-05 1.120602e-03 8.375255e-04 8.035701e-04 5.171574e-
05 1.879300e-03 -9.016462e-04 7.063402e-04 -2.457916e-03
x.PC53 1.290423e-03 -1.087137e-03 1.293389e-03 -1.210309e-03 -5.271222e-
    5.103381e-04 -1.643399e-03 5.861473e-04 8.474110e-04
x.PC54 -5.709473e-04 -1.387551e-04 1.262928e-04 -1.443320e-04 3.289509e-
04 -5.449429e-04 -1.463580e-03 6.898551e-04 1.542088e-03
x.PC55 2.728525e-04 8.176695e-04 -1.283277e-03 4.353178e-04 -8.185500e-
04 6.827984e-04 1.196302e-04 -6.352861e-04 7.043853e-05
x.PC56 -7.618535e-04 1.068274e-03 -1.790414e-03 5.851301e-04 -4.065230e-
04 -1.458194e-03 -4.661223e-04 -8.717289e-04 1.240850e-03
x.PC57 8.619587e-04 3.780482e-04 -7.356277e-04 -8.123623e-04 8.133631e-
06 -1.174719e-04 -4.753934e-04 1.188013e-03 -3.606721e-04
x.PC58 -1.431458e-03 4.040887e-05 -1.748745e-03 -3.165725e-04 8.970403e-
04 2.282440e-04 -1.068134e-03 -4.729584e-04 -8.300757e-05
x.PC59 3.605837e-04 -1.411481e-03 -3.110365e-04 1.672886e-04 -8.362464e-
04 4.041423e-04 1.490517e-03 -1.516912e-03 -1.208214e-04
x.PC60 4.360581e-05 1.386374e-03 4.527804e-04 1.657609e-04 3.930531e-
04 \quad 7.949157e - 04 \quad -6.779219e - 04 \quad 1.534896e - 04 \quad -2.759175e - 05
x.PC61 -8.208552e-04 -4.234144e-04 1.819755e-04 -1.042028e-03 -6.505080e-
04 -5.627861e-04 9.164937e-04 -1.252149e-04 1.228905e-03
x.PC62 -1.060943e-03 -9.595903e-04 3.764583e-04 -6.844337e-04 -7.718333e-
05 -3.784368e-04 -1.668065e-03 5.145767e-04 -9.763197e-04
```

```
x.PC63 -6.870847e-04 2.425455e-04 -4.951114e-04 1.193382e-03 -7.779851e-
04 -2.134689e-04 -5.538440e-04 1.067232e-03 -1.424514e-03
x.PC64 6.367102e-04 -8.593747e-04 2.784483e-04 2.415564e-04 7.300438e-
04 5.379370e-04 -6.600842e-04 2.101985e-03 -3.380602e-04
x.PC65 -3.850057e-04 -6.491557e-04 -2.590467e-04 -8.559171e-05 -9.868030e-
04 8.744928e-04 2.831127e-04 1.151256e-03 -9.110795e-04
x.PC66 -1.801796e-03 6.212179e-04 -6.602041e-04 1.044381e-03 2.323403e-
04 1.435639e-03 1.603816e-03 1.321660e-03 -6.980142e-04
x.PC67 -1.143843e-03 7.621598e-04 -2.592112e-04 3.361786e-04 -1.325246e-
03 7.109396e-04 6.732327e-04 8.310102e-05 9.817985e-04
x.PC68 3.801735e-04 -1.195014e-03 -1.970394e-05 -2.396079e-04 1.096595e-
03 -5.644461e-04 -1.483346e-04 8.221582e-04 5.388019e-04
x.PC69 -4.361366e-04 8.253972e-04 3.387084e-04 -5.728079e-04 -8.664646e-
04 -3.630215e-04 -1.911101e-03 1.140363e-04 4.025668e-05
x.PC70 8.718086e-04 2.316106e-04 4.767394e-04 -2.927070e-04 1.328595e-
03 1.743674e-04 3.572512e-04 -1.380665e-03 -1.288407e-03
x.PC71 1.703474e-03 -1.561851e-03 3.445950e-04 5.513612e-04 -6.081005e-
05 -3.088916e-04 -1.023820e-03 -1.373929e-03 2.433157e-03
x.PC72 -8.833088e-05 2.690338e-04 -1.504146e-06 -7.249416e-04 -5.377645e-
04 -2.126803e-04 5.158194e-04 2.959013e-04 2.972643e-04
x.PC73 -2.085675e-04 2.388393e-04 -4.364385e-04 5.438729e-04 1.617319e-
03 -9.128383e-04 2.386427e-04 1.964922e-03 -6.945483e-04
x.PC74 7.839293e-04 -3.448978e-04 1.237467e-03 2.818160e-04 4.154917e-
04 -1.297938e-03 -8.053102e-04 2.618844e-04 1.064547e-03
Proportion of trace:
  LD1 LD2 LD3
                      LD4
                            LD5
                                   LD6
                                          LD7
                                                 LD8
                                                         T<sub>D</sub>9
0.4376 0.2054 0.0917 0.0798 0.0665 0.0543 0.0365 0.0181 0.0101
> lda.pred <- predict(lda.fit, dat.pca.test)</pre>
> #Error rate:
> mean(lda.pred$class != dat.pca.test$y)
[1] 0.2166667
> #LDA using full dimensions
> #Could not run with columns x.px1, x.px29
> flda.train <- subset(dat.train, select = -c(x.px1, x.px29))</pre>
> system.time(lda.fit.full<-lda(y~., data=flda.train))
  user system elapsed
   3.39
          0.01 3.39
> lda.fit.full
Call:
lda(y ~ ., data = flda.train)
Prior probabilities of groups:
                   1
                                9
                    8
0.10250000 0.08916667 0.11333333 0.10166667 0.08416667 0.13166667
0.09583333 0.09083333 0.09333333 0.09750000
Group means:
      x.px2 x.px3 x.px4 x.px5
                                               х.рхб
                                                         x.px7
        x.px9
x.px8
                 x.px10
                            x.px11 x.px12
                                                         x.px13
x.px14
```

```
0.20325203\ 1.05691057\ 0.78048780\ 0.60162602\ 0.78048780\ 1.0487805
9.5040650 16.1219512 28.9024390 57.59349593 83.84552846 68.17073171
50.91056911
x.px15 x.px16 x.px17 x.px18 x.px19 x.px20 x.px21 x.px22 x.px23 x.px24 x.px25 x.px26 x.px
0 48.8617886 57.1463415 84.4634146 77.7886179 46.260163 18.317073
12.4796748 7.1056911 1.7560976 0.43902439 0.15447154 0.024390244
0.000000000
x.px28 x.px30 x.px31 x.px32 x.px33 x.px34
x.px35 x.px36 x.px37 x.px38 x.px39 x.px40
x.px41
0 0.9837398 0.227642276 3.73170732 3.28455285 2.7804878 6.0406504
24.8943089 62.7967480 1.013008e+02 126.09756098 145.45528455 174.6341463
178.243902
                                                           x.px47
     x.px42 x.px43 x.px44 x.px45 x.px46
x.px48 x.px49 x.px50 x.px51 x.px52 x.px53 x.px54
0 175.7642276 172.56911 182.569106 179.813008 155.926829 141.878049
116.081301 84.804878 41.024390 12.0650407 5.1788618 3.1219512 1.63414634
      x.px55 x.px56 x.px57 x.px58 x.px59 x.px60
x.px61 x.px62 x.px63 x.px64 x.px65 x.px66
x.px67
0 0.325203252 1.674796748 0.0000000 0.699186992 4.62601626 4.43902439
5.3902439 39.7967480 95.7642276 120.7235772 135.93495935 141.77235772
x.px68 x.px69 x.px70 x.px71 x.px72 x.px73 x.px74 x.px75 x.px76 x.px77 x.px78 x.px78 x.px80
0 152.7398374 155.593496 160.504065 158.691057 161.390244 155.967480
148.772358 145.113821 140.300813 130.560976 113.300813 68.2195122
16.2113821
  x.px81 x.px82 x.px83 x.px84 x.px85 x.px86
x.px87 x.px88 x.px89 x.px90 x.px91 x.px92
x.px93
0 5.73983740 3.62601626 1.60975610 1.2357724 0.0000000 1.17886179
4.10569106 5.4227642 22.4227642 94.5203252 126.7073171 129.8373984
144.68292683
x.px94 x.px95 x.px96 x.px97 x.px98 x.px99
x.px100 x.px101 x.px102 x.px103 x.px104 x.px105 x.px106
0 1.513008e+02 1.528943e+02 151.73983740 157.910569 152.5447154 150.349593
154.804878 154.9186992 150.113821 153.325203 148.04878 140.097561
x.px107 x.px108 x.px109 x.px110 x.px111 x.px112 x.px113 x.px114 x.px115 x.px116 x.px117 x.px118 x.px119
0 114.9593496 55.5447154 8.959349593 4.666666667 2.72357724 0.68292683
0.0000000 1.5284553 4.3414634 7.3902439 53.7560976 121.6910569 139.5447154
x.px120 x.px121 x.px122 x.px123 x.px124 x.px125 x.px126 x.px127 x.px128 x.px129 x.px130 x.px131 x.px132
0 138.08943089 148.6991870 1.518130e+02 1.486423e+02 146.235772 145.967480
148.512195 149.203252 149.016260 145.138211 143.81301 149.056911
147.691057
    x.px133 x.px134 x.px135 x.px136 x.px137 x.px138
x.px139 x.px140 x.px141 x.px142 x.px143 x.px144 x.px145
0 144.528455 137.292683 131.016260 85.4878049 22.74796748 5.25203252
3.89430894 \ 0.1707317 \ 0.1382114 \ 1.75609756 \ 4.48780488 \ 15.2764228
```

87.0243902

```
x.px146 x.px147 x.px148 x.px149 x.px150 x.px151 x.px152 x.px153 x.px154 x.px155 x.px156 x.px157
0 136.1219512 140.8048780 139.3414634 149.69105691 151.2845528 151.5365854
149.37398374 146.382114 147.5447154 152.422764 150.048780 145.333333
    x.px158 x.px159 x.px160 x.px161 x.px162 x.px163
x.px164 x.px165 x.px166 x.px167 x.px168 x.px169
x.px170
0 147.544715 151.42276 147.032520 144.032520 137.365854 137.463415
113.4308943 46.40650407 7.2682927 4.065040650 0.2113821 0.195121951
     x.px171 x.px172 x.px173 x.px174 x.px175
x.px176 x.px177 x.px178 x.px179 x.px180 x.px181
x.px182
0 5.10569106 29.430894309 109.0000000 142.7317073 1.439837e+02
1.461138e+02 156.22764228 157.95121951 152.47967480 152.41463415
149.75609756 152.040650
   x.px183 x.px184 x.px185 x.px186 x.px187 x.px188 x.px189
x.px190 x.px191 x.px192 x.px193 x.px194 x.px195
x.px196
0 150.77236 150.691057 153.36585 150.91870 151.81301 155.27642 148.951220
140.642276 138.756098 127.1382114 62.284553 13.8048780 4.39024390
0.6504065
    x.px197 x.px198 x.px199 x.px200 x.px201
x.px202 x.px203 x.px204 x.px205 x.px206 x.px207
x.px208
0.0.552845528 \quad 2.69105691 \quad 5.67479675 \quad 44.560975610 \quad 112.52845528
1.401301e+02 146.15447154 152.29268293 157.5934959 150.55284553
143.6991870 144.7317073
    x.px209 x.px210 x.px211 x.px212 x.px213 x.px214
x.px215 x.px216 x.px217 x.px218 x.px219 x.px220 x.px221
x.px222
0 144.1300813 144.528455 143.715447 144.90244 144.76423 144.69919
142.04065 150.56911 149.30081 148.552846 138.349593 127.1788618
72.74796748 22.121951
     x.px223 x.px224 x.px225 x.px226 x.px227
x.px228 x.px229 x.px230 x.px231 x.px232 x.px233
x.px234
0 \quad 5.20325203 \quad 1.2357724 \quad 0.756097561 \quad 1.154471545 \quad 9.154471545
56.536585366 113.61788618 131.46341463 143.69105691 155.53658537
163.6829268 153.4471545
     x.px235 x.px236 x.px237 x.px238 x.px239 x.px240
x.px241 x.px242 x.px243 x.px244 x.px245 x.px246 x.px247
x.px248
0 149.9186992 152.4227642 152.910569 147.634146 150.62602 152.17886
149.33333 144.59350 145.62602 154.34146 154.07317 153.121951 132.032520
121.6016260
x.px249 x.px250 x.px251 x.px252 x.px253 x.px254 x.px255 x.px256 x.px257 x.px258 x.px259 x.px260
0 82.268293 34.008130 4.95934959 1.6666667 0.75609756 2.58536585
14.47967480 65.504065041 1.100325e+02 1.235610e+02 1.360000e+02
147.62601626
    x.px261 x.px262 x.px263 x.px264 x.px265 x.px266
x.px267 x.px268 x.px269 x.px270 x.px271 x.px272 x.px273
x.px274
```

```
0 163.8536585 159.1626016 154.1382114 151.008130 150.959350 146.02439
145.61789 148.61789 151.17073 149.82114 148.62602 160.50407 151.72358
142.666667
             x.px276 x.px277 x.px278
    x.px275
                                          x.px279 x.px280
x.px281
                                         x.px285
        x.px282 x.px283 x.px284
                                                     x.px286
x.px287
0 127.130081 116.3008130 84.65041 36.219512 9.46341463 1.9593496
0.918699187 2.569105691 13.29268293 54.16260163 88.83739837 105.3658537
121.9024390
     x.px288 x.px289 x.px290 x.px291 x.px292 x.px293
x.px294 x.px295 x.px296 x.px297 x.px298 x.px299 x.px300
0 128.1951220 157.3577236 167.9674797 155.032520 155.95122 153.64228
149.60163 149.56911 151.27642 153.73984 152.2927 153.00000 161.11382
135.74797
    x.px302 x.px303 x.px304 x.px305 x.px306
x.px308 x.px309 x.px310 x.px311 x.px312 x.px313
x.px314
0\ 122.373984\ 108.292683\ 89.1219512\ 63.62601626\ 32.195122\ 11.33333333
0.6666667 1.26016260 1.15447154 8.54471545 31.5365854 48.2764228
62.7235772
     x.px315 x.px316
                        x.px317 x.px318 x.px319 x.px320
x.px321 x.px322 x.px323 x.px324 x.px325 x.px326 x.px327
x.px328
0 77.8455285 93.8861789 153.2926829 166.325203 155.55285 152.75610
153.63415 152.6504 151.83740 154.98374 155.76423 151.43089 152.27642
160.32520
   x.px329 x.px330 x.px331 x.px332 x.px333 x.px334
x.px335 x.px336 x.px337 x.px338 x.px339 x.px340
x.px341
0 120.32520 82.333333 67.869919 56.0569106 39.59350 17.878049
4.308943 0.5853659 1.23577236 1.2276423 8.1951220 20.9918699
28.6341463
     x.px342 x.px343 x.px344 x.px345 x.px346 x.px347
x.px348 x.px349 x.px350 x.px351 x.px352 x.px353 x.px354
x.px355
0 35.2357724 39.991870 68.8617886 150.243902 169.14634 157.406504
153.95935 150.34146 151.97561 152.46341 150.62602 151.50407 153.39837
151.67480
   x.px356 x.px357 x.px358 x.px359
                                        x.px360 x.px361
x.px362 x.px363 x.px364 x.px365 x.px366 x.px367
x.px368
0 161.86179 112.63415 50.878049 36.77236 32.1382114 25.21951
14.430894 5.2764228 1.3658537 0.00000000 0.3495935 6.0731707
15.4878049
     x.px369 x.px370 x.px371 x.px372 x.px373 x.px374
x.px375 x.px376 x.px377 x.px378 x.px379 x.px380 x.px381
x.px382
0 18.3252033 21.4878049 24.9349593 57.414634 147.21138 170.560976
159.62602 158.96748 154.62602 151.43902 150.91057 151.91870 152.55285
157.16260
   x.px383 x.px384 x.px385 x.px386
                                        x.px387
                                                   x.px388
x.px389 x.px390 x.px391 x.px392 x.px393 x.px394
```

x.px395

```
0 158.67480 163.30894 104.796748 35.975610 20.333333 19.9024390
15.10569 11.479675 4.6260163 0.3333333 0.00000000 0.01626016
3.9024390
    x.px396 x.px397 x.px398 x.px399 x.px400 x.px401
x.px402 x.px403 x.px404 x.px405 x.px406 x.px407 x.px408
x.px409
0 11.089431 13.8292683 15.9593496 17.788618 53.495935 144.76423
168.61789 161.80488 158.2195 155.04878 155.21138 151.90244 152.34959
156.87805
   x.px410 x.px411 x.px412 x.px413 x.px414 x.px415
x.px416 x.px417 x.px418 x.px419 x.px420 x.px421 x.px422
0 155.90244 156.42276 164.20325 101.333333 25.406504 13.113821
16.6991870 12.78862 10.26016 4.1219512 0.1951220 0.02439024
0.1544715 0.9918699
    x.px424 x.px425 x.px426 x.px427 x.px428 x.px429
x.px430 x.px431 x.px432 x.px433 x.px434 x.px435 x.px436
x.px437
  6.674797 9.3983740 11.5365854 16.048780 53.130081 144.12195
168.34146 163.27642 158.12195 157.30081 154.82114 151.21138 154.16260
157.08130
   x.px438 x.px439 x.px440 x.px441 x.px442 x.px443
x.px444 x.px445 x.px446 x.px447 x.px448
                                               x.px449
x.px450
0 152.41463 155.2927 163.16260 103.520325 25.886179 10.731707
12.2601626 8.1869919 5.95935 3.2439024 0.0000000 0.02439024
0.44715447
    x.px451 x.px452 x.px453 x.px454 x.px455 x.px456
x.px457 x.px458 x.px459 x.px460 x.px461 x.px462 x.px463
x.px464
   0.9837398 5.902439 8.3495935
                                9.9756098 14.86992 53.959350
147.83740 168.75610 163.05691 160.47967 159.98374 158.04878 157.78049
158.21138
   x.px465 x.px466 x.px467 x.px468 x.px469 x.px470 x.px471
x.px472 x.px473 x.px474 x.px475 x.px476 x.px477
x.px478
0 158.80488 157.3333 156.4228 163.43902 109.723577 25.23577 7.487805
9.2276423 7.2195122 5.715447 3.747967 0.0000000 0.02439024
0.8211382
    x.px479 x.px480 x.px481 x.px482 x.px483
x.px485 x.px486 x.px487 x.px488 x.px489 x.px490 x.px491
x.px492
0 1.178862 6.03252 8.0325203 10.4634146 14.74797 55.471545
149.31707 168.21951 162.91870 162.12195 160.33333 160.82114 160.37398
158.82114
  x.px493 x.px494 x.px495 x.px496 x.px497 x.px498 x.px499
x.px500 x.px501 x.px502 x.px503 x.px504 x.px505 x.px506
0 159.7073 158.2358 157.0081 162.86992 116.60163 26.121951 7.609756
9.2357724 7.373984 5.878049 4.268293 0.0000000 0.07317073
1.105691
    x.px507 x.px508 x.px509 x.px510 x.px511 x.px512
x.px513 x.px514 x.px515 x.px516 x.px517 x.px518 x.px519
```

x.px520

```
0 1.3333333 6.1869919 8.1869919 10.8130081 15.910569 60.121951
154.72358 167.46341 163.49593 162.68293 159.49593 159.83740 159.93496
159.60163
  x.px521 x.px522 x.px523 x.px524 x.px525 x.px526 x.px527
x.px528 x.px529 x.px530 x.px531 x.px532 x.px533 x.px534
0 159.2846 158.0569 158.09756 163.01626 123.292683 28.65854 7.447154
9.2520325 7.487805 6.03252 4.609756 0.0000000 0.64227642 1.056911
     x.px535 x.px536 x.px537 x.px538 x.px539 x.px540
x.px541 x.px542 x.px543 x.px544 x.px545 x.px546 x.px547
x.px548
0 \quad 1.30894309 \quad 6.7073171 \quad 8.5365854 \quad 10.9430894 \quad 16.536585 \quad 66.268293
159.41463 166.27642 163.03252 165.88618 165.12195 162.24390 162.13008
161.49593
   x.px549 x.px550 x.px551 x.px552 x.px553 x.px554 x.px555
x.px556 x.px557 x.px558 x.px559 x.px560 x.px561 x.px562
0 160.16260 161.52846 160.38211 164.44715 130.30894 35.15447 7.739837
9.1056911 7.471545 6.5203252 4.772358 0.2845528 0.6585366
0.6504065
     x.px563 x.px564 x.px565 x.px566 x.px567
x.px569 x.px570 x.px571 x.px572 x.px573 x.px574 x.px575
x.px576
   1.6585366 6.7235772 8.7479675 10.8211382 16.414634 74.073171
161.00813 166.65041 164.47154 166.18699 164.33333 162.60163 162.11382
x.px577 x.px578 x.px579 x.px580 x.px581 x.px582 x.px584 x.px585 x.px586 x.px587 x.px588 x.px589
                                                 x.px582 x.px583
0 161.43089 161.69919 161.13008 164.00000 133.284553 44.186992 8.276423
9.0000000 7.512195 6.6829268 4.447154 0.2764228 0.000000
      x.px590 x.px591 x.px592 x.px593 x.px594 x.px595
x.px596 x.px597 x.px598 x.px599 x.px600 x.px601 x.px602
x.px603
0 0.008130081 1.4552846 6.0813008 7.5203252 9.5365854 17.211382
82.544715 165.21951 169.04878 167.72358 169.81301 168.2114 164.87805
165.47967
   x.px604 x.px605 x.px606 x.px607 x.px608 x.px609 x.px610
x.px611 x.px612 x.px613 x.px614 x.px615 x.px616
x.px617
0 163.00000 164.13821 164.54472 161.34146 162.77236 137.642276 52.585366
7.674797 7.9512195 6.016260 5.3089431 4.097561 0.0000000
x.px618 x.px619 x.px620 x.px621 x.px622 x.px623 x.px624 x.px625 x.px626 x.px627 x.px628 x.px629 x.px630
x.px631
0 0.02439024 1.3577236 4.3577236 6.5772358 8.3252033 18.707317
89.292683 167.65854 169.30894 167.87805 170.36585 168.04878 167.10569
166.70732
   x.px632 x.px633 x.px634 x.px635 x.px636 x.px637
x.px638 x.px639 x.px640 x.px641 x.px642 x.px643
x.px644
0 165.56911 165.593496 165.918699 164.040650 164.36585 142.520325
59.934959 8.951220 6.6747967 5.4552846 5.0975610 4.097561
0.0000000
    x.px645 x.px646 x.px647 x.px648 x.px649 x.px650
x.px651 x.px652 x.px653 x.px654 x.px655 x.px656 x.px657
```

```
0 0.0000000 0.02439024 1.2601626 3.9674797 5.66666667 7.8455285
20.910569 88.65854 164.276423 168.487805 168.691057 167.617886 166.308943
    x.px658 x.px659 x.px660 x.px661 x.px662 x.px663
x.px664 x.px665 x.px666 x.px667 x.px668 x.px669
x.px670
0 167.422764 167.918699 163.772358 164.308943 165.325203 162.439024
162.731707 143.682927 63.804878 9.861789 6.1463415 5.536585
5.1056911
   x.px671 x.px672 x.px673 x.px674 x.px675 x.px676
x.px677 x.px678 x.px679 x.px680 x.px681 x.px682 x.px683
x.px684
0 3.926829 0.0000000 0.0000000 0.01626016 1.9024390 3.943089
5.8617886 8.130081 22.577236 90.861789 161.26829 167.37398 167.86179
171.195122
            x.px686 x.px687 x.px688 x.px689 x.px690
    x.px685
x.px691 x.px692 x.px693 x.px694 x.px695 x.px696 x.px697
0 169.894309 169.130081 167.27642 163.853659 166.910569 166.593496
159.552846 160.268293 143.463415 67.40650 11.2845528 6.5121951
5.804878
    x.px698 x.px699 x.px700 x.px701 x.px702 x.px703
x.px704 x.px705 x.px706 x.px707
                                         x.px708
                                                    x.px709
x.px710
0 \quad 5.5609756 \quad 4.146341 \quad 0.0000000 \quad 0.0000000 \quad 0.02439024 \quad 0.7235772
1.4552846 3.5203252 5.8130081 23.5691057 90.4471545 161.2520325
163.284553
    x.px711 x.px712 x.px713 x.px714 x.px715
                                                      x.px716
x.px717 x.px718 x.px719 x.px720 x.px721 x.px722
x.px723
0 162.552846 167.91870 165.861789 165.626016 163.0162602 161.5365854
161.0325203 159.1463415 157.162602 156.747967 137.3170732 67.9756098
13.5528455
     x.px724 x.px725 x.px726 x.px727 x.px728 x.px729
x.px730 x.px731 x.px732 x.px733 x.px734 x.px735
x.px736
0 4.3170732 2.8943089 2.8536585 1.6260163 0.0000000 0.0000000
0.016260163 0.008130081 0.37398374 2.4390244 5.2357724 26.9756098
93.9024390
    x.px737 x.px738 x.px739 x.px740 x.px741
x.px742 x.px743 x.px744 x.px745 x.px746 x.px747
x.px748
0 164.0650407 168.5203252 172.8617886 175.6016260 176.63414634
178.41463415 178.17073171 175.34959350 174.926829 172.9756098 170.6016260
164.390244
    x.px749 x.px750 x.px751 x.px752 x.px753 x.px754
x.px755 x.px756 x.px757 x.px758 x.px759 x.px760 x.px761
0\ 145.349593 \quad 73.252033 \quad 15.0325203 \quad 4.5121951 \quad 2.17886179 \quad 2.07317073
0.699187 0.0000000 0.00000000 0.032520325 0.04878049 0.2113821 1.8943089
    x.px762 x.px763 x.px764 x.px765 x.px766 x.px767 x.px768
x.px769 x.px770 x.px771 x.px772 x.px773 x.px774
0 2.3739837 12.2276423 42.878049 74.138211 91.268293 98.813008 105.341463
110.29268 115.8292683 1.187236e+02 113.93495935 111.7235772 103.97560976
    x.px775 x.px776 x.px777 x.px778 x.px779 x.px780
x.px781 x.px782 x.px783 x.px784
```

0 95.8617886 85.357724 68.6260163 30.8048780 9.1138211 2.6829268 1.89430894 1.796748 0.4552846 0.000000000 [reached getOption("max.print") -- omitted 9 rows]

Coefficients of linear discriminants:

LD3 LD4 LD9 LD5 LD6 LD7 LD8 -1.844786e+00 -4.037873e-03 -8.528366e-01 6.740005e-01 x.px2 7.535051e-01 -8.172915e-01 -4.392631e-01 -1.811275e+00 1.665180e-01 -1.134707e-01 5.445598e-02 4.328841e-01 -6.604356e-02 3.521699e-01 -1.355346e-01 -4.512568e-01 1.019148e+00 -2.602046e-013.783115e-01 9.348333e-04 -2.648278e-01 2.655506e-01 -1.487369e-01 -1.483734e-01 1.563853e-01 -2.411258e-01 -3.966517e-024.564789e-02 2.567906e-01 1.501304e-01 -1.425565e-02 z.px5 1.474674e-01 -1.414241e-01 8.908180e-02 -1.386653e-01 -5.871245e-02-2.725224e-02 -5.278634e-02 1.570201e-02 -7.113146e-02 -1.478800e-01 -8.623460e-03 7.191596e-02 1.935980e-01 8.143766e-02 -4.847158e-02 8.541307e-03 2.087966e-02 1.079669e-02 x.px7 1.203616e-03 -5.730074e-03 -2.607066e-02 2.972946e-02 6.905391e-02 -1.194125e-02 2.976404e-02 3.541821e-02 5.614898e-03 9.323305e-04 7.061749e-03 3.835407e-02 -2.286035e-02 -1.491257e-02 x.px9 -3.850316e-03 -2.965662e-03 -1.110535e-02 -8.900022e-03 1.044667e-02 5.730694e-03 -1.504064e-02 3.786834e-03 2.171743e-02x.px10 -3.572908e-05 8.057090e-03 -3.883048e-03 2.812802e-03 1.652958e-03 -1.306186e-03 1.187835e-03 -5.551328e-03 4.547312e-03 x.px11 -4.321561e-03 -7.026190e-04 -1.060683e-03 7.750243e-04 1.502062e-03 2.885851e-03 -3.912983e-03 4.947988e-03 4.020164e-03 x.px12 5.992387e-03 2.634649e-03 -7.284430e-03 1.650631e-03 4.816437e-03 -2.518435e-03 5.710161e-03 -4.425785e-03 3.769784e-03 x.px13 -9.354820e-03 1.249012e-03 5.401735e-04 -4.510416e-03 7.059224e-04 7.000884e-03 2.438130e-03 1.081358e-03 -2.468926e-03 x.px14 6.876721e-03 4.108693e-03 -8.656657e-03 3.616610e-03 1.302871e-03 5.322998e-03 -5.277576e-03 -5.383361e-03 2.209791e-03 x.px15 2.064860e-03 -3.544115e-03 2.542576e-03 -1.969878e-03 2.566069e-03 6.226144e-04 1.334437e-02 7.910518e-03 1.869843e-03 x.px16 -2.763991e-03 -3.668945e-03 -1.236991e-03 2.122065e-04 -2.263439e-03 -7.270403e-03 -1.372256e-02 -1.929073e-03 -3.055207e-03 x.px17 -9.421707e-04 5.394593e-03 8.217776e-03 8.549296e-04 -5.017657e-03 -9.598547e-05 9.716332e-04 1.031075e-03 -5.544065e-03x.px18 -1.194196e-03 -3.601010e-04 5.105439e-04 -1.190580e-03 4.437760e-04 -3.938005e-04 -2.361898e-03 -3.113122e-03 8.505830e-04 x.px19 -3.255842e-03 -4.478114e-04 1.014522e-02 -4.347644e-03 -3.857181e-03 -3.340406e-03 3.715068e-03 5.816336e-03 4.492012e-05 x.px20 5.818774e-03 1.142879e-02 -2.557298e-03 1.367751e-03 1.792642e-03 9.946796e-04 1.470984e-03 -7.680326e-03 2.327033e-03 x.px21 5.226127e-04 -7.981208e-03 5.913669e-03 -5.237614e-03 -2.663307e-02 8.081108e-03 1.233225e-02 5.901132e-03 4.064950e-04 x.px22 -1.398313e-02 1.327693e-02 -1.493192e-02 1.806886e-02 1.476083e-02 -1.579959e-02 -1.831231e-02 -9.056198e-03 -7.759556e-032.337628e-02 4.870220e-03 6.122679e-02 -2.190081e-02 x.px23 6.658460e-03 2.494689e-02 -2.165827e-02 3.795278e-03 2.307492e-02 x.px24 -2.327058e-02 -2.189416e-02 -5.175348e-02 5.631193e-02 4.163341e-02 -3.987152e-02 -2.347278e-02 -2.666740e-02 -3.034086e-02

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x.px25 4.495672e-02 -2.750208e-02 -6.355267e-02 -1.452140e-01 -
1.400253e-01 -3.469049e-02 1.293531e-01 1.288087e-01 5.846121e-02
x.px26 -3.475960e-01 2.338307e-02 1.107384e-01 -2.530375e-01
3.370393e-01 3.541646e-02 -2.864284e-01 3.920468e-01 -1.200237e-01
x.px27 -1.310161e+00 -2.761083e+00 -2.283970e+00 -8.633838e-01
9.103597e-02 7.185804e-01 1.508048e+00 5.704842e-01 -1.736237e+00
x.px28 -7.863326e+00 -5.290929e+00 -4.279239e+00 3.153657e+00
1.214026e+00 -6.792294e+00 -8.871871e+00 -6.651148e+00 -7.286776e-01
x.px30 4.741850e+00 4.178500e+00 1.773066e+00 -1.316503e+00
1.445883e-01 -3.654040e-01 -1.430926e+00 -4.242582e-01 -2.479359e-01
x.px31 -1.011670e-01 3.019089e-01 8.488192e-02 -2.115874e-01
1.483879e-01 4.892900e-01 -5.532539e-02 -8.449230e-01 -6.565123e-02
x.px32 -1.950096e-01 -2.596298e-01 2.865041e-01 3.753703e-02 -
2.498521e-01 -7.523159e-02 1.299336e-03 4.459145e-01 2.117353e-01
x.px33 -2.308550e-02 -8.532438e-02 -1.364582e-01 1.236269e-02
3.459217e-02 4.709440e-02 -9.907168e-02 -5.909759e-02 -2.898736e-02
       3.685717e-02 -3.492798e-02 -6.780137e-02 -2.440016e-02
2.529287e-02 1.166927e-02 -1.262455e-02 4.384624e-05 -4.666590e-02
x.px35 -3.142528e-03 -7.541589e-03 -1.149702e-02 1.466060e-02 -
1.047115e-02 -1.344410e-02 -6.527935e-03 1.291100e-03 -5.553256e-03
x.px36 -2.303162e-03 -9.202345e-03 -4.882589e-03 2.577073e-03
7.743072e-04 -2.366132e-03 -2.332711e-03 -5.124464e-03 -6.450101e-03
x.px37 6.461788e-04 9.506148e-03 2.709849e-03 -1.329965e-03 -
8.177438e-03 -3.250575e-03 8.482653e-05 -6.620572e-03 -8.023462e-04
x.px38 -3.868426e-03 -5.822953e-04 -6.186726e-03 -7.556878e-03 -
2.697117e-03 3.709802e-03 -1.002733e-02 4.293682e-03 -3.707237e-03
x.px39 -5.513927e-03 5.734768e-03 -5.110713e-03 -3.770667e-03
3.123351e-03 -6.559747e-04 7.830225e-03 -3.102418e-03 2.614083e-03
x.px40 -2.535243e-04 3.294192e-03 3.989658e-03 8.334341e-04 -
2.771547e-03 5.054339e-04 -5.023931e-03 -5.844009e-03 -7.764932e-03
x.px41 -8.255194e-03 2.869219e-03 -1.126343e-03 4.022465e-03 -
6.722161e-03 -2.409345e-03 -2.467442e-03 3.778198e-03 -3.651644e-04
x.px42 1.951937e-03 -3.229615e-03 3.719952e-03 -1.972297e-03
3.859050e-03 -5.219814e-03 3.543770e-04 4.738560e-03 1.258363e-03
x.px43 -4.957248e-03 5.405047e-03 -1.855836e-03 1.077327e-03 -
1.222552e-03 1.879527e-03 -7.100722e-03 -3.029636e-03 -5.931132e-03
x.px44 4.673268e-03 -6.052308e-03 2.927776e-03 -4.570787e-03 -
3.372576e-03 4.510894e-03 1.085135e-02 -2.311615e-03 4.138165e-03
x.px45 -2.256612e-03 -3.297817e-03 -4.191356e-03 -2.836581e-04
4.226778e-04 -1.190430e-04 1.068409e-03 -1.929955e-03 1.309395e-03
x.px46 1.464526e-03 -1.906807e-03 -1.325884e-03 4.549236e-03
1.453882e-03 5.684926e-03 1.219162e-03 -7.663796e-03 2.710354e-03
x.px47 3.529008e-03 -5.787945e-03 4.079950e-03 6.198953e-03 -
4.007415e-03 -1.595199e-03 -3.643680e-03 -3.931676e-03 3.362054e-03
        2.216203e-04 1.667345e-03 -1.321981e-03 -1.015967e-03
7.140260e-03 2.791222e-03 -1.708310e-03 -4.309429e-04 2.205871e-03
x.px49 -4.016387e-03 -4.145579e-03 9.399239e-04 7.268557e-03 -
1.550822e-03 -5.419795e-03 5.681755e-03 -6.870023e-05 -5.482030e-04
x.px50 1.700539e-04 -5.724348e-04 1.379168e-02 -1.335808e-02 -
2.409969e-03 1.611160e-02 -1.366777e-04 1.682145e-02 5.358785e-03
x.px51 1.597174e-02 -5.374674e-03 -1.095643e-02 1.129971e-02
6.635107e-03 -2.058684e-02 -1.198119e-02 -7.217427e-03 -1.749079e-02
x.px52 1.757556e-02 2.071152e-02 7.348488e-03 -4.915006e-02 -
4.043002e-02 3.267607e-02 4.868717e-02 1.041891e-02 2.182464e-02
```

```
x.px53 -5.196403e-02 2.156698e-02 2.454116e-02 9.612975e-02
8.028754e-02 -2.189845e-02 -4.272304e-02 -6.133314e-02 -5.114588e-02
       9.694915e-02 -1.863410e-02 -3.851270e-03 -1.319437e-01 -
1.190877e-01 6.196351e-02 7.823966e-02 4.882124e-02 6.546352e-02
x.px55 -2.711203e-03 2.099633e-03 7.428114e-02 1.806693e-01
8.534024e-02 -1.138815e-01 2.668752e-02 -1.052079e-01 -1.140301e-01
       4.612700e+00 2.611647e+00 2.448696e+00 -1.827758e+00 -
9.556437e-01 3.788989e+00 5.454199e+00 4.208649e+00 8.988045e-01
       6.927554e+00 4.316691e+00 -2.333351e+00 -3.922387e+00
4.777861e+00 -1.178340e+01 -9.020286e-01 -1.378504e+00 -7.504000e+00
x.px58 -6.120361e-02 -3.363288e+00 1.910708e+00 2.606406e+00 -
2.680765e-01 1.939264e+00 9.206408e-01 8.022190e-01 4.716763e+00
        3.587597e-01 -2.367979e-01 -4.940910e-01 -2.781320e-01 -
x.px59
1.019234e-01 9.056832e-02 2.413756e-01 3.399188e-01 -2.615482e-02
x.px60 7.546700e-02 1.493677e-02 -6.487781e-02 -3.691965e-02 -
1.243437e-02 -1.244470e-01  1.047136e-01  -3.714296e-02  1.238540e-01
x.px61 -3.600913e-02 -3.240583e-02 4.721639e-02 4.468795e-02 -
3.536052e-02 -1.379485e-02 3.101712e-02 2.081356e-02 3.109728e-02
       1.108219e-02 -3.047422e-03 -3.627594e-03 -2.167673e-02
x.px62
1.791127e-02 2.189099e-02 2.563628e-03 9.308908e-03 8.447146e-03
x.px63 -5.676362e-03 3.266854e-03 7.541292e-03 -9.680898e-03
6.816224e-03 6.548848e-03 -5.345840e-03 1.309395e-02 2.460521e-03
x.px64 5.212443e-03 -5.887487e-04 -1.327967e-03 -2.489572e-03
3.808928e-03 2.899127e-03 -1.974567e-03 8.693377e-04 1.139340e-03
x.px65 -1.939948e-03 -2.077944e-03 -1.274967e-03 6.377368e-03
3.825712e-03 2.406630e-03 -5.196092e-03 7.735967e-04 -2.114271e-03
       3.642783e-03 -1.272583e-03 -2.931898e-04 9.129539e-04
x.px66
7.020219e-03 1.429520e-03 5.701563e-03 -4.613315e-03 5.793955e-03
x.px67 3.590900e-03 -1.078546e-03 -2.095017e-04 4.797385e-03
2.614590e-03 3.441587e-03 -5.427433e-03 -7.938376e-04 2.482102e-03
x.px68 -1.826765e-03 -3.284488e-03 2.819120e-03 -4.604124e-03 -
1.652279e-03 1.273797e-04 3.559835e-03 5.465581e-03 4.395679e-03
       7.287872e-03 -6.564041e-03 2.292067e-03 7.162245e-03
x.px69
1.173970e-03 -2.006001e-03 8.393333e-04 -4.318973e-03 -5.865009e-03
x.px70 -6.479704e-04 -7.736137e-04 -6.192175e-05 -4.115586e-03 -
2.854861e-03 5.441222e-03 2.819800e-03 -5.199860e-03 -1.068294e-02
x.px71 -1.796453e-03 -3.413957e-03 2.150779e-03 -1.815680e-03
3.322932e-03 3.378941e-03 -1.299526e-03 8.493682e-03 1.002063e-02
x.px72 -4.400511e-03 4.703329e-03 -1.437373e-03 -6.497297e-05
5.975517e-03 -4.212875e-03 3.234220e-03 4.633385e-03 -1.853582e-03
x.px73 2.026774e-03 -2.023900e-03 -4.728604e-03 1.816345e-03 -
9.908535e-03 1.676048e-03 -1.217051e-03 3.276830e-03 3.573054e-03
x.px74 -4.352494e-03 8.641014e-03 -3.350168e-03 -4.522051e-03
5.646129e-03 -9.546403e-03 -1.402625e-03 4.264376e-03 -6.090886e-03
x.px75 4.979256e-04 3.704332e-04 2.538295e-03 -2.843614e-03 -
7.942014e-04 -3.163856e-03 -4.789977e-03 -5.797399e-04 7.651609e-03
x.px76 -3.811203e-03 5.880720e-03 -1.178000e-04 -4.607472e-03 -
5.274989e-03 -1.396678e-03 2.860620e-03 4.128583e-03 -6.824252e-03
x.px77 -1.130786e-03 -3.326278e-03 2.585131e-03 3.469359e-03
7.547411e-04 -1.348667e-03 -7.955784e-03 -8.425060e-03 7.886910e-03
x.px78 5.852739e-03 4.440312e-03 -5.768573e-04 5.574628e-03 -
5.895595e-03 -2.140415e-03 1.077685e-02 5.982661e-03 -3.504151e-03
x.px79 5.581302e-03 -4.756629e-03 -1.181772e-02 1.376894e-02 -
2.955090e-03 -8.909506e-03 5.532758e-03 -1.427206e-02 1.197226e-04
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x.px80 -2.817028e-02 3.582585e-03 -6.779017e-03 -4.350235e-03
1.119376e-02 1.019655e-02 -1.715201e-03 1.674853e-02 2.680496e-03
       3.589663e-02 -1.089945e-02 1.706728e-02 -1.838030e-02 -
6.431835e-03 -1.389963e-02 -1.645447e-02 2.412867e-02 1.819282e-02
x.px82 -6.412078e-02 1.866502e-02 -1.216873e-02 1.148263e-01
7.218425e-02 -6.726244e-02 -1.425285e-02 -6.044657e-02 -5.828546e-02
       4.800771e-02 6.394358e-04 -7.503691e-02 -1.482746e-01 -
7.633048e-02 6.516382e-02 -1.689112e-02 7.891293e-02 1.128868e-01
x.px84 -2.628281e-01 -3.896160e-02 -1.693021e-02 6.568560e-01 -
5.829072e-02 -9.792776e-02 2.073148e-01 -1.839746e-01 -3.946620e-01
       3.017281e+00 -1.576046e+00 9.002642e-01 -2.238254e+00 -
x.px85
3.714176e+00 7.655779e+00 1.068024e+00 -1.092568e-01 8.703844e-01
x.px86 -4.299262e-01 1.414056e+00 -1.777063e+00 -1.759615e+00
3.228653e-01 -1.298704e+00 -5.107108e-01 -6.475471e-01 -2.790601e+00
x.px87 -2.220124e-01 1.458028e-01 1.634763e-01 1.141472e-01
3.243124e-01 -7.847301e-02 -1.766109e-01 -2.276428e-01 2.508708e-02
x.px88 -9.787030e-02 3.150948e-02 4.568437e-02 1.872044e-02
1.555906e-03 6.307786e-02 -2.299317e-02 9.664759e-02 -9.339020e-02
       8.150952e-03 2.609079e-02 2.008537e-03 1.131182e-02
4.057883e-03 -2.169446e-02 -8.151285e-03 -2.400710e-02 2.527059e-02
x.px90 -4.291328e-04 3.733709e-03 -4.378866e-03 6.862618e-03
6.764811e-03 -8.102729e-03 4.220207e-03 -2.090243e-02 -1.590186e-03
x.px91 -3.028338e-03 -6.838499e-03 1.345556e-04 5.454534e-03 -
5.193610e-03 -4.466614e-03 9.116005e-03 -7.547309e-03 3.028466e-04
x.px92 -6.852283e-03 5.251311e-03 6.446800e-03 -8.450625e-03 -
5.510250e-03 6.678517e-04 -1.054875e-02 6.999031e-03 -1.700485e-03
       1.575547e-02 -7.894414e-03 -4.439448e-04 -8.619925e-04
x.px93
6.587386e-03 -7.023842e-03 9.024365e-03 2.410301e-03 3.722078e-03
x.px94 8.571859e-06 4.488080e-03 4.445445e-03 -4.311568e-03 -
1.116783e-02 -3.456462e-03 1.818746e-03 7.605137e-03 -5.793240e-03
       1.325899e-04 3.102161e-03 6.334097e-03 5.779961e-06
x.px95
4.989183e-04 5.254136e-03 5.036310e-03 -2.320574e-03 5.739949e-03
       6.929939e-03 9.753986e-03 -1.780892e-03 4.846123e-03
x.px96
1.764818e-03 -2.447647e-03 -1.083202e-03 -1.338501e-03 7.828495e-04
x.px97 -7.955889e-03 -1.433142e-03 3.555542e-04 1.020933e-02 -
1.833423e-03 -6.072923e-03 -8.496551e-03 -1.265738e-02 -6.763298e-03
x.px98 -3.677774e-03 2.486010e-03 3.203208e-03 -2.532878e-03
2.422746e-03 5.296259e-03 -1.829561e-03 4.240290e-03 7.890496e-03
x.px99 5.567090e-03 3.862013e-03 4.961409e-03 -1.267271e-03 -
2.457511e-03 -1.477516e-03 -6.142368e-03 8.509784e-04 -9.569101e-03
x.px100 2.175370e-03 -6.600739e-04 -3.650175e-04 -5.180666e-03
2.175567e-03 -3.991371e-04 3.672823e-03 7.501469e-03 1.101151e-02
x.px101 -4.246638e-03 -6.323474e-03 3.558357e-04 -5.069639e-03
3.552275e-03 4.452750e-03 8.981578e-03 2.717126e-03 3.807808e-04
x.px102 5.499649e-03 -7.077311e-04 -4.723244e-04 -2.337059e-03
6.286553e-05 9.394584e-03 6.258327e-03 -4.499409e-03 1.080145e-03
x.px103 -7.777330e-03 -3.433585e-03 -5.490673e-03 2.761954e-03
6.010944e-03 -2.543935e-03 6.139794e-03 2.250634e-03 -4.465186e-03
x.px104 -4.972446e-03 7.661969e-04 -3.233003e-03 4.152644e-03 -
1.688447e-04 9.761275e-03 -9.889392e-03 -9.118751e-03 -1.789257e-03
x.px105 2.473158e-03 -2.483742e-03 7.592055e-04 9.498928e-03 -
2.376592e-03 -3.744320e-03 8.513148e-03 1.209677e-03 3.014502e-03
x.px106 -9.938245e-04 -2.338506e-03 -6.458488e-03 -2.341117e-03
8.172076e-04 6.304179e-04 -9.469738e-04 4.395390e-03 2.986749e-03
```

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x.px107 -1.605098e-03 -9.347781e-03 -5.771875e-04 -8.810373e-03
1.758232e-03 5.862147e-03 6.768180e-03 5.227593e-03 6.860053e-03
x.px108 4.557691e-03 3.067859e-03 1.597358e-02 1.617433e-03 -
3.004728e-03 -4.964760e-03 -5.631889e-03 5.798062e-03 -6.124928e-03
x.px109 -3.283562e-04 6.764649e-03 2.237986e-02 -3.031127e-03 -
3.260665e-03 3.520397e-03 -7.158832e-03 -2.271113e-02 -1.363446e-02
x.px110 3.562873e-02 -4.020065e-03 6.595136e-03 -2.909815e-02 -
2.847936e-02 2.726842e-02 -2.240742e-03 2.230445e-02 1.847146e-02
x.px111 8.606444e-03 -2.002458e-03 5.316862e-03 1.294973e-02
1.790502e-03 -4.044445e-02  1.502966e-02 -2.760875e-02 -1.232748e-02
x.px112 -2.765953e-02 1.060800e-02 3.003406e-02 3.332979e-02
2.831954e-02 -2.382200e-03 -3.158869e-03 1.157654e-01 4.022148e-02
x.px113 -6.123084e+00 3.213119e+00 -3.890274e+00 3.866745e-02
3.397054e+00 -5.785310e+00 -2.403742e+00 -1.796957e+00 8.973669e-01
[ reached getOption("max.print") -- omitted 671 rows ]
Proportion of trace:
        LD2
               LD3
                       LD4
                              LD5
                                     LD6
  LD1
                                           LD7
                                                   LD8
                                                          LD9
0.4176\ 0.1493\ 0.1123\ 0.0994\ 0.0674\ 0.0621\ 0.0457\ 0.0274\ 0.0187
> lda.pred.full <- predict(lda.fit.full, dat.test)</pre>
> #Error rate higher than with PCA:
> mean(lda.pred.full$class != dat.test$y)
[1] 0.355
> #QDA using PCA
> system.time(qda.fit<-qda(y~., data=dat.pca.train))</pre>
  user system elapsed
          0.00 0.02
  0.03
> qda.fit
Call:
qda(y ~ ., data = dat.pca.train)
Prior probabilities of groups:
                                         3
                   1
0.10250000 0.08916667 0.113333333 0.10166667 0.08416667 0.13166667
0.09583333 0.09083333 0.09333333 0.09750000
Group means:
                                                                x.PC6
                                         x.PC4
                                                    x.PC5
                   x.PC2
                              x.PC3
                         x.PC9 x.PC10 x.PC11
            x.PC8
                                                     x.PC12
0 - 764.05710 \qquad 693.36939 - 155.55580 - 148.46180 \qquad -7.337393 \qquad 707.22224 -
105.75837 11.7634545 -16.2946515 -34.74666 77.72095 -2.327815 -
64.72897
     95.66339 1398.06749 -182.66459 327.68519 288.971748 -364.26354
282.88772 159.3187987 340.9250460 54.77037 -7.98132 21.388956 -
72.87123
2 -904.56984 -259.88791 490.04439 -203.44607 235.953419 -98.61471
26.40454 -46.7625018 -29.2220470 -99.17127 44.13156 29.387096 -
12.02044
3 -141.18251 1110.99435 -288.24086 139.22419 -189.055011 -127.56917
20.08120 -22.6706425 -281.6182581 -99.88930 -49.95557 2.249283
86.22974
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4 - 1073.55749 - 182.30826 \ 435.43897 - 22.48462 - 13.713699 - 325.21008 -
223.04340 26.6256347 15.9469066 124.76117 -62.22953 15.865455
69.41020
5 1453.49562 -219.73259 104.35957 -200.03629 76.698670 60.90032 -
47.14059
               32.1681772 15.5127439 -17.92852 8.78871 9.292916
95.23817
6 -633.65537 -80.02712 337.71923 -215.92841 65.490308
                                                                                                    53.73187 -
22.19901 -0.2329146 -50.6236613 43.79859 -89.31235 -45.590898
32.46572
7 1382.29575 -466.61103 171.43298 559.03541 -209.058056 122.77092
23.68542 -359.3659630 38.2899642 37.34411 69.53084 -29.786297
8 - 148.59048 - 826.48541 36.98883 - 44.84459 - 614.841129 - 10.62753
286.58053 277.6409359 -0.8156656 68.00780 22.65358 24.460768 -
119.43522
9 355.62452 -1105.24138 -982.36737 -28.39332 293.524886 -121.23112 -
217.53740 -77.2510130 13.1171495 -20.55721 -30.44072 -30.717887 -
66.02989
         x.PC14
                          x.PC15 x.PC16
                                                            x.PC17 x.PC18
                                                                                                 x.PC19
x.PC20 x.PC21 x.PC22 x.PC23 x.PC24 x.PC25 x.PC26
x.PC27
25.834634 -15.683478
      6.853842 49.446089 -27.86143 -42.173484 -21.58218 -1.865776 -
21.967576 8.167367 25.382195 6.771001 -10.236557 47.797976 -
4.468072 -0.380031
2 18.134310 27.195676 18.63724 -10.598201 55.72810 65.444250
45.446948 31.739627 7.118714 14.176084 -24.022566 9.911452
1.637830 21.549056
   1.944539 -23.802504 44.91180 60.376923 61.86947 28.381057
28.817075 -21.801958 -49.427580 23.503856 -23.093350 -33.969003 -
38.710249 19.273580
4 -2.845603 -38.777858 23.73091 -37.842113 -95.27665 -38.510429 -
4.963525 -14.783413 -30.055466 -56.425072 20.777737 -18.333841 13.590875
17.360106
5 -28.488257 -26.835217 -94.69423 -22.945256 61.98943 -25.608461
8.299244 -21.966630 -15.793708 22.789544 29.101908 -7.190621 5.978456
1.159357
6 33.277830 -7.499987 35.82298 16.322987 13.86073 -69.452395
4.742326 -15.387490 38.546798 8.074732 20.960993 6.913245 -16.418731
-62.988210
7 31.597896 85.997623 50.37479 14.102621 -20.39702 -5.083443
13.273376 6.396868 35.725912 -24.358498 -8.912932 13.256769
21.854226 12.769635
8 - 36.257772 - 33.075149 - 32.39927 \quad 39.673921 \quad 10.47836 \quad 67.134337 - 10.47836 \quad 67.13437 - 10.47836 \quad 67.13437 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.47837 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 - 10.4787 -
91.244295 -3.762691 -15.973205 1.516351 -9.782938 -20.665305
3.022961 18.232222
      9.914896 -15.991490 39.50807 -19.840067 -31.70164 -15.977746
17.740782 32.377022 -11.376797 -7.432347 3.373452 22.054862 -
11.533962 -12.300182
        x.PC28 x.PC29 x.PC30 x.PC31
                                                                                     x.PC32 x.PC33
                 x.PC35 x.PC36 x.PC37 x.PC38 x.PC39
x.PC34
x.PC40
```

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0 \quad 30.426380 \quad 1.371630 \quad -7.2120726 \quad -20.2841109 \quad -7.0916226 \quad 13.575761
6.786120 - 0.6662605 - 17.930032 11.7555530 0.5663023 1.027203 -
3.3904127
1 21.025941 9.344218 4.4348680 26.6456527 -6.2335716 11.916089
1.276021 31.3592138 -2.712494 -1.8103055 12.5980331 -11.219077
5.8962463
2 \ -19.568090 \ -18.013744 \ 17.4673176 \ -9.1741413 \ 18.3475947 \ -10.434934 \ -
1.259149 - 8.8444783 - 24.733036 - 7.3661594 - 19.5822038 17.004890 -
2.1491633
3 - 23.104582 - 5.052624 - 0.7376129 - 0.1568662
                                                                                              0.5638363 -29.446862 -
13.480023 - 19.0734018 \quad 15.257512 \quad -4.7280894 \quad -30.5350359 \quad 4.734392 \quad -
                         1.601478 -18.9897359 -15.1980617 3.0522619 -9.858398
4 -9.309736
15.987383 10.3954406 -12.012298 -10.1550535 -12.6831419 -19.138311
0.5085038
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18.253155 1.4507386 5.211583 -17.7981942 -5.2533864 8.834056 -
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1.1468529
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16.462709 -8.6451248 -19.072292 9.9354788 -4.7936343 -3.364994
19.9917029
       1.744293 35.399932 18.8124481 10.0430153 18.3201260 0.243495
11.049195 0.7156720 31.717814 11.3206278 17.5680864 2.827444
4.6883934
9 -34.775322 1.476922 12.4887060 -9.1851081 25.7009081 -4.811144 -
11.328299 17.2506564 3.515687 0.9491449 1.0947153 -8.211064 -
13.7772779
            x.PC41
                               x.PC42 x.PC43
                                                                           x.PC44
                                                                                                x.PC45
                                                                                                                      x.PC46
                     x.PC48 x.PC49
x.PC47
                                                                     x.PC50
                                                                                              x.PC51
                                                                                                                      x.PC52
x.PC53
0 -2.7089264 12.484422 -4.736564 6.523969 -1.7100349 3.732052
6.8421103 - 2.9905119 \quad 3.031015 - 15.747692864 \quad 0.9959531 \quad 0.4132903 - 12.9905119 \quad 0.4132900 - 12.9905119 \quad 0.4132900 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.990000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 12.99000 - 
7.0091869
1 \quad 19.0127552 \quad 11.732709 \quad 3.535813 \quad 2.585805 \quad 6.7746773 \quad -7.810791 \quad -
4.1568575 -23.4155459 -5.629362 -7.845227214 15.1615604 -16.9268592
4.4173575
2 10.4969225 10.906544 16.997721
                                                                        1.899589 5.0473956 -1.705347
1.5467194 - 8.8472443 - 16.699360 - 16.914821108 - 2.7712593 6.6160867
3.7707787
3 -5.6598022 -17.319495  5.013454 -18.014022 -8.0774039 -5.728943 -
13.6150831 20.5804481 14.637666 19.208634628 -18.2797027 21.4893045
3.5186994
4 - 17.4094789 - 3.463938 - 6.196157 10.015211 - 2.2228522 1.317471 -
```

0.3150766 20.3760436 -4.249328 1.399131655 6.2177219 13.0478638 -

5 2.2245937 -13.564381 27.642408 13.671814 -4.5870910 -6.981977 23.1104150 0.3628822 2.245435 13.716805080 -0.7432098 -4.1749285 -

6 -8.4665511 -6.933485 -40.804437 -10.575284 -4.7089046 14.373720 -11.5682168 -7.0540216 11.545559 14.556208861 8.7444890 -21.4967304

4.1988622

16.8389091

0.6935902

```
7 -8.8195020 22.713491 -32.336188 -26.954978 -0.8569414 1.404662
1.3101801 0.9129498 11.176423 -9.543881506 -8.4692482 1.3961451
1.6293294
8 8.3646045 -6.960586 13.097341 2.613008 4.5005420 6.170967
3.1702289 6.8193134 -14.525323 -1.870620792 -9.9550466 -2.0325047 -
 2.4413927
9 \quad -0.2840812 \quad -4.847003 \quad 2.474342 \quad 13.249126 \quad 7.3895701 \quad -1.876719 \quad 
14.8142110 -5.1424872 -1.110130 0.005374994 11.8304281 1.0961149
21.7784140
                                  x.PC54 x.PC55 x.PC56 x.PC57 x.PC58
                                                                                                                                                                                                                                                                                                                             x.PC59
x.PC60
                                                            x.PC61 x.PC62 x.PC63 x.PC64 x.PC65
0 -1.0180187 -3.5882662 5.5239500 -5.172675 -10.0669269 -0.9038855 -
2.56444829 0.58598224 2.1205643 9.566402 -3.937092 -0.7819553 -
1.581355
1 \quad 3.7980125 \quad -6.0192179 \quad -0.3919504 \quad -1.207581 \quad 5.6551614 \quad 7.8150661 \quad -1.207581 
11.11654503 7.46718380 8.4968384 -2.010329 1.578382
                                                                                                                                                                                                                                                                                                       3.1643922 -
 3.609001
                   7.5289703 -7.7803211 -7.0485245 9.419235
                                                                                                                                                                                                                                             0.8612474 -16.2616888
4.61127752 0.03907355 4.5232888 2.720235 12.235023 3.6803323
8.177804
 3 -1.6443968 4.2636283 -10.6418748 -6.674595 -0.3085238
                                                                                                                                                                                                                                                                                                        2.3285450
6.61797773 -7.01868596 -0.5247082 3.937541 4.914695 5.6979732
10.775997
4 - 8.4138862 - 0.2944623 - 4.1470330 - 1.252446 8.7798335
                                                                                                                                                                                                                                                                                                        0.3831331
4.02221096 - 4.46378452 6.2486074 - 3.050565 - 7.907850 - 3.6700269 -
4.503034
5 - 13.8742640 3.3888876 -2.8661476 -2.236215 -4.9295679 10.9801321 -
4.01064045 \qquad 0.97648370 \quad -13.3624536 \quad -2.606208 \quad -2.385560 \qquad 0.8814345
11.028453
                                                                                                                                                                                                                                           1.6961744 0.2385126
                 9.2761969 0.3597750 13.0141949 -7.044355
1.50862087 8.76929745 -7.7424993 -13.598481 -10.618747 -11.9293890 -
 7.896427
                    9.2610532 -0.2667769
                                                                                                                              9.2625108 4.164463
                                                                                                                                                                                                                                           9.6147010 -9.5311297
2.20117879 - 11.86499100 \quad 4.2422538 \quad 4.099797 \quad 6.901972 \quad -6.6513982 \quad -6.6513
8.808596
8 -1.0766650 15.1701033 14.6957713 7.161339 6.7663347
                                                                                                                                                                                                                                                                                                        1.9286563 -
0.94154579 1.96078322 -1.0426614 6.509262 -5.798756
                                                                                                                                                                                                                                                                                                         6.0502101
1.533344
 9 -0.1554645 -5.0741580 -14.1971788 2.843231 -13.2918240 1.9176958 -
0.08682429 2.92022313 2.5963603 -6.017914 2.954978 1.8168999 -
12.285306
                             x.PC67
                                                                                 x.PC68
                                                                                                                                               x.PC69
                                                                                                                                                                                                           x.PC70
                                                                                                                                                                                                                                                                  x.PC71
                                                                                                                                                                                                                                                                                                                         x.PC72
x.PC73
                                                 x.PC74
            1.429914 -3.31886266 3.3212827 -3.0210364 -2.532040 0.1932055
2.0005665 5.7539210
1 - 2.979899 - 7.04768639 - 2.5384711 - 3.3613944 - 4.748624 - 0.5213181 -
0.3384956 0.3443229
2 -1.126958 5.49197131 1.9747392 -4.3398997 -11.574341 2.0982575
12.6053342 1.2976511
 3 6.017167 -4.46406594 -0.8790432 -1.4794235 1.551159 -3.9144256 -
5.3571188 -4.2244288
4 \quad -7.013861 \quad -5.04057031 \qquad 3.2471341 \quad 14.1368063 \quad -8.597618 \quad -1.5113857 \quad -
3.8667446 -5.4831166
```

```
1.139487 0.04989391 -13.9266015 5.0923210 -5.083219 0.8025129
3.9247816 -5.5048930
   4.456981 0.29478398 0.8352892 -0.5638588 12.724475 1.2544699 -
6.5123723 5.6645875
7 -10.538076 6.21727665 2.7954719 2.2626387 9.221793 -5.8887337
7.7557313 6.4780741
  9.315196 -7.33199780 6.4233113 -8.9464222 -3.489707 3.2344084 -
6.3659546 -8.1738655
9 -2.706854 0.53538815 3.8807207 0.7673845 6.684379 2.3406726 -
7.5525374 4.9216078
> qda.pred <- predict(qda.fit, dat.pca.test)</pre>
> #Error rate
> mean(qda.pred$class != dat.pca.test$y)
[1] 0.2633333
> #QDA using full dimension
> #Some group is too small for QDA. Tried running with character
> #type for y-values. It seems that the problem is with another
> #column
> qda.train <- dat.train</pre>
> gda.train$y <- as.character(dat.train$y)</pre>
> system.time(qda.fit.full<-qda(y~., data=qda.train))
Error in qda.default(x, grouping, ...) :
  some group is too small for 'qda'
Timing stopped at: 0.08 0 0.08
> #Part III: Deep Learning
> xx train <- dat[1:60000, 1:784]
> yy train <- to categorical(dat[1:60000,785],10)</pre>
> xx test <- dat[60001:70000,1:784]</pre>
> yy_test <- dat[60001:70000,785]</pre>
> xx train <- xx train/255
> xx test <- xx test/255
> xx train<-array reshape(as.matrix(xx train),c(60000,784))</pre>
> xx test<-array reshape(as.matrix(xx test),c(10000,784))</pre>
> modelnn <- keras model sequential()</pre>
2023-05-18 09:28:47.569104: I
tensorflow/core/platform/cpu feature guard.cc:193] This TensorFlow binary
is optimized with oneAPI Deep Neural Network Library (oneDNN) to use the
following CPU instructions in performance-critical operations: AVX AVX2
To enable them in other operations, rebuild TensorFlow with the
appropriate compiler flags.
> modelnn %>%
    layer dense (units = 256, activation = "relu",
                input shape = c(784)) %>%
    layer_dropout(rate = 0.4) %>%
    layer dense(units = 128, activation = "relu") %>%
    layer dropout(rate = 0.3) %>%
    layer dense(units = 10, activation = "softmax")
> modelnn %>% compile(loss = "categorical crossentropy",
                      optimizer = optimizer rmsprop(), metrics =
c("accuracy")
+ )
> system.time(
+ history <- modelnn %>%
```

```
fit(xx_train, yy_train, epochs = 30, batch size = 128,
       validation split = 0.2)
+ )
Epoch 1/30
375/375 [============== ] - 7s 17ms/step - loss: 0.6812 -
accuracy: 0.7575 - val loss: 0.4544 - val accuracy: 0.8294
Epoch 2/30
375/375 [============ ] - 5s 13ms/step - loss: 0.4721 -
accuracy: 0.8275 - val loss: 0.4161 - val accuracy: 0.8474
Epoch 3/30
375/375 [============ ] - 5s 13ms/step - loss: 0.4310 -
accuracy: 0.8448 - val loss: 0.3813 - val accuracy: 0.8548
Epoch 4/30
accuracy: 0.8551 - val loss: 0.3487 - val accuracy: 0.8707
Epoch 5/30
375/375 [============= ] - 4s 12ms/step - loss: 0.3886 -
accuracy: 0.8600 - val loss: 0.3607 - val_accuracy: 0.8732
Epoch 6/30
accuracy: 0.8659 - val loss: 0.3665 - val accuracy: 0.8673
Epoch 7/30
375/375 [============ ] - 4s 12ms/step - loss: 0.3630 -
accuracy: 0.8690 - val loss: 0.3614 - val accuracy: 0.8712
Epoch 8/30
375/375 [============ ] - 5s 12ms/step - loss: 0.3545 -
accuracy: 0.8721 - val loss: 0.3362 - val accuracy: 0.8800
Epoch 9/30
375/375 [============= ] - 5s 12ms/step - loss: 0.3474 -
accuracy: 0.8744 - val loss: 0.3418 - val accuracy: 0.8797
Epoch 10/30
375/375 [============= ] - 5s 12ms/step - loss: 0.3397 -
accuracy: 0.8767 - val loss: 0.3473 - val accuracy: 0.8788
Epoch 11/30
accuracy: 0.8795 - val loss: 0.3284 - val accuracy: 0.8842
Epoch 12/30
accuracy: 0.8824 - val loss: 0.3269 - val accuracy: 0.8843
Epoch 13/30
accuracy: 0.8839 - val loss: 0.3427 - val accuracy: 0.8845
Epoch 14/30
accuracy: 0.8845 - val loss: 0.3264 - val accuracy: 0.8878
Epoch 15/30
accuracy: 0.8850 - val loss: 0.3316 - val accuracy: 0.8838
Epoch 16/30
375/375 [============ ] - 4s 12ms/step - loss: 0.3132 -
accuracy: 0.8869 - val loss: 0.3247 - val accuracy: 0.8867
Epoch 17/30
375/375 [============ ] - 4s 12ms/step - loss: 0.3089 -
accuracy: 0.8870 - val loss: 0.3305 - val accuracy: 0.8888
```

```
Epoch 18/30
accuracy: 0.8875 - val loss: 0.3379 - val accuracy: 0.8878
Epoch 19/30
accuracy: 0.8905 - val loss: 0.3406 - val accuracy: 0.8866
Epoch 20/30
375/375 [============ ] - 4s 11ms/step - loss: 0.3033 -
accuracy: 0.8907 - val loss: 0.3280 - val accuracy: 0.8908
Epoch 21/30
375/375 [============ ] - 4s 11ms/step - loss: 0.3006 -
accuracy: 0.8920 - val loss: 0.3330 - val accuracy: 0.8871
Epoch 22/30
accuracy: 0.8934 - val loss: 0.3351 - val accuracy: 0.8921
Epoch 23/30
375/375 [============= ] - 4s 11ms/step - loss: 0.2967 -
accuracy: 0.8939 - val loss: 0.3413 - val_accuracy: 0.8870
Epoch 24/30
accuracy: 0.8940 - val loss: 0.3269 - val accuracy: 0.8878
Epoch 25/30
375/375 [============= ] - 4s 11ms/step - loss: 0.2856 -
accuracy: 0.8964 - val loss: 0.3346 - val accuracy: 0.8914
Epoch 26/30
accuracy: 0.8959 - val loss: 0.3363 - val accuracy: 0.8898
Epoch 27/30
375/375 [============= ] - 4s 11ms/step - loss: 0.2828 -
accuracy: 0.8979 - val loss: 0.3379 - val accuracy: 0.8928
Epoch 28/30
accuracy: 0.8968 - val loss: 0.3475 - val accuracy: 0.8911
Epoch 29/30
accuracy: 0.8980 - val loss: 0.3451 - val accuracy: 0.8928
Epoch 30/30
accuracy: 0.9000 - val loss: 0.3308 - val accuracy: 0.8941
 user system elapsed
       5.28 134.03
 52.25
> accuracy <- function(pred, truth)</pre>
+ mean(drop(as.numeric(pred)) == drop(truth))
> Error =
+ 1-modelnn %>% predict(xx test) %>% k argmax() %>% accuracy(yy test)
313/313 [============ ] - 1s 3ms/step
> #Error rate:
> Error
[1] 0.1151
> #Multinomial Logistic Regression
> #Too many weights. Using pca data instead.
> mn.train <- dat.pca.train</pre>
> system.time(mn test <- multinom(y ~ ., data = mn.train))</pre>
# weights: 760 (675 variable)
```

```
initial value 2763.102112
iter 10 value 725.025834
iter 20 value 561.262430
iter 30 value 520.839954
iter 40 value 509.910104
iter 50 value 503.940932
iter 60 value 501.270286
iter 70 value 500.112692
iter 80 value 499.772312
iter 90 value 499.608408
iter 100 value 499.550384
final value 499.550384
stopped after 100 iterations
  user system elapsed
  1.29 0.00 1.30
> mn.train$ClassPredicted <- predict(mn test,
                                 newdata = dat.pca.test, "class")
> tab <- table(mn.train$Class, mn.train$ClassPredicted)</pre>
> #DNN with PCA
> xx train s <- dat.train[1:1200,1:784]</pre>
> yy train s <- to categorical(dat.train[1:1200,785],10)</pre>
> xx test s <- dat.test[1:600,1:784]</pre>
> yy test s <- dat.test[1:600,785]</pre>
> xx train s <- xx train s/255
> xx test s <- xx test s/255
> xx train s<-array reshape(as.matrix(xx train s),c(1200,784))
> xx test s<-array reshape(as.matrix(xx test s),c(600,784))</pre>
> modelnn s <- keras model sequential()</pre>
> modelnn s %>%
   layer dense(units = 256, activation = "relu",
               input shape = c(784)) %>%
   layer dropout (rate = 0.4) %>%
   layer dense(units = 128, activation = "relu") %>%
   layer dropout(rate = 0.3) %>%
   layer dense(units = 10, activation = "softmax")
> modelnn s %>% compile(loss = "categorical crossentropy",
                     optimizer = optimizer rmsprop(), metrics =
c("accuracy")
+ )
> system.time(
  history <- modelnn s %>%
      fit(xx train s, yy train s, epochs = 30, batch size = 128,
         validation split = 0.2)
+ )
Epoch 1/30
accuracy: 0.3438 - val loss: 1.3484 - val accuracy: 0.4958
Epoch 2/30
8/8 [============= ] - Os 34ms/step - loss: 1.2405 -
accuracy: 0.5573 - val loss: 0.9875 - val accuracy: 0.6458
Epoch 3/30
8/8 [=========== ] - Os 34ms/step - loss: 1.0918 -
accuracy: 0.6125 - val loss: 0.7944 - val accuracy: 0.7917
Epoch 4/30
```

```
8/8 [============ ] - Os 34ms/step - loss: 0.9506 -
accuracy: 0.6781 - val loss: 0.7422 - val_accuracy: 0.7833
Epoch 5/30
8/8 [=========== ] - Os 34ms/step - loss: 0.8670 -
accuracy: 0.6906 - val loss: 0.7143 - val accuracy: 0.8000
Epoch 6/30
accuracy: 0.7115 - val loss: 0.6715 - val accuracy: 0.7792
Epoch 7/30
8/8 [============= ] - Os 36ms/step - loss: 0.7568 -
accuracy: 0.7188 - val loss: 0.6381 - val accuracy: 0.8042
Epoch 8/30
8/8 [=========== ] - Os 34ms/step - loss: 0.7320 -
accuracy: 0.7406 - val loss: 0.6151 - val accuracy: 0.7958
Epoch 9/30
8/8 [========== ] - Os 31ms/step - loss: 0.6531 -
accuracy: 0.7688 - val loss: 0.7037 - val accuracy: 0.7875
Epoch 10/30
accuracy: 0.7573 - val loss: 0.5898 - val accuracy: 0.8042
Epoch 11/30
accuracy: 0.7792 - val loss: 0.6133 - val accuracy: 0.8125
Epoch 12/30
8/8 [=========== ] - Os 32ms/step - loss: 0.5984 -
accuracy: 0.7823 - val_loss: 0.5527 - val_accuracy: 0.8417
Epoch 13/30
8/8 [========== ] - 0s 34ms/step - loss: 0.5701 -
accuracy: 0.7875 - val loss: 0.6119 - val accuracy: 0.7792
Epoch 14/30
8/8 [========= ] - 0s 32ms/step - loss: 0.5315 -
accuracy: 0.8115 - val loss: 0.5525 - val accuracy: 0.8292
Epoch 15/30
8/8 [============ ] - Os 36ms/step - loss: 0.5307 -
accuracy: 0.8167 - val loss: 0.5666 - val accuracy: 0.8292
Epoch 16/30
8/8 [=========== ] - Os 34ms/step - loss: 0.5449 -
accuracy: 0.8062 - val loss: 0.5852 - val accuracy: 0.8375
Epoch 17/30
8/8 [========== ] - 0s 34ms/step - loss: 0.4824 -
accuracy: 0.8281 - val loss: 0.7037 - val accuracy: 0.7750
Epoch 18/30
8/8 [========== ] - Os 32ms/step - loss: 0.4972 -
accuracy: 0.8271 - val loss: 0.5218 - val accuracy: 0.8542
Epoch 19/30
8/8 [=========== ] - Os 34ms/step - loss: 0.4380 -
accuracy: 0.8406 - val loss: 0.6496 - val accuracy: 0.8000
Epoch 20/30
8/8 [========== ] - 0s 32ms/step - loss: 0.4608 -
accuracy: 0.8302 - val loss: 0.5903 - val accuracy: 0.8583
Epoch 21/30
8/8 [=========== ] - Os 34ms/step - loss: 0.4508 -
accuracy: 0.8469 - val_loss: 0.5512 - val_accuracy: 0.8542
Epoch 22/30
```

```
accuracy: 0.8479 - val loss: 0.5973 - val_accuracy: 0.8333
Epoch 23/30
8/8 [========= ] - Os 36ms/step - loss: 0.4044 -
accuracy: 0.8656 - val loss: 0.5495 - val accuracy: 0.8375
Epoch 24/30
8/8 [============= ] - Os 32ms/step - loss: 0.3781 -
accuracy: 0.8583 - val loss: 0.5717 - val accuracy: 0.8333
Epoch 25/30
8/8 [========== ] - Os 32ms/step - loss: 0.3762 -
accuracy: 0.8687 - val loss: 0.5201 - val accuracy: 0.8417
Epoch 26/30
accuracy: 0.8469 - val loss: 0.5409 - val accuracy: 0.8667
Epoch 27/30
8/8 [============= ] - Os 37ms/step - loss: 0.3784 -
accuracy: 0.8677 - val loss: 0.5966 - val accuracy: 0.8583
Epoch 28/30
8/8 [============= ] - 0s 32ms/step - loss: 0.3370 -
accuracy: 0.8740 - val loss: 0.5662 - val accuracy: 0.8667
Epoch 29/30
8/8 [============ ] - Os 32ms/step - loss: 0.3242 -
accuracy: 0.8823 - val loss: 0.5549 - val accuracy: 0.8375
Epoch 30/30
accuracy: 0.8844 - val loss: 0.5479 - val accuracy: 0.8625
  user system elapsed
  2.27 0.14 8.53
> #Elapsed time is 7.4s
> accuracy_s <- function(pred, truth)</pre>
+ mean(drop(as.numeric(pred)) == drop(truth))
> Error =
+ 1-modelnn s %>% predict(xx test) %>% k argmax() %>% accuracy(yy test)
> #Error rate
> Error
[1] 0.1935
```