Discrete Structures II

Spring 2021

Homework III

1. Which of the following languages is/are recursive?

   (a) $L_1 = \{< M >: 0 \in L(M)\}$

   (b) $L_2 = \{< M >: L(M) \text{ is context-free}\}$

   (c) $L_3 = \{< M >: L(M) = \phi\}$

   (d) $L_4 = \{< M >: L_d \subset L(M)\}$

   (e) $L_5 = \{< M >: L(M) \text{ is not r.e.}\}$

2. Which of the following problems is/are decidable?

   (a) Given a TM M; does M accept a string that starts with a 0?

   (b) Given a TM M; does M accept a string of length > 10?

   (c) Given a TM M; is $L(M)$ empty?

3. Is the set of non-recursive languages closed under complement? Prove your answer.

4. Is the set of non r.e languages closed under complement? Justify.

5. Is the set of non r.e languages closed under union or intersection? Prove your answer.

6. Consider the language $L_{ne} = \{< M >: L(M) \neq \phi\}$ (defined in 9.7.3). Is L_{ne} recursive? What can you say about its complement? Prove your answer.

7. Prove that $4SAT \propto 5SAT$.

8. Prove that $6SAT \propto 4SAT$.

9. In the Guard Cover problem we are given a graph G and a positive integer k and asked whether G has a set S of k or less vertices such that every vertex of G can be reached from a vertex of S by a path of at most two edges. Show that Guard Cover is NP-Complete by reduction from Vertex Cover.

10. In the Maximum Common Edge Subgraph problem ($MCES$) the input consists of two graphs G_1 and G_2 and an integer $k > 0$ and the posed question is whether there is a subgraph of G_1 consisting of k edges that is isomorphic to a subgraph of G_2. Prove that $MCES$ is NP-hard (by reduction from a suitable problem of your choice).

11. Show that Dominating Set $\propto$ Set Cover.