group 2

2023-09-16

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1. simulating Data

```
#install.packages(c("fields", "MASS"))
library(fields)  # for Matern and rdist functions
```

```
## Warning: package 'fields' was built under R version 4.2.3
```

```
## Loading required package: spam
```

```
## Warning: package 'spam' was built under R version 4.2.3
```

```
## Spam version 2.9-1 (2022-08-07) is loaded.
## Type 'help( Spam)' or 'demo( spam)' for a short introduction
## and overview of this package.
## Help for individual functions is also obtained by adding the
## suffix '.spam' to the function name, e.g. 'help( chol.spam)'.
```

```
##
## Attaching package: 'spam'
```

```
## The following objects are masked from 'package:base':
##
##     backsolve, forwardsolve
```

```
## Loading required package: viridisLite
```

```
##
## Try help(fields) to get started.
```

```
library(MASS)     # for rmvnorm function
```

```
## Warning: package 'MASS' was built under R version 4.2.3
```

```r
# Define the sequence of coordinates
coords_xygrid = seq(0, 10, length=10)

# Number of simulations
num_simulations <- 1000

# Store results for each simulation
z.grid_list <- vector("list", num_simulations)

for(sim in 1:num_simulations) {
  # Compute the Matern covariance matrix
  Sigma_grid = fields::Matern(rdist(coords_xygrid), alpha=1, nu=0.5, phi=1.0)

  # Simulate z.grid based on the Gaussian process with Matern covariance
  z.grid_list[[sim]] = rmvnorm(1, mu = rep(5, 10), Sigma = Sigma_grid)
  tempL = rep(0,1000)
  tempL[sim] = mean(z.grid_list[[sim]])
}
```

#2. Definning necessary functions

```
#a) Covariance Function:
#We'll use an exponential covariance function



# Define c(0; theta) function
c0_function <- function(phi) {
  return(1)
}

#b) c(0; ϑ) Function:
#This function represents the covariance at lag 0.

# Define covariance function
cov_function <- function(si, sj, theta) {
  # Ensure that inputs are numeric
  if (!is.numeric(si) || !is.numeric(sj) || !is.numeric(theta)) {
    stop("All inputs to cov_function must be numeric.")
  }

  # Compute the distance
  distance <- abs(si - sj)

  # Check if distance is NA
  if (any(is.na(distance))) {
    stop("Computed distance is NA.")
  }

  # Compute the covariance
  covariance <- exp(-theta * distance)

  # Check if covariance is NA
  if (any(is.na(covariance))) {
    stop("Computed covariance is NA.")
  }

  return(covariance)
}
```

3. Estimating Parameters using the vicchia Likelihood

```r
# Vecchia approximation likelihood
vecchia_likelihood <- function(params, z.grid, coords) {
  mu = params[1] # the mean of the Gaussian process
  phi = params[2] # parameter related to the covariance function.
  #print(paste("z.grid_list",z.grid_list))

  n <- length(z.grid)
  #print(paste("n",n))


  QV = 0
  for (i in 1:n) {
    # For simplicity, let's use the first two observations as neighbors (adjust as needed)
    if (i == 1) {
          neighbors = 1
      } else if (i == 2) {
          neighbors = 1:2
      } else {
          neighbors = (i-2):i
      }
    z_neighbors = z.grid[neighbors]
    #print(paste("z_neighbors:", z_neighbors))
    r = cov_function(z.grid[i], z_neighbors, phi)
    #print(paste("r:", r))
    Sigma = matrix(cov_function(z_neighbors, z_neighbors, phi), length(z_neighbors), length(z_ne
ighbors))

    #print(paste("sigma:", Sigma))
    Sigma_inv = ginv(Sigma)
    #print(paste("Sigma_inv:", Sigma_inv))

    term1_value = c0_function(phi) - t(r) %*% Sigma_inv %*% r
    if (is.na(term1_value) || term1_value <= 0) {
      term1_value = 1e10  # Return a large value to avoid optimization issues
    }
    #print(paste("term1_value:", term1_value))

    term1 = log(term1_value)
    #print(paste('term1',term1))

    term2_numerator = z.grid[i] - mu - t(r) %*% Sigma_inv %*% (z_neighbors - mu)
    if (is.na(term2_numerator) || term2_numerator <= 0) {
      term2_numerator = 1e10
    }
    #print(paste('term2_numerator',term2_numerator))

    term2_denominator = c0_function(phi) - t(r) %*% Sigma_inv %*% r
    if (is.na(term2_denominator) ||term2_denominator <= 0) {
      term2_denominator = 1e10  # Return a large value to avoid optimization issues
    }
    #print(paste('term2_denominator',term2_denominator))

    term2 = term2_numerator^2 / term2_denominator
```

```
    QV = QV - term1 - term2
    #print(paste("QV",QV))
  }
  return(-QV)
}


results_matrix <- matrix(0, nrow= 1000 , ncol=2)
#loop over each simulation
for (sim in 1:1000){
  initial_params <- c(mean(z.grid_list[[sim]]), 90)
  lower_bounds <- c(0, 0)
  upper_bounds <- c(10, 180)
  # Run optimization for the current simulation
  result <- optim(par = initial_params, fn = vecchia_likelihood, method = "L-BFGS-B",lower=lower
_bounds, upper=upper_bounds, z.grid = z.grid_list[[sim]], coords = coords_xygrid)

   # Check for convergence
  if(result$convergence != 0) {
    print(paste("Warning: Optimization did not converge for simulation", sim, "with convergence
code", result$convergence))
  }


  mu_estimated = result$par[1]
  phi_estimated = result$par[2]

# Print the estimated parameters with their names
  print(paste('mu:', mu_estimated, 'phi', phi_estimated))#
  results_matrix[sim, ] <- result$par

}
```

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## [1] "mu: 5.10783417483148 phi 180"
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