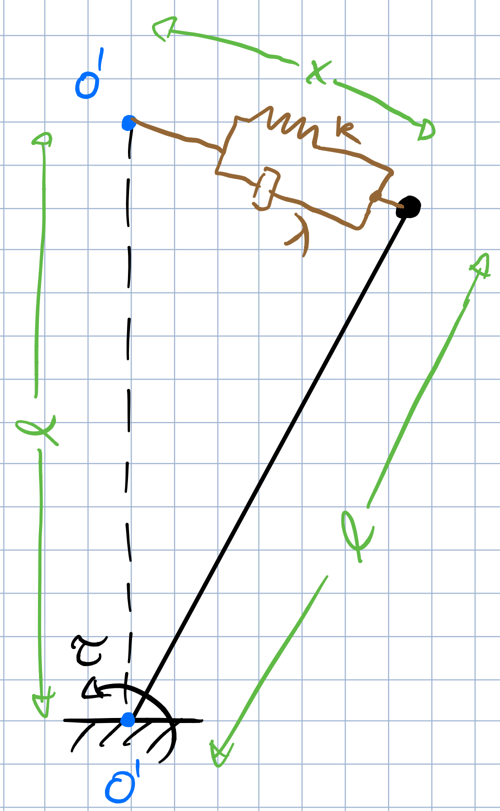
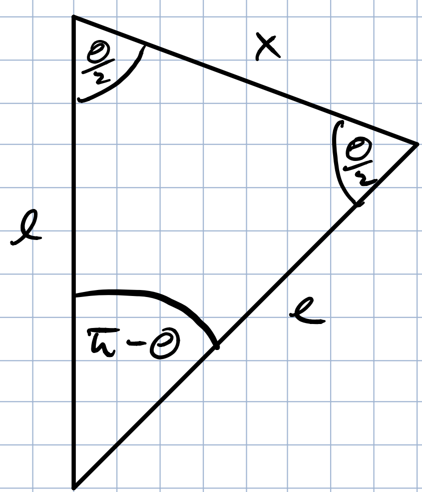
Virtual Model Control – Pendulum



- Stabilisation of "upside down" pendulum by using "virtual" spring & dashpot
- Refer moments generated by spring & dashpot (O') to motor (O)
- θ +-ve ccw

  $\theta = 0$ @ pendulum hanging

Isosceles triangle: (mounts @ l above O!)



Extension

$$x = 2l \sin\left(\frac{\pi - \theta}{2}\right) = 2l \cos\left(\frac{\theta}{2}\right)$$

$$\dot{x} = -l\dot{\theta}\sin\left(\frac{\theta}{2}\right)$$

Force from spring & dashpot $\perp$ to rod:

$$F_{s\perp} = F_s \cdot \sin\left(\frac{\theta}{2}\right) = 2kl\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$\therefore F_{s\perp} = kl\sin(\theta)$$

and

$$F_{d\perp} = -\lambda l \dot{\theta} \sin\left(\frac{\theta}{2}\right)$$

$\Rightarrow$ Equivalent moment $\rightarrow$ Torque:

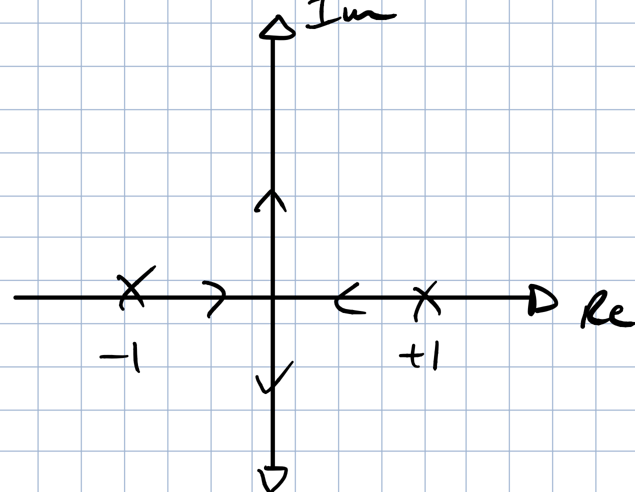
$$\tau = \left(F_{s\perp} + F_{d\perp}\right) l$$

$$\therefore \quad \tau = l^2\left[k\sin(\theta) - \lambda\dot{\theta}\sin\left(\frac{\theta}{2}\right)\right]$$

Spring-only controller <u>cannot</u> stabilise system, since...

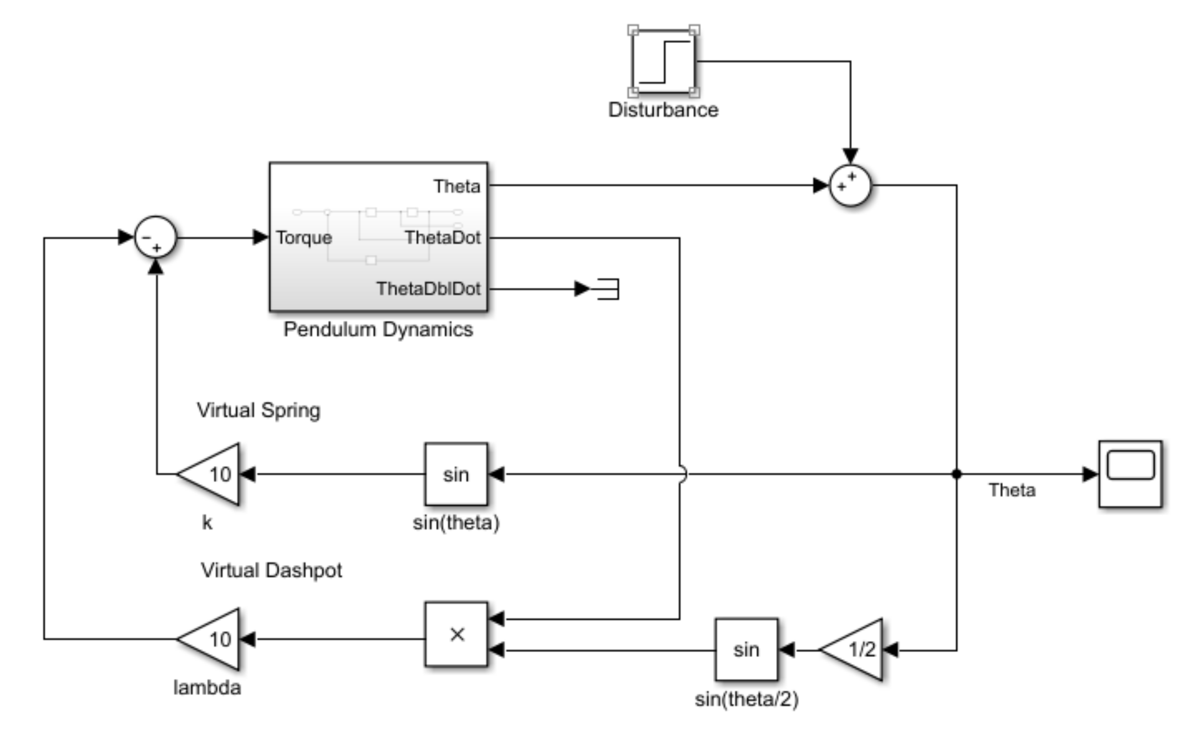
Linearised pendulum model (unstable equilibrium, $\theta_0 = 180°$) : $\frac{\bar{\theta}}{\bar{\tau}} = \frac{1}{s^2 - 1}$

↳ Root Locus:



↳ Spring is effectively non-linear proportional controller, thus only ~~unstable~~ marginally stable! Need dashpot ↳ "D" controller!

MatLAB Simulation:



Initial condition: $\Theta = 175^\circ$ + Step disturbance @ $t = 10s$