Quantum Communication Protocols Utilizing Higher Dimensional Embedding

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Abstract

We propose a novel Quantum Dense Coding protocol, 'Embedded Dense Coding'. Embedded Dense Coding utilizes higher dimensional quantum state embedding to maximize the information contained within a single quantum state for a given entanglement resource. This protocol has fidelity advantages when subject to specific kinds of noise and shows promise as a more efficient communication protocol.

Background

- Unlike traditional communication with 1s and 0s, quantum communication uses qubits, which can exist in a superposition of 0 and 1, enabling richer forms of data encoding.
- Quantum entanglement allows qubits to share a correlated state, which can be used advantageously in communication protocols.

Qubit Dense Coding utilizes these concepts by allowing Alice (the sender) to **transmit two classical bits** to Bob (the receiver) by sending **only one qubit**, utilizing entanglement and local quantum operations.

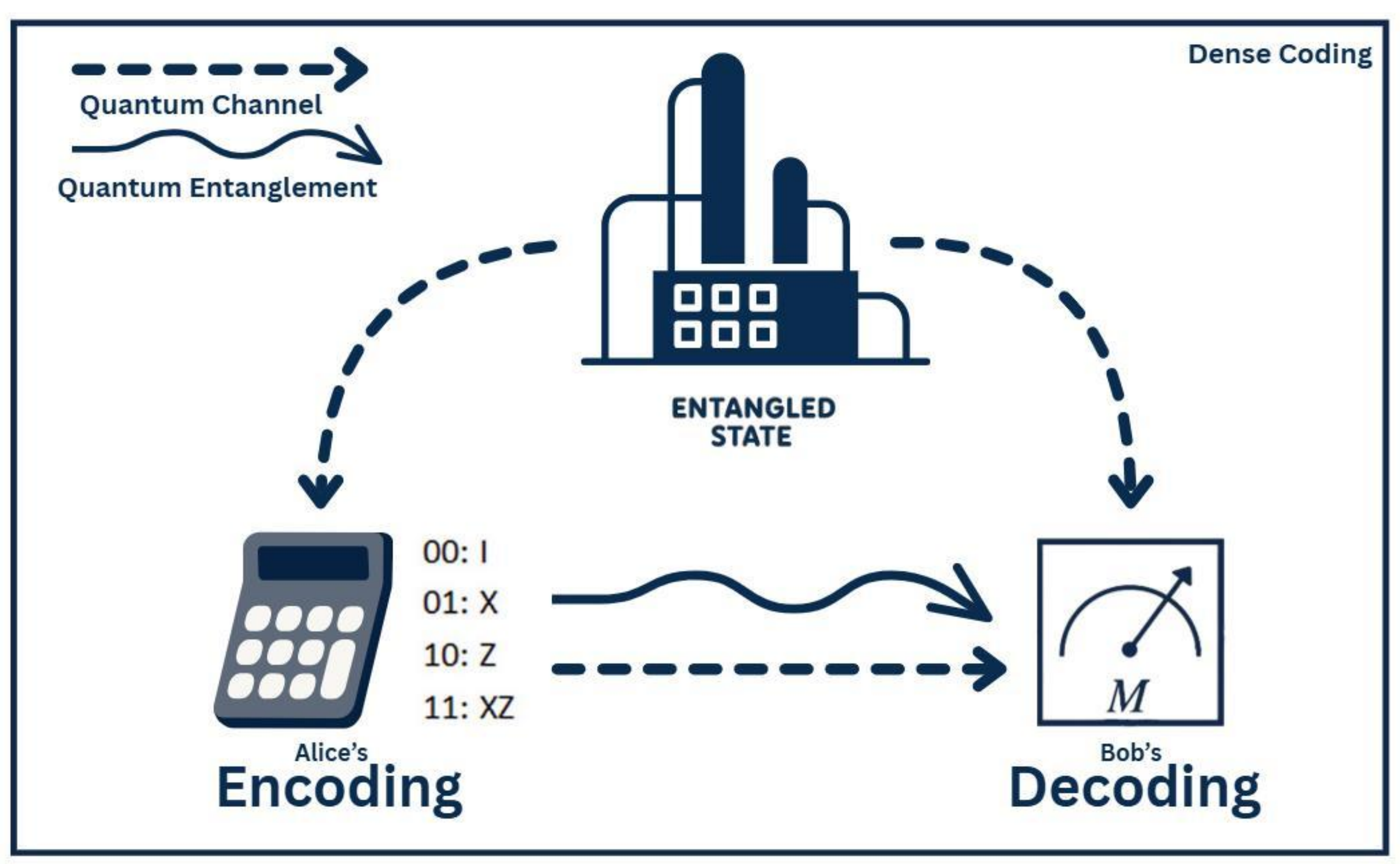

Figure 1: Alice chooses to apply a Z gate, an X gate, both, or neither, resulting in 4 distinct outputs. By doing this, she transmits 2 bits of information by sending only 1 qubit.

Dense Coding can be impractical:

- Perfect entanglement is hard to achieve and distribute over real-world noisy channels.
- Current classical systems are robust, less costly, and easier to scale.

References

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Embedded Dense Coding

Higher Dimensional Embedding

- A quantum state in Hilbert space $\mathcal{H}_A$ can be embedded into a higher dimensional space $\mathcal{H}_B$ via isometric embedding.
- In embedded dense coding, two parties, Alice and Bob, share a entangled state in some lower dimensional quantum state.
- Alice and Bob are able to embed their joint quantum state into a higher dimension which allows them to encode more information.

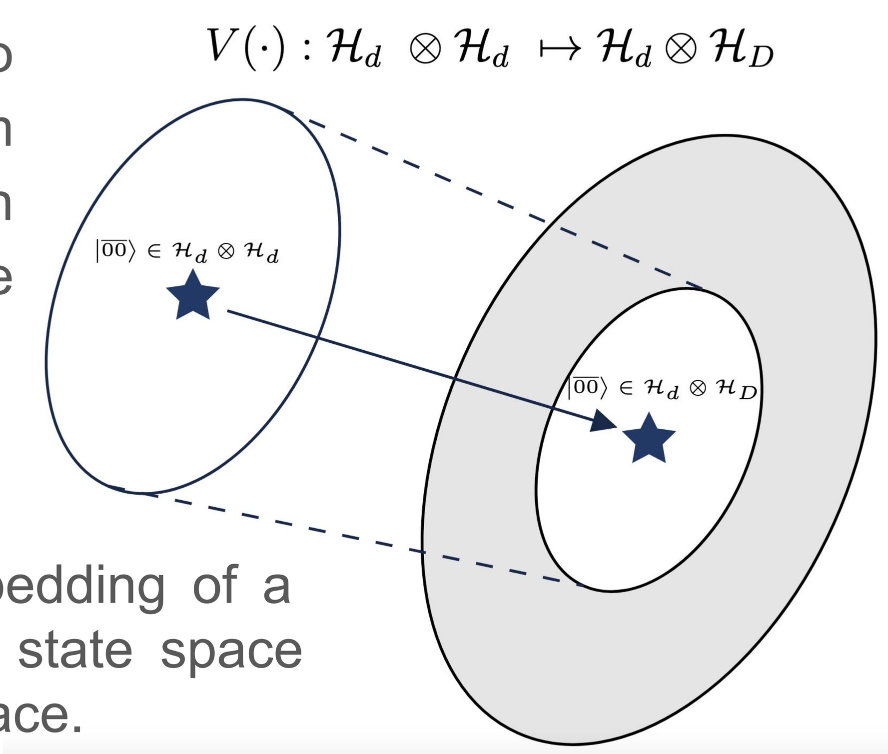

Figure 2: A Isometric embedding of a lower dimensional quantum state space into a higher dimensional space.

Embedded Dense Coding Protocol

- In typical d dimensional dense coding protocol Bob encodes their message by applying one of d Z gates and one of d X gates.
- In EDC, Bob performs one of d Z gates pre embedding, and performs one of D X gates post embedding. This results in d ✕ D unique encodings as opposed to d ✕ d.
- Alice can measure the joint state and recover the encoded message from the D dimensional quantum state.

Theorem: The success probability of a EDC protocol, with a maximally entangled resource, subject to dephasing noise with probability *p* is given by

$$p_{succ} = 1 - \frac{pD(d-1)}{dD - d}$$

Here we can see that higher dimensional embedding reduces the effect of environmental dephasing noise.

This protocol allows Alice and Bob to communicate more messages through one physical quantum state, while utilizing the same entanglement resource as in the typical dense coding scheme. This results in a potentially more efficient use of a given resource, and a potentially faster transfer of information.

Quantum Computer Performance

The protocol was tested on IBM Quantum Computers

- No native support for qudits
- Simulate qudit behavior using encoding into pairs of qubits
- Two alternative Dense Coding protocols
  - Embedding both particles in entangled pair
  - Only embedding Alice's particle to a higher dimension

$$|00\rangle \to |0\rangle \quad |01\rangle \to |1\rangle$$
$$|10\rangle \to |2\rangle \quad |11\rangle \to |3\rangle$$

Figure 3: Example of a pair of qubits being encoded into a 4-state qudit

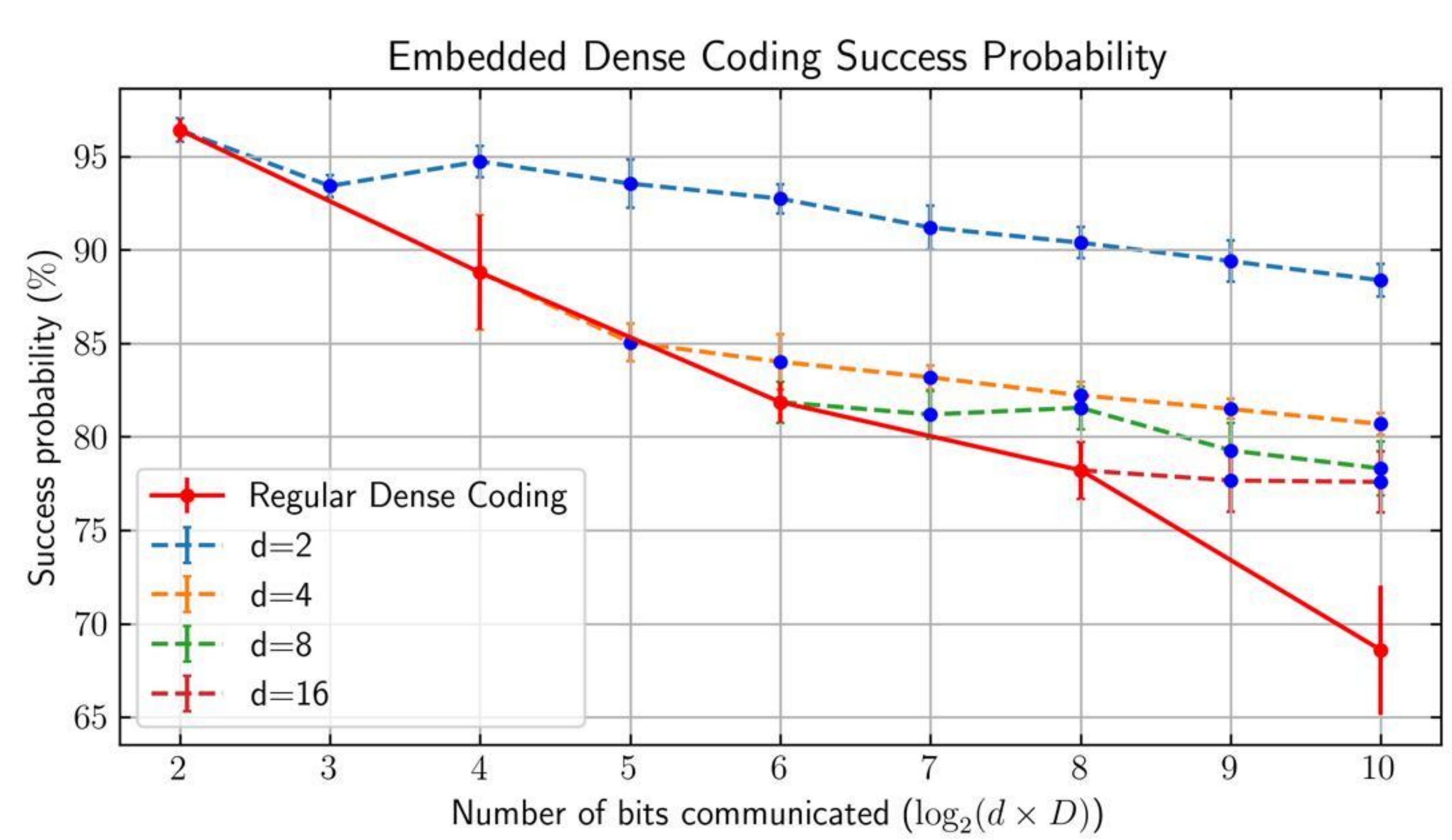

Figure 5: EDC protocol on IBM Torino Quantum Computer with 10,000 shots for six separate trials.

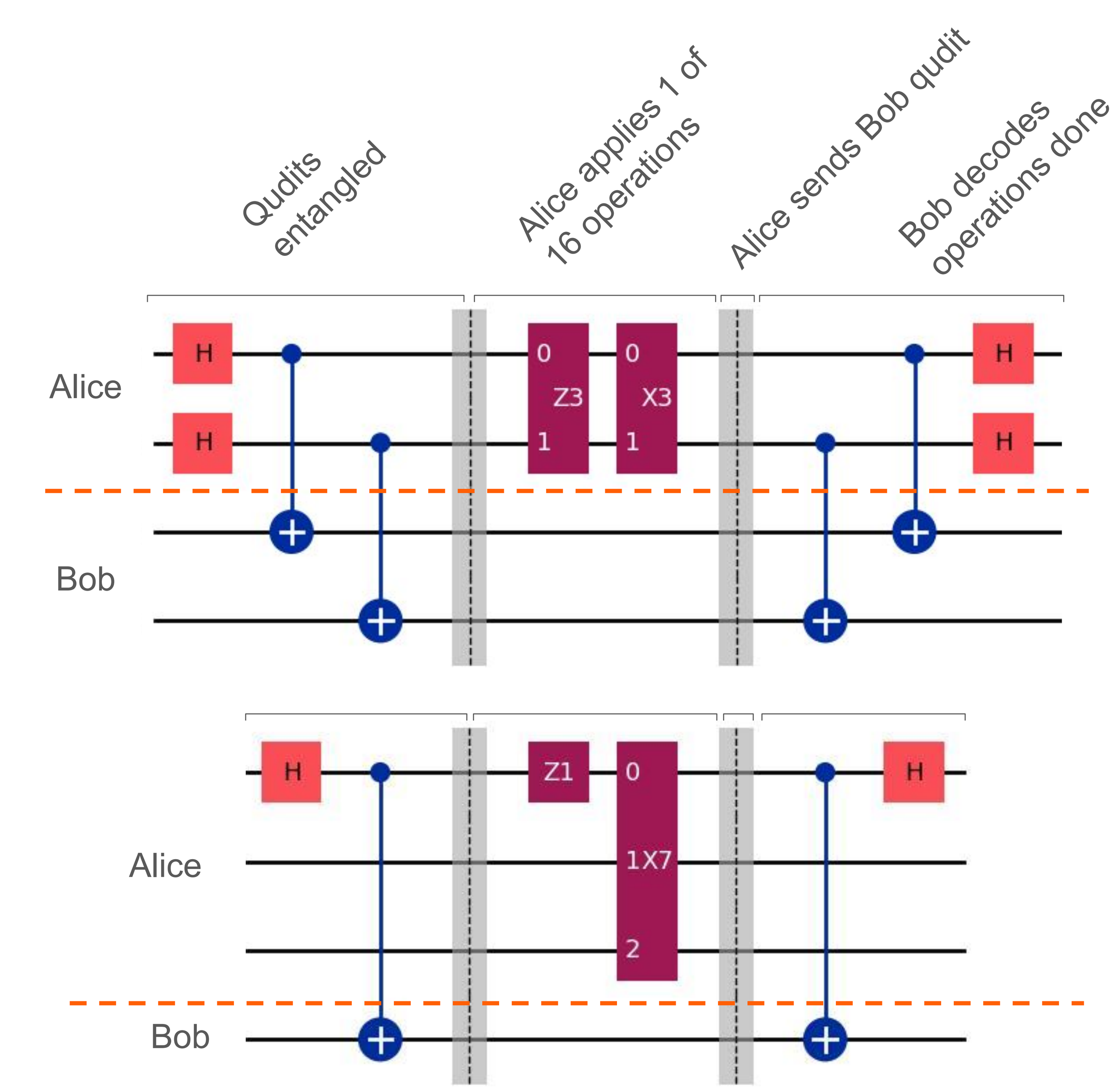

Figure 4:
Top: Dense Coding Protocol using d=D=4 qudits. Alice transmits 4 bits of information by sending one d=4 qudit.
Bottom: Embedded Dense Coding Protocol using d=2, D=8. 4 bits are transmitted by sending one d=8 qudit, using less entanglement.