Tutorial 10 – Further DP Examples

These are various DP examples, several of which come from the Winston textbook.

Strawberries

The owner of a chain of three grocery stores has purchased five crates of fresh strawberries. The estimated potential sales of the strawberries before spoilage differs among the three stores. Therefore, the owner wishes to know how she should allocate the five crates to the three stores to maximize expected profit.

The owner does not wish to split crates between stores but she is willing to distribute zero crates to any of her stores. The following table gives the estimated expected profit at each store when it is allocated various numbers of crates:

Crates	Store 1	Store 2	Store 3
0	$0	$0	$0
1	$3	$5	$4
2	$7	$10	$6
3	$9	$11	$11
4	$12	$11	$12
5	$13	$11	$12

How many of the five crates should be assigned to each of the three stores to maximize the total expected profit?

Democracy

A State consists of three cities with populations 1.2 million people, 1.4 million people and 400,000 people. The House of Representatives consists of three representatives. Given proportional representation, City 1 should have $d_1 = 3(1.2/3) = 1.2$ representatives; City 2 should have $d_2 = 1.4$ representatives; and City 3 should have $d_3 = 0.4$ representatives. Since each city must receive an integral number of representatives, this is impossible.

The State has therefore decided to allocate x_i representatives to city i, where the allocation should minimize the maximum discrepancy between the desired and actual number of representatives received by a city. How many representatives should each city receive?

Altitude Sickness

Joe Cougar needs to drive from A to J. Due to a medical condition he wants to avoid high altitude. The following figure gives the maximum altitude of roads between intermediate cities on the way to J.
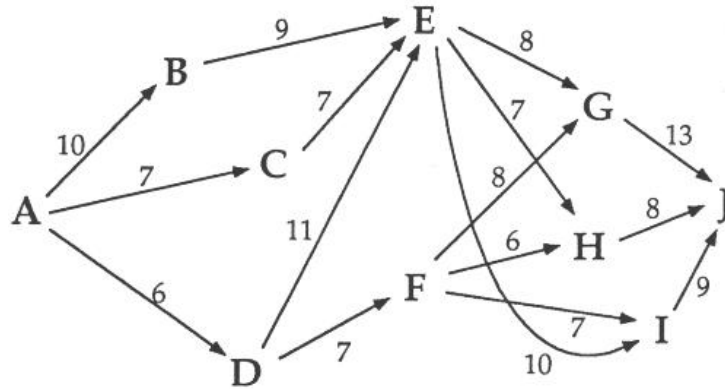
What route minimizes the maximum altitude of Joe's journey?

Betting Strategy

Suppose Michelle currently has $2 and is allowed to play a game of chance three times. If she bets b dollars on a play of the game then with probability 0.4 she wins b dollars while with probability 0.6 she loses b dollars. (Each bet must be in a whole number of dollars. She can choose to bet $0 on a game.) Suppose Michelle wants to maximise her probability of having at least $5 after the three games. What strategy of bets should she use to achieve this?

Advertising

A firm is planning its advertising strategy for a period of four weeks. In each week the sales level will be either *High* or Low and the firm will receive profits on sales of $800 or $600, respectively.

If the sales were High in the previous week then there is a 60% chance that sales will be High again in the current week if they do not advertise in the current week or 80% if they do advertise. If the sales were Low in the previous week then there is a 20% chance that sales will be High in the current week if they do not advertise in the current week or 60% if they do advertise.

The cost of advertising in one week is $70. An extra cost of $80 is incurred if the level of sales (and thus production) is changed from one week to the next.

What advertising strategy should the firm pursue?