DMFS - problemset 2

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Indhold

1

Matrix

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	1	0	1	0	0	0
2	0	0	0	1	0	1	0	0	0	1
3	0	0	0	0	0	1	0	0	1	0
4	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Figur 1: Representation matrix of relation S

Transitive matrix

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	1	1	1	1	1	1
2	0	0	0	1	0	1	0	1	0	1
3	0	0	0	0	0	1	0	0	1	0
4	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Figur 2: Transitive closure of relation S

Reflexive matrix

	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	0	1	0	1	0	1	0	1	0	1
3	0	0	1	0	0	1	0	0	1	0
4	0	0	0	1	0	0	0	1	0	0
5	0	0	0	0	1	0	0	0	0	1
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	0	1

Figur 3: Reflexive closure of relation S

1.1 a

The matrix is constructed with rows representing changes in i and columns representing changes in j. If $(a_i, b_j) \in S$, then the position (i,j) is set to 1 and vice versa: if $(a_i, b_j) \notin S$, then position (i,j) is set to 0.

This is represented on the graph as: if vertex i has an edge to vertex j, then the corresponding element is set to 1.

1.2 b

When denoting the transitive closure of the relation S the following needs to be true:
$\forall\, a, b, c \;\; (a, b) \in T \wedge (b, c) \in T \Rightarrow (a, c) \in T$

When looking at the graph, this expression means that if a vertex have a path to another vertex, an edge should be added connection the two.
Ex: 2 has an edge to 4 which has an edge to 8. Hence in row 2, columns 4 and 8 should be set to 1.

1.3 c

When denoting the reflexive closure of the relation T the following needs to be true:
$\forall\, a \;\; (a, a) \in R$

This is represented as all vertices looping back to themselves on the graph. In the context of matrices, reflexive closure can be denoted as having all the middle-diagonal elements be 1.

1.4 d

For the relation R we have:
${}_aR_b \; if \;\; and \;\; only \;\; if \;\; a|b$
Meaning: if $b \, mod \, a = 0$ then $(a, b) \in R$
This relation have been discussed previous in the course, for example in example 4 on p. 128 in KBR.

2

2.1 a

Let j be the number of children $j = 12$ and let m be the amount of collected mushrooms $m = 77$.
We let n_i denote the amount of mushrooms each child has collected. We know from the assignment description that $n_i \geq 1$. If all children had collected a different amount of mushrooms we have that:
$n = \sum_{i=1}^{j} n_i = 1 + 2 + ... + 12 = 78$
Noticing $n > m$ we have according to the pigeonhole principle, that at least two children must have collected the same amount of mushroom.

To clarify: To get $n = m$ we have to subtract 1 from n, meaning one child n_i must collect one less mushroom $(n_i - 1)$. If we try this with an arbitrary child, lets say the 4th child we get that $n_4 - 1 = n_3$. Hence at least two children (in this example n_4 and n_3) must have collected the same amount of mushrooms.

2.2 b

Let m denote the number of children $m = 12$
We have that if a prime number $p|n$ then $p\, mod\, (n+1) = 1$ meaning $p \not| (n+1)$. With this knowledge we can conclude that two consecutive numbers n and $n+1$ have no prime in common, also known as being relative primes.

This can be used to construct n holes of consecutive numbers that we know are relatively prime:
$\{1, 2\}, \{3, 4\}...\{21, 22\}$. We get that $n = 11$
It follows by the pigeon hole principle that because $m > n$ at least two of the 12 randomly picked numbers will be in the same subset. This is the equivalent to two children drawing sheets with relatively primes.

3

3.1 a

Tabel 1: $(p \Rightarrow (q \wedge r)) \Rightarrow ((q \vee \sim p) \wedge (r \vee \sim p))$

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$\sim q$	$q \vee \sim q$	$r \vee \sim p$	$(q \vee \sim p) \vee (r \vee \sim p)$	$(p \Rightarrow (q \wedge r)) \Rightarrow ((q \vee \sim p) \wedge (r \vee \sim p$
T	T	T	T	T	F	T	T	T	**T**
T	T	F	F	F	F	T	F	F	**T**
T	F	T	F	F	F	F	T	F	**T**
T	F	F	F	F	F	F	F	F	**T**
F	T	T	T	T	T	T	T	T	**T**
F	T	F	F	T	T	T	T	T	**T**
F	F	T	F	T	T	T	T	T	**T**
F	F	F	F	T	T	T	T	T	**T**

We see that the statement is true for all possible values of its propositional variables, therefore it's a tautology.

Table explanation:

1. $p \wedge r$: The conjunction between q and r is True when both q and r are True, as seen in rows 1 and 5.
2. $p \Rightarrow (q \wedge r)$: p implies $(q \wedge r)$ is only False when p is True and $(q \wedge r)$ is False, as seen in row 2, 3 and 4.
3. $q \vee \sim p$: The disjunction between q and $\sim p$ is False only when both are False, as seen in row 3 and 4.
4. $r \vee \sim p$: The disjunction between r and $\sim p$ is False only when both are False, as seen in row 2 and 5.
5. $(q \vee \sim p) \wedge (r \vee \sim p)$: The conjunction is True when both are True, as seen in rows 2, 3 and 4.
6. $(p \Rightarrow (q \wedge r)) \Rightarrow ((q \vee \sim p) \wedge (r \vee \sim p))$: $(p \Rightarrow (q \wedge r))$ implies $((q \vee \sim p) \wedge (r \vee \sim p))$ is only False when $(p \Rightarrow (q \wedge r))$ is True and $(q \vee \sim q) \wedge (r \vee \sim p)$ is False. This is not the case since they are equivalent
$(p \Rightarrow (q \wedge r)) \equiv ((q \vee \sim p) \wedge (r \vee \sim p))$. Hence the tautology.

3.2 b

Tabel 2: $((p \wedge q) \Rightarrow r) \Rightarrow ((r \vee \sim p) \wedge (r \vee \sim q))$

p	q	r	$(p \wedge q)$	$((p \wedge q) \Rightarrow r)$	$\sim p$	$r \vee \sim p$	$\sim q$	$r \vee \sim q$	$((r \vee \sim p) \wedge (r \vee \sim q))$
T	T	T	T	T	F	T	F	T	T
T	F	F	F	T	F	F	T	T	F

We see that when p,q and r is True. $((p \wedge q) \Rightarrow r)$ is True because p and q is both True, and the conjunction of the to implies something True. $((r \vee \sim p) \wedge (r \vee \sim q))$ is also True since both $((r \vee \sim p)$ and $(r \vee \sim q))$ is True.

We see that when p is True, but q and r is False. Then $((p \wedge q) \Rightarrow r)$ is True, but it implies something false $((r \vee \sim p) \wedge (r \vee \sim q))$. Therefore in this case $((p \wedge q) \Rightarrow r) \Rightarrow ((r \vee \sim p) \wedge (r \vee \sim q))$ is False.

Since the statement can be either True or False it's a contingency.