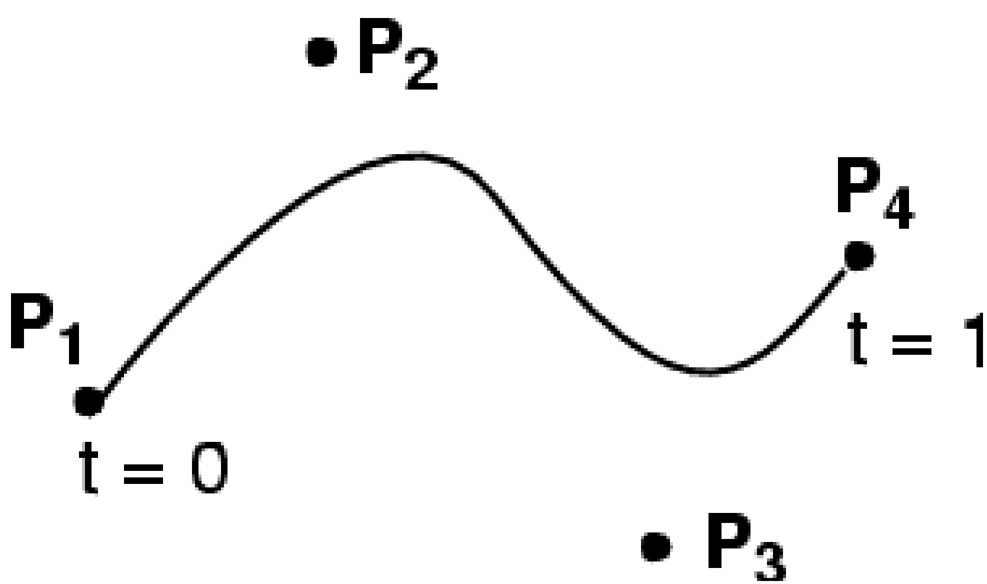
6.837 Computer Graphics

Curve Properties & Conversion, Surface Representations

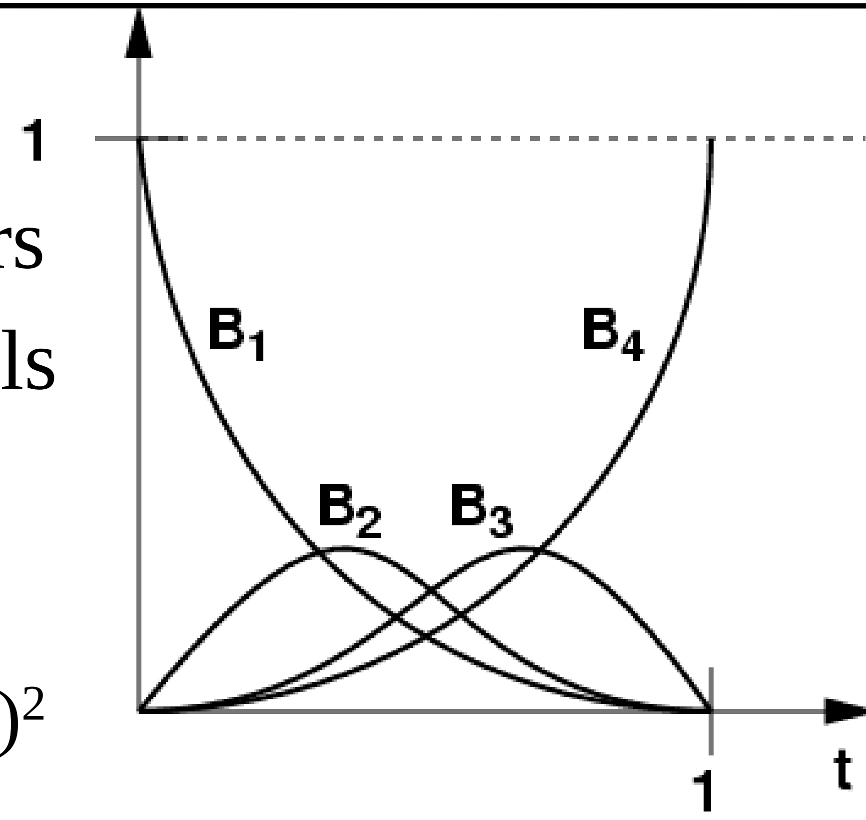
Cubic Bezier Splines

- $$P(t) = (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t) P_3 + t^3 P_4$$



Bernstein Polynomials

- For Bézier curves, the basis polynomials/vectors are Bernstein polynomials



- For cubic Bezier curve:

  $B_1(t)=(1-t)^3$ $B_2(t)=3t(1-t)^2$

  $B_3(t)=3t^2(1-t)$ $B_4(t)=t^3$

  (careful with indices, many authors start at 0)

- Defined for any degree

General Spline Formulation

$$Q(t) = \mathbf{GBT(t)} \; = \text{Geometry } \mathbf{G} \cdot \text{Spline Basis } \mathbf{B} \cdot \text{Power Basis } \mathbf{T(t)}$$

- Geometry: control points coordinates assembled into a matrix (P_1, P_2, …, P_{n+1})
- Power basis: the monomials 1, t, t^2, ...
- Cubic Bézier:

$$\mathrm{P(t)} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} =$$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{pmatrix} \begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ t \\ t^2 \\ t^3 \end{pmatrix}$$

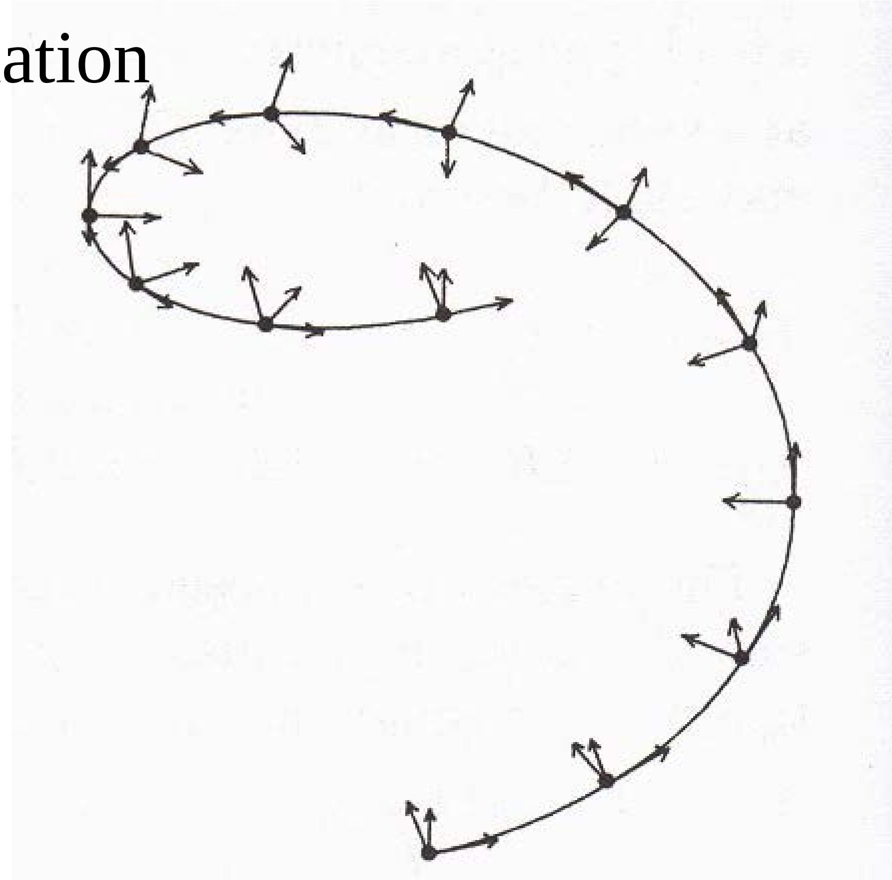
Questions?

The Plan for Today

- Differential Properties of Curves & Continuity
- B-Splines
- Surfaces
  - Tensor Product Splines
  - Subdivision Surfaces
  - Procedural Surfaces
  - Other

Differential Properties of Curves

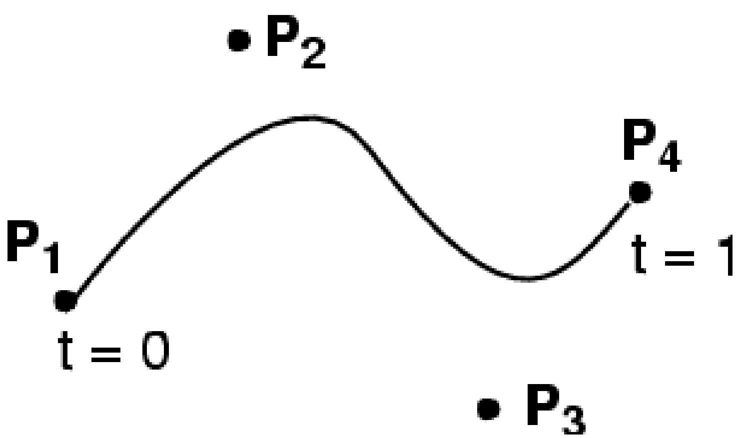
- Motivation
  - Compute normal for surfaces
  - Compute velocity for animation
  - Analyze smoothness



Velocity

- First derivative w.r.t. *t*
- Can you compute this for Bezier curves?

$$
\begin{aligned}
P(t) = \quad & (1-t)^3 && P_1 \\
+ \quad & 3t(1-t)^2 && P_2 \\
+ \quad & 3t^2(1-t) && P_3 \\
+ \quad & t^3 && P_4
\end{aligned}
$$



- You know how to differentiate polynomials...

Velocity

- First derivative w.r.t. *t*
- Can you compute this for Bezier curves?

$$
\begin{aligned}
P(t) = \quad & (1-t)^3 && P_1 \\
+ \quad & 3t(1-t)^2 && P_2 \\
+ \quad & 3t^2(1-t) && P_3 \\
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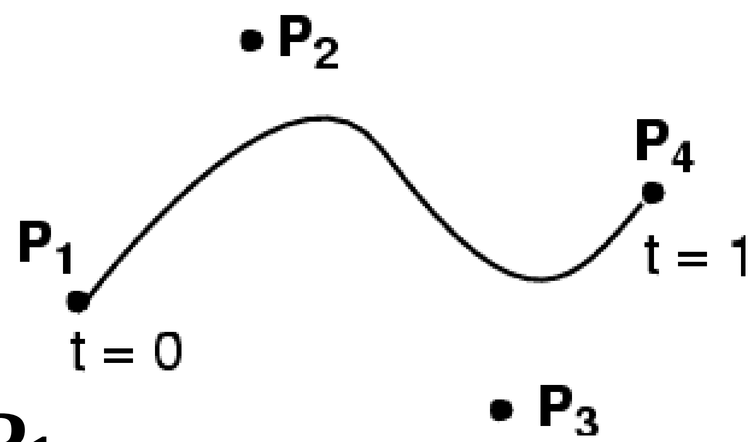
\end{aligned}
$$



- $$
\begin{aligned}
\mathbf{P'(t)} = \quad & \mathbf{-3(1-t)^2} && \mathbf{P_1} \\
+ \quad & \mathbf{[3(1-t)^2 - 6t(1-t)]} && \mathbf{P_2} \\
+ \quad & \mathbf{[6t(1-t) - 3t^2]} && \mathbf{P_3} \\
+ \quad & \mathbf{3t^2} && \mathbf{P_4}
\end{aligned}
$$

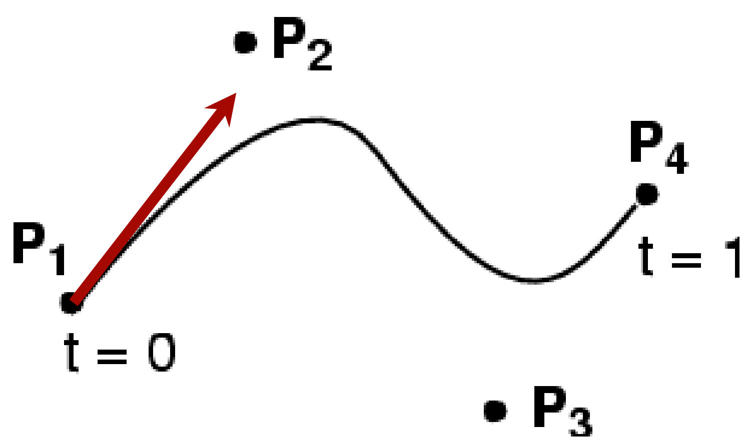
Sanity check: t=0; t=1

Linearity?

- Differentiation is a linear operation
  - (f+g)’=f’+g’
  - (af)’=a f’
- This means that the derivative of the basis is enough to know the derivative of any spline.
- Can be done with matrices
  - Trivial in monomial basis
  - But get lower-order polynomials

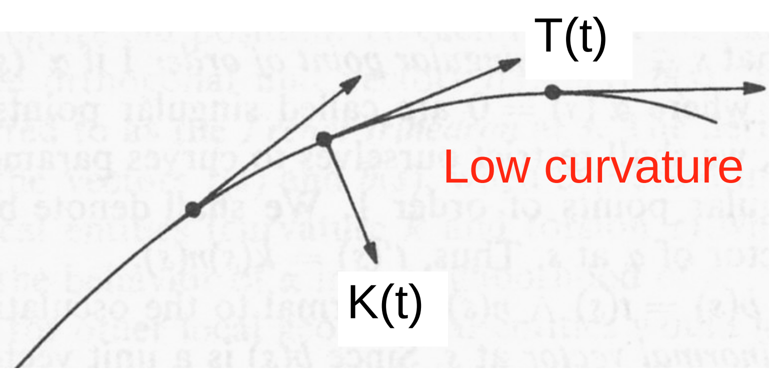
Tangent Vector

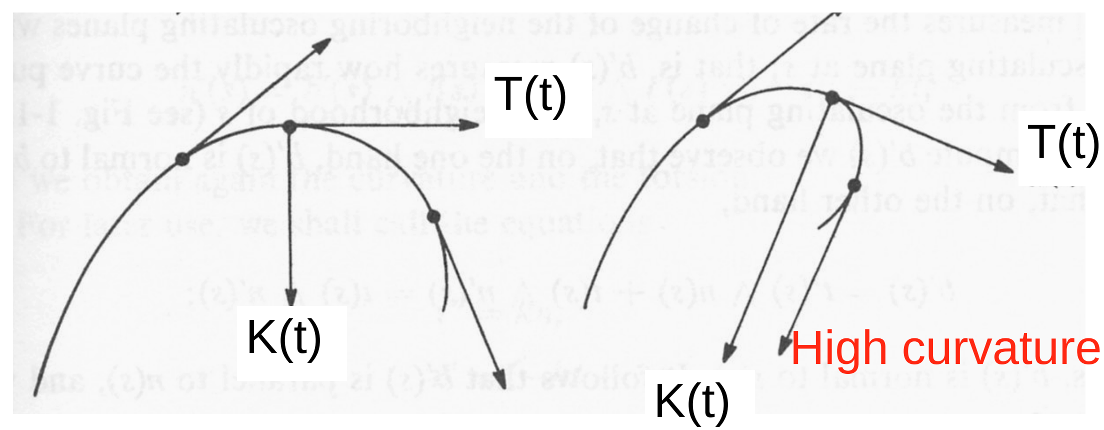
- The tangent to the curve P(t) can be defined as $T(t)=P'(t)/||P'(t)||$
  – normalized velocity, $||T(t)|| = 1$
- This provides us with one orientation for swept surfaces later



Curvature Vector

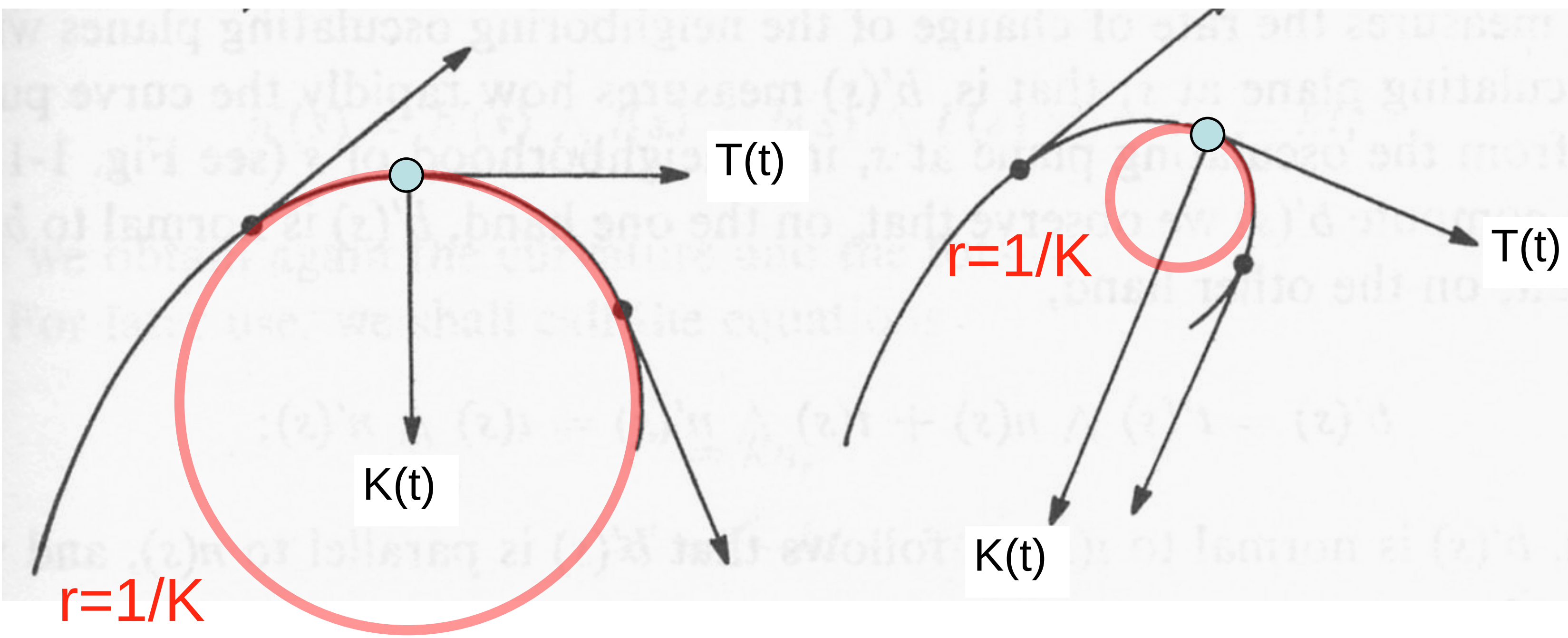
- Derivative of unit tangent
  - K(t)=T'(t)
  - Magnitude ||K(t)|| is constant for a circle
  - Zero for a straight line
- Always orthogonal to tangent, ie. $K \cdot T = 0$





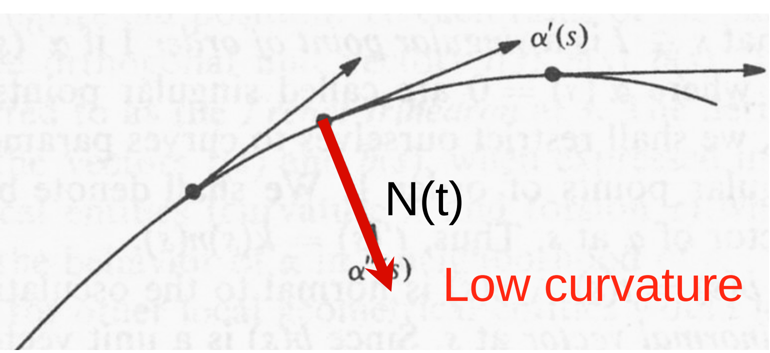
Geometric Interpretation

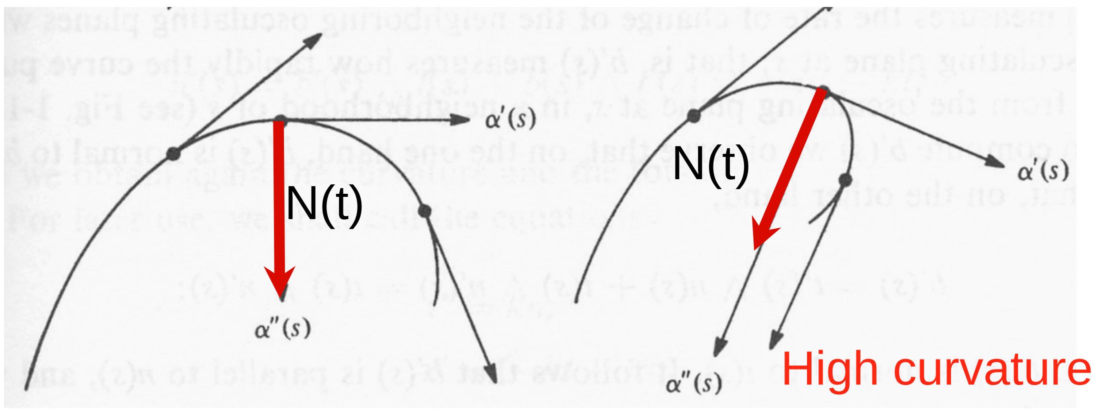
- K is zero for a line, constant for circle
  - What constant? 1/r
- 1/||K(t)|| is the radius of the circle that touches P(t) at *t* and has the same curvature as the curve



Curve Normal

- Normalized curvature: $T'(t)/||T'(t)||$

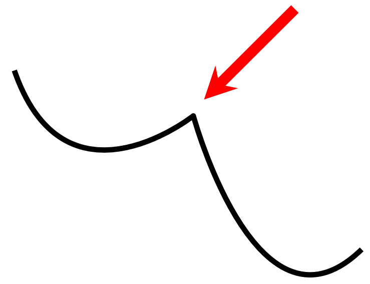




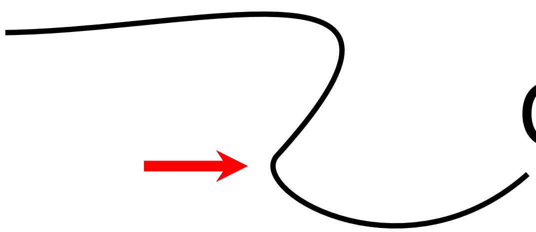
Questions?

Orders of Continuity

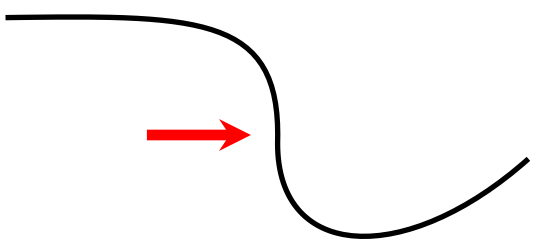
- C^0 = continuous
  - The seam can be a sharp kink
- G^1 = geometric continuity
  - Tangents **point to the same direction** at the seam
- C^1 = parametric continuity
  - Tangents **are the same** at the seam, implies G^1
- C^2 = curvature continuity
  - Tangents and their derivatives are the same



C^0



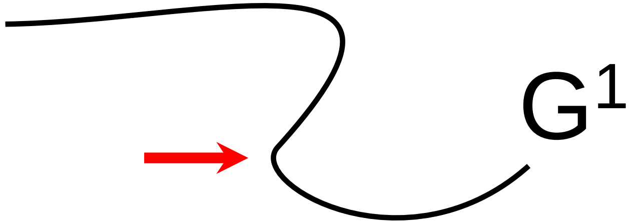
G^1

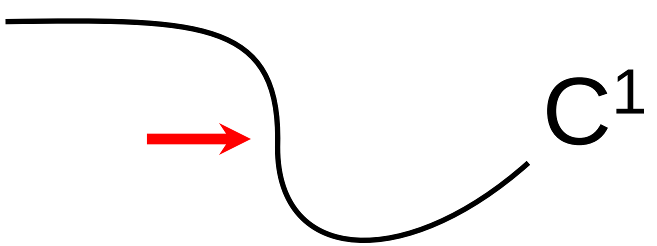


C^1

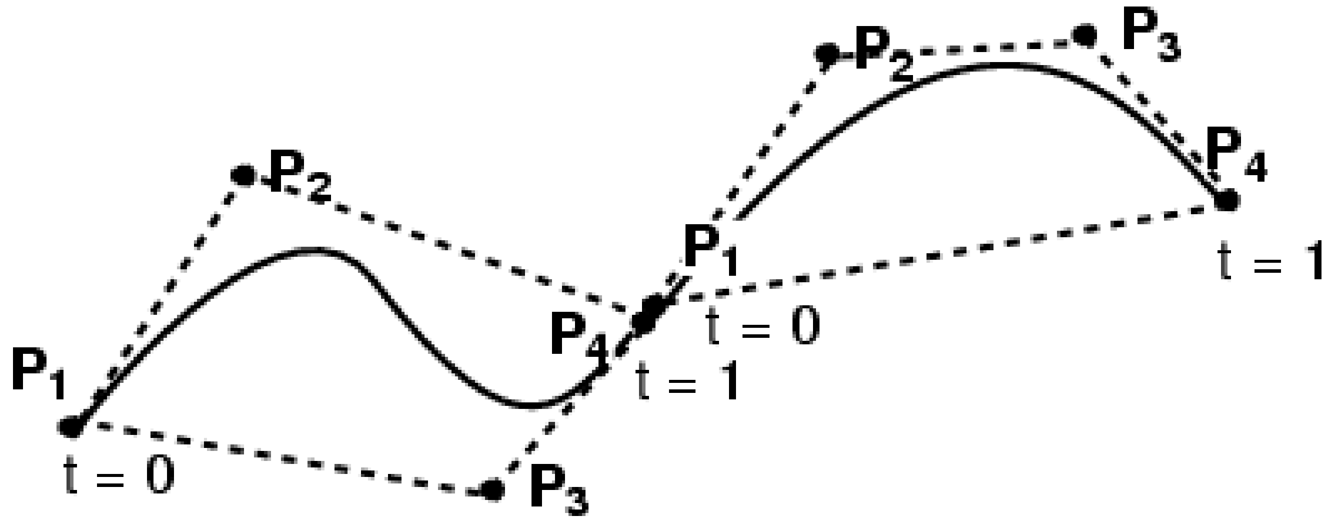
Orders of Continuity

- G^1 = geometric continuity
  - Tangents **point to the same direction** at the seam
  - good enough for modeling
- C^1 = parametric continuity
  - Tangents **are the same** at the seam, implies G^1
  - often necessary for animation



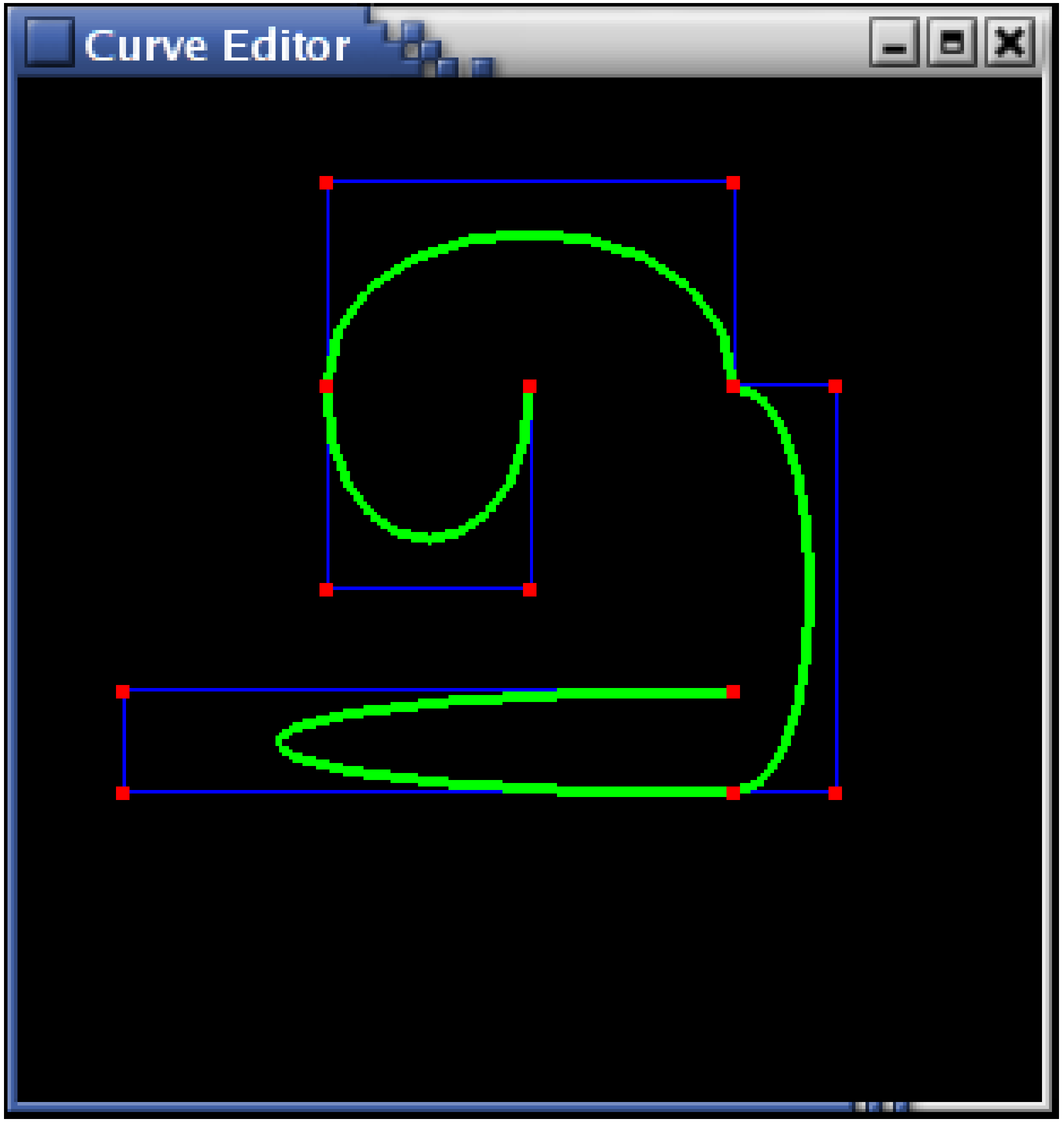


Connecting Cubic Bézier Curves



- How can we guarantee C^0 continuity?
- How can we guarantee G^1 continuity?
- How can we guarantee C^1 continuity?
- C^2 and above gets difficult

Connecting Cubic Bézier Curves

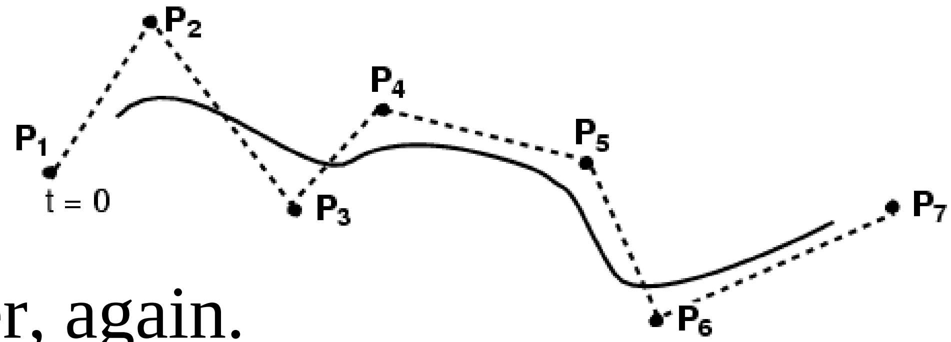


- Where is this curve
  - C^0 continuous?
  - G^1 continuous?
  - C^1 continuous?
- What's the relationship between:
  - the # of control points, and the # of cubic Bézier subcurves?

Questions?

Cubic B-Splines

- $\geq$ 4 control points
- Locally cubic
  - – Cubics chained together, again.



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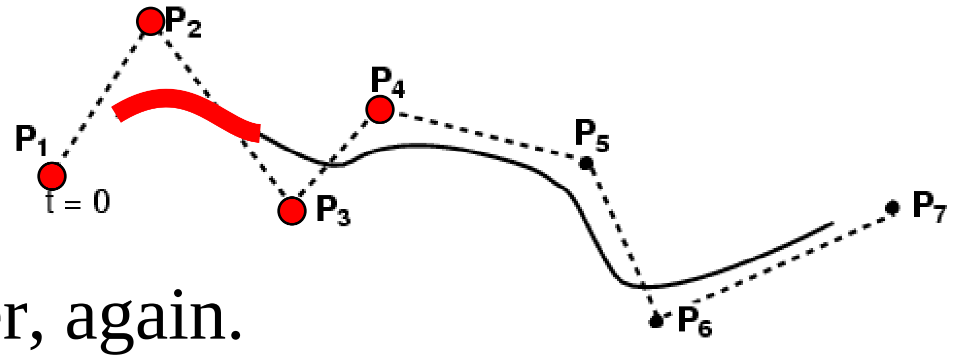

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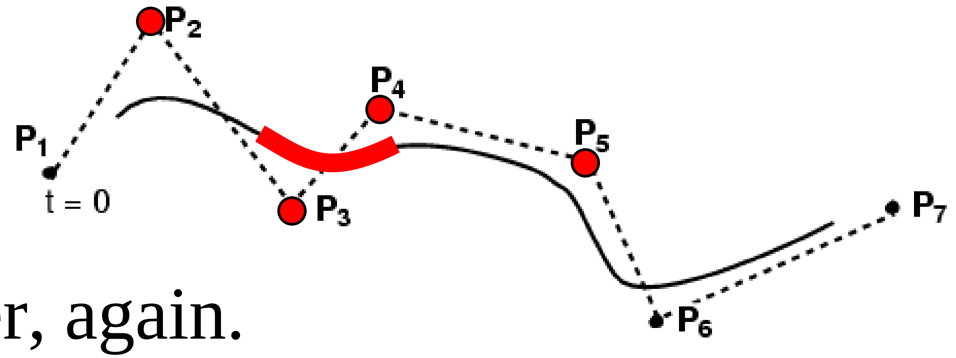

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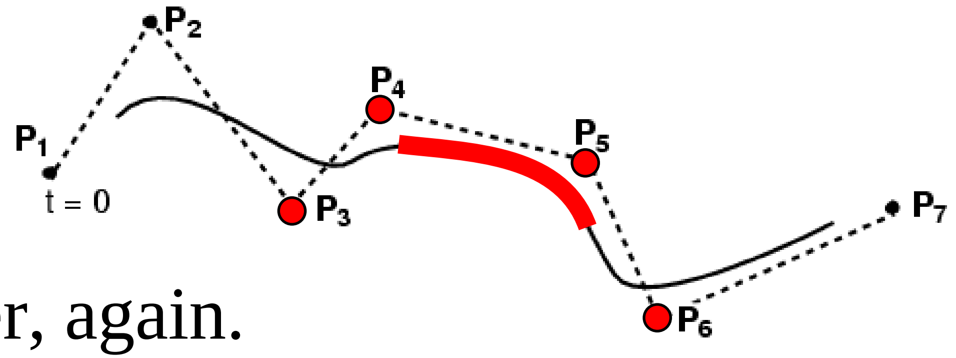

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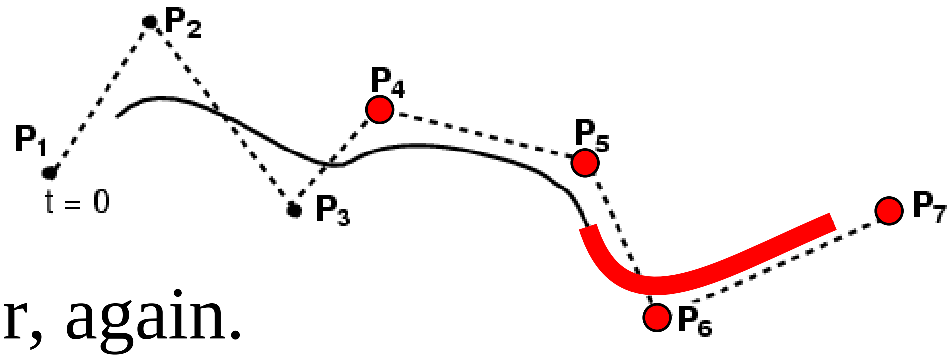
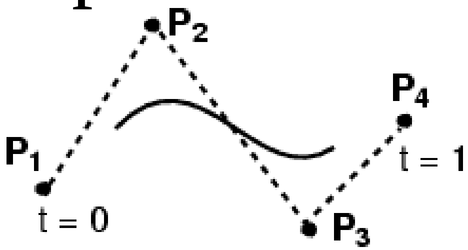
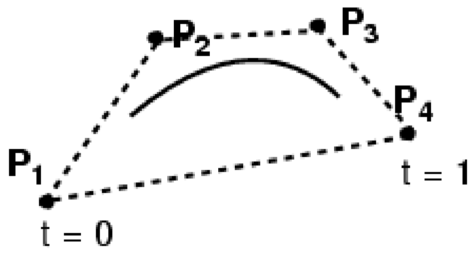
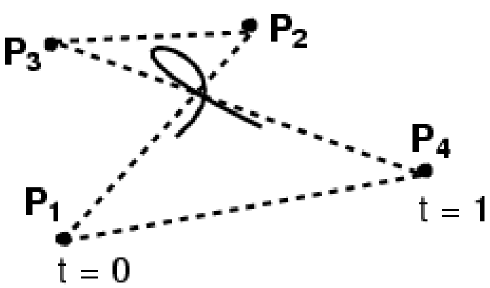


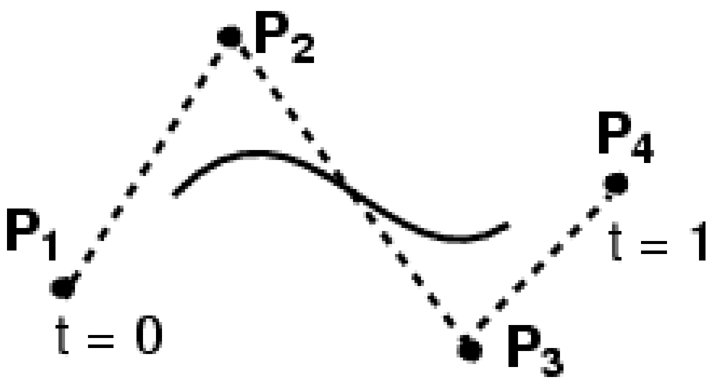
- Curve is not constrained to pass through any control points

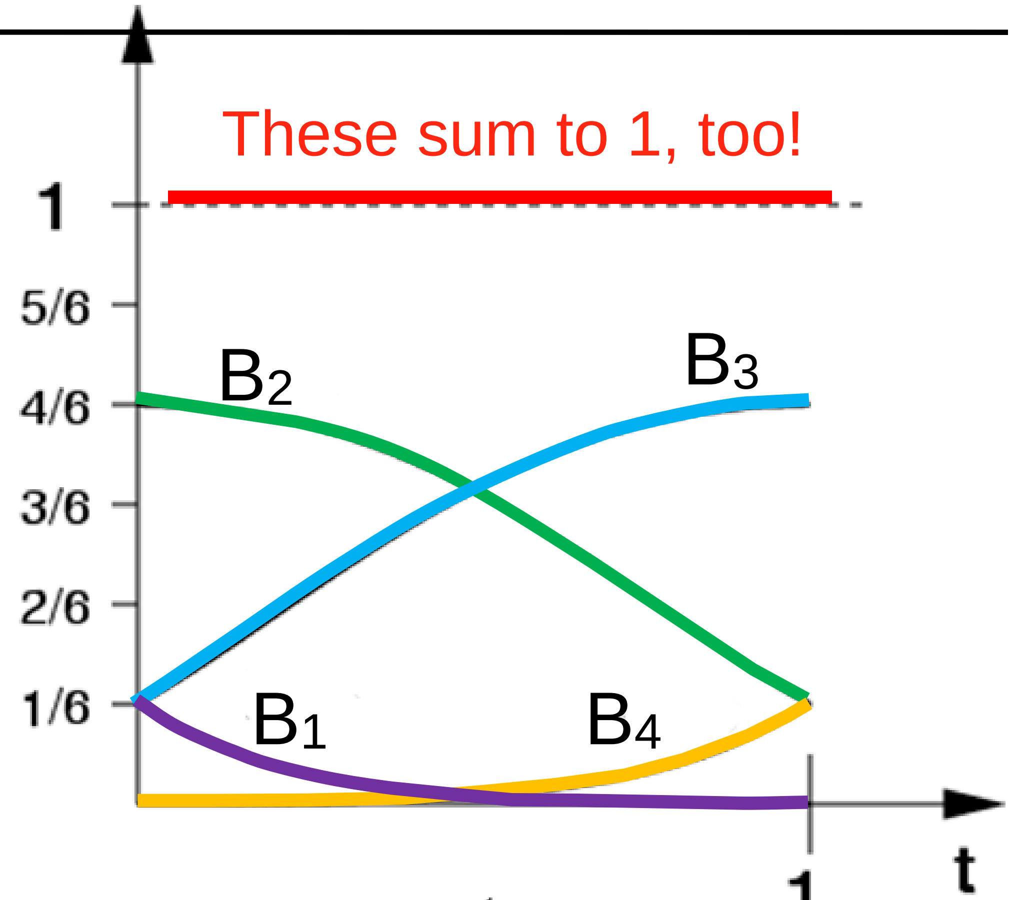






Cubic B-Splines: Basis





A B-Spline curve is also bounded by the convex hull of its control points.

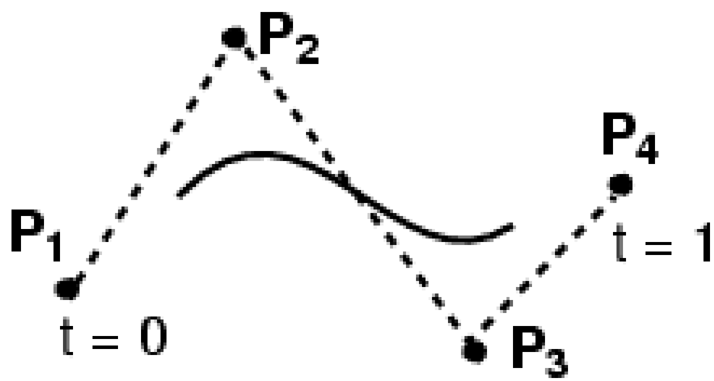
$$B_1(t) = \tfrac{1}{6}(1-t)^3$$

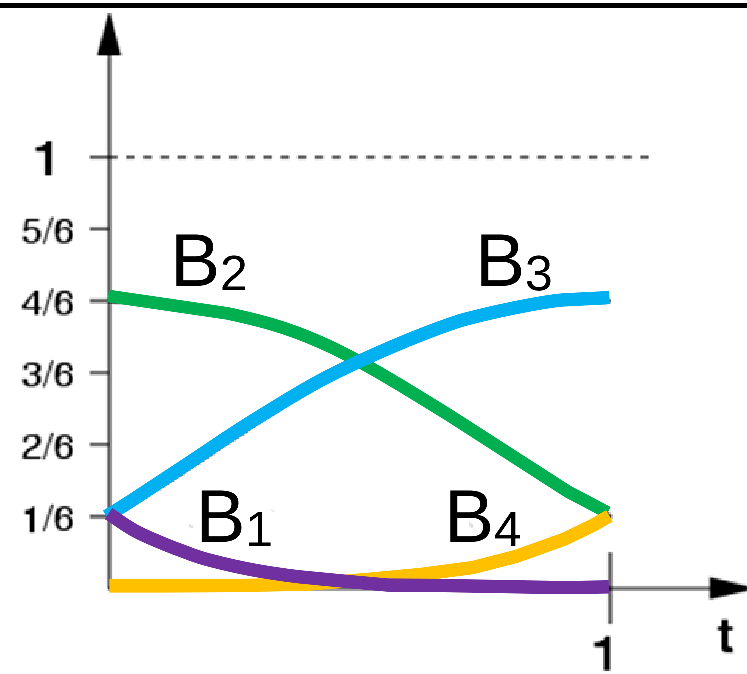
$$B_2(t) = \frac{1}{6}(3t^3 - 6t^2 + 4)$$

$$B_3(t) = \tfrac{1}{6}(-3t^3 + 3t^2 + 3t + 1)$$

$$B_4(t) = \frac{1}{6}t^3$$

Cubic B-Splines: Basis





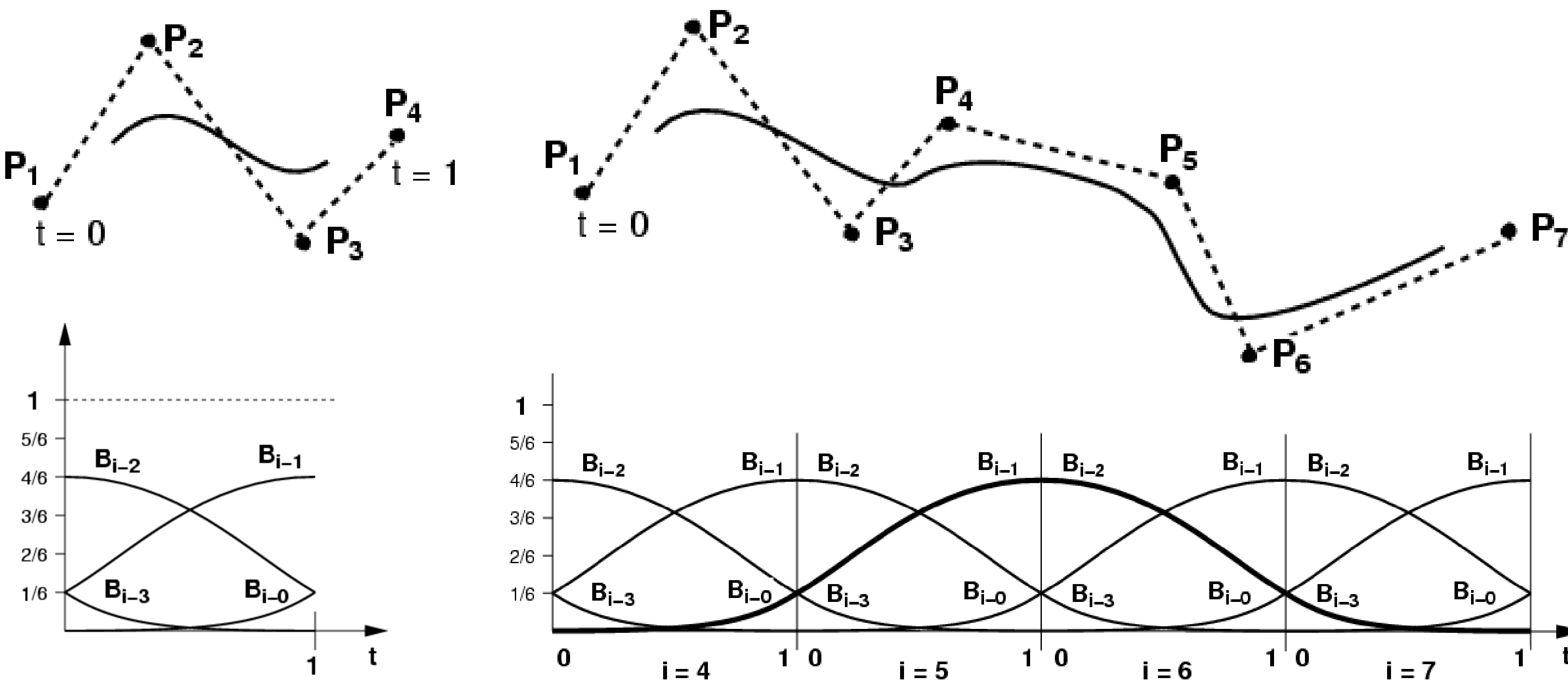
$$Q(t) = \frac{(1-t)^3}{6} P_1 \quad + \frac{3t^3 - 6t^2 + 4}{6} P_2 \quad + \frac{-3t^3 + 3t^2 + 3t + 1}{6} P_3 \quad + \frac{t^3}{6} P_4$$

$$Q(t) = \mathbf{GBT(t)}$$

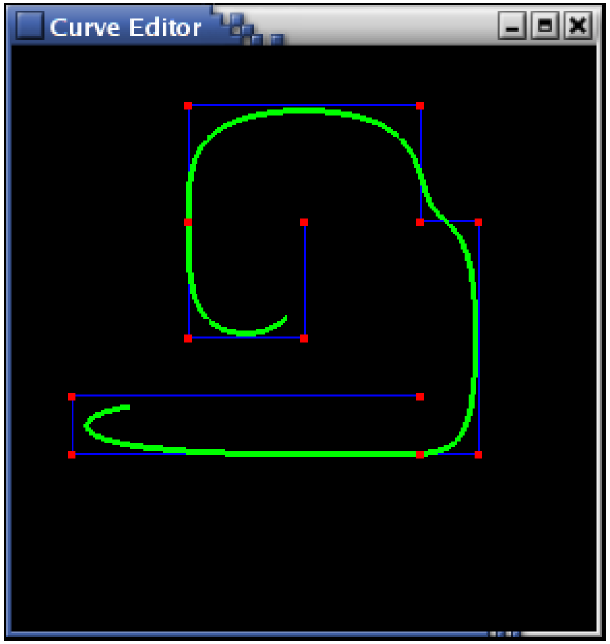
$$\boxed{B_{B-Spline} = \frac{1}{6} \begin{pmatrix} 1 & -3 & 3 & -1 \\ 4 & 0 & -6 & 3 \\ 1 & 3 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

Cubic B-Splines

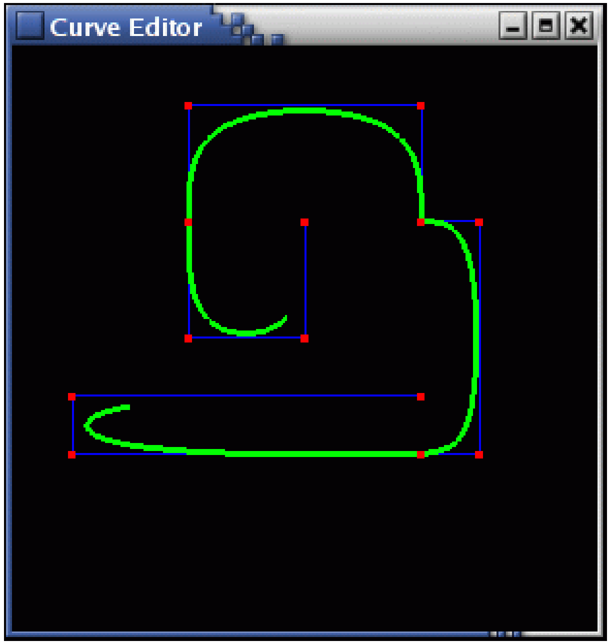
- Local control (windowing)
- Automatically C^2, and no need to match tangents!



B-Spline Curve Control Points

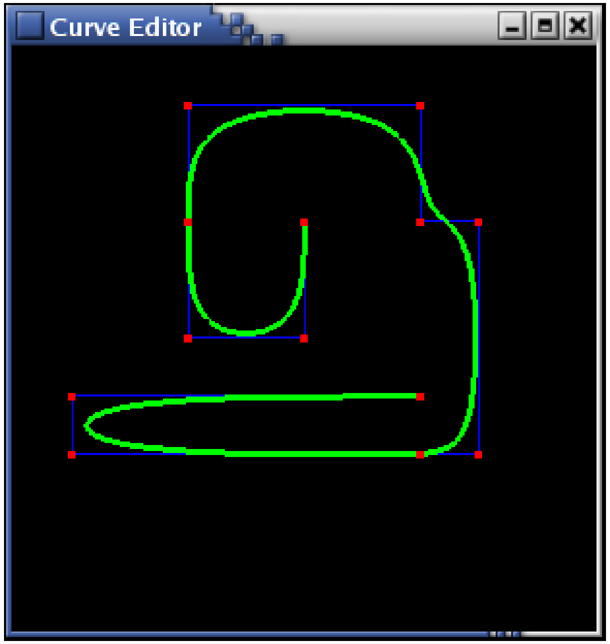


Default B-Spline



B-Spline with derivative discontinuity

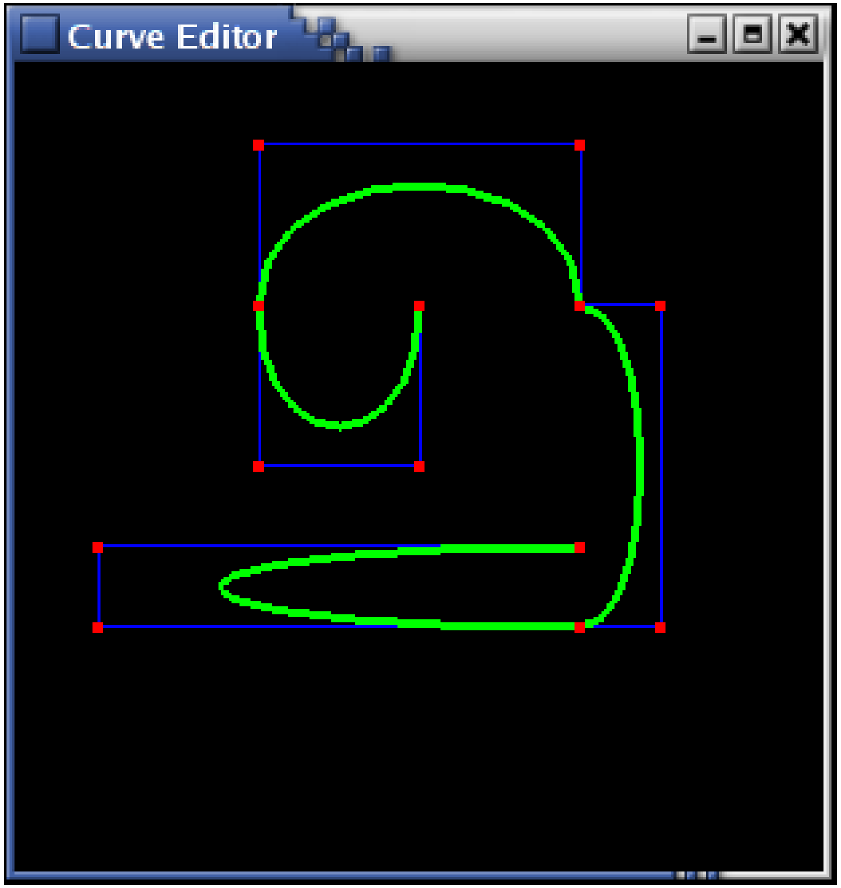
Repeat interior control point



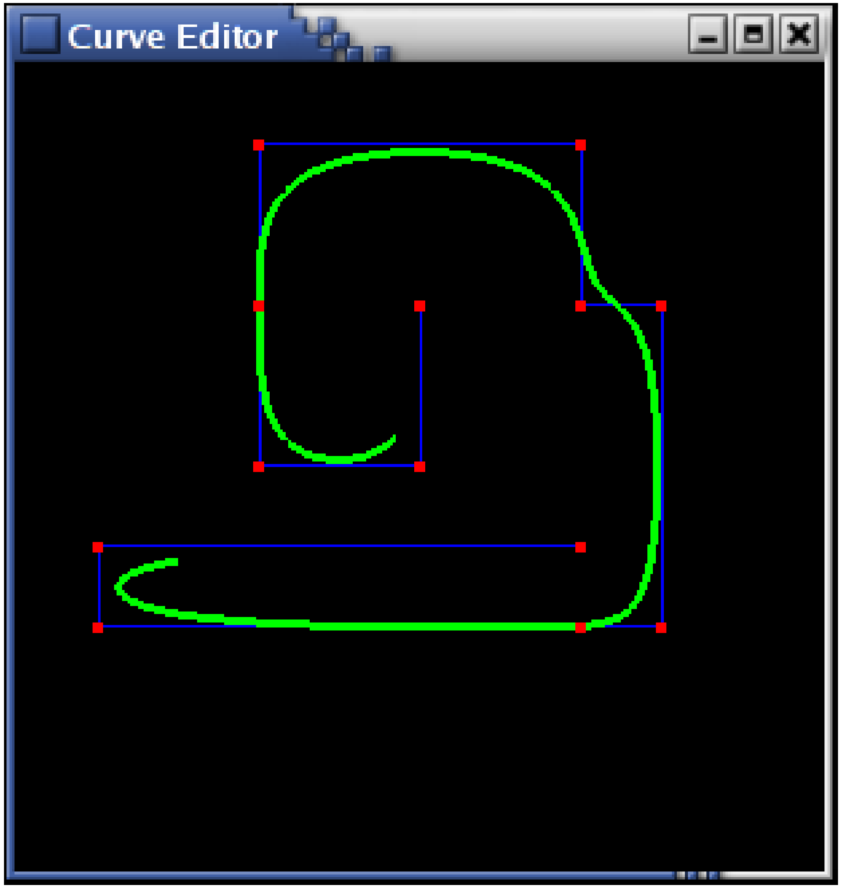
B-Spline which passes through end points

Repeat end points

Bézier ≠ B-Spline



Bézier



B-Spline

But both are cubics, so one can be converted into the other!

Converting between Bézier & BSpline

$Q(t) = \mathbf{GBT(t)}$ = Geometry $\mathbf{G}$ · Spline Basis $\mathbf{B}$ · Power Basis $\mathbf{T(t)}$

- Simple with the basis matrices!
  - Note that this only works for a single segment of 4 control points

$$B_{Bezier} = \begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $P(t) = \mathbf{G}\,\mathbf{B}_1\,\mathbf{T}(t) =$ $\mathbf{G}\,\mathbf{B}_1\,(\mathbf{B}_2^{-1}\mathbf{B}_2)\,\mathbf{T}(t) =$ $(\mathbf{G}\,\mathbf{B}_1\,\mathbf{B}_2^{-1})\,\mathbf{B}_2\,\mathbf{T}(t)$

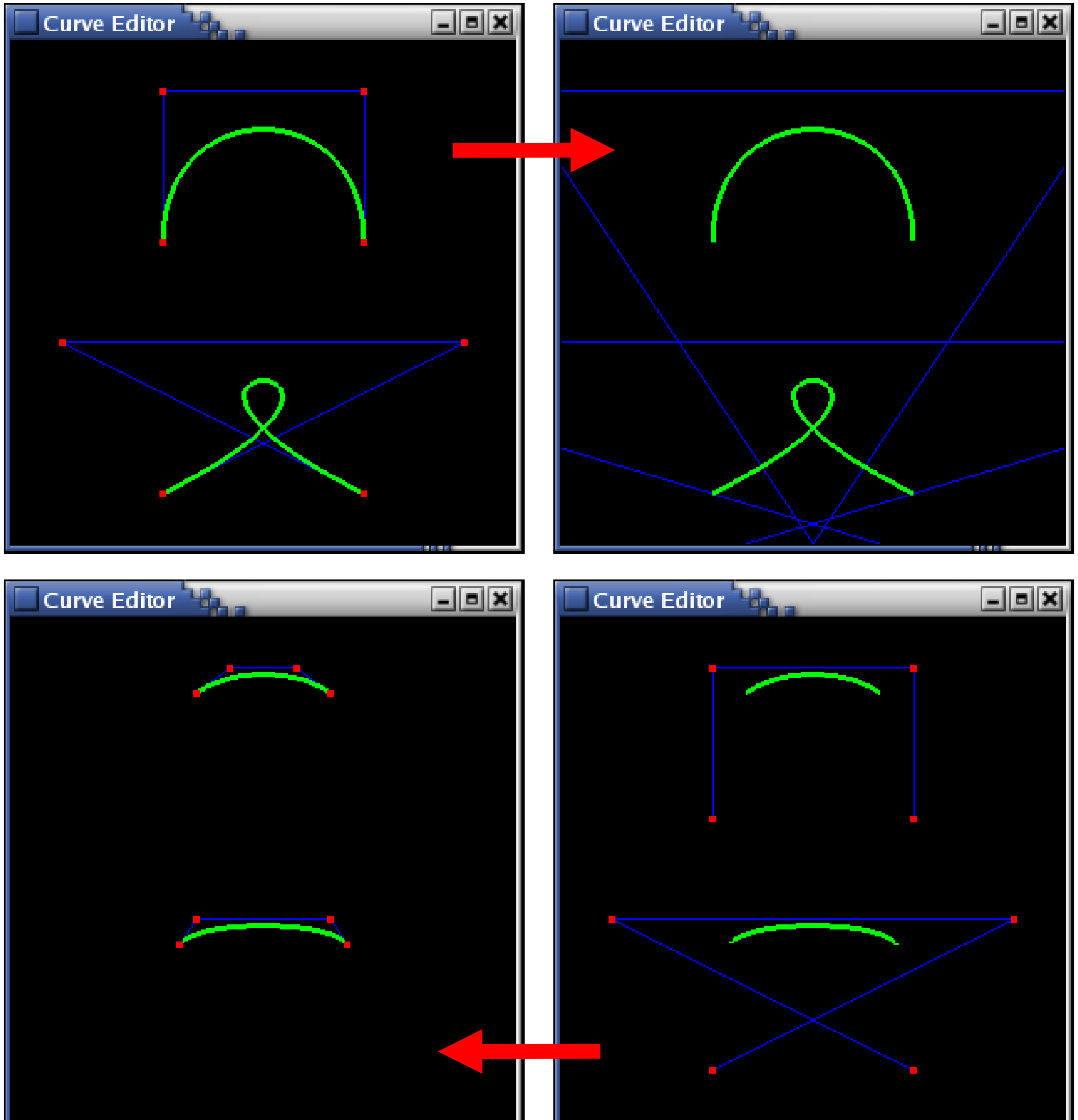
$$B_{B-Spline} = \frac{1}{6}\begin{pmatrix} 1 & -3 & 3 & -1 \\ 4 & 0 & -6 & 3 \\ 1 & 3 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $\mathbf{G}\,\mathbf{B}_1\,\mathbf{B}_2^{-1}$ are the control points for the segment in new basis.

Converting between Bézier & B-Spline

original control points as Bézier

new BSpline control points to match Bézier



new Bézier control points to match B-Spline

original control points as B-Spline

NURBS (Generalized B-Splines)

- Rational cubics
  - Use homogeneous coordinates, just add *w* !
    - Provides an extra weight parameter to control points

- NURBS: Non-Uniform Rational B-Spline
  - **non-uniform** = different spacing between the blending functions, a.k.a. “knots”
  - **rational** = ratio of cubic polynomials (instead of just cubic)
    - implemented by adding the homogeneous coordinate *w* into the control points.

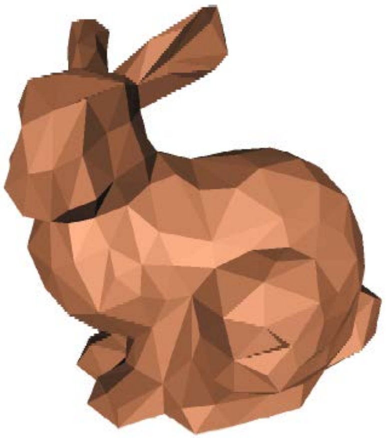
Questions?

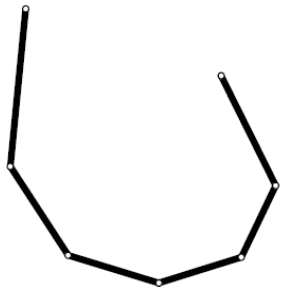
Representing Surfaces

- Triangle meshes
  - Surface analogue of polylines, this is what GPUs draw
- **Tensor Product Splines**
  - Surface analogue of spline curves
- **Subdivision surfaces**
- **Implicit surfaces**
  - $f(x,y,z)=0$
- **Procedural**
  - e.g. surfaces of revolution, generalized cylinder
- From volume data (medical images, etc.)

Triangle Meshes

- What you've used so far in Assignment 0
- Triangle represented by 3 vertices
- **Pro**: simple, can be rendered directly
- **Cons**: not smooth, needs many triangles to approximate smooth surfaces (tessellation)



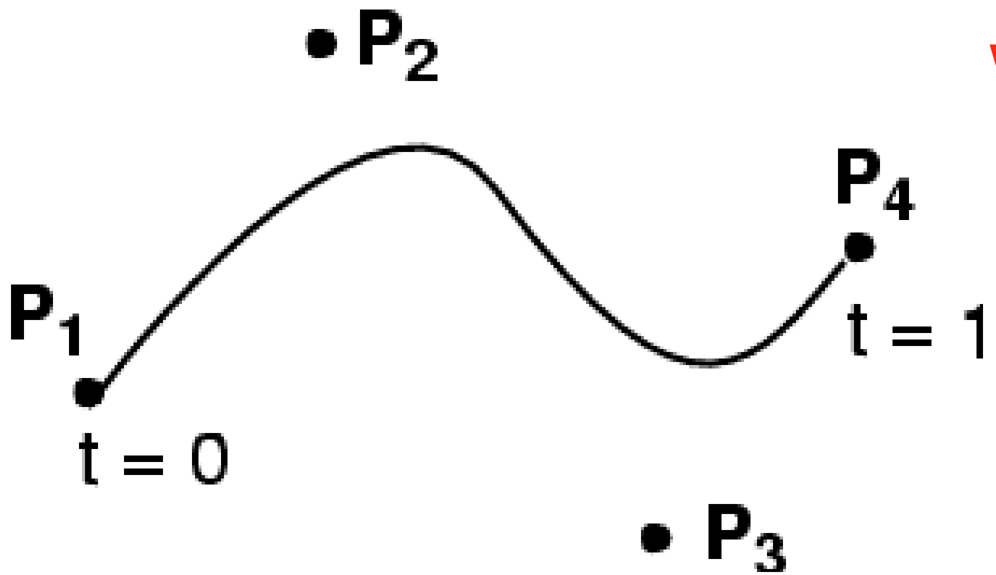


Smooth Surfaces?

- $P(t) = (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t) P_3 + t^3 P_4$

What's the dimensionality of a curve? 1D!

What about a surface?

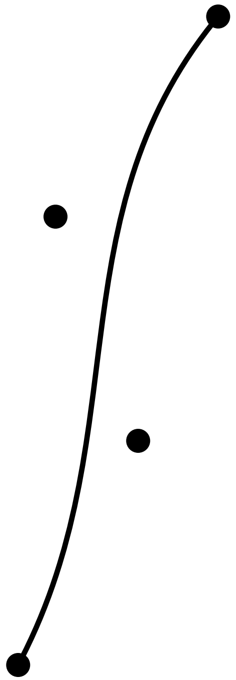


How to Build Them? Here's an Idea

- $P(u) = (1-u)^3 P_1 + 3u(1-u)^2 P_2 + 3u^2(1-u) P_3 + u^3 P_4$

$$
\begin{aligned}
P(u) = \quad & (1-u)^3 && P_1 \\
+ \quad & 3u(1-u)^2 && P_2 \\
+ \quad & 3u^2(1-u) && P_3 \\
+ \quad & u^3 && P_4
\end{aligned}
$$

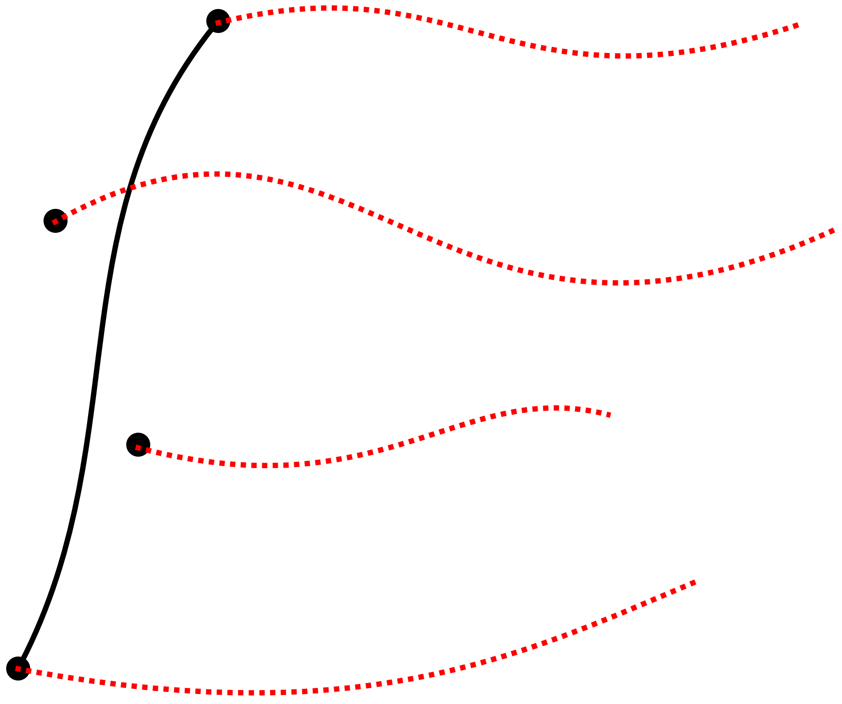
(Note! We relabeled *t* to *u*)



How to Build Them? Here's an Idea

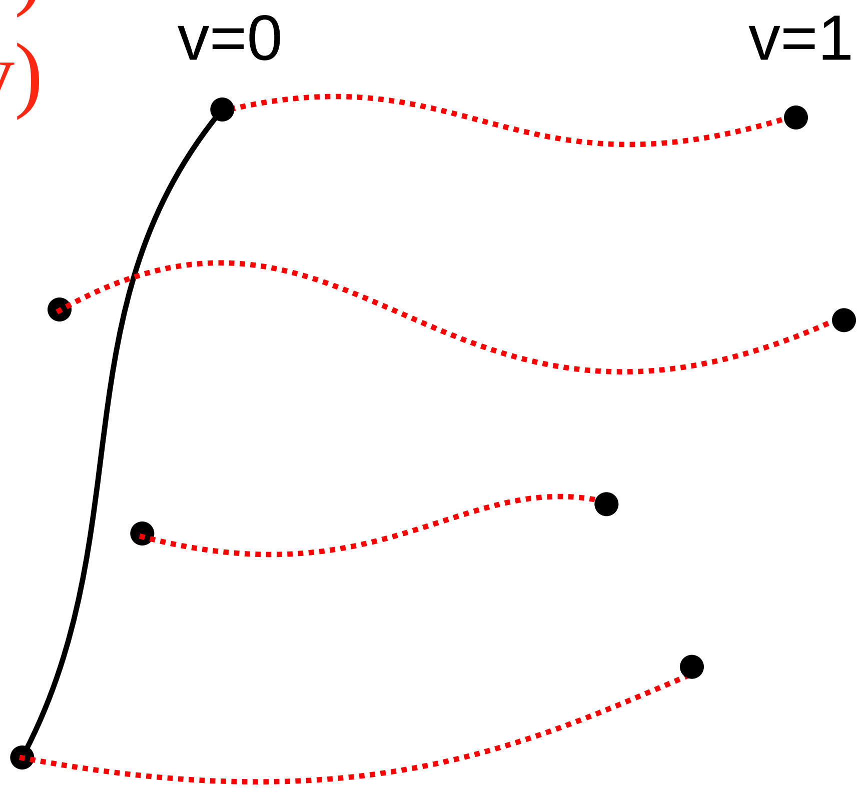
- $P(u) = (1-u)^3 \; P_1$
  $+ \; 3u(1-u)^2 \; P_2$
  $+ \; 3u^2(1-u) \; P_3$
  $+ \; u^3 \; P_4$

(Note! We relabeled *t* to *u*)



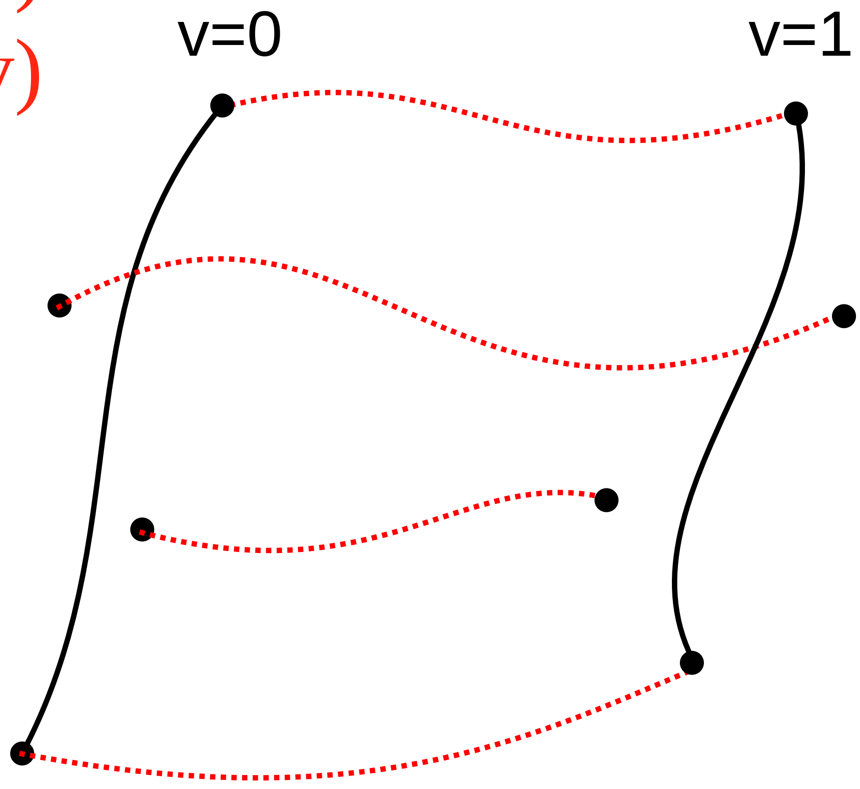
Here's an Idea

- $P(u, \textcolor{red}{v}) = (1-u)^3 \; P_1\textcolor{red}{(v)}$
  $+ \; 3u(1-u)^2 \; P_2\textcolor{red}{(v)}$
  $+ \; 3u^2(1-u) \; P_3\textcolor{red}{(v)}$
  $+ \; u^3 \; P_4\textcolor{red}{(v)}$
- Let's make
  the P_is move along
  curves!



Here's an Idea

- $P(u, v) = (1-u)^3 \quad P_1(v)$
  $+ \quad 3u(1-u)^2 \quad P_2(v)$
  $+ \quad 3u^2(1-u) \quad P_3(v)$
  $+ \quad u^3 \quad P_4(v)$
- Let's make the P_is move along curves!

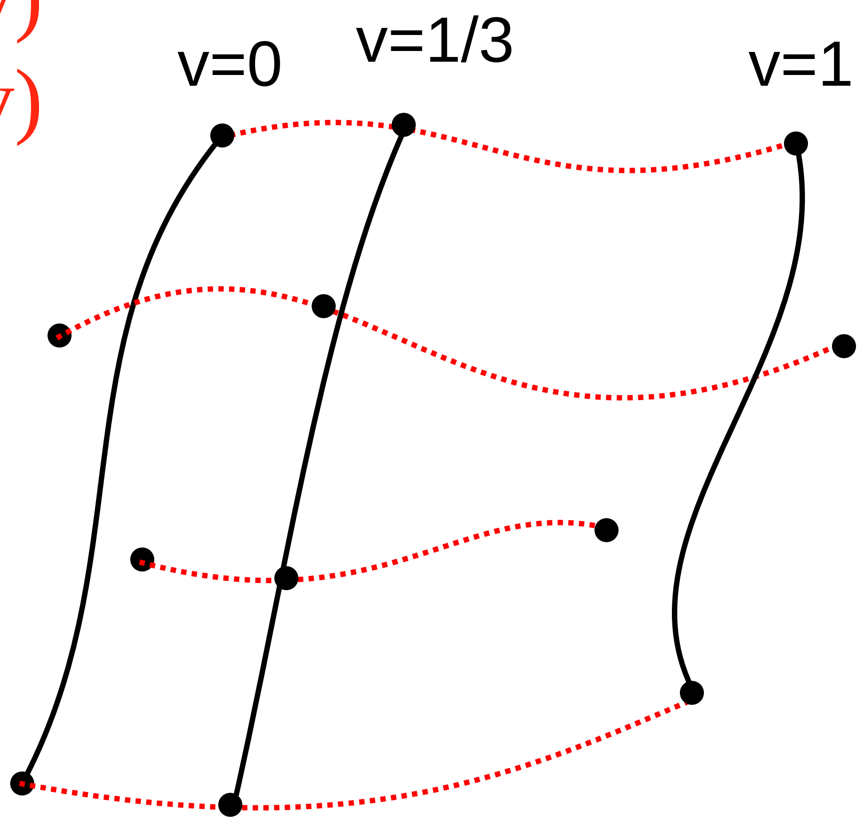


Here's an Idea

- $P(u, v) = (1-u)^3 \; P_1(v) + 3u(1-u)^2 \; P_2(v) + 3u^2(1-u) \; P_3(v) + u^3 \; P_4(v)$

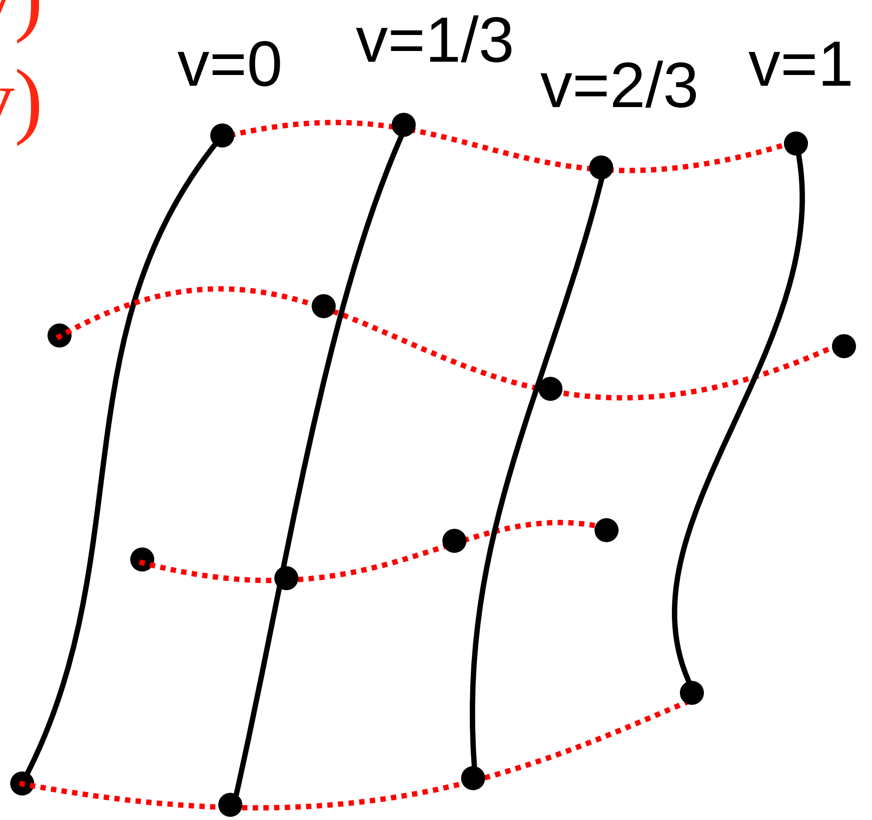
$$
\begin{aligned}
P(u, v) = \; & (1-u)^3 && P_1(v) \\
+ \; & 3u(1-u)^2 && P_2(v) \\
+ \; & 3u^2(1-u) && P_3(v) \\
+ \; & u^3 && P_4(v)
\end{aligned}
$$

- Let's make the P_is move along curves!



Here's an Idea

- $P(u, v) = (1-u)^3 \quad P_1(v)$
  $+ \quad 3u(1-u)^2 \quad P_2(v)$
  $+ \quad 3u^2(1-u) \quad P_3(v)$
  $+ \quad u^3 \quad P_4(v)$
- Let's make the P_is move along curves!



Here's an Idea

- $P(u, v) = (1-u)^3 \quad P_1(v)$
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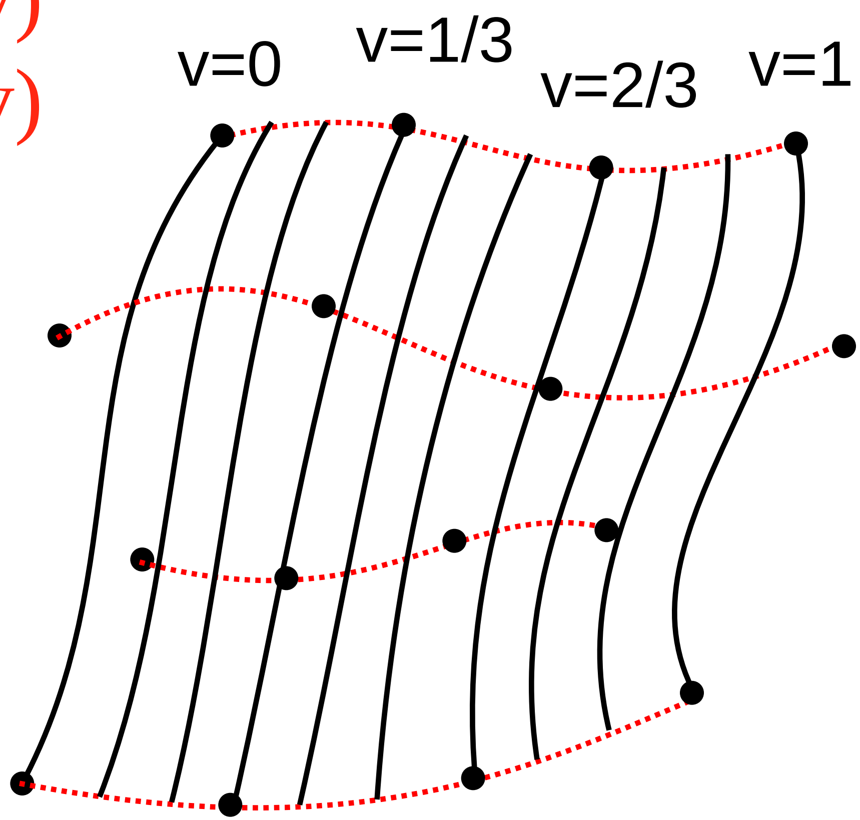
- Let's make the P_is move along curves!



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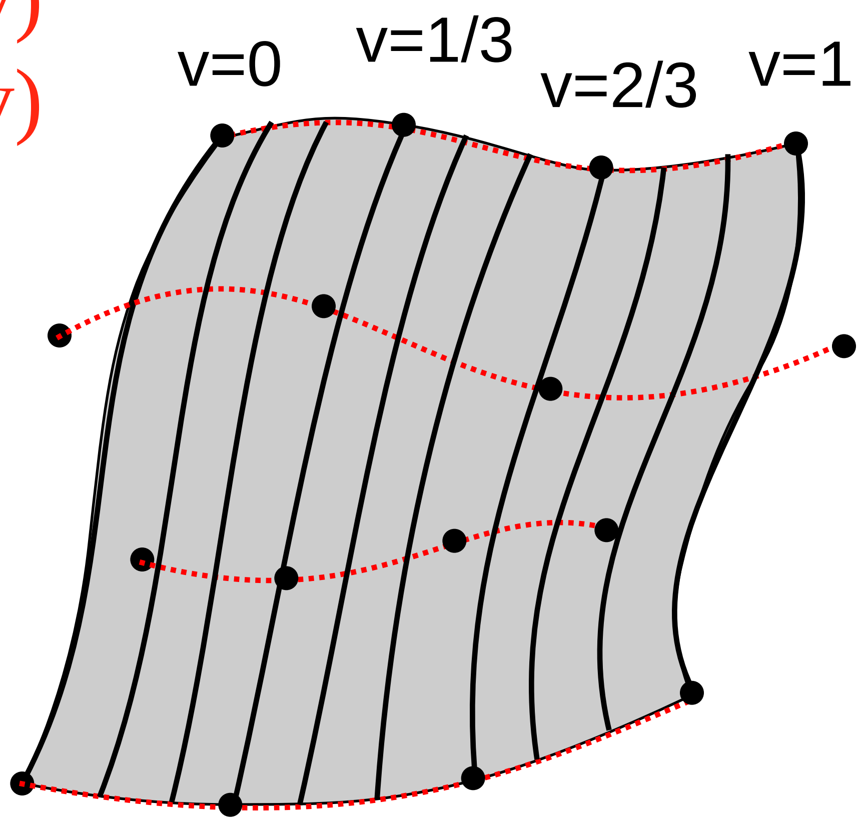
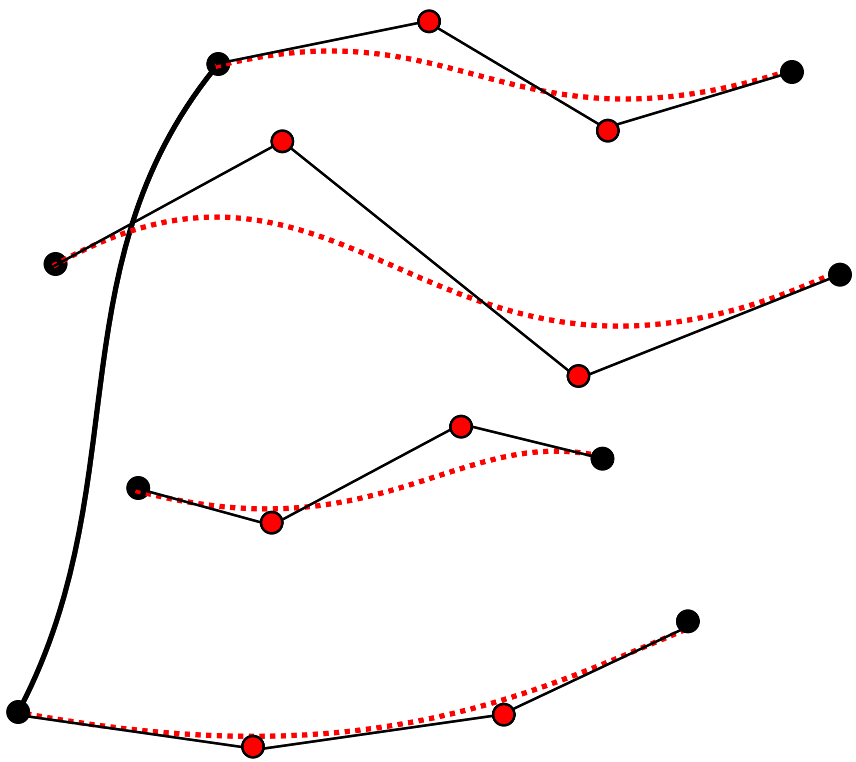
  $+ \quad u^3 \quad P_4(v)$
- Let's make the P_is move along curves!

A 2D surface patch!



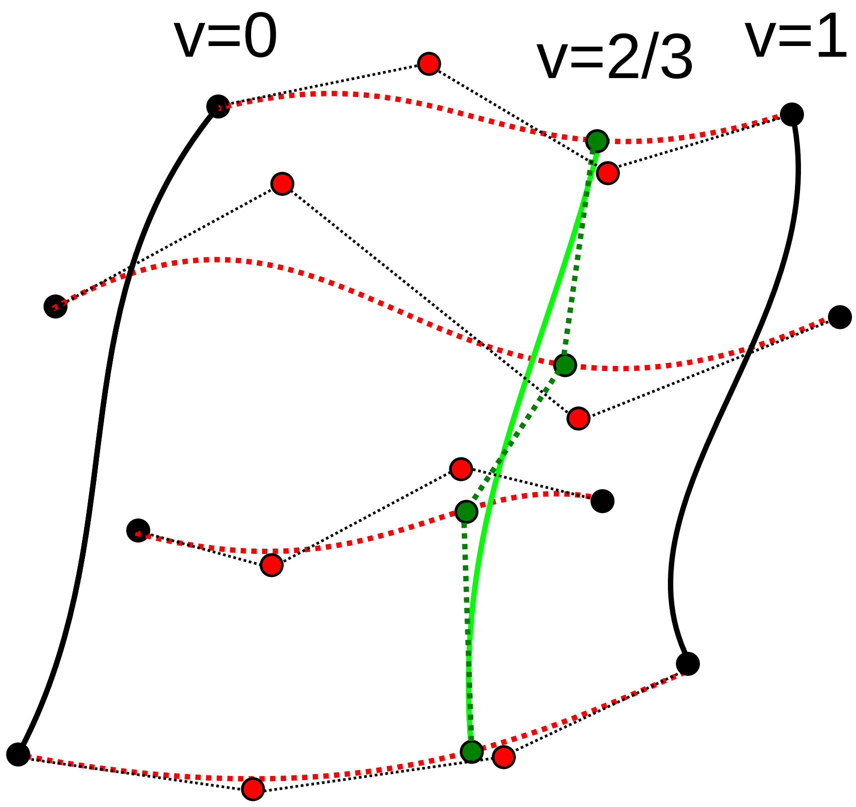
Tensor Product Bézier Patches

- In the previous, P_is were just some curves
- What if we make **them** Bézier curves?



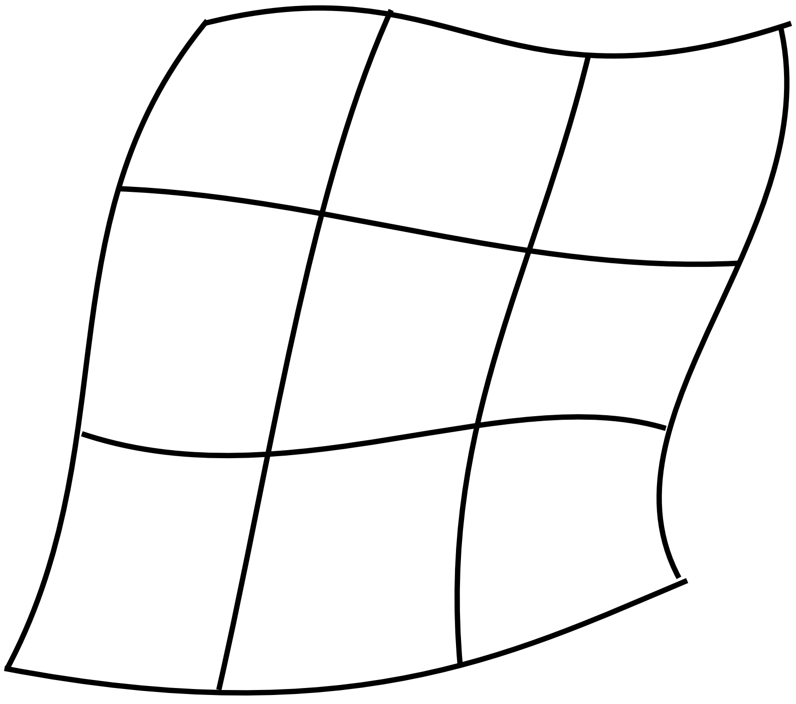
Tensor Product Bézier Patches

- In the previous, P_is were just some curves
- What if we make **them** Bézier curves?
- Each u=const. **and** v=const. curve is a Bézier curve!
- Note that the boundary control points (except corners) are NOT interpolated!



Tensor Product Bézier Patches

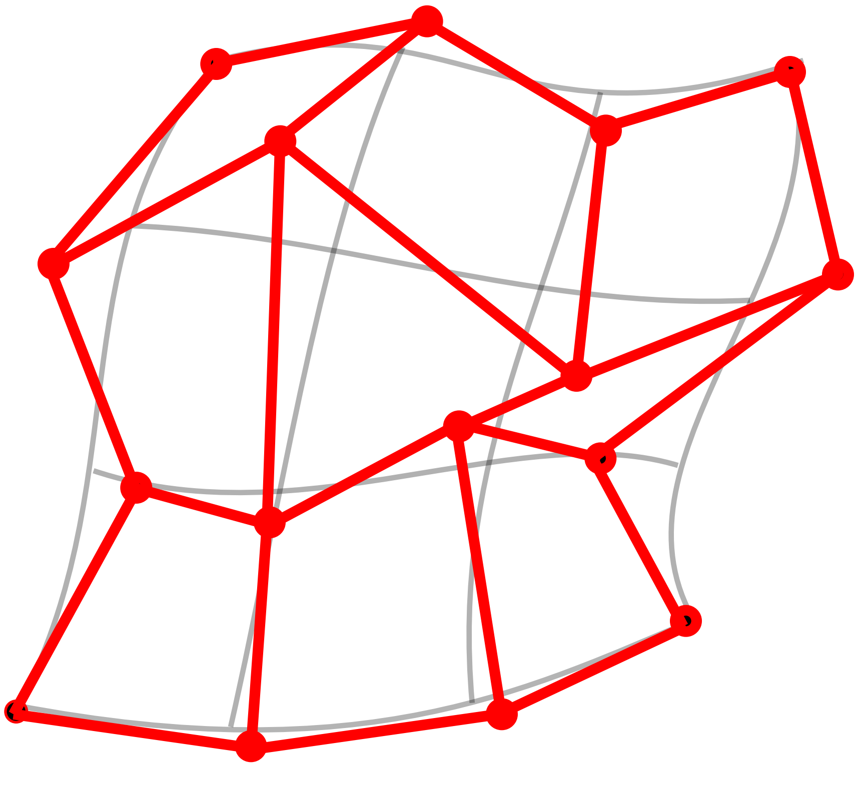
A **bicubic Bézier surface**



Tensor Product Bézier Patches

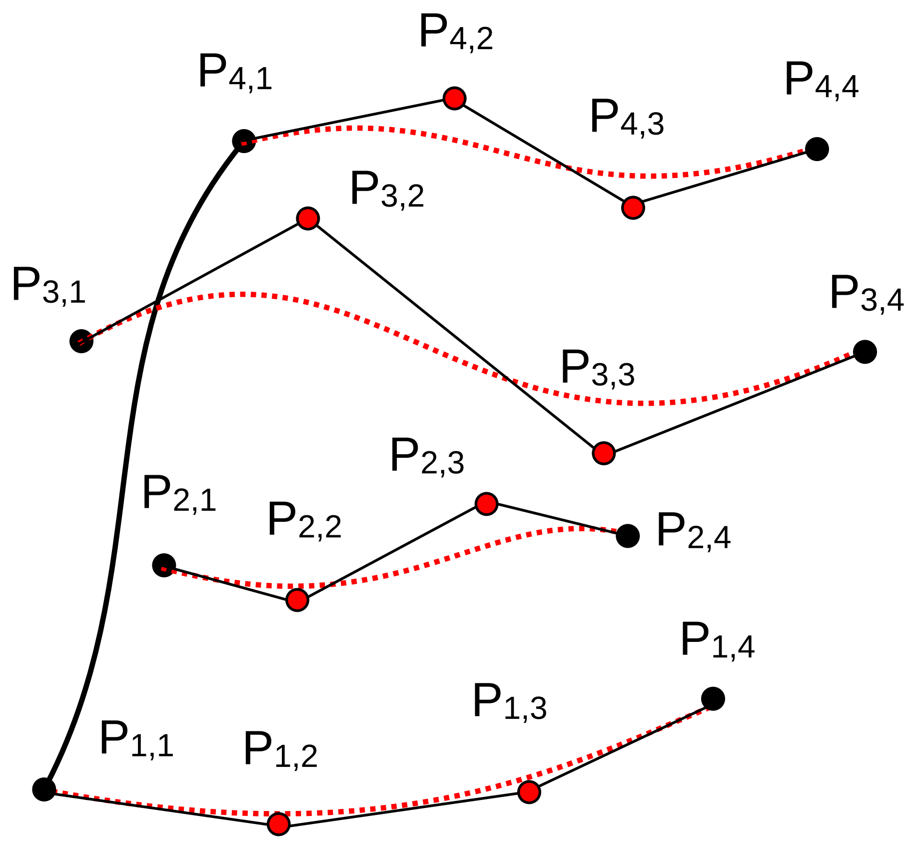
The “Control Mesh”

16 control points



Bicubics, Tensor Product

- $$P(u,v) = B_1(u) * P_1(v) + B_2(u) * P_2(v) + B_3(u) * P_3(v) + B_4(u) * P_4(v)$$
- $$P_i(v) = B_1(v) * P_{i,1} + B_2(v) * P_{i,2} + B_3(v) * P_{i,3} + B_4(v) * P_{i,4}$$



Bicubics, Tensor Product

- $P(u,v) = B_1(u) * P_1(v)$
  $+ B_2(u) * P_2(v)$
  $+ B_3(u) * P_3(v)$
  $+ B_4(u) * P_4(v)$
- $P_i(v) = B_1(v) * P_{i,1}$
  $+ B_2(v) * P_{i,2}$
  $+ B_3(v) * P_{i,3}$
  $+ B_4(v) * P_{i,4}$

$$P(u,v) =$$

$$\sum_{i=1}^{4} B_i(u) \left[\sum_{j=1}^{4} P_{i,j}\, B_j(v) \right]$$

$$= \sum_{i=1}^{4} \sum_{j=1}^{4} P_{i,j}\, B_{i,j}(u,v)$$

$$B_{i,j}(u,v) = B_i(u) B_j(v)$$

Bicubics, Tensor Product

- P(u,v) = B1(u) * P_1(v)
  - \+ B2(u) * P_2(v)
  - \+ B3(u) * P_3(v)
  - \+ B4(u) * P_4(v)
- Pi(v) = B1(v) * $P_{i,1}$
  - \+ B2(v) * $P_{i,2}$
  - \+ B3(v) * $P_{i,3}$
  - \+ B4(v) * $P_{i,4}$

$$P(u,v) =$$

16 control points $P_{i,j}$

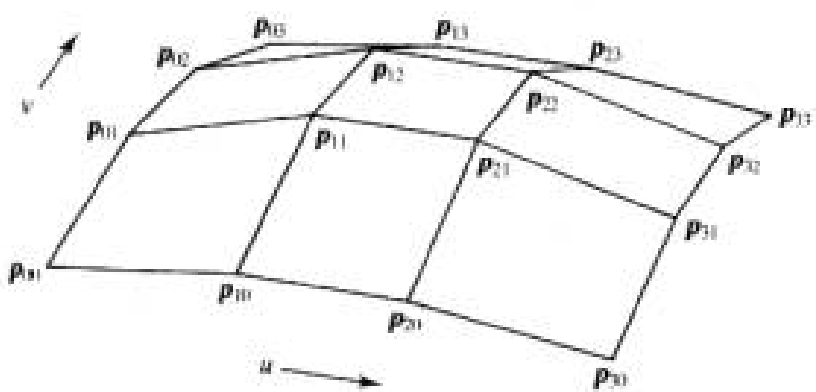
16 2D basis functions $B_{i,j}$

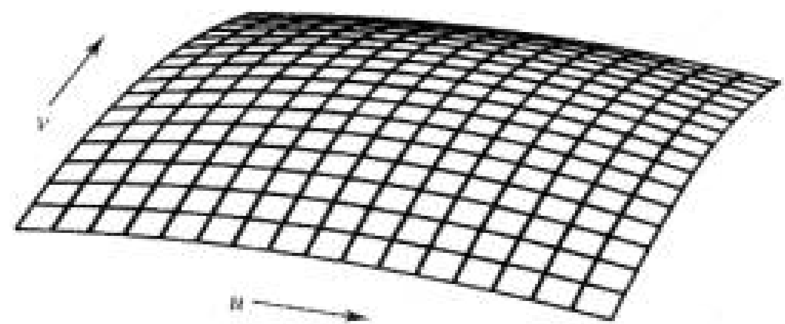
$$= \sum_{i=1}^{4} \sum_{j=1}^{4} \boxed{P_{i,j}} B_{i,j}(u,v)$$

$$\boxed{B_{i,j}(u,v) = B_i(u) B_j(v)}$$

Recap: Tensor Bézier Patches

- Parametric surface P(*u*,*v*) is a bicubic polynomial of two variables *u* & *v*
- Defined by 4x4=16 control points $P_{1,1}$, $P_{1,2}$.... $P_{4,4}$
- Interpolates 4 corners, approximates others
- Basis are product of two Bernstein polynomials: $B_1(u)B_1(v)$; $B_1(u)B_2(v)$;... $B_4(u)B_4(v)$

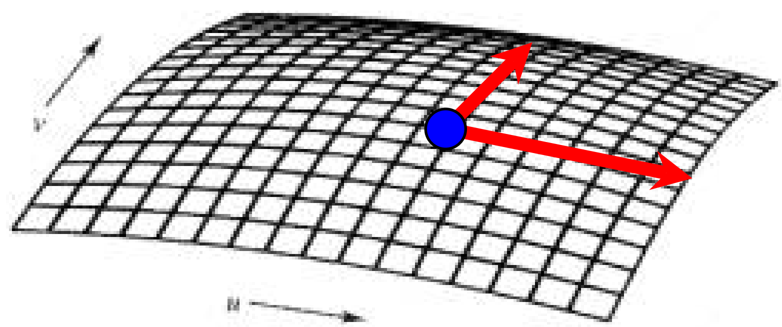




Questions?

Tangents and Normals for Patches

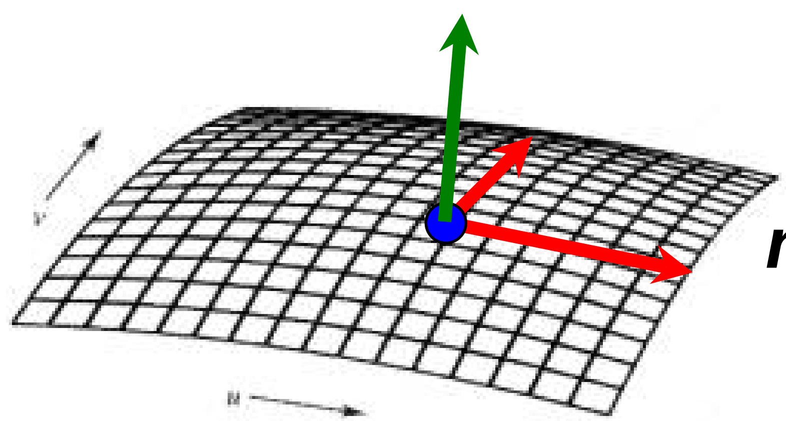
- P(u,v) is a 3D point specified by *u*, *v*
- The partial derivatives $\partial P/\partial u$ and $\partial P/\partial v$ are 3D vectors
  - Both are tangent to surface at P



Tangents and Normals for Patches

- P(u,v) is a 3D point specified by *u*, *v*
- The partial derivatives $\partial P/\partial u$ and $\partial P/\partial v$ are 3D vectors
  - Both are tangent to surface at P
  - Normal is perpendicular to both, i.e.,

$$n = (\partial P/\partial u) \times (\partial P/\partial v)$$



***n* is usually not unit, so must normalize!**

Questions?

Recap: Matrix Notation for Curves

- Cubic Bézier in matrix notation

point on curve
(2x1 vector)

Canonical
"power basis"

$$\mathrm{P}(\mathrm{t}) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} =$$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{pmatrix} \begin{pmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ t \\ t^2 \\ t^3 \end{pmatrix}$$

"Geometry matrix"
of control points $P_1..P_4$
(2 x 4)

"Spline matrix"
(Bernstein)

Hardcore: Matrix Notation for Patches

- Not required, but convenient!

x coordinate of surface at (*u*,*v*)

$$P(u,v) = \sum_{i=1}^{4} B_i(u) \left[\sum_{j=1}^{4} P_{i,j}\, B_j(v) \right]$$

$$P^x(u,v) =$$

Column vector of basis functions (*v*)

$$(B_1(u), \ldots, B_4(u)) \begin{pmatrix} P^x_{1,1} & \cdots & P^x_{1,4} \\ \vdots & & \vdots \\ P^x_{4,1} & \cdots & P^x_{4,4} \end{pmatrix} \begin{pmatrix} B_1(v) \\ \vdots \\ B_4(v) \end{pmatrix}$$

Row vector of basis functions (*u*)

4x4 matrix of *x* coordinates of the control points

Hardcore: Matrix Notation for Patches

- Curves:

$$P(t) = \boldsymbol{G}\,\boldsymbol{B}\,\boldsymbol{T}(t)$$

- Surfaces:

$$P^x(u,v) = \boldsymbol{T}(u)^{\mathrm{T}}\,\boldsymbol{B}^{\mathrm{T}}\,\boldsymbol{G}^x\,\boldsymbol{B}\,\boldsymbol{T}(v)$$

A separate 4x4 geometry matrix for x, y, z

- **T** = power basis
  B = spline matrix
  G = geometry matrix

Super Hardcore: Tensor Notation

- You can stack the $\boldsymbol{G}^x$, $\boldsymbol{G}^y$, $\boldsymbol{G}^z$ matrices into a geometry **tensor** of control points
  - I.e., $G^k_{i,j}$ = the k^{th} coordinate of control point $P_{i,j}$
  - A cube of numbers!

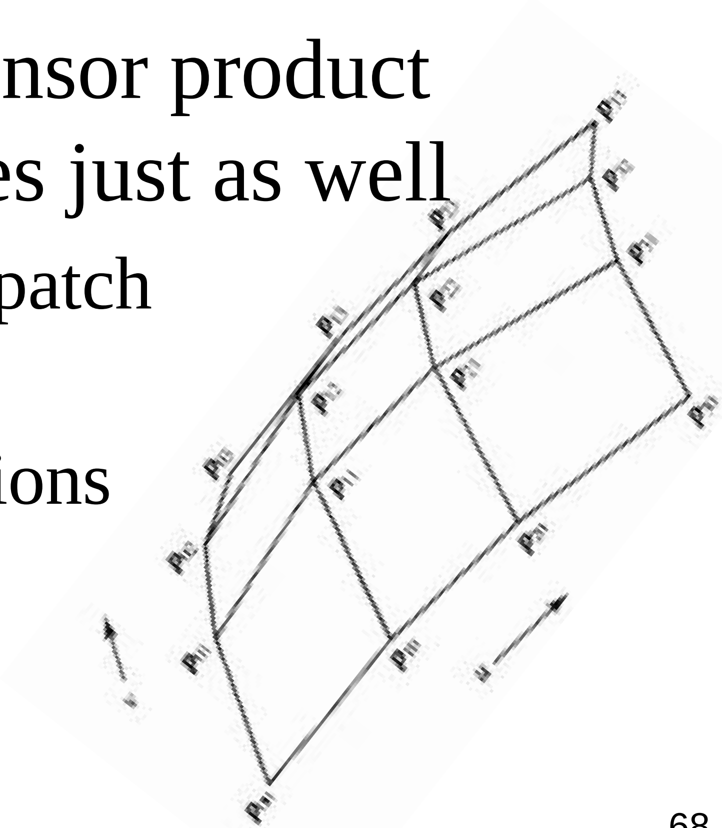
$$P^k(u,v) = \boldsymbol{T}^l(u)\,\boldsymbol{B}_l^i\,\boldsymbol{G}_{ij}^k\,\boldsymbol{B}_m^j\,\boldsymbol{T}^m(v)$$

- "Definitely not required, but nice!
  - See http://en.wikipedia.org/wiki/Multilinear_algebra

Tensor Product B-Spline Patches

- Bézier and B-Spline curves are both cubics
  - Can change between representations using matrices

- Consequently, you can build tensor product surface patches out of B-Splines just as well
  - Still 4x4 control points for each patch
  - 2D basis functions are pairwise products of B-Spline basis functions
  - Yes, simple!

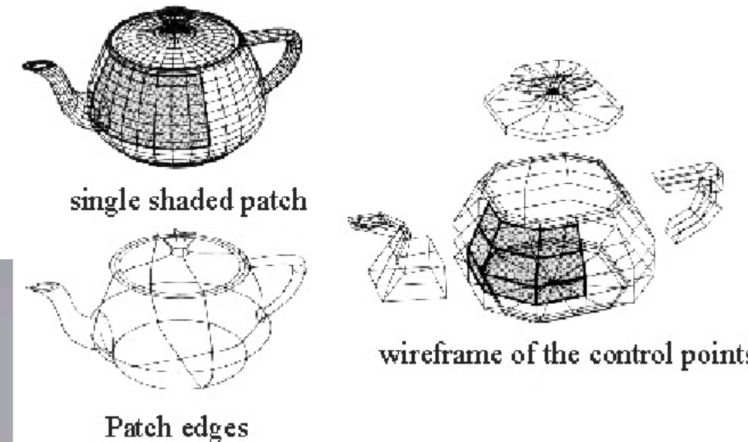


Tensor Product Spline Patches

- Pros
  - Smooth
  - Defined by reasonably small set of points
- Cons
  - Harder to render (usually converted to triangles)
  - Tricky to ensure continuity at patch boundaries
- Extensions
  - Rational splines: Splines in homogeneous coordinates
  - NURBS: Non-Uniform Rational B-Splines
    - Like curves: ratio of polynomials, non-uniform location of control points, etc.

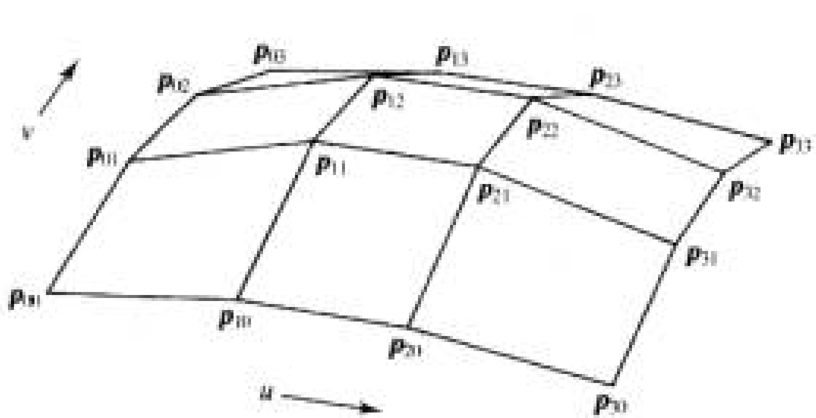
Utah Teapot: Tensor Bézier Splines

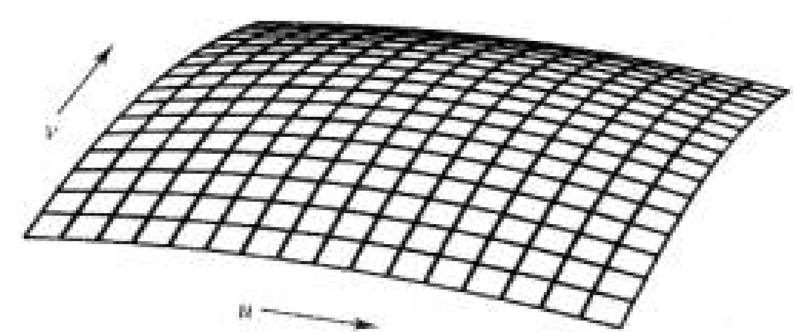
- Designed by Martin Newell



Cool: Displacement Mapping

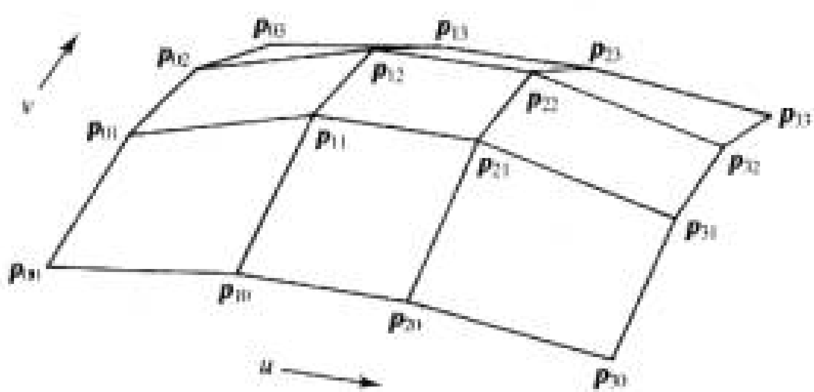
- Not all surfaces are smooth...

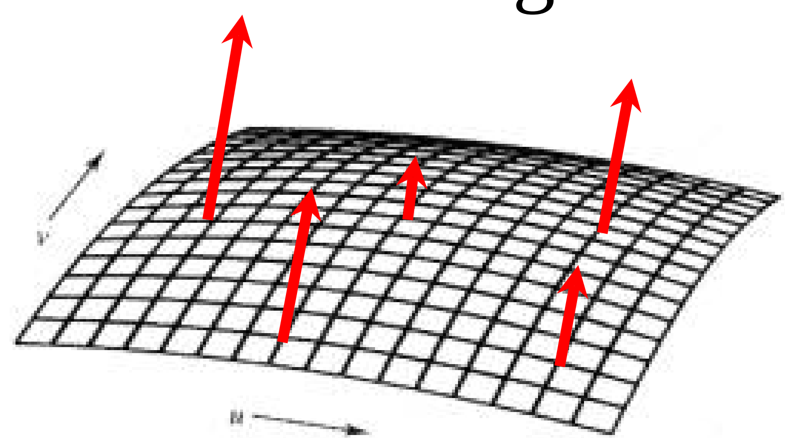




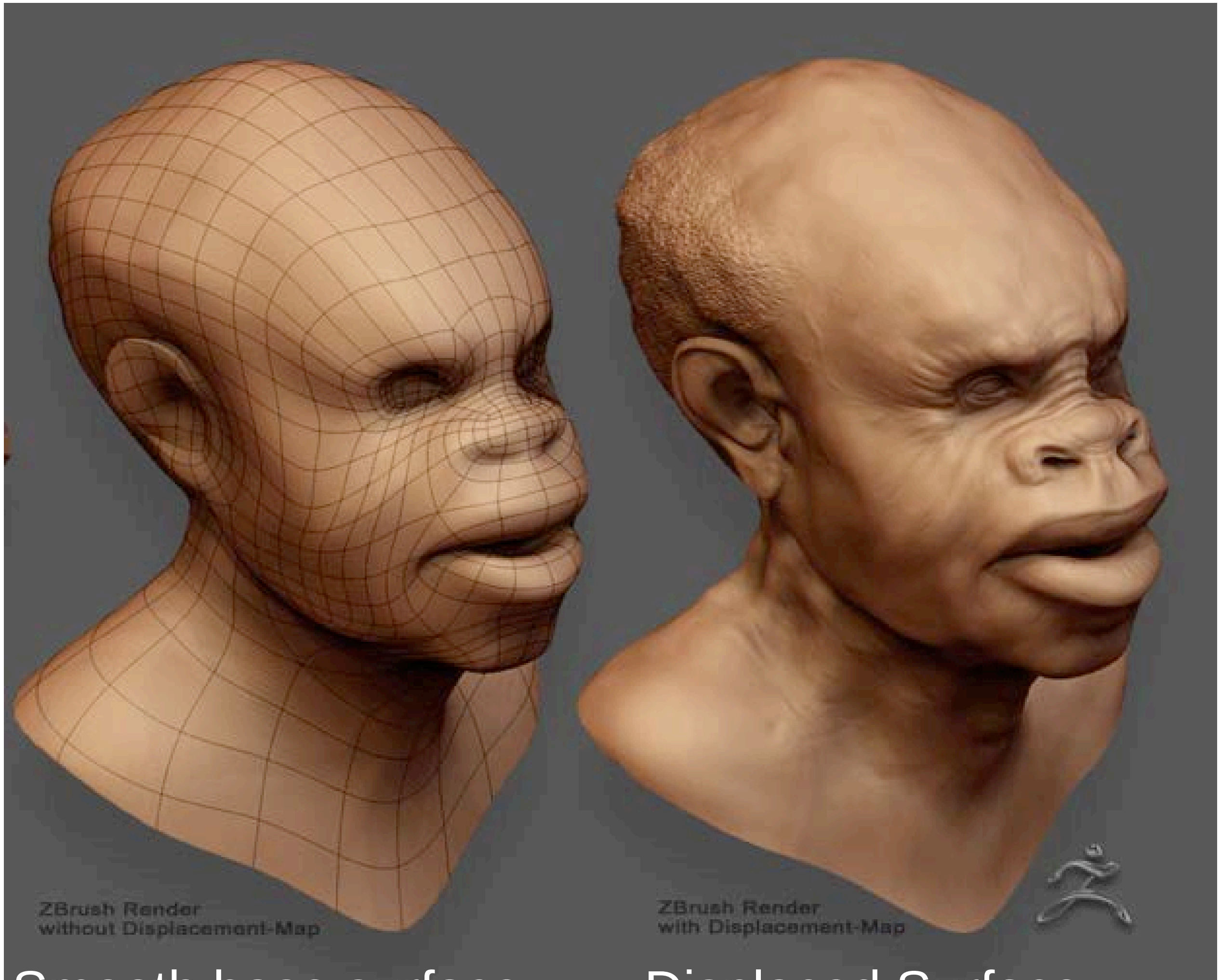
Cool: Displacement Mapping

- Not all surfaces are smooth...
- "Paint" displacements on a smooth surface
  - – For example, in the direction of normal
- Tessellate smooth patch into fine grid, then add displacement D(u,v) to vertices
- Heavily used in movies, more and more in games





Displacement Mapping Example



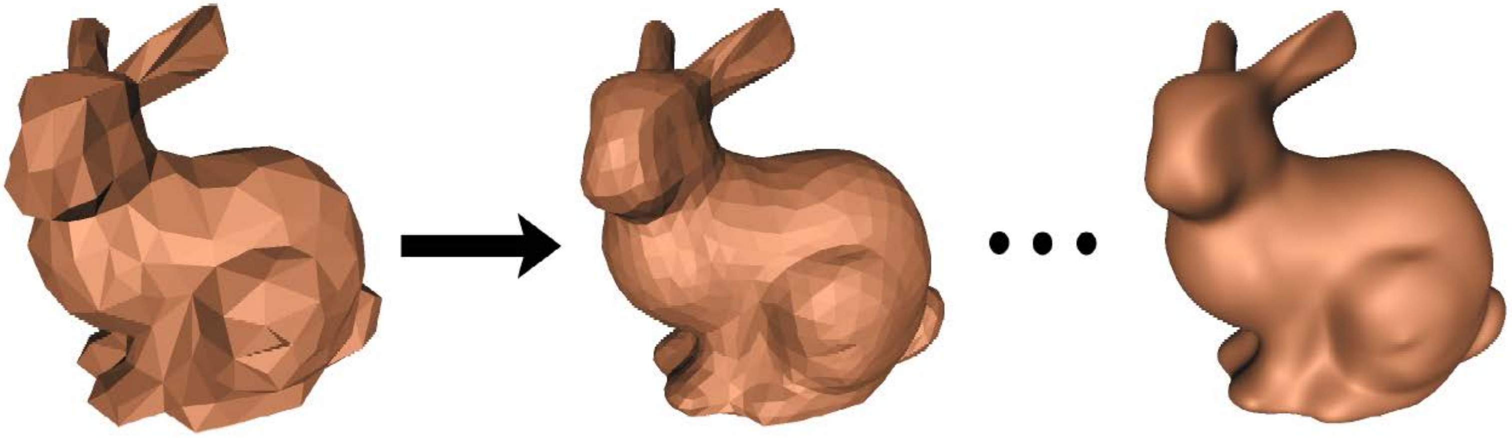
Smooth base surface Displaced Surface

zbrushcentral

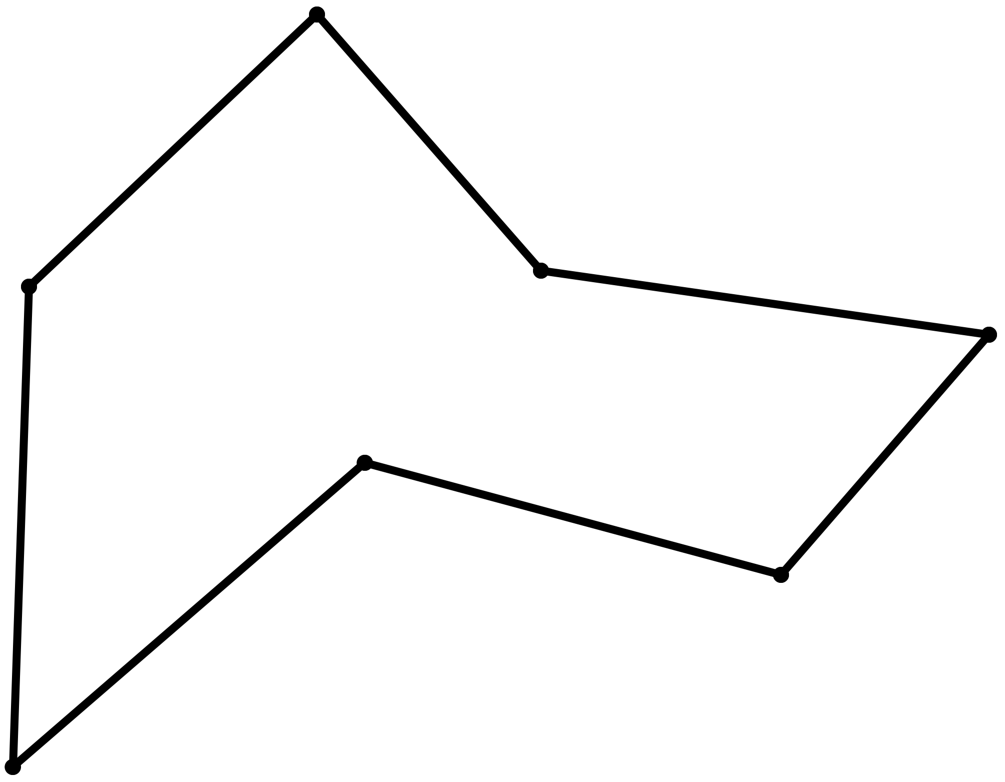
Questions?

Subdivision Surfaces

- Start with polygonal mesh
- Subdivide into larger number of polygons, smooth result after each subdivision
  - Lots of ways to do this.
- The limit surface is smooth!

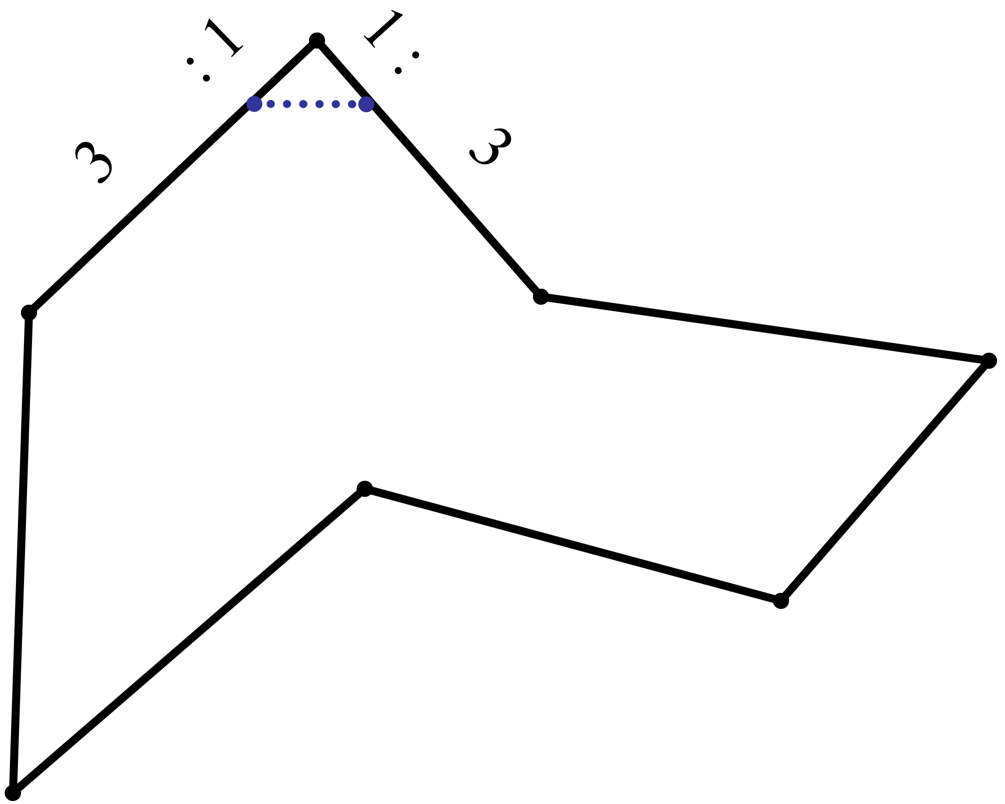


Corner Cutting



Slide by Adi Levin

Corner Cutting



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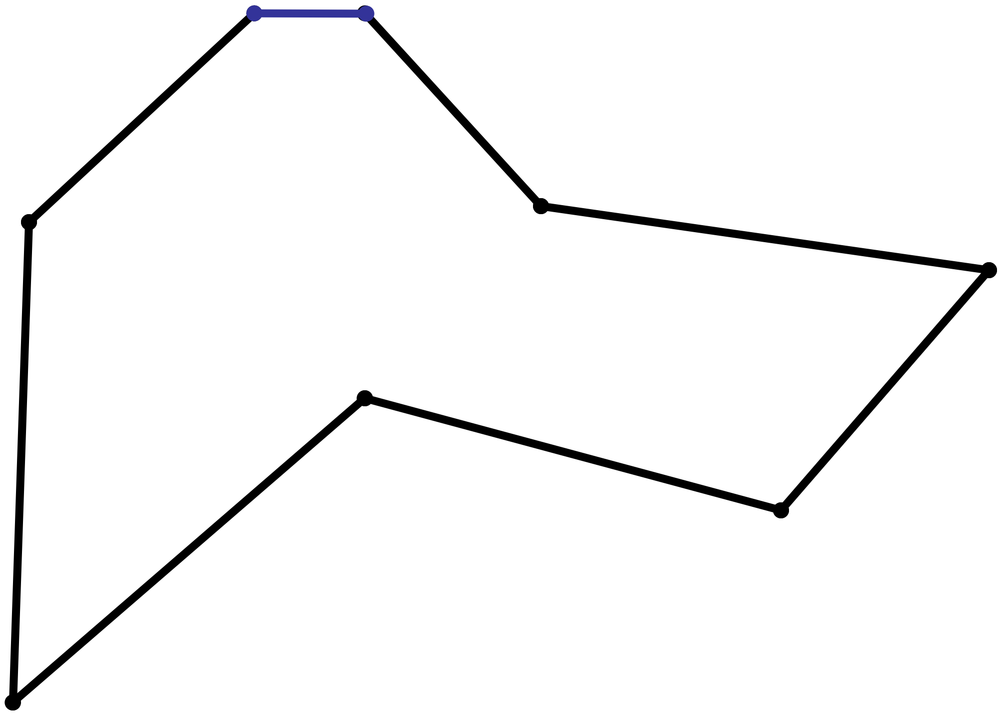
Corner Cutting



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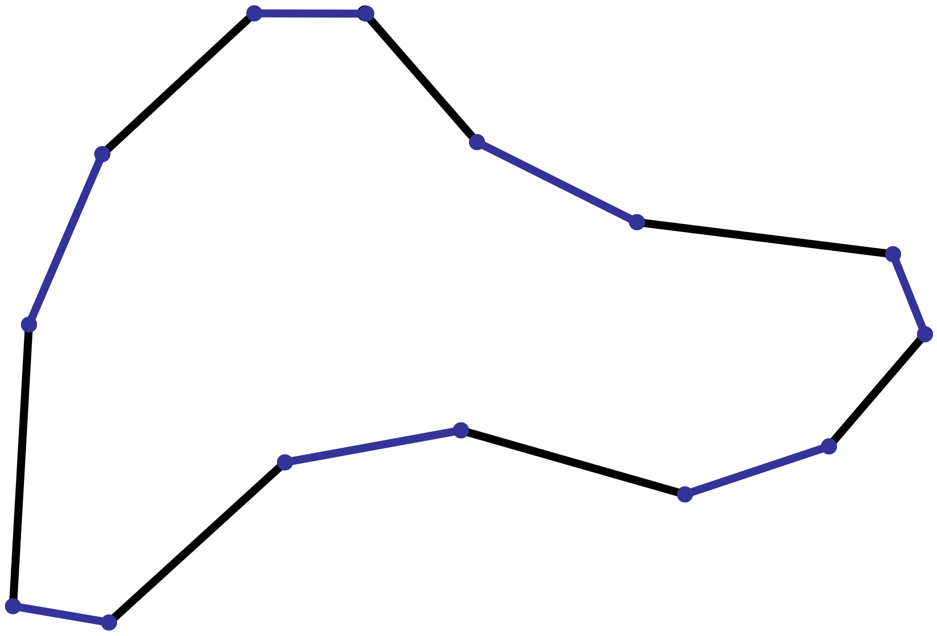
Corner Cutting



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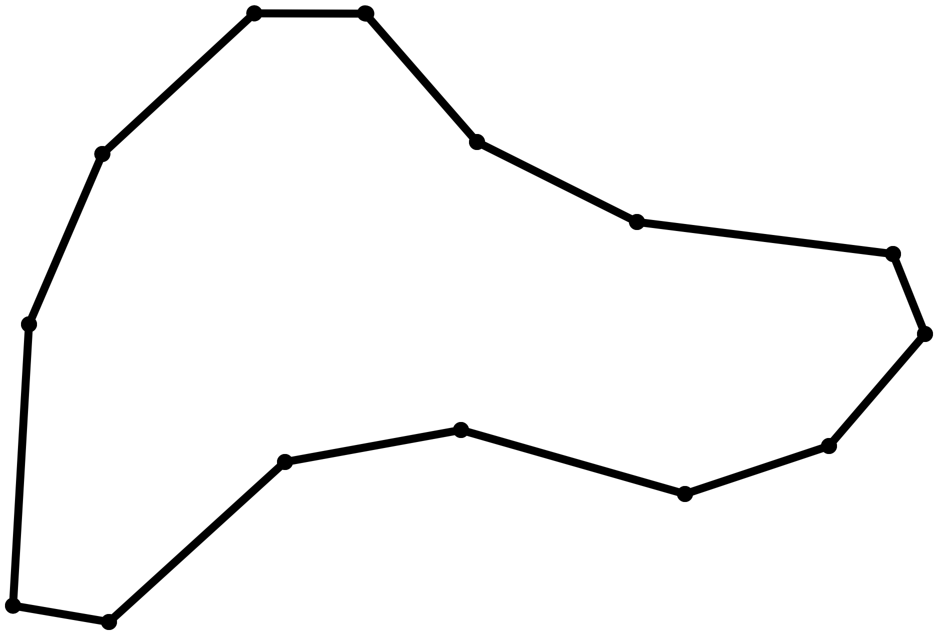

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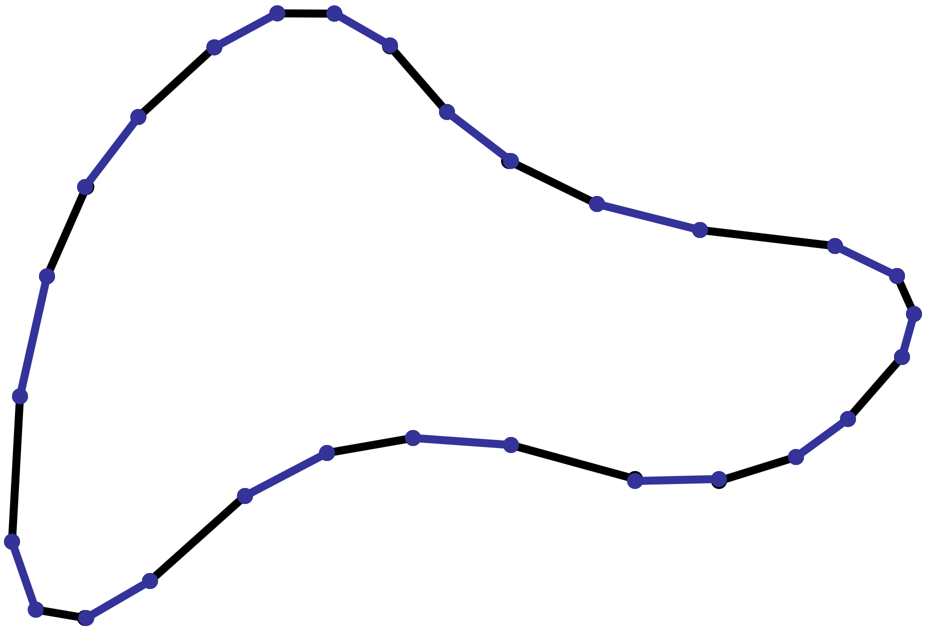
Corner Cutting



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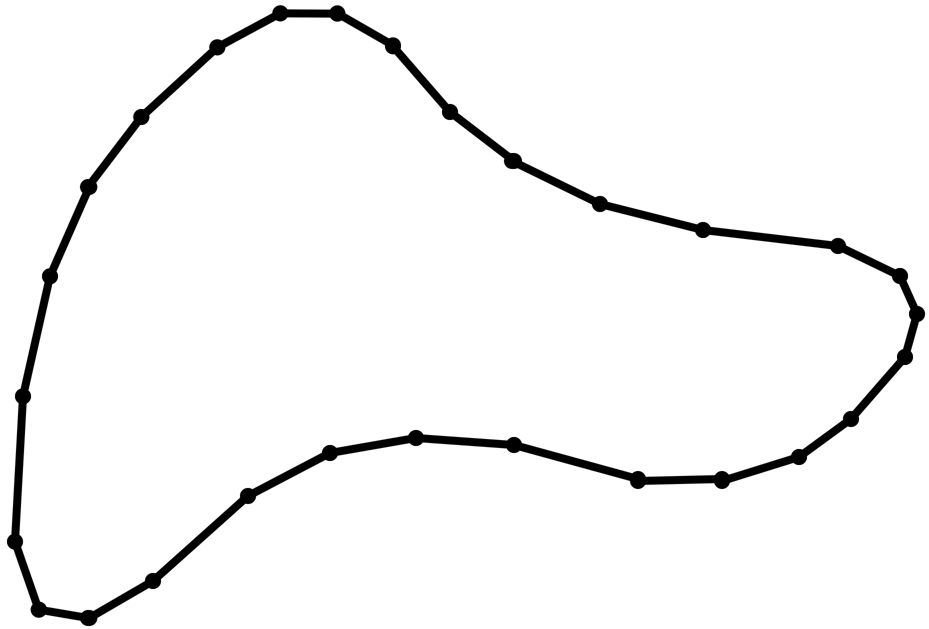

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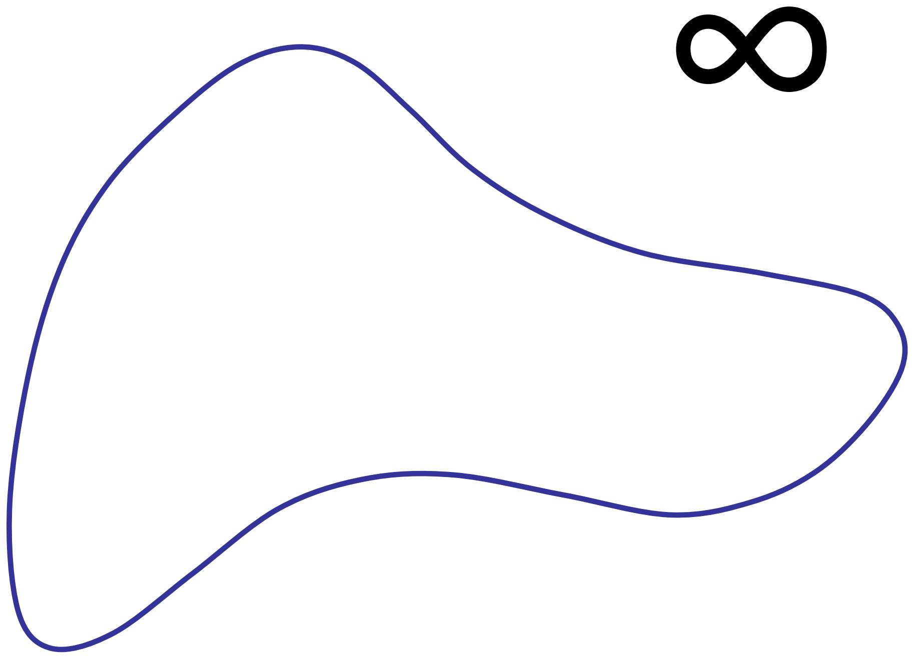
Corner Cutting



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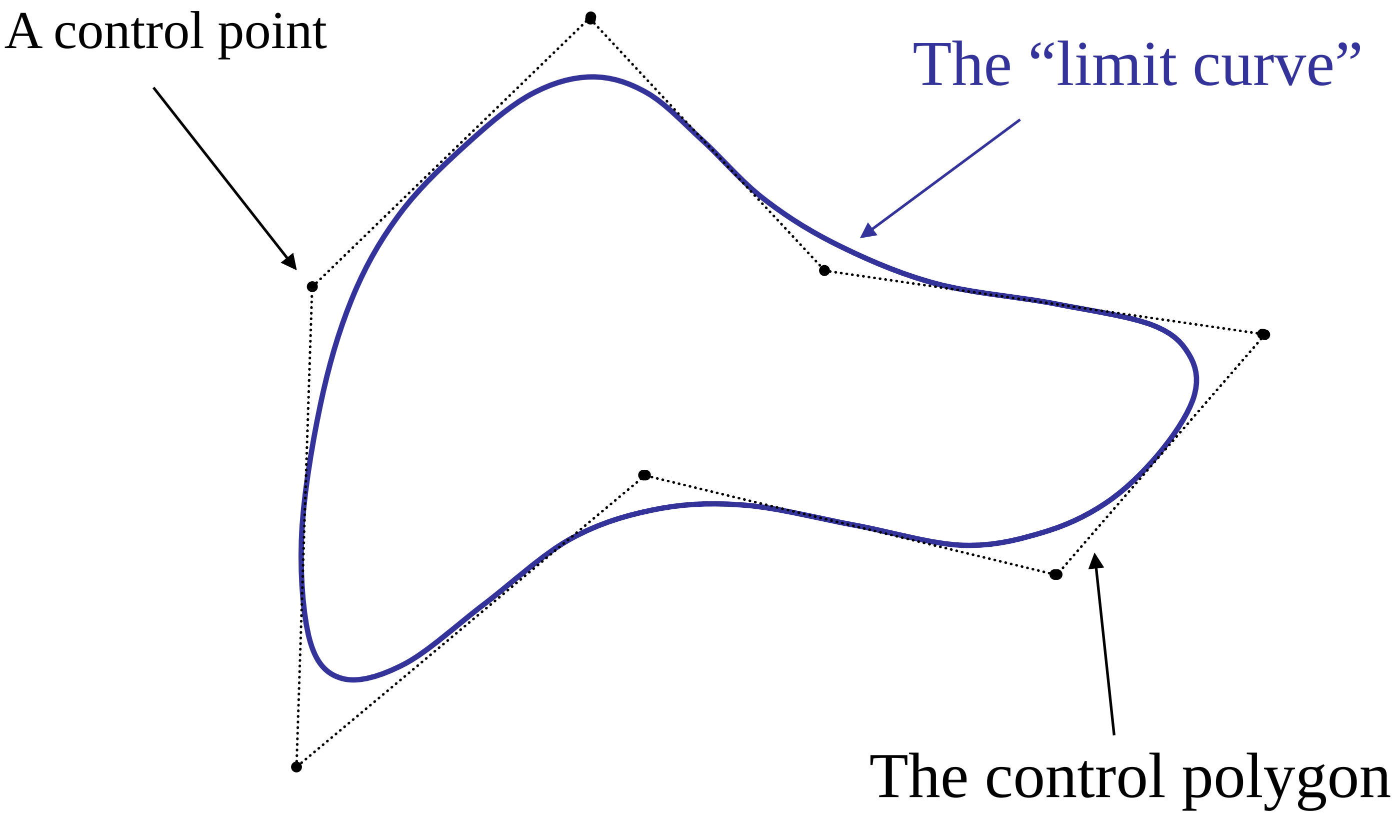
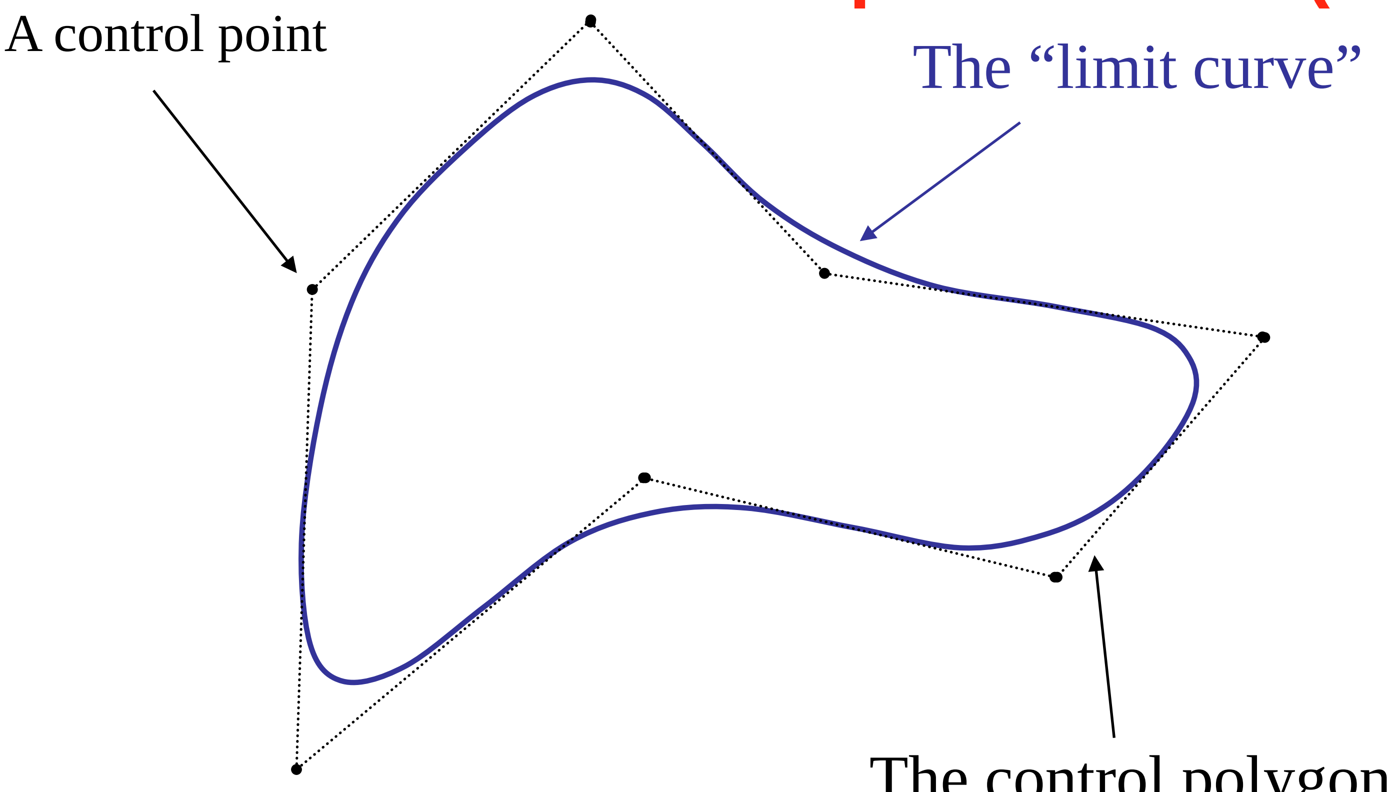
Corner Cutting



Slide by Adi Levin

Corner Cutting

It turns out corner cutting (Chaikin's Algorithm) produces a quadratic B-Spline curve! (Magic!)



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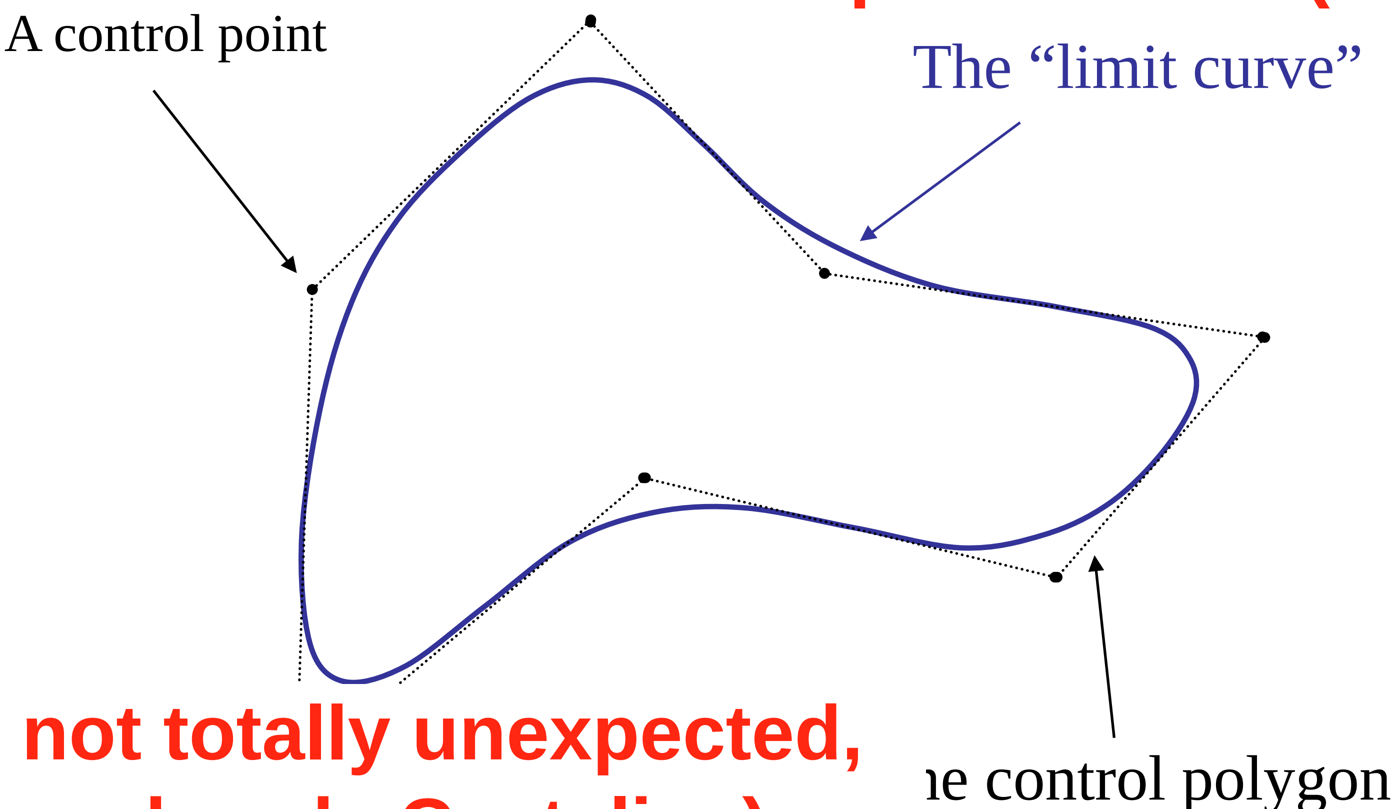
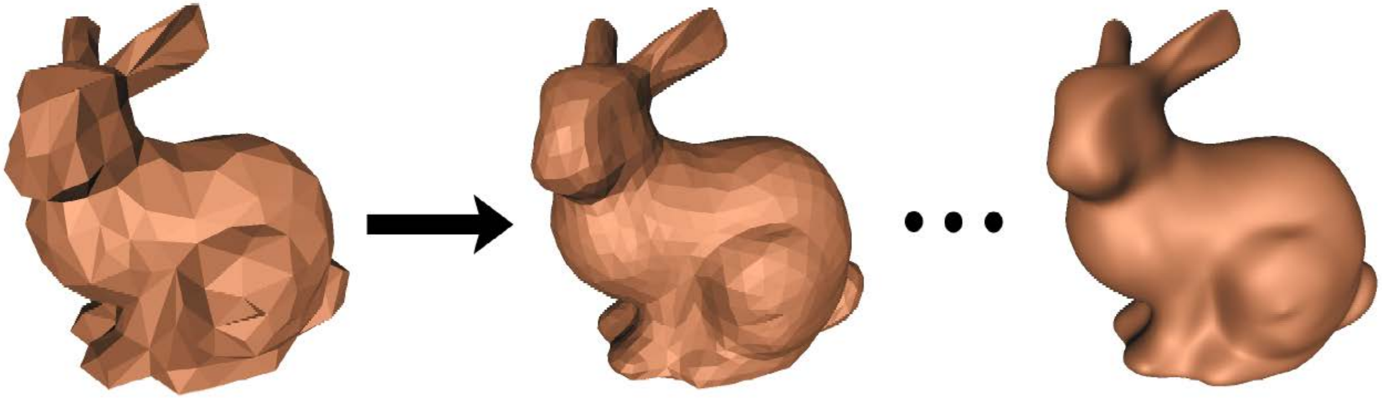


(Well, not totally unexpected, remember de Casteljau)

Slide by Adi Levin

Subdivision Curves and Surfaces

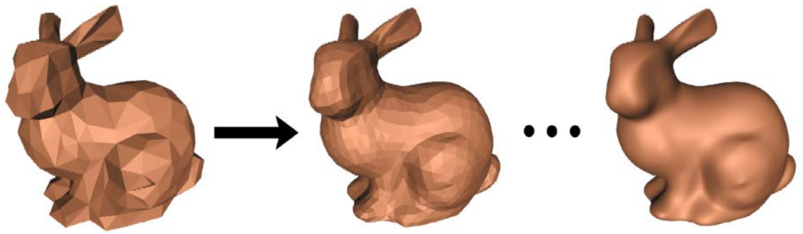
- Idea: cut corners to smooth
- Add points and compute weighted average of neighbors
- Same for surfaces
  - Special case for irregular vertices
    - vertex with more or less than 6 neighbors in a triangle mesh



Warren et al.

Subdivision Curves and Surfaces

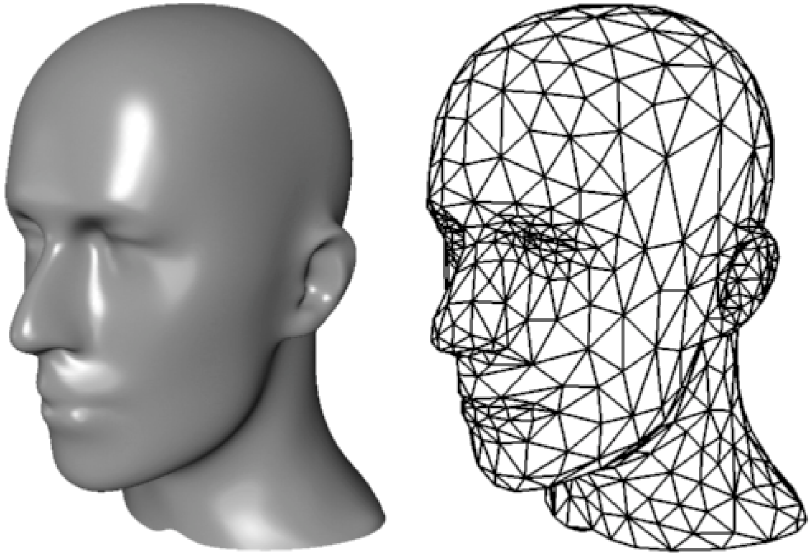
- Advantages
  - Arbitrary topology
  - Smooth at boundaries
  - Level of detail, scalable
  - Simple representation
  - Numerical stability, well-behaved meshes
  - Code simplicity
- Little disadvantage:
  - Procedural definition
  - Not parametric
  - Tricky at special vertices



Warren et al.

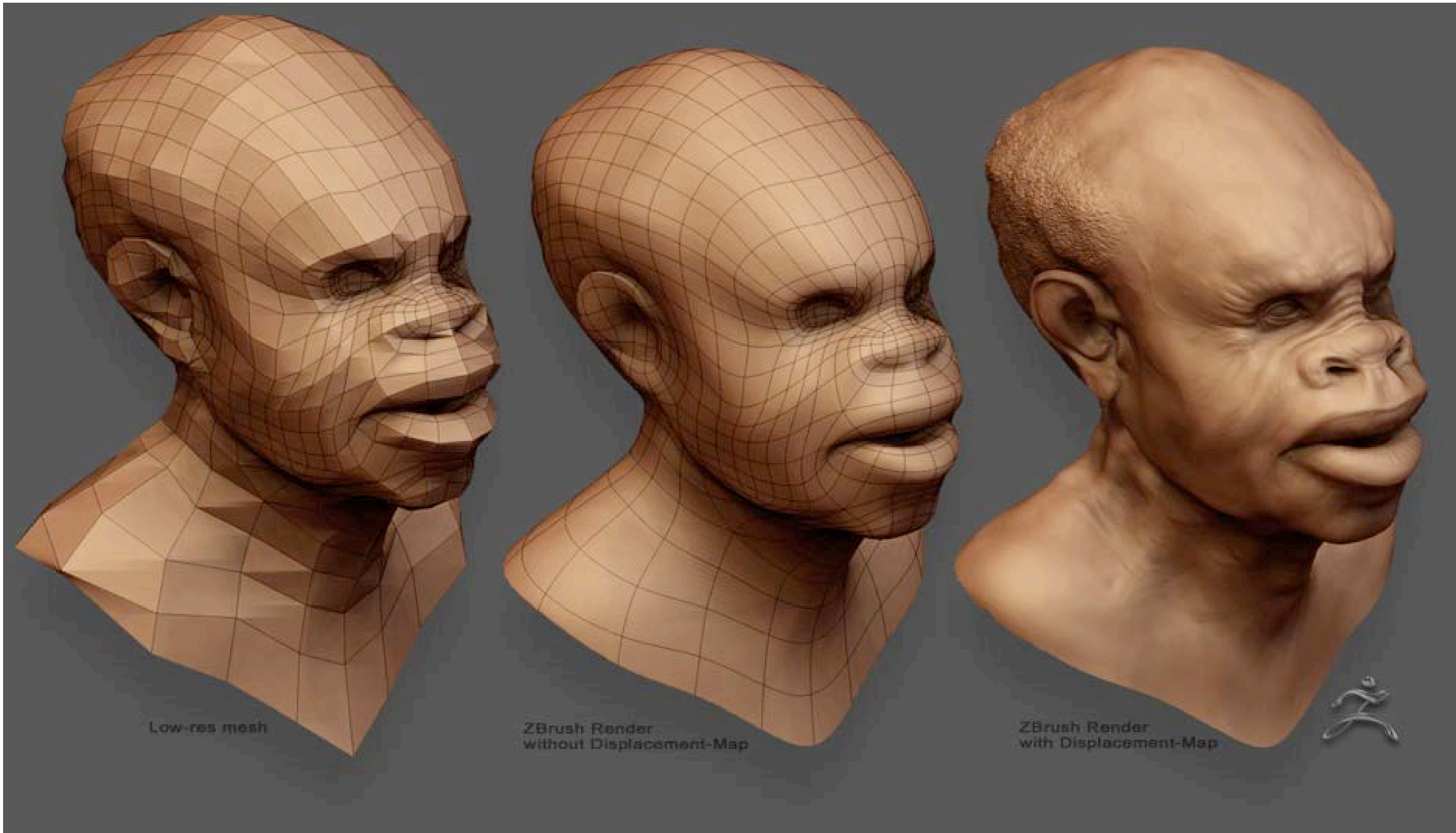
Flavors of Subdivision Surfaces

- Catmull-Clark
  - Quads and triangles
  - Generalizes bicubics to arbitrary topology!
- Loop, Butterfly
  - Triangles
- Doo-Sabin, sqrt(3), biquartic...
  - and a whole host of others
- Used **everywhere** in movie and game modeling!
- See http://www.cs.nyu.edu/~dzorin/sig00course/



Leif Kobbelt

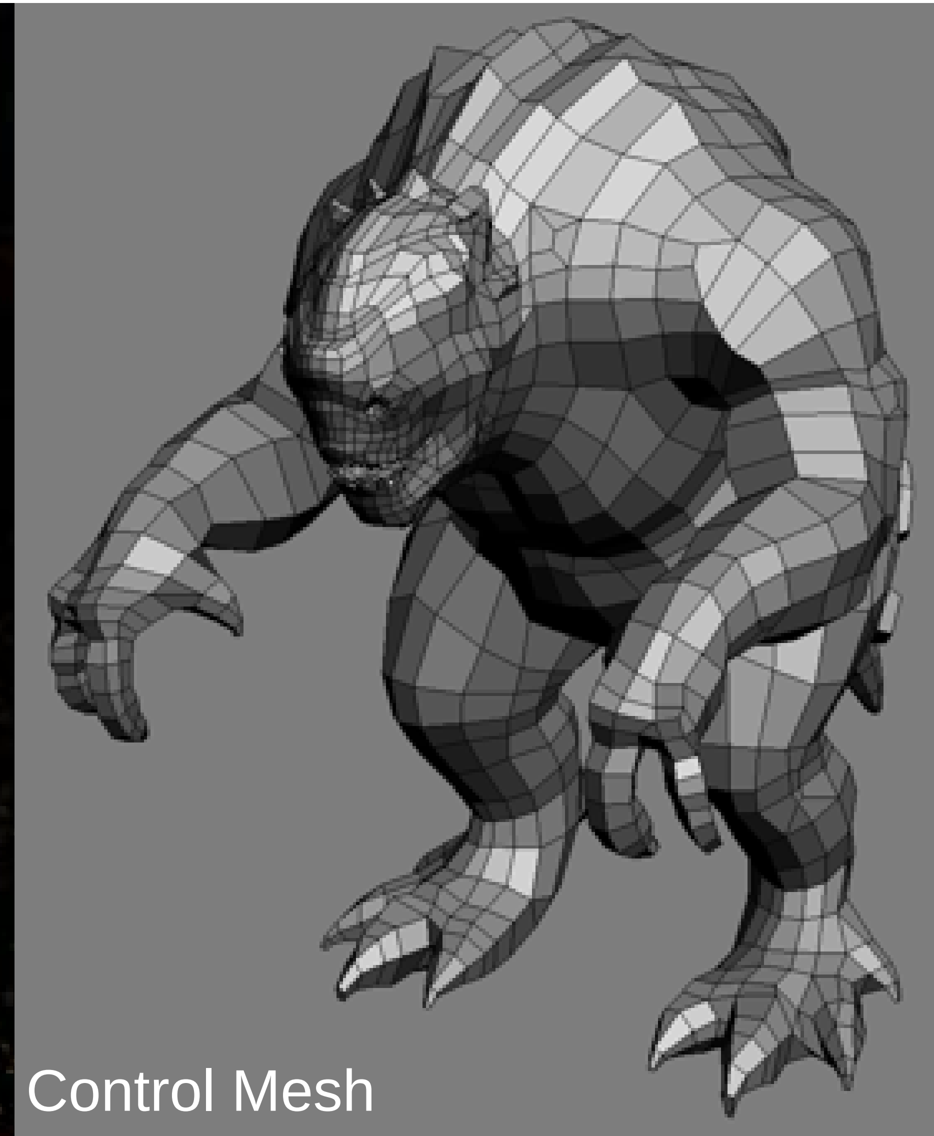
Subdivision + Displacement



zbrushcentral

Subdivision + Displacement

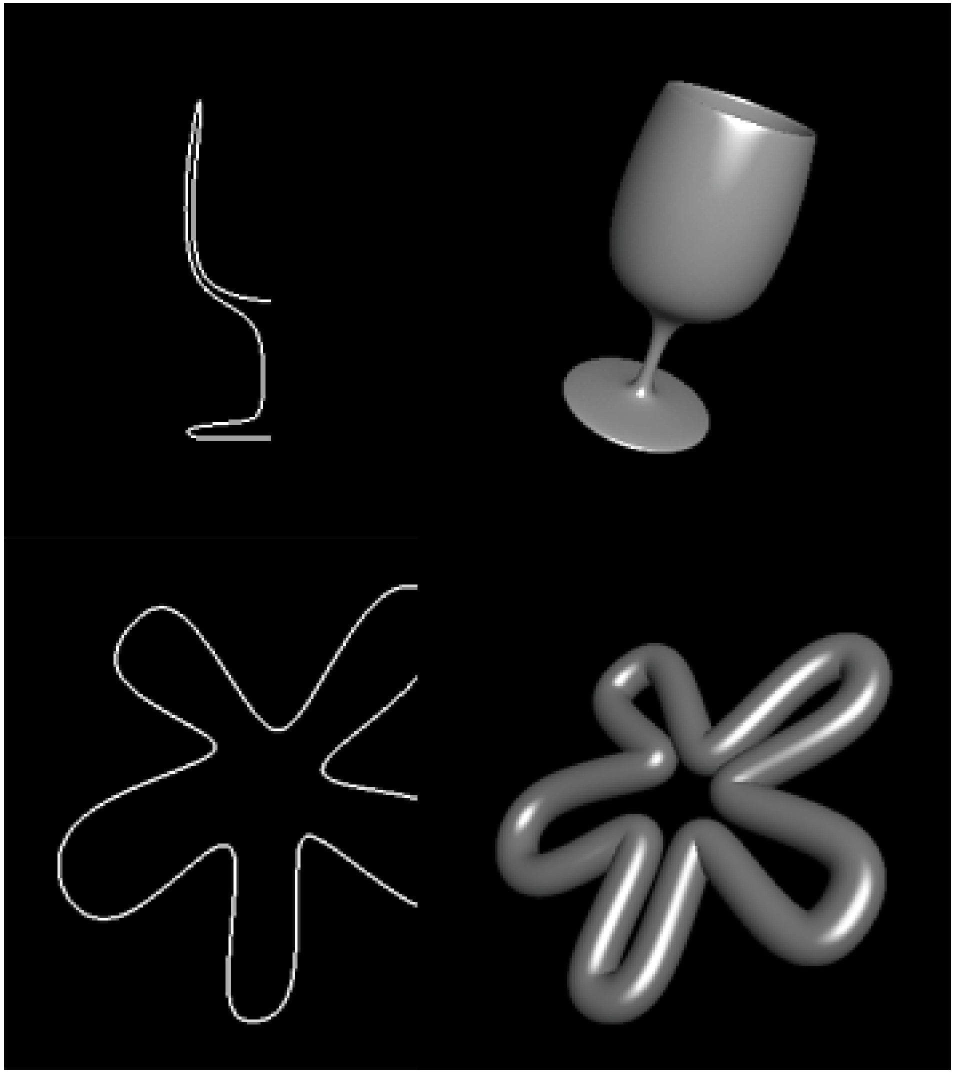
Epic Games



Questions?

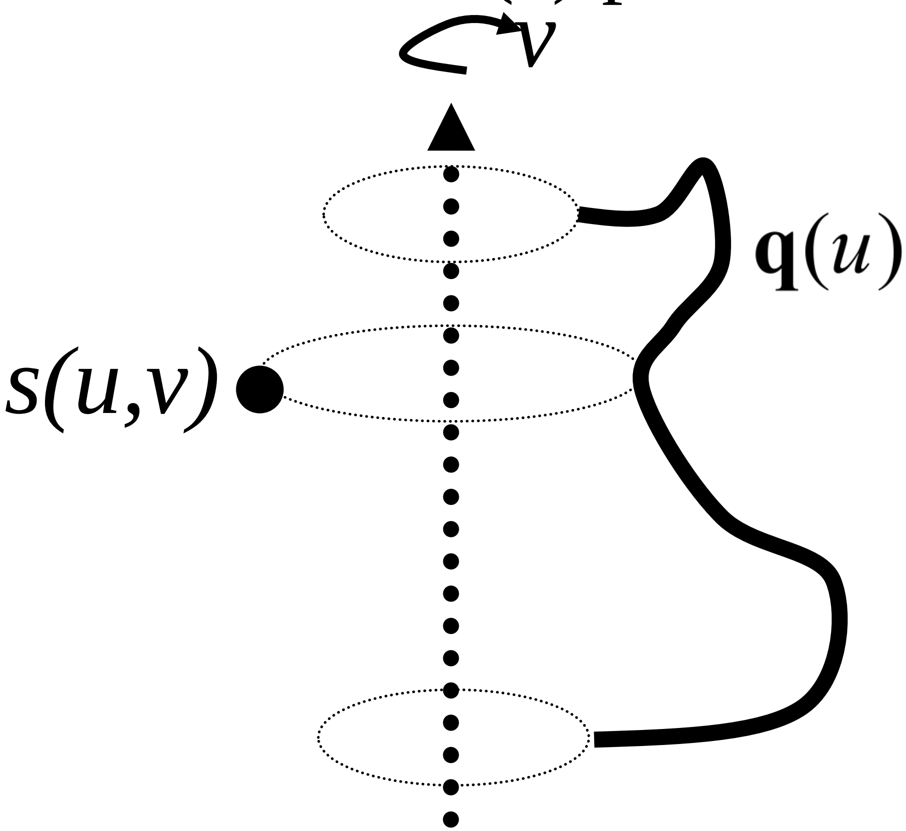
Specialized Procedural Definitions

- Surfaces of revolution
  - Rotate given 2D profile curve
- Generalized cylinders
  - Given 2D profile and 3D curve, sweep the profile along the 3D curve
- **Assignment 1!**



Surface of Revolution

- 2D curve q(*u*) provides one dimension
  - Note: works also with 3D curve
- Rotation R(*v*) provides 2nd dimension



$\boldsymbol{s}(u,v)=\boldsymbol{R}(v)\boldsymbol{q}(u)$

where ***R*** is a matrix,

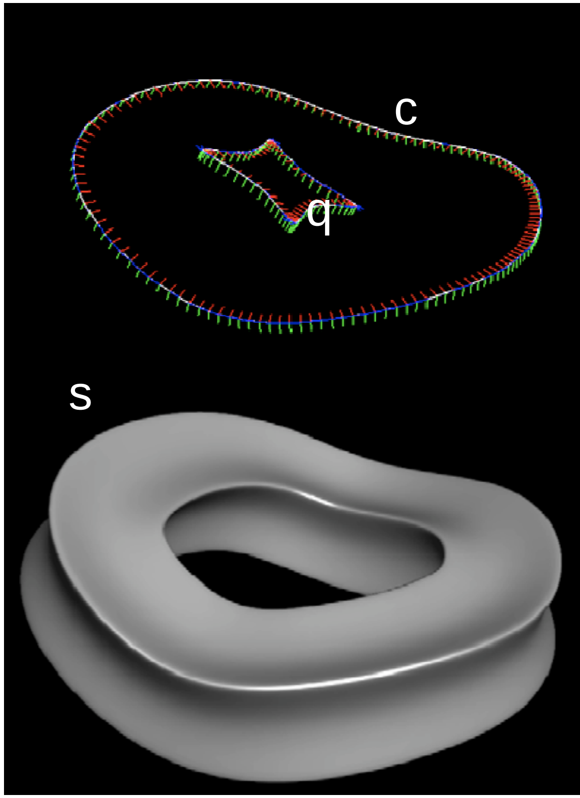
q a vector,

and ***s*** is a point on

the surface

General Swept Surfaces

- Trace out surface by moving a profile curve along a trajectory.
  - profile curve **q**(*u*) provides one dim
  - trajectory **c**(*u*) provides the other
- Surface of revolution can be seen as a special case where trajectory is a circle

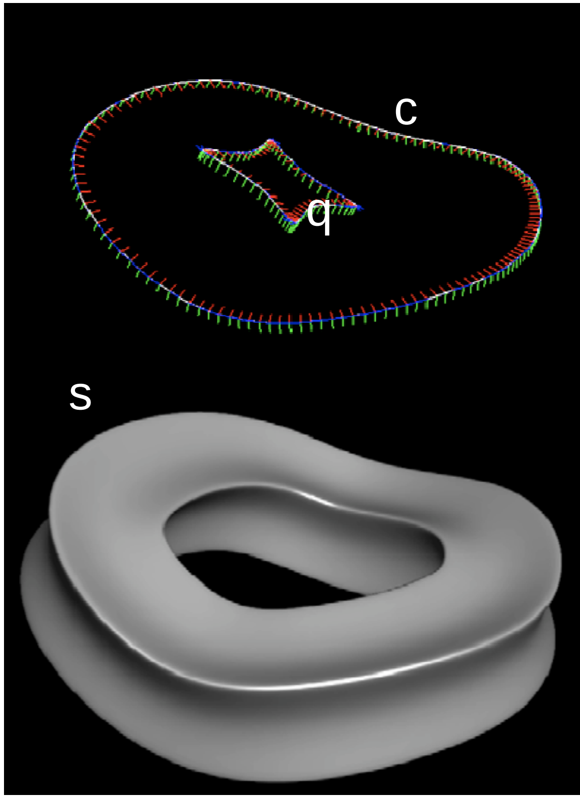


$$\mathbf{s}(u,v)=\mathbf{M}(\mathbf{c}(v))\mathbf{q}(u)$$

where ***M*** is a matrix that depends on the trajectory ***c***

General Swept Surfaces

- How do we get **M**?
  - Translation is easy, given by ***c****(v)*
  - What about orientation?
- Orientation options:
  - Align profile curve with an axis.
  - **Better**: Align profile curve with frame that "follows" the curve

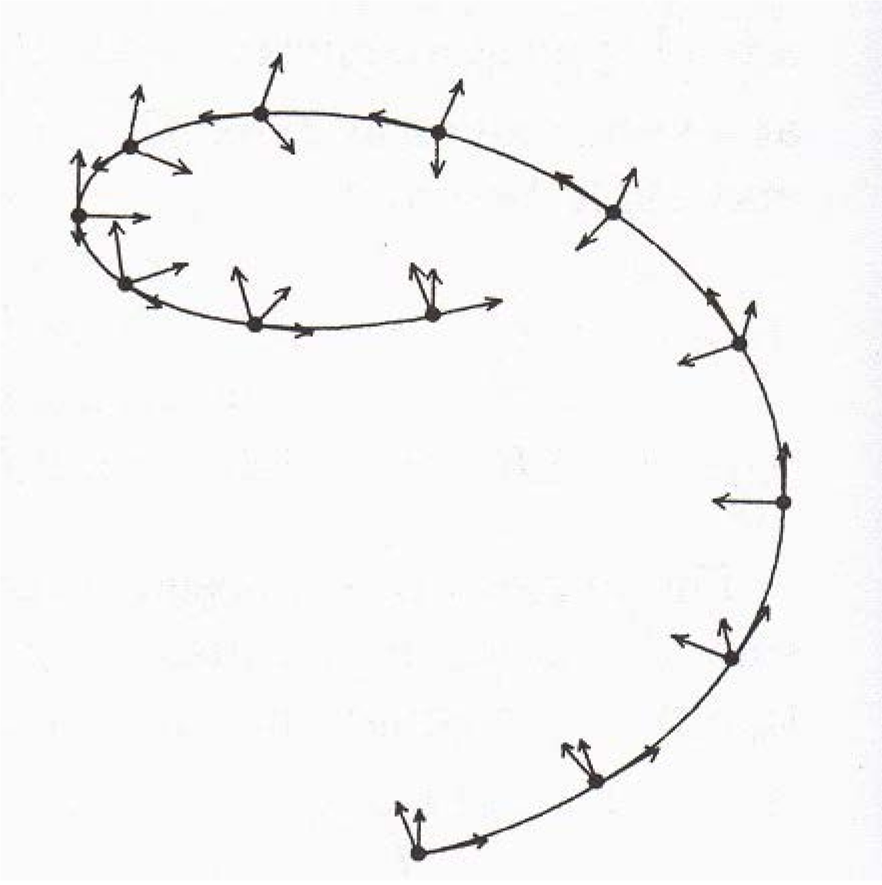


$$\mathbf{s}(u,v)=\mathbf{M}(\mathbf{c}(v))\mathbf{q}(u)$$

where ***M*** is a matrix that depends on the trajectory ***c***

Frames on Curves: Frenet Frame

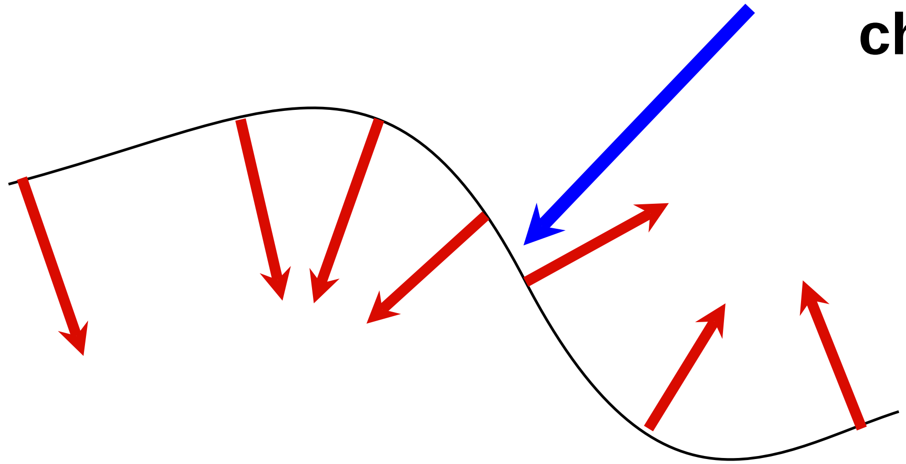
- Frame defined by 1st (tangent), 2nd and 3rd derivatives of a 3D curve
- Looks like a good idea for swept surfaces...



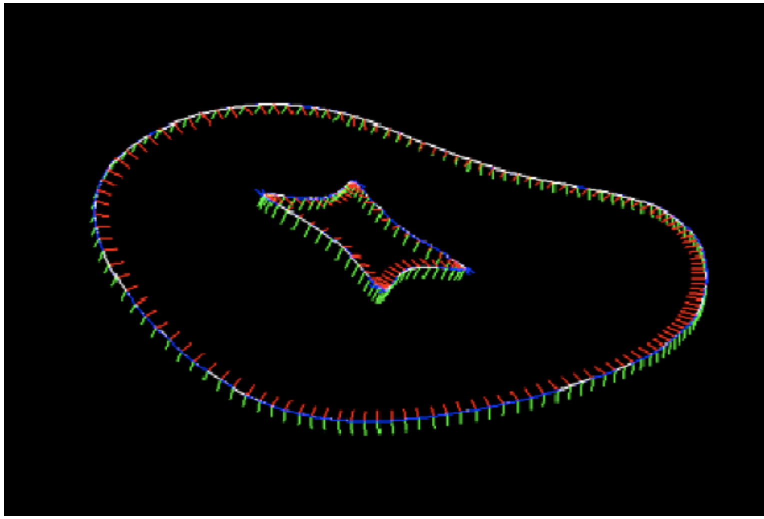
Frenet: Problem at Inflection!

- Normal flips!
- Bad to define a smooth swept surface

An inflection is a point where curvature changes sign



Smooth Frames on Curves



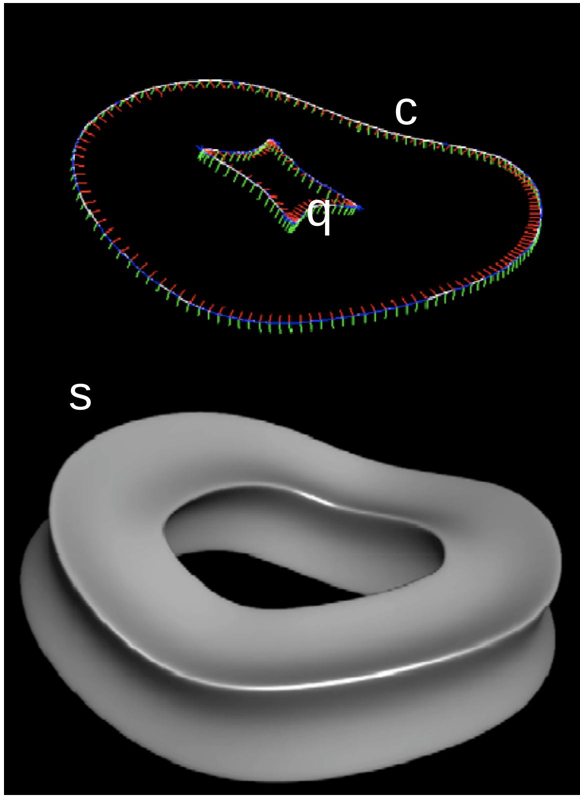
- Build triplet of vectors
  - include tangent (it is reliable)
  - orthonormal
  - coherent over the curve
- Idea:
  - use cross product to create orthogonal vectors
  - exploit discretization of curve
  - use previous frame to bootstrap orientation
  - **See Assignment 1 instructions!**

Normals for Swept Surfaces

- Need partial derivatives w.r.t. both *u* and *v*

$$n = (\partial \mathbf{s}/\partial u) \times (\partial \mathbf{s}/\partial v)$$

  - *Remember to normalize!*
- One given by tangent of profile curve, the other by tangent of trajectory



$$\mathbf{s}(u,v)=\mathbf{M}(\mathbf{c}(v))\mathbf{q}(u)$$

where ***M*** is a matrix that depends on the trajectory ***c***

Questions?

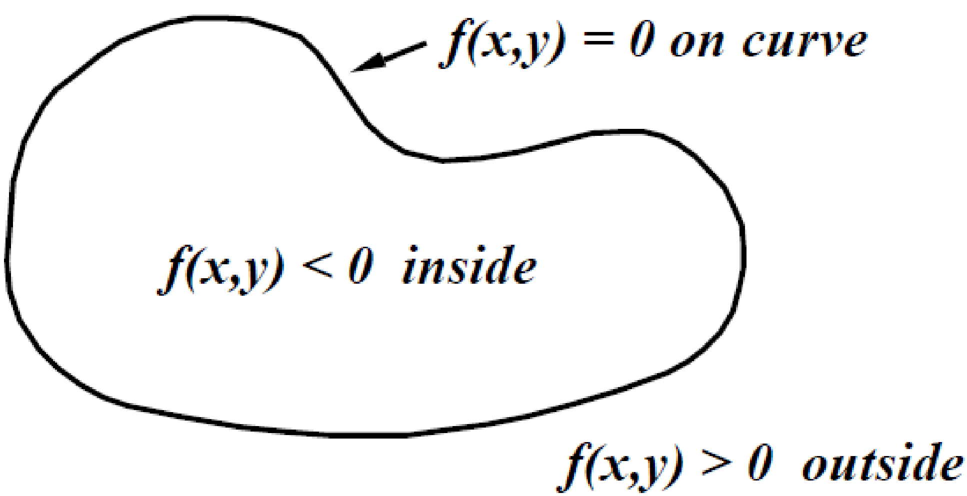
Implicit Surfaces

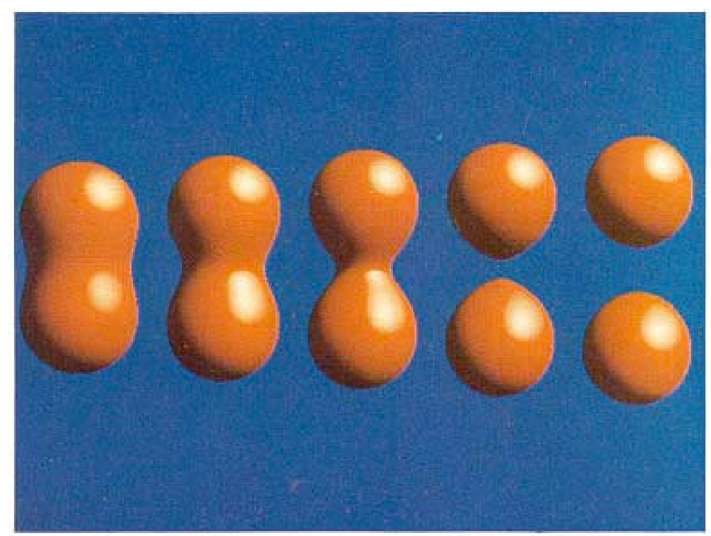
- Surface defined implicitly by a function

  $f(x, y, z) = 0$ (on surface)

  $f(x, y, z) < 0$ (inside)

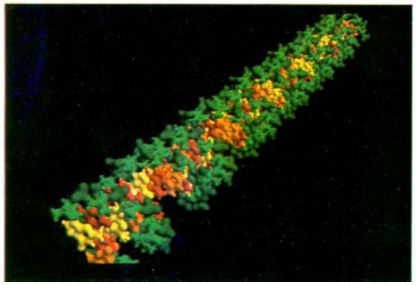
  $f(x, y, z) > 0$ (outside)





From Blinn 1982

Implicit Surfaces

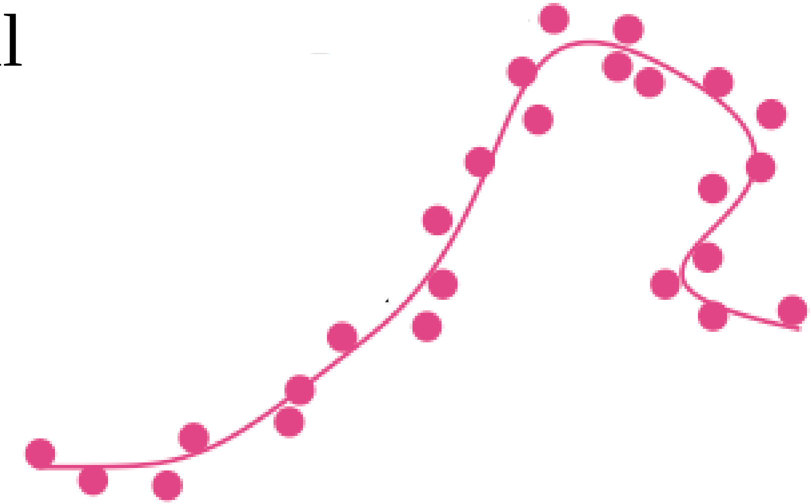


- Pros:
  - Efficient check whether point is inside
  - Efficient Boolean operations
  - Can handle weird topology for animation
  - Easy to do sketchy modeling
- Cons:
  - Does not allow us to easily generate a point on the surface

Questions?

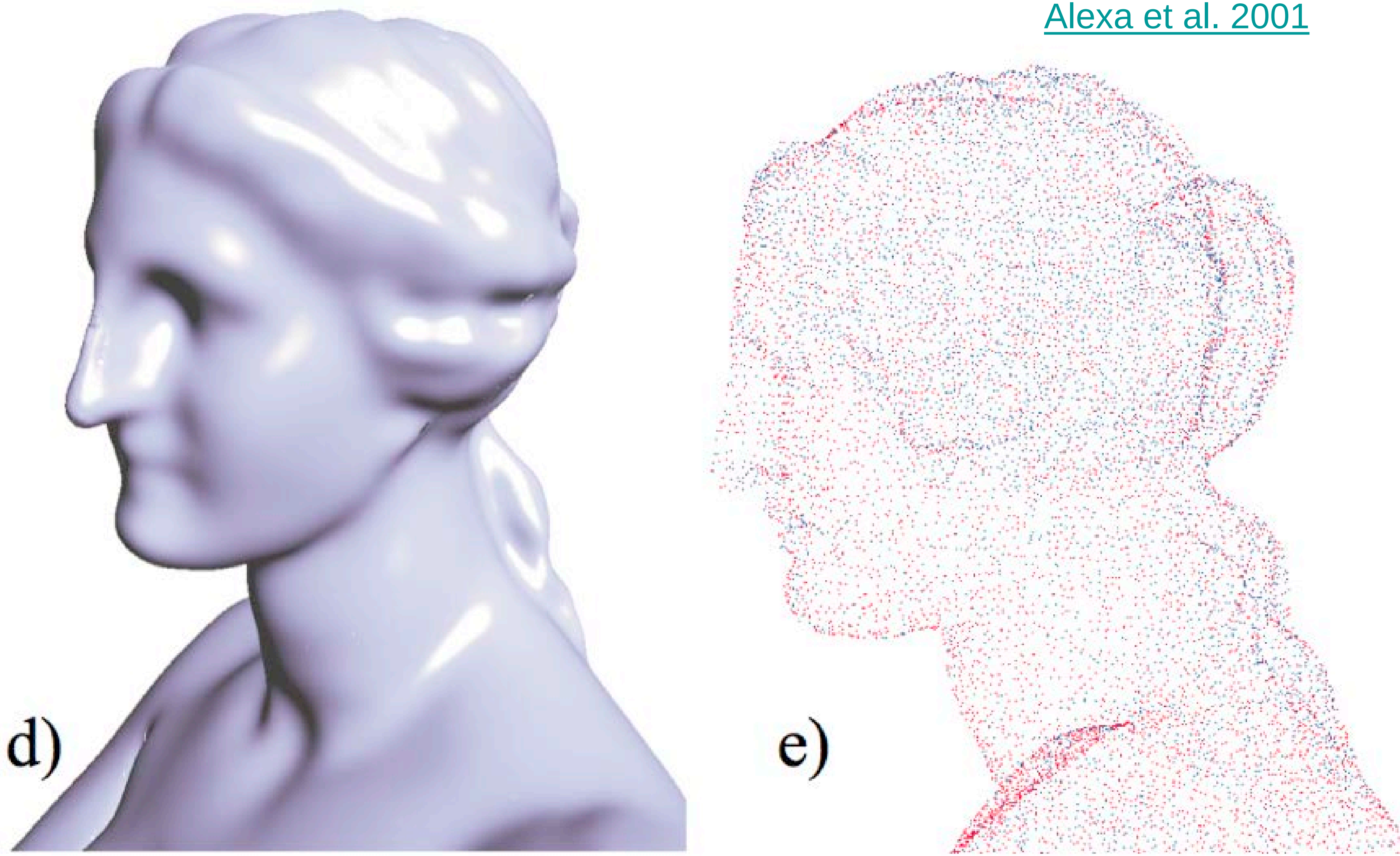
Point Set Surfaces

- Given only a noisy 3D point cloud (no connectivity), can you define a reasonable surface using only the points?
  - Laser range scans only give you points, so this is potentially useful



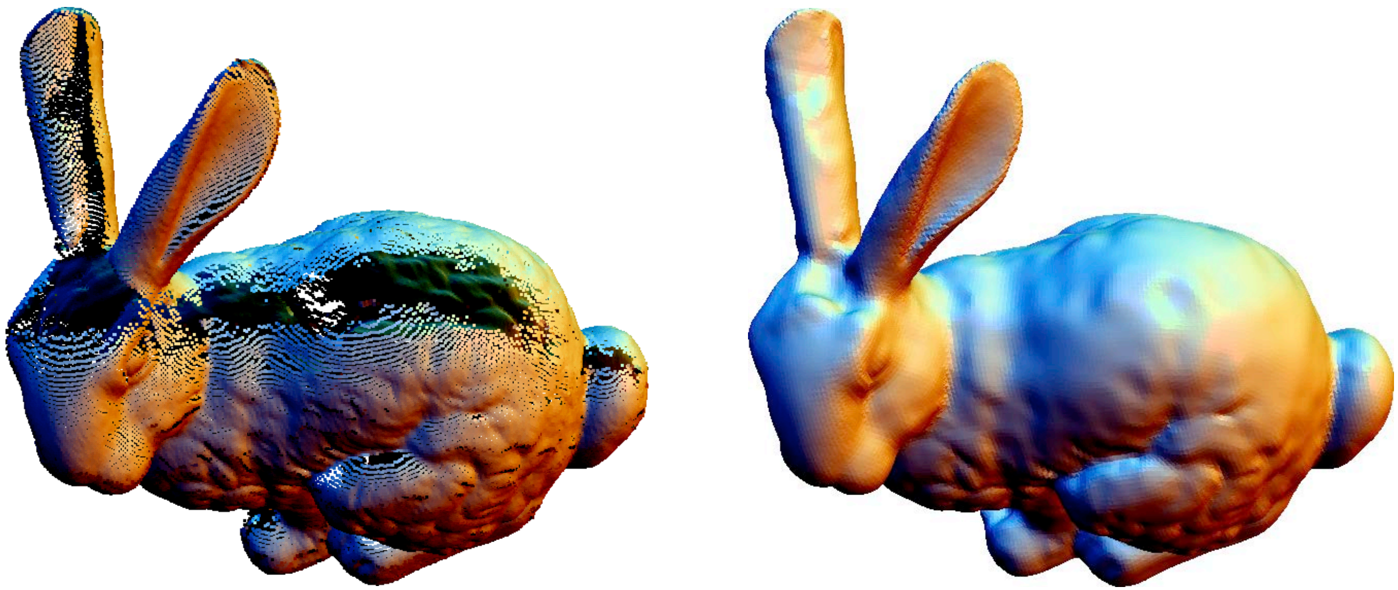
Point Set Surfaces

Alexa et al. 2001



Point Set Surfaces

- Modern take on implicit surfaces
- Cool math: Moving Least Squares (MLS), partitions of unity, etc.



Ohtake et al. 2003

- Not required in this class, but nice to know.

Questions?

That's All for Today

- Further reading
  - Buss, Chapters 7 & 8
- Subvision curves and surfaces
  - http://www.cs.nyu.edu/~dzorin/sig00course/