6.837 Introduction to Computer Graphics

Quiz 1

Thursday October 19, 2006 2:40-4pm One sheet of notes (2 pages) allowed

Name:

1	/ 10
2	/ 22
3	/ 18
Total	/ 50

- 1 Transformations [/10]
- 1.1 Linearity [/3]

What does it mean for a transformation or an operator to be linear? [

/3

1.2 Homogeneous coordinates and IFS [/7]

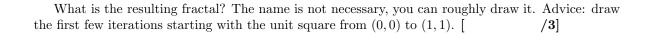
Consider the 2D IFS (Iterated Function System) defined in 2D by

$$A = \cup f_i(A)$$

That is, this fractal is the set of points A that is equal to the union of its transformed versions by the transformations f_i . The f_i are described by the following matrices in homogeneous coordinates

$$f_0 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_1 = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_2 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

Explain the effect of each of the three transformations (e.g. translation by something followed by a rotation by another thing). [/4]



2 Curves and surfaces [/22]

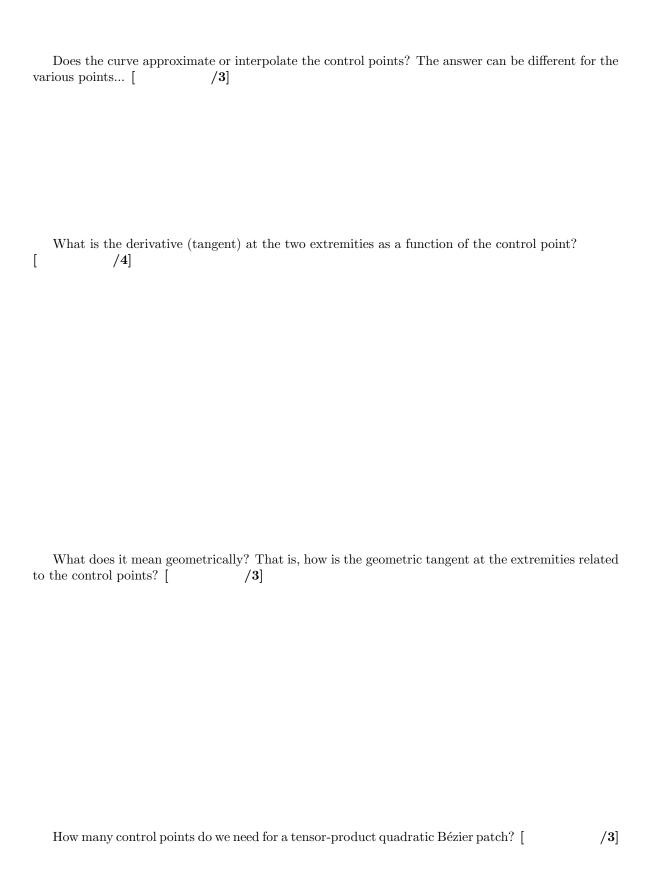
In class, we have focused on cubic Bézier splines. However, one can similarly define quadratic Bézier splines using the Bernstein polynomials:

$$B_1(t) = (1-t)^2$$
 $B_2(t) = 2t(1-t)$ $B_3(t) = t^2$

How many control points do we need for a quadratic Bézier spline? [/1]

Prove that the weights defined by these basis functions always sum to one. [/4]

Why is it critical for splines that the weights sum to one? [/4]



3	Animation [/18]		
3.1	Particles [/10]		
initi	sider a simplified 1D versals conditions are $x(0) = W$ hat is the rest length o	d and $\frac{dx}{dt}(0) = 0$.	uation: $\frac{d^2x}{dt^2} = -kx$ where x is a s /1]	calar function. Th
	Describe the system after and $\frac{dx}{dt}(0) = 0$, which yo		ntegration with time step h (that $/3$]	t is, give the value
I	Describe the system after	two step of Euler i	ntegration with time step h . [/3]

3.2 Quaternion [/8]

Let q_1 and q_2 be two unit quaternions. Prove that $(q_1q_2)^* = q_2^*q_1^*$.

First, prove this using quaternion algebra. Recall that $(d; \vec{u})^* = (d; -\vec{u})$ and $(d, \vec{u})(d', \vec{u}') = (dd' - \vec{u}.\vec{u}'; d\vec{u}' + d'\vec{u} + \vec{u} \times \vec{u}')$. [/5]

Second, give a geometric or matrix argument. [

4 EXTRA CREDIT

4.1 Easy extra credit

Consider a general multivariable linear first-order ODE of the form $\frac{dX}{dt} = MX$ where X is an n-dimensional vector and M is an $n \times n$ matrix.

/3]

Derive the implicit Euler integration for this case. That is, express X(t+h).

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What limits the stability of the method, that is, how does the maximum stable time step h relate to properties of the matrix?

4.3 Even more fun extra credit

Now consider a general first-order multivariate ODE of the type $\frac{dX}{dt} = f(X)$ where f is an arbitrary smooth function. How do you adapt the above implicit integration scheme to this situation?