# 6.837 Introduction to Computer Graphics
## Quiz 1
Thursday October 19, 2006 2:40-4pm
One sheet of notes (2 pages) allowed

**Name:**

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# 1 Transformations [ /10]

## 1.1 Linearity [ /3]

What does it mean for a transformation or an operator to be linear? [ **/3**]

## 1.2 Homogeneous coordinates and IFS [ /7]

Consider the 2D IFS (Iterated Function System) defined in 2D by

$$A = \cup f_i(A)$$

That is, this fractal is the set of points $A$ that is equal to the union of its transformed versions by the transformations $f_i$. The $f_i$ are described by the following matrices in homogeneous coordinates

$$f_0 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_1 = \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad f_2 = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

Explain the effect of each of the three transformations (e.g. translation by something followed by a rotation by another thing). [ **/4**]

What is the resulting fractal? The name is not necessary, you can roughly draw it. Advice: draw the first few iterations starting with the unit square from $(0,0)$ to $(1,1)$. [ **/3**]

## 2 Curves and surfaces [ /22]

In class, we have focused on cubic Bézier splines. However, one can similarly define quadratic Bézier splines using the Bernstein polynomials:

$$B_1(t) = (1-t)^2 \quad B_2(t) = 2t(1-t) \quad B_3(t) = t^2$$

How many control points do we need for a quadratic Bézier spline? [ **/1**]

Prove that the weights defined by these basis functions always sum to one. [ **/4**]

Why is it critical for splines that the weights sum to one? [ **/4**]

Does the curve approximate or interpolate the control points? The answer can be different for the various points... [ **/3**]

What is the derivative (tangent) at the two extremities as a function of the control point? [ **/4**]

What does it mean geometrically? That is, how is the geometric tangent at the extremities related to the control points? [ **/3**]

How many control points do we need for a tensor-product quadratic Bézier patch? [ **/3**]

# 3 Animation [ /18]

## 3.1 Particles [ /10]

Consider a simplified 1D version of the spring equation: $\frac{d^2x}{dt^2} = -kx$ where $x$ is a scalar function. The initials conditions are $x(0) = d$ and $\frac{dx}{dt}(0) = 0$.

What is the rest length of this spring? [ **/1**]

Describe the system after one step of Euler integration with time step $h$ (that is, give the values of x and $\frac{dx}{dt}(0) = 0$, which you can note $v$). [ **/3**]

Describe the system after two step of Euler integration with time step $h$. [ **/3**]

For which value of $h$ does the length of $x$ increase after two iterations compared to the initial length? That is, when do we have $|x_2| > |x_0|$, where $x_i$ is the value after $i$ iterations. [ **/3**]

### 3.2 Quaternion [ /8]

Let $q_1$ and $q_2$ be two unit quaternions. Prove that $(q_1 q_2)^* = q_2^* q_1^*$.

First, prove this using quaternion algebra. Recall that $(d; \vec{u})^* = (d; -\vec{u})$ and $(d, \vec{u})(d', \vec{u}') = (dd' - \vec{u}.\vec{u}'; d\vec{u}' + d'\vec{u} + \vec{u} \times \vec{u}')$. [ **/5**]

Second, give a geometric or matrix argument. [ **/3**]

# 4 EXTRA CREDIT

## 4.1 Easy extra credit

Consider a general multivariable linear first-order ODE of the form $\frac{dX}{dt} = MX$ where $X$ is an $n$-dimensional vector and M is an $n \times n$ matrix.

Derive the implicit Euler integration for this case. That is, express $X(t+h)$.

## 4.2 Harder extra credit

What limits the stability of the method, that is, how does the maximum stable time step $h$ relate to properties of the matrix?

## 4.3 Even more fun extra credit

Now consider a general first-order multivariate ODE of the type $\frac{dX}{dt} = f(X)$ where $f$ is an arbitrary smooth function. How do you adapt the above implicit integration scheme to this situation?