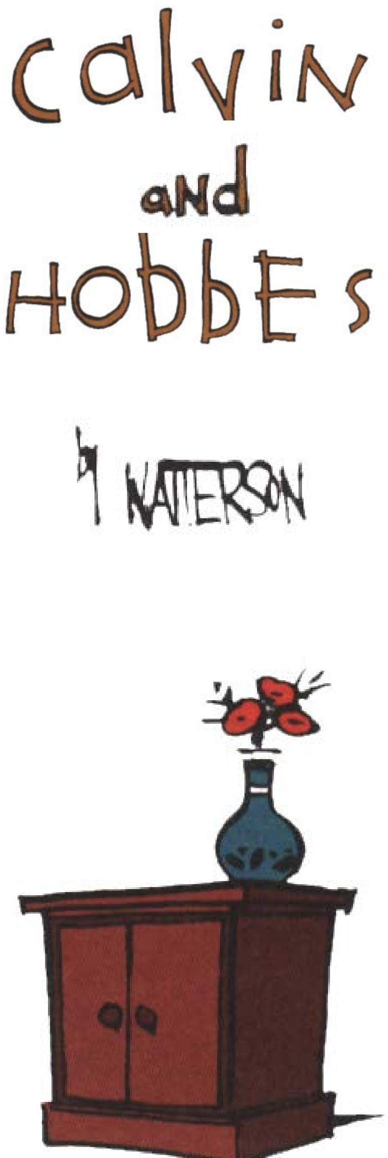
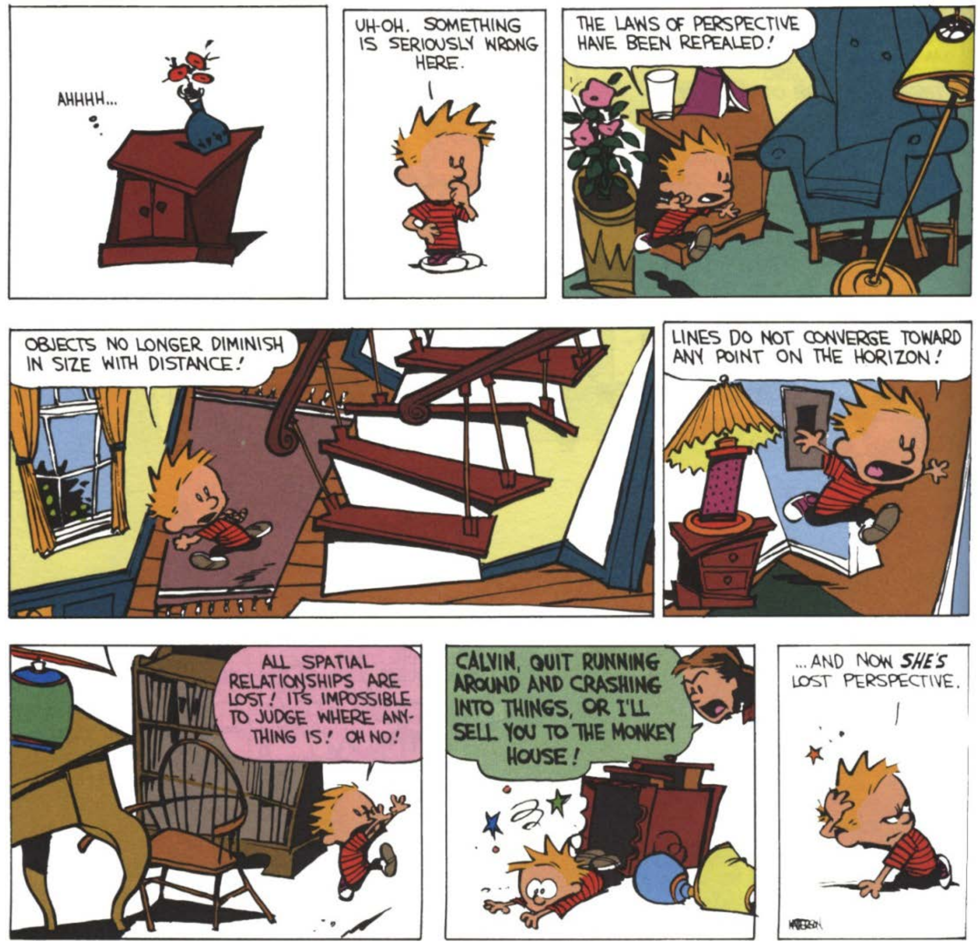
Graphics Pipeline & Rasterization II



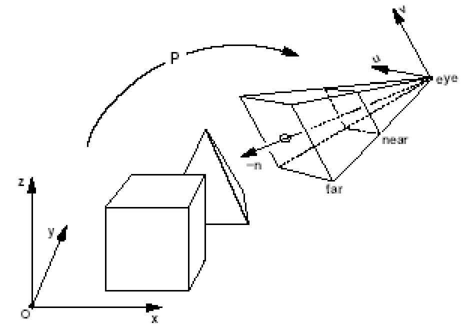


MIT EECS 6.837
Computer Graphics

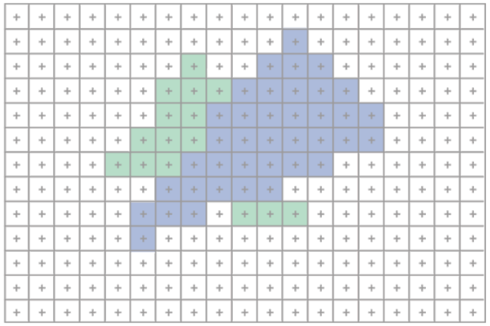
Wojciech Matusik

Modern Graphics Pipeline

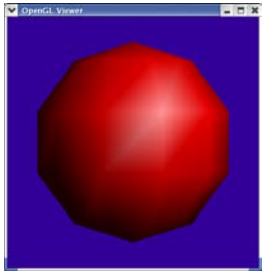
- Project vertices to 2D (image)



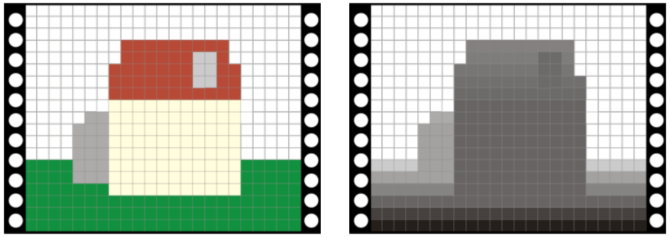
- Rasterize triangle: find which pixels should be lit



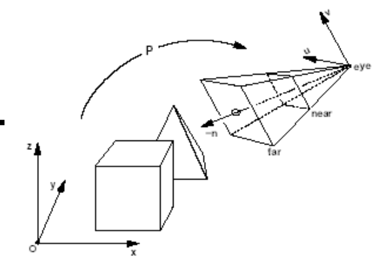
- Compute per-pixel color



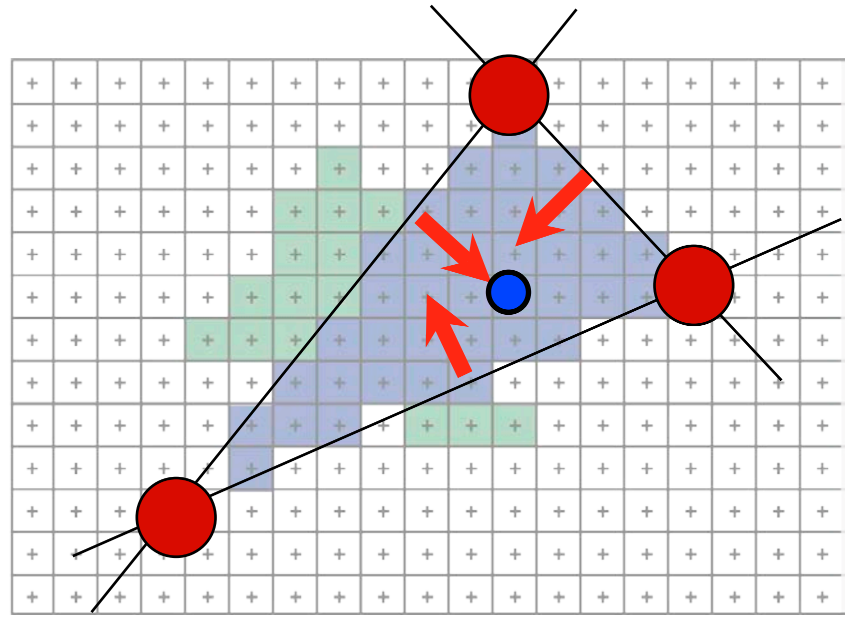
- Test visibility (Z-buffer), update frame buffer color



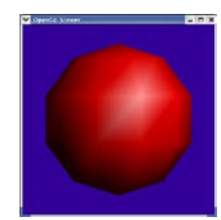
Modern Graphics Pipeline



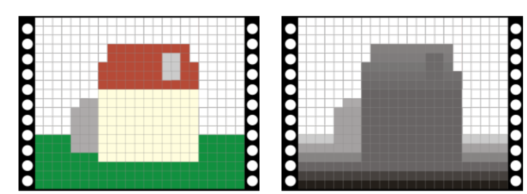
- Project vertices to 2D (image)



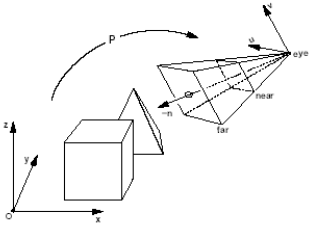
- Rasterize triangle: find which pixels should be lit
  - For each pixel, test 3 edge equations
    - if all pass, draw pixel



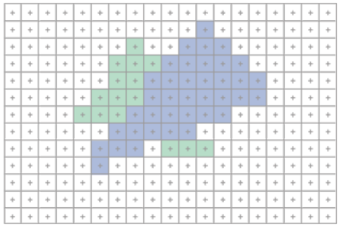
- Compute per-pixel color
- Test visibility (Z-buffer), update frame buffer color



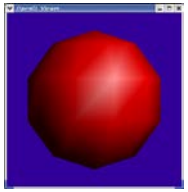
Modern Graphics Pipeline



- Perform projection of vertices
- Rasterize triangle: find which pixels should be lit

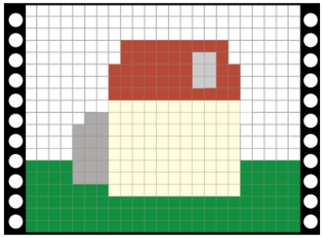


- Compute per-pixel color

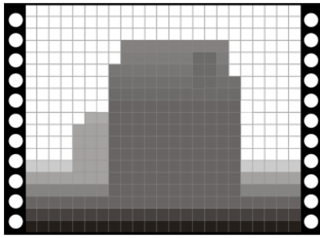


- Test visibility, update frame buffer color
  - Store minimum distance to camera for each pixel in “Z-buffer”
    - ~same as t_{min} in ray casting!
  - if new_z < zbuffer[x,y] ...

```
– if new_z < zbuffer[x,y]
       zbuffer[x,y]=new_z
       framebuffer[x,y]=new_color
```



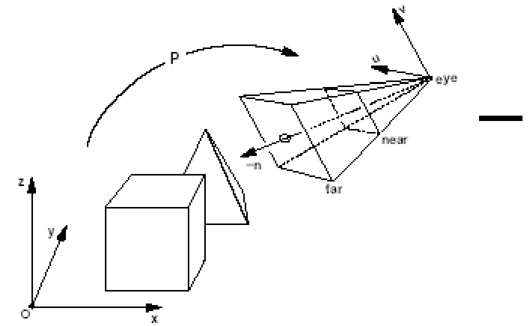
frame buffer

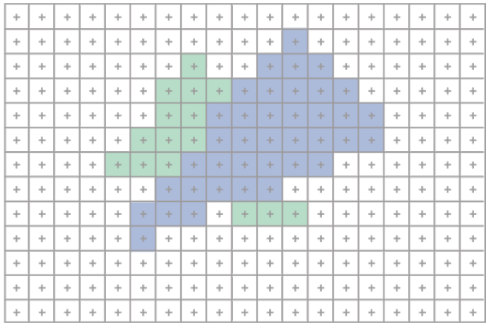


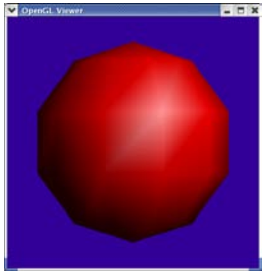
Z buffer

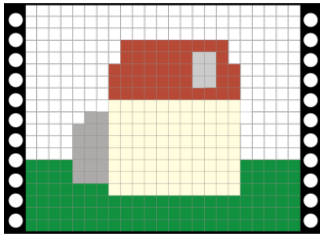
Modern Graphics Pipeline

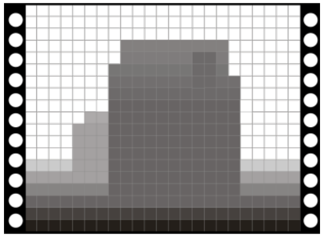
```
For each triangle
   transform into eye space
   (perform projection)
   setup 3 edge equations
   for each pixel x,y
      if passes all edge equations
         compute z
         if z<zbuffer[x,y]
            zbuffer[x,y]=z
            framebuffer[x,y]=shade()
```



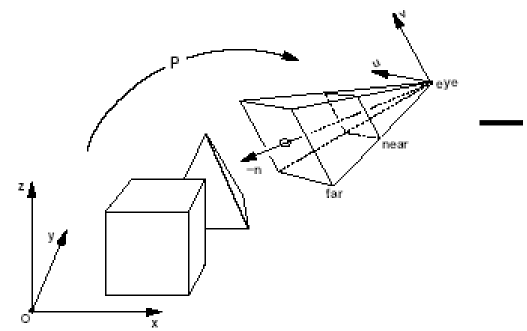




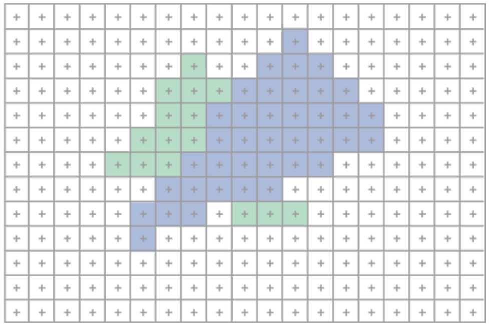


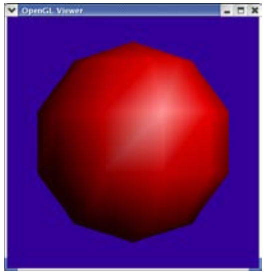


Modern Graphics Pipeline

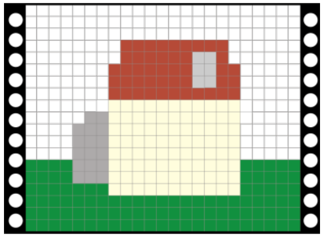


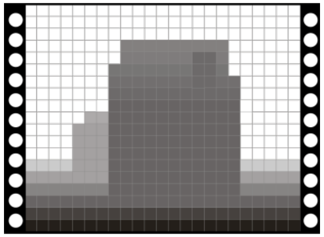

```
For each triangle
   transform into eye space
   (perform projection)
   setup 3 edge equations
   for each pixel x,y
      if passes all edge equations
         compute z
         if z<zbuffer[x,y]
            zbuffer[x,y]=z
            framebuffer[x,y]=shade()
```





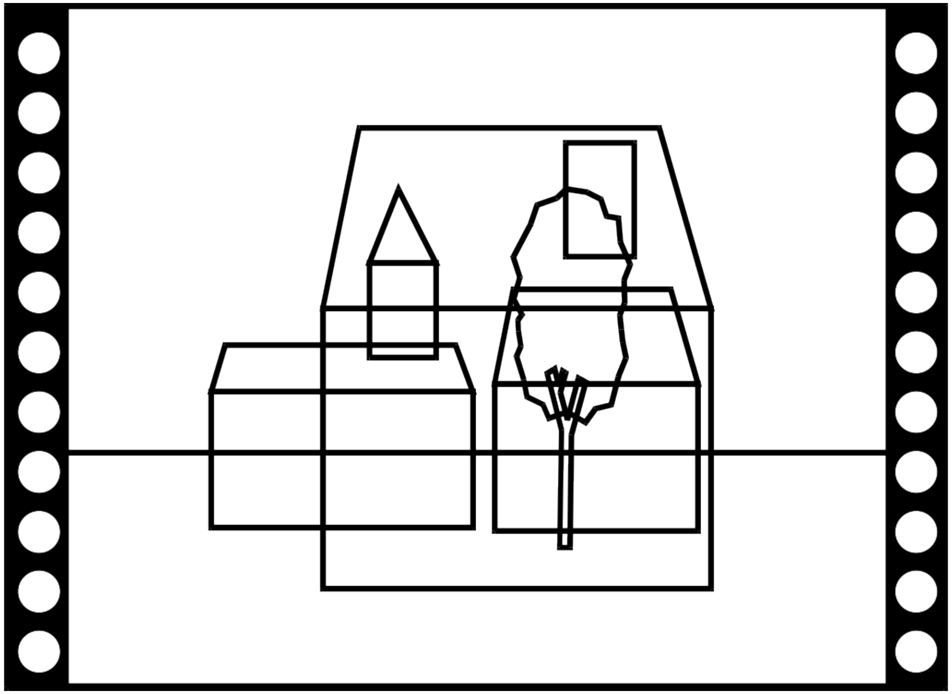
Questions?

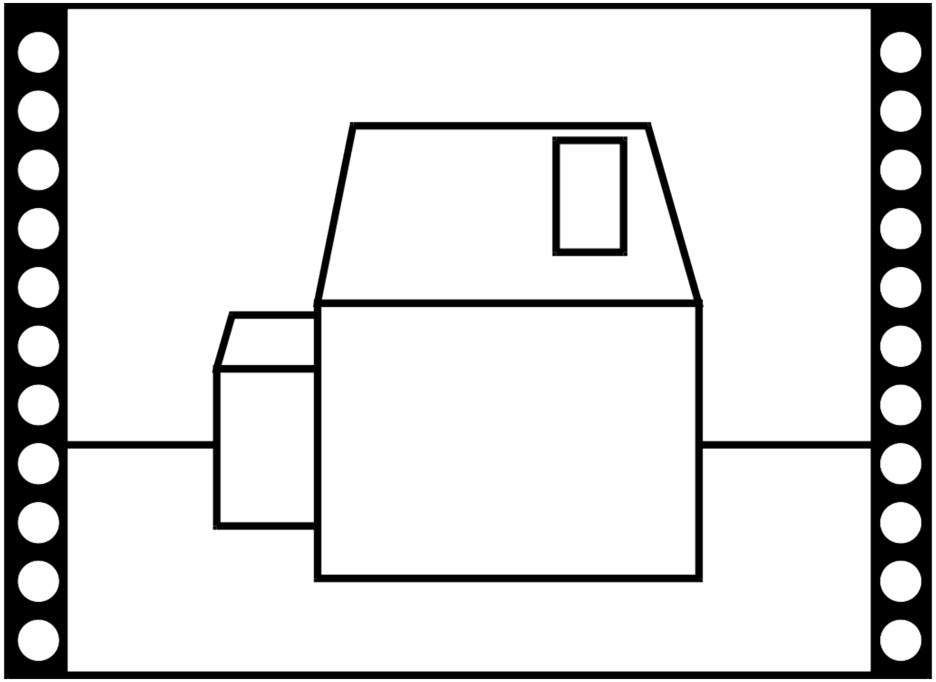




Visibility

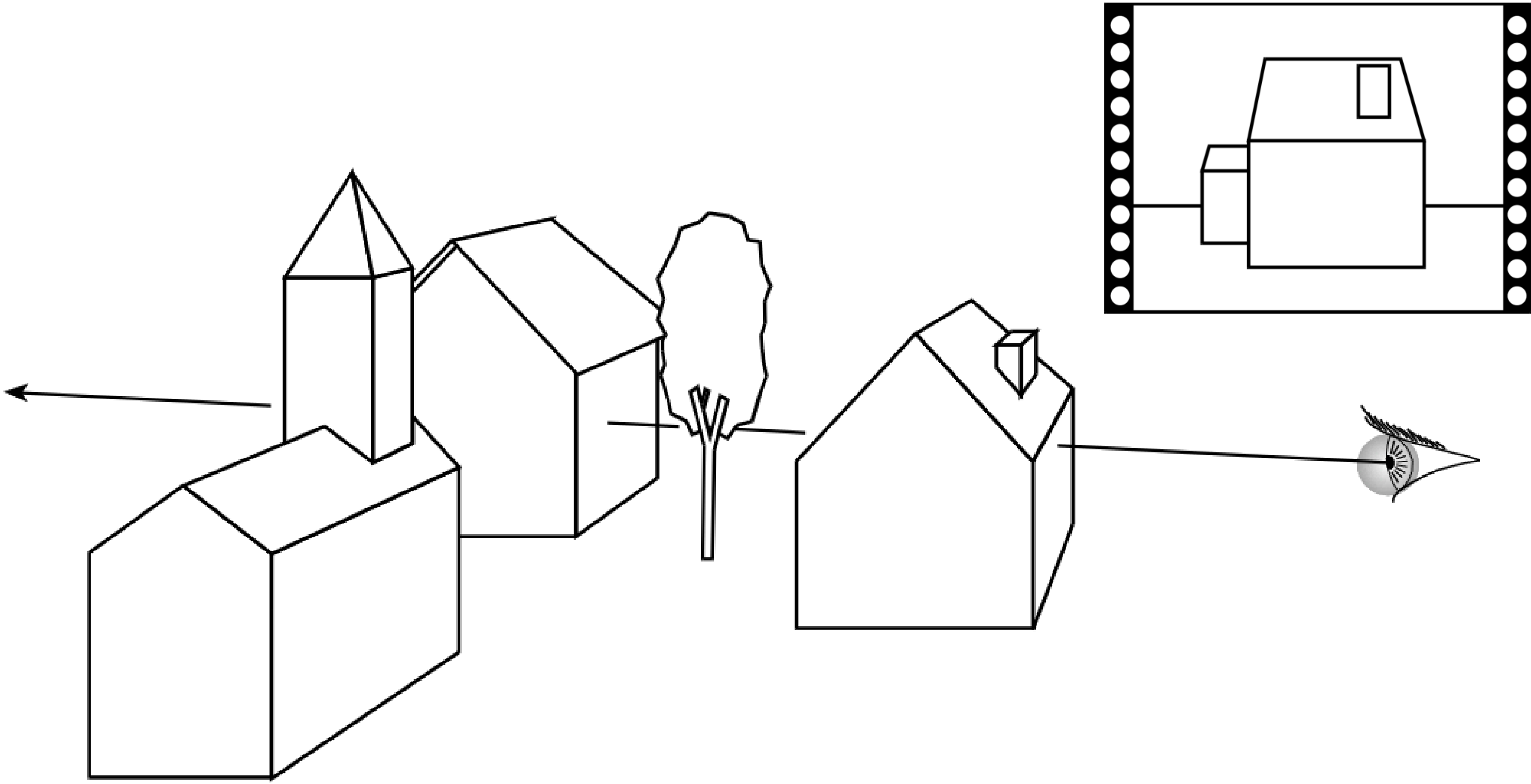
- How do we know which parts are visible/in front?





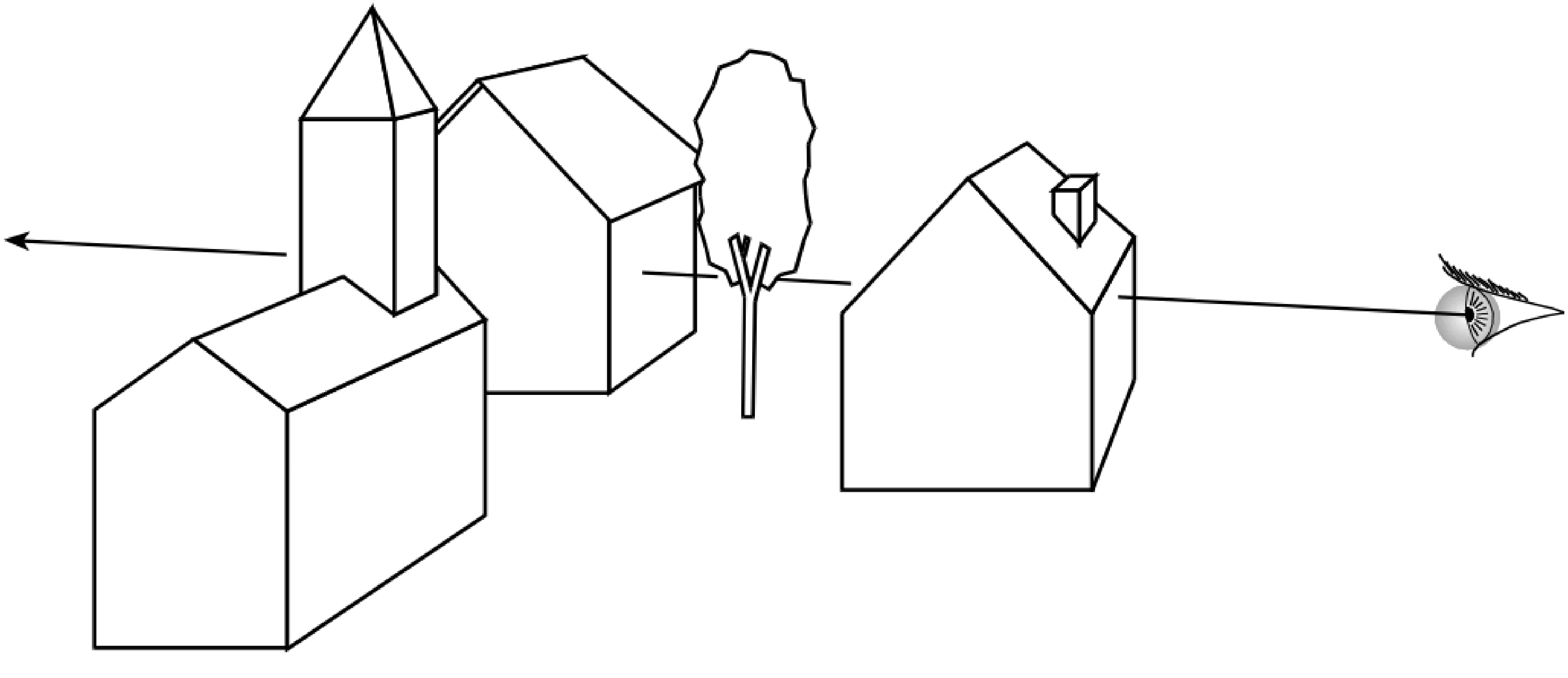
Ray Casting

- Maintain intersection with closest object



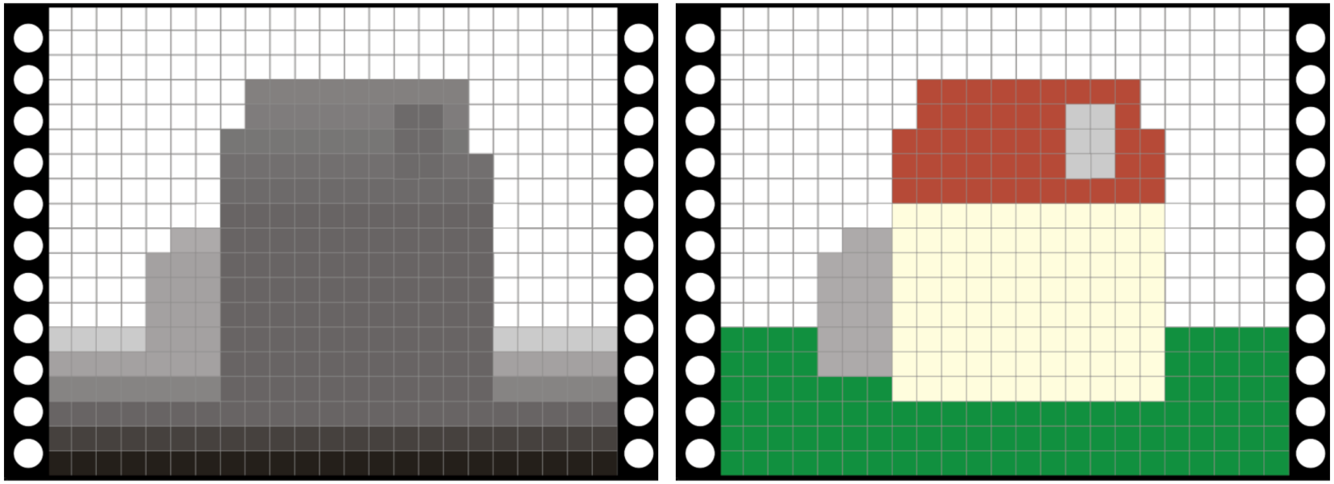
Visibility

- In ray casting, use intersection with closest t
- Now we have swapped the loops (pixel, object)
- What do we do?

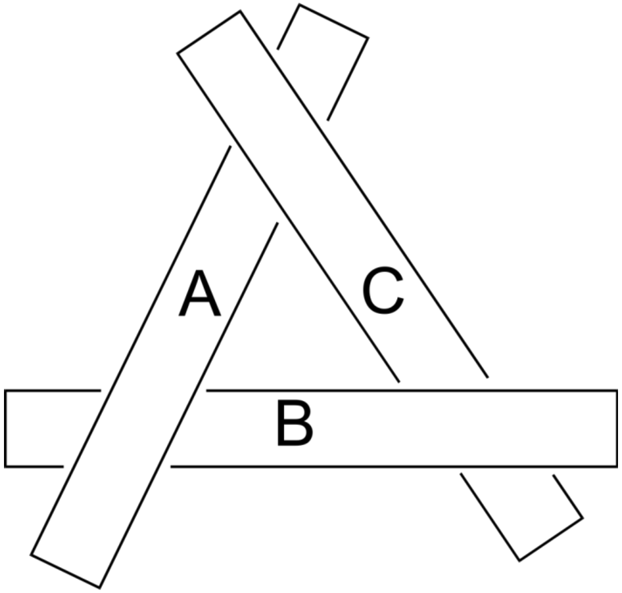


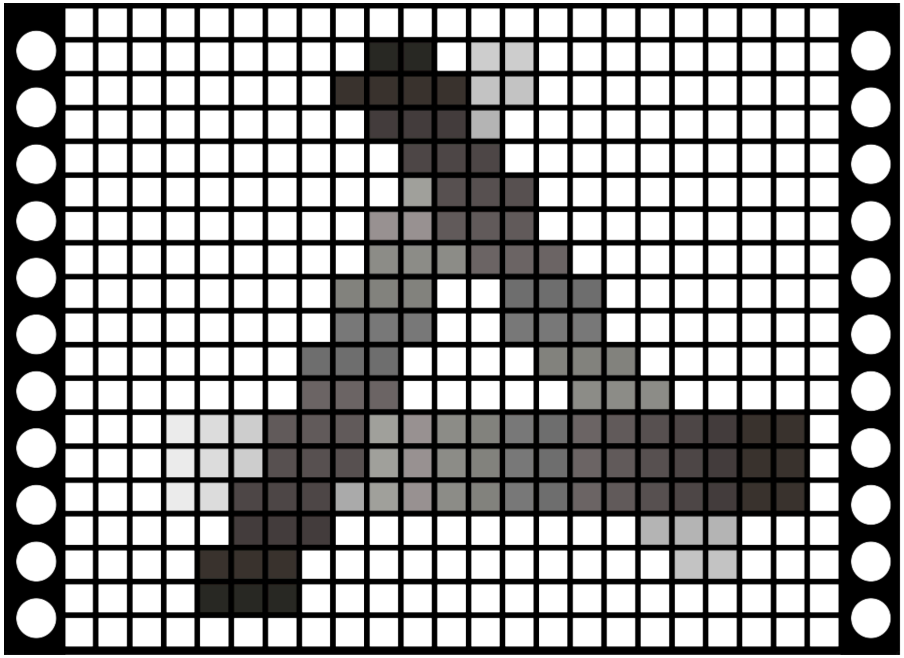
Z buffer

- In addition to frame buffer (R, G, B)
- Store distance to camera (*z*-buffer)
- Pixel is updated only if *newz* is closer than *z*-buffer value



Works for hard cases!

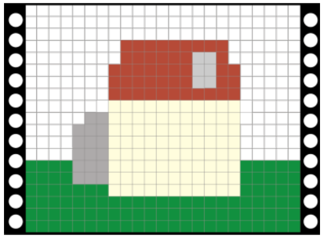


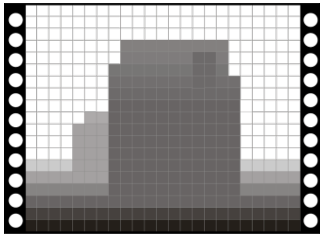


Interpolation in Screen Space

- How do we get that Z value for each pixel?
  - We only know z at the vertices...
  - (Remember, screen-space z is actually z’/w’)
  - Must interpolate from vertices into triangle interior

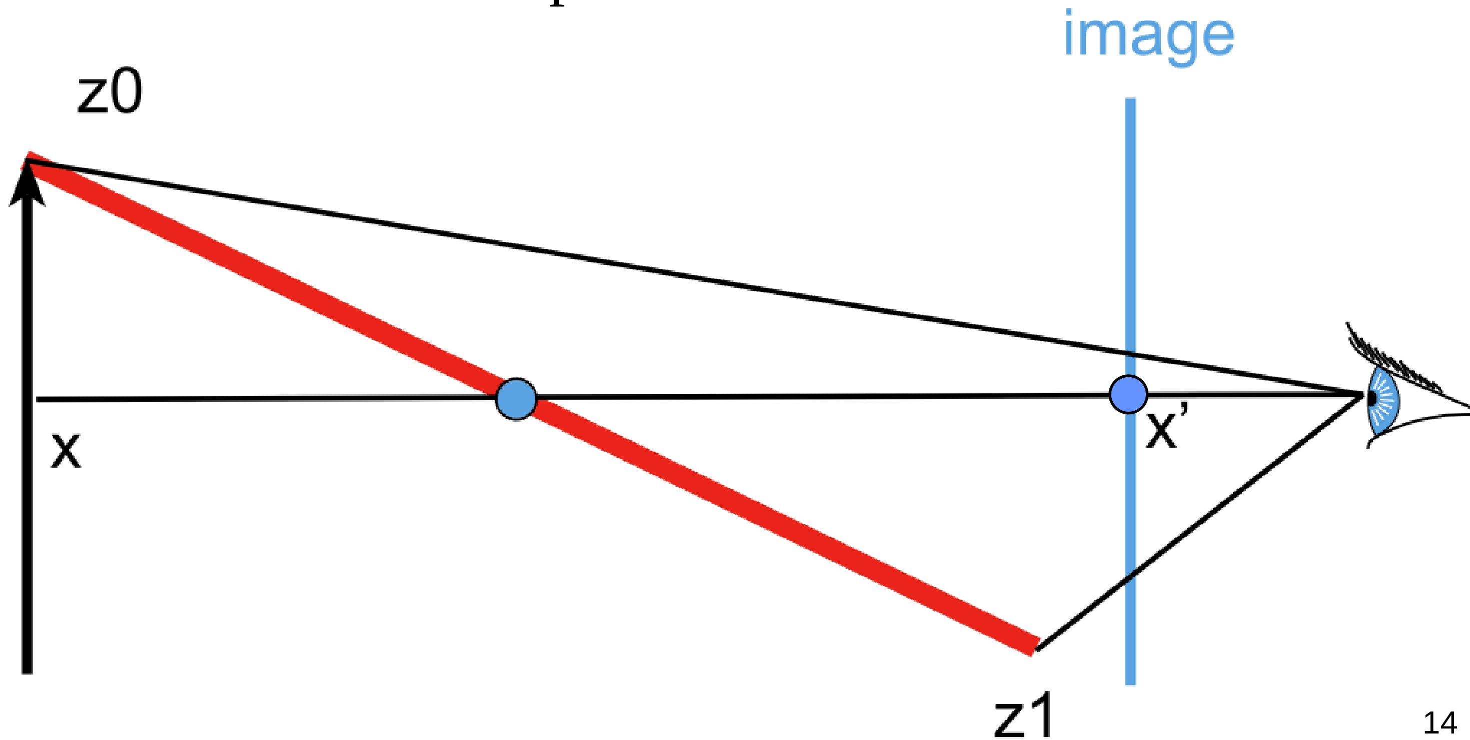
```
For each triangle
   for each pixel (x,y)
      if passes all edge equations
         compute z
         if z<zbuffer[x,y]
            zbuffer[x,y]=z
            framebuffer[x,y]=shade()
```



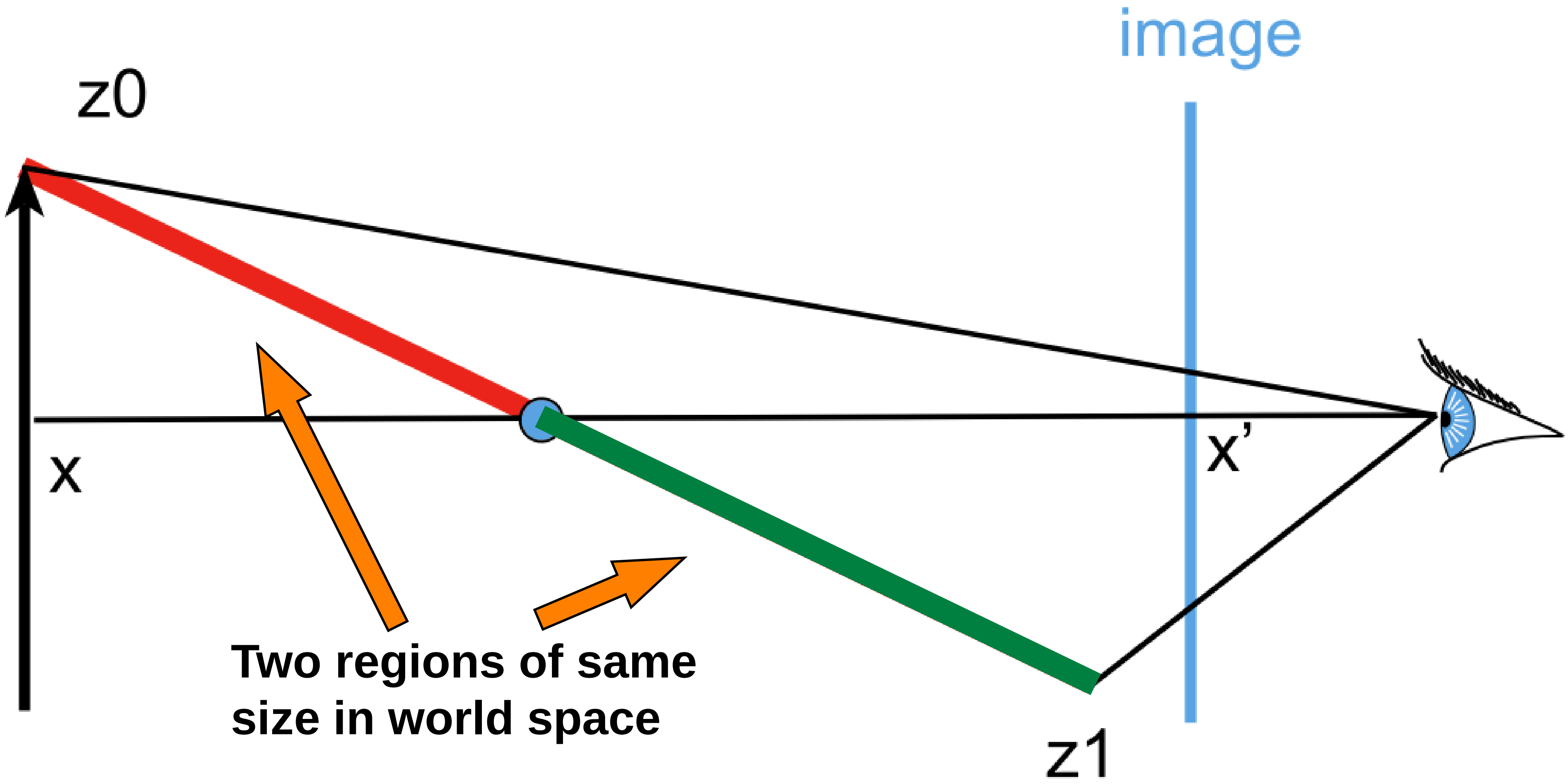


Interpolation in Screen Space

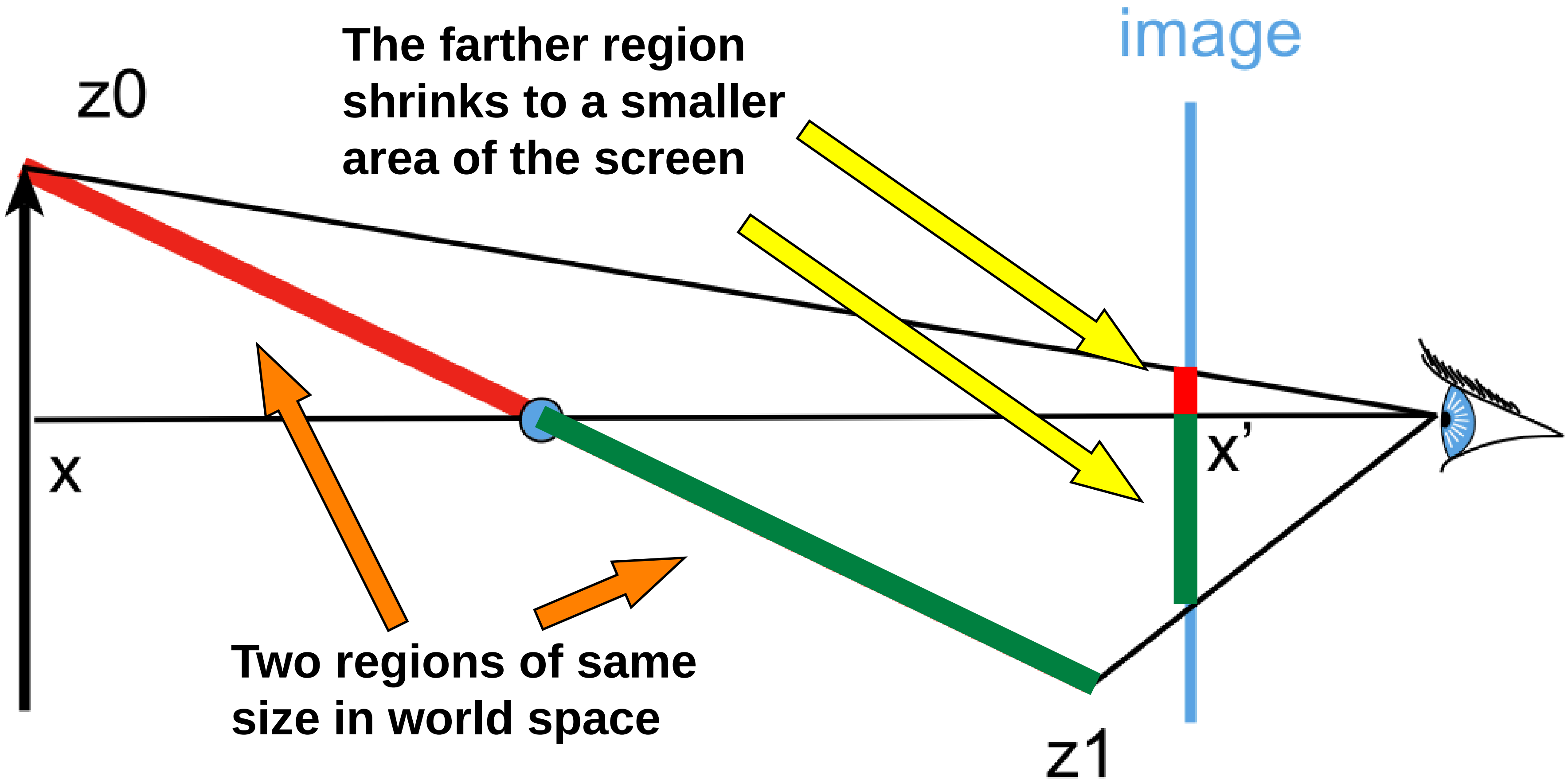
- Also need to interpolate color, normals, texture coordinates, etc. between vertices
  - We did this with barycentrics in ray casting
    - Linear interpolation in object space
  - Is this the same as linear interpolation on the screen?



Interpolation in Screen Space



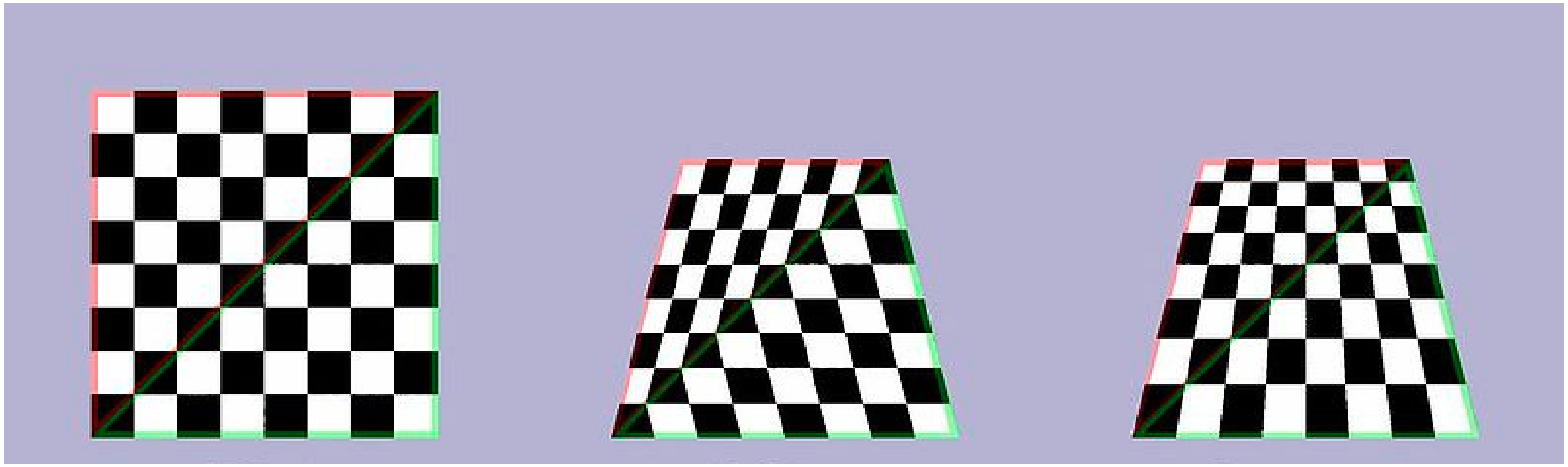
Interpolation in Screen Space



Nope, Not the Same

- Linear variation in world space does not yield linear variation in screen space due to projection
  - Think of looking at a checkerboard at a steep angle; all squares are the same size on the plane, but not on screen

Image: Wikipedia



Head-on view

linear screen-space (“Gouraud”) interpolation **BAD**

Perspective-correct Interpolation

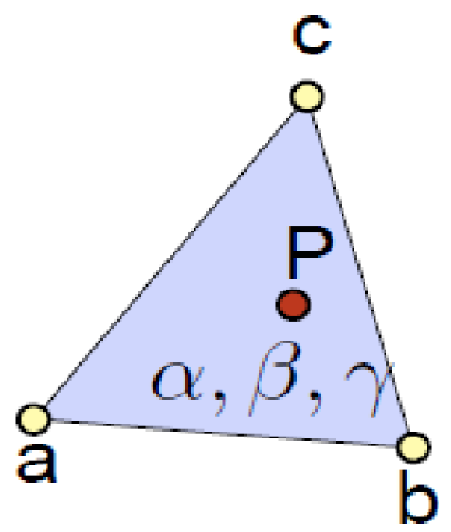
Back to the basics: Barycentrics

- Barycentric coordinates for a triangle (**a**, **b**, **c**)

$$P(\alpha, \beta, \gamma) = \alpha \boldsymbol{a} + \beta \boldsymbol{b} + \gamma \boldsymbol{c}$$

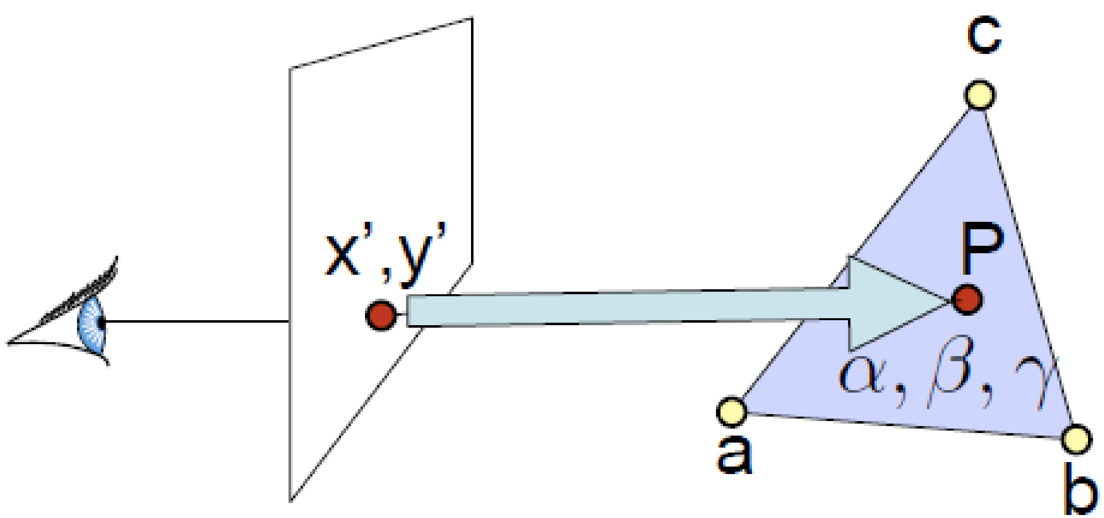
  – Remember, $\alpha + \beta + \gamma = 1, \quad \alpha, \beta, \gamma \geq 0$

- Barycentrics are very general:
  – Work for x, y, z, u, v, r, g, b
  – Anything that varies linearly in object space
  – including z



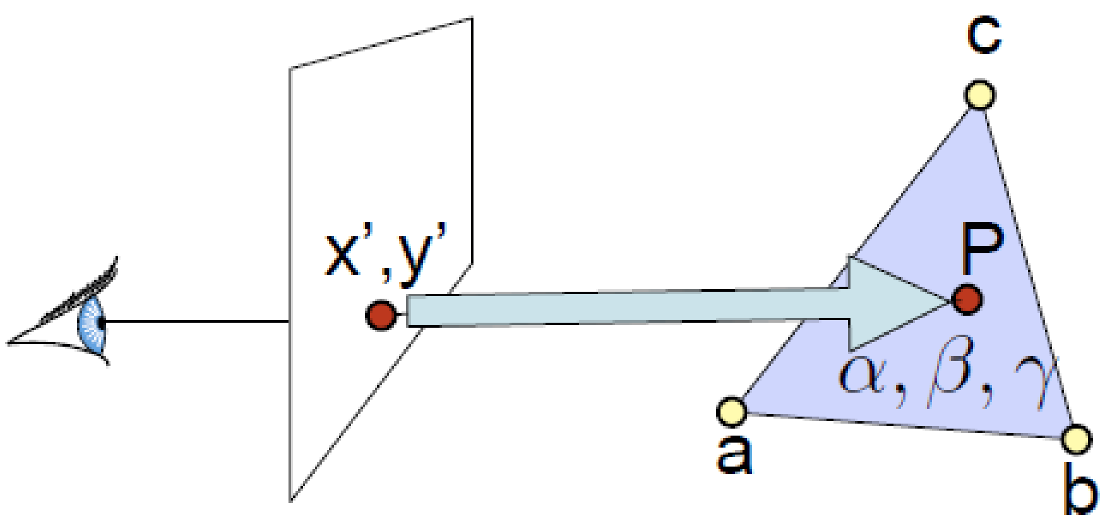
Basic strategy

- Given screen-space x', y'
- Compute barycentric coordinates
- Interpolate anything specified at the three vertices



Basic strategy

- How to make it work
  - start by computing x’, y’ given barycentrics
  - invert
- Later: shortcut barycentrics, directly build interpolants



From barycentric to screen-space

- Barycentric coordinates for a triangle (**a**, **b**, **c**)

$$P(\alpha, \beta, \gamma) = \alpha \boldsymbol{a} + \beta \boldsymbol{b} + \gamma \boldsymbol{c}$$

  – Remember, $\alpha + \beta + \gamma = 1, \quad \alpha, \beta, \gamma \geq 0$

- Let’s project point P by projection matrix **C**

$$\begin{aligned} \boldsymbol{C}P &= \boldsymbol{C}(\alpha \boldsymbol{a} + \beta \boldsymbol{b} + \gamma \boldsymbol{c}) \\ &= \alpha \boldsymbol{C}\boldsymbol{a} + \beta \boldsymbol{C}\boldsymbol{b} + \gamma \boldsymbol{C}\boldsymbol{c} \\ &= \alpha \boldsymbol{a}' + \beta \boldsymbol{b}' + \gamma \boldsymbol{c}' \end{aligned}$$

a’, **b**’, **c**’ are the projected homogeneous vertices before division by w

Projection

- Let’s use simple formulation of projection going from 3D homogeneous coordinates to 2D homogeneous coordinates

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- No crazy near-far or storage of 1/z
- We use ’ for screen space coordinates

From barycentric to screen-space

- From previous slides:

$$P' = CP = \alpha \mathbf{a}' + \beta \mathbf{b}' + \gamma \mathbf{c}'$$

a', **b**', **c**' are the projected homogeneous vertices

- Seems to suggest it's linear in screen space. But it's homogenous coordinates

From barycentric to screen-space

- From previous slides:

$$P' = CP = \alpha\mathbf{a}' + \beta\mathbf{b}' + \gamma\mathbf{c}'$$

a', **b**', **c**' are the projected homogeneous vertices

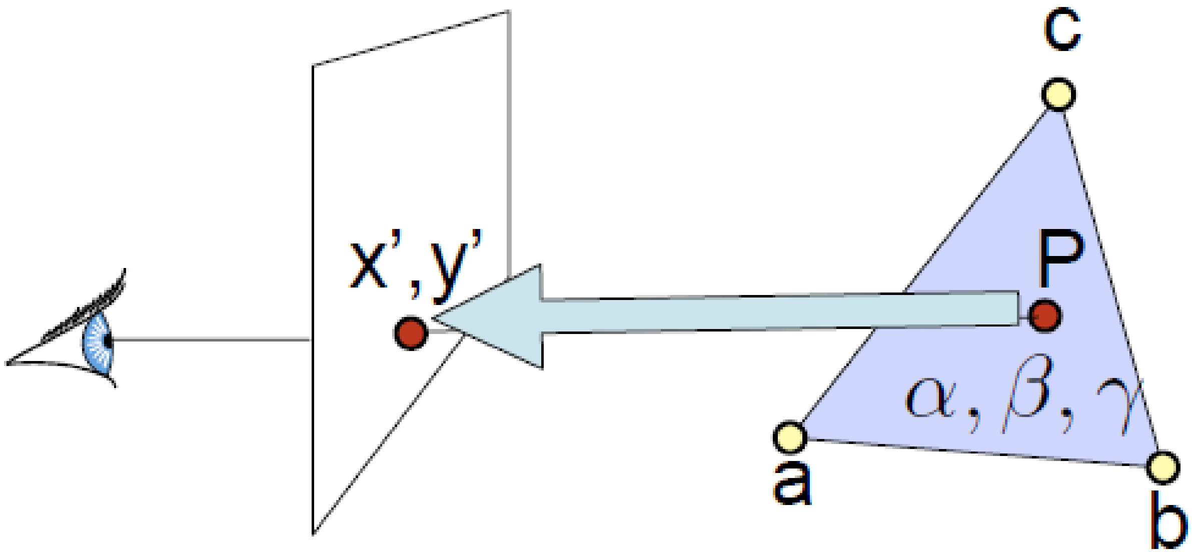
- Seems to suggest it's linear in screen space. But it's homogenous coordinates
- After division by *w*, the *(x,y)* screen coordinates are

$$(P'_x/P'_w, P'y/P'w) =$$
$$\left(\frac{\alpha a'_x + \beta b'_x + \gamma c'_x}{\alpha a'_w + \beta b'_w + \gamma c'_w}, \frac{\alpha a'_y + \beta b'_y + \gamma c'_y}{\alpha a'_w + \beta b'_w + \gamma c'_w}\right)$$

Recap: barycentric to screen-space

projective equivalence (up to scale) | 2D homogenous coordinates | projected vertices | barycentric coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \begin{pmatrix} P'_x \\ P'_y \\ P'_w \end{pmatrix} = \begin{pmatrix} a'_x & b'_x & c'_x \\ a'_y & b'_y & c'_y \\ a'_z & b'_z & c'_z \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$



From screen-space to barycentric

projective equivalence (up to scale) | 2D homogenous coordinates | projected vertices | barycentric coordinates

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \begin{pmatrix} P'_x \\ P'_y \\ P'_w \end{pmatrix} = \begin{pmatrix} a'_x & b'_x & c'_x \\ a'_y & b'_y & c'_y \\ a'_z & b'_z & c'_z \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

- It’s a projective mapping from the barycentrics onto screen coordinates!
  - Represented by a 3x3 matrix
- We’ll take the inverse mapping to get from (x, y, 1) to the barycentrics!

From Screen to Barycentrics

projective equivalence

$$\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix} \sim \begin{pmatrix} a'_x & b'_x & c'_x \\ a'_y & b'_y & c'_y \\ a'_w & b'_w & c'_w \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- Recipe
  - Compute projected homogeneous coordinates **a**’, **b**’, **c**’
  - Put them in the columns of a matrix, invert it
  - Multiply screen coordinates (x, y, 1) by inverse matrix
  - **Then divide by the sum of the resulting coordinates**
    - This ensures the result is sums to one like barycentrics should
  - Then interpolate value (e.g. Z) from vertices using them!

From Screen to Barycentrics

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \sim \begin{pmatrix} a'_x & b'_x & c'_x \\ a'_y & b'_y & c'_y \\ a'_w & b'_w & c'_w \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- Notes:
  - matrix is inverted once per triangle
  - can be used to interpolate z, color, texture coordinates, etc.

Pseudocode – Rasterization

```
For every triangle
    ComputeProjection
    Compute interpolation matrix
    Compute bbox, clip bbox to screen limits
    For all pixels x,y in bbox
        Test edge functions
        If all Ei>0
            compute barycentrics
            interpolate z from vertices
            if z < zbuffer[x,y ]
                interpolate UV coordinates from vertices
                look up texture color kd
                Framebuffer[x,y ] = kd      //or more complex shader
```

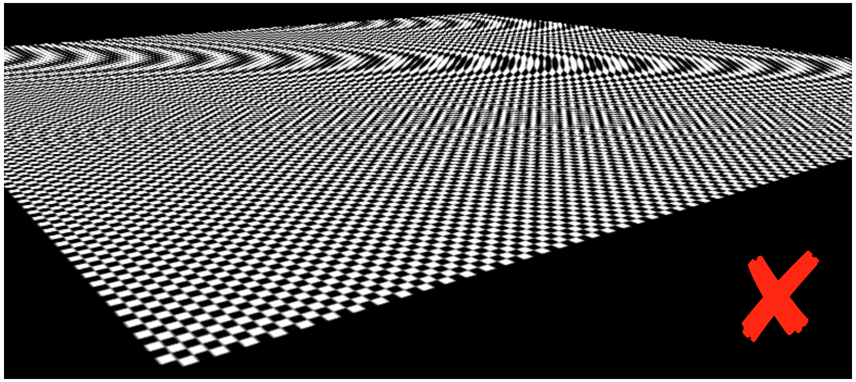
Pseudocode – Rasterization

```
For every triangle
    ComputeProjection
    Compute interpolation matrix
    Compute bbox, clip bbox to screen limits
    For all pixels x,y in bbox
        Test edge functions
        If all Ei>0
            compute barycentrics
            interpolate z from vertices
            if z < zbuffer[x,y ]
                interpolate UV coordinates from vertices
                look up texture color kd
                Framebuffer[x,y ] = kd        //or more complex shader
```

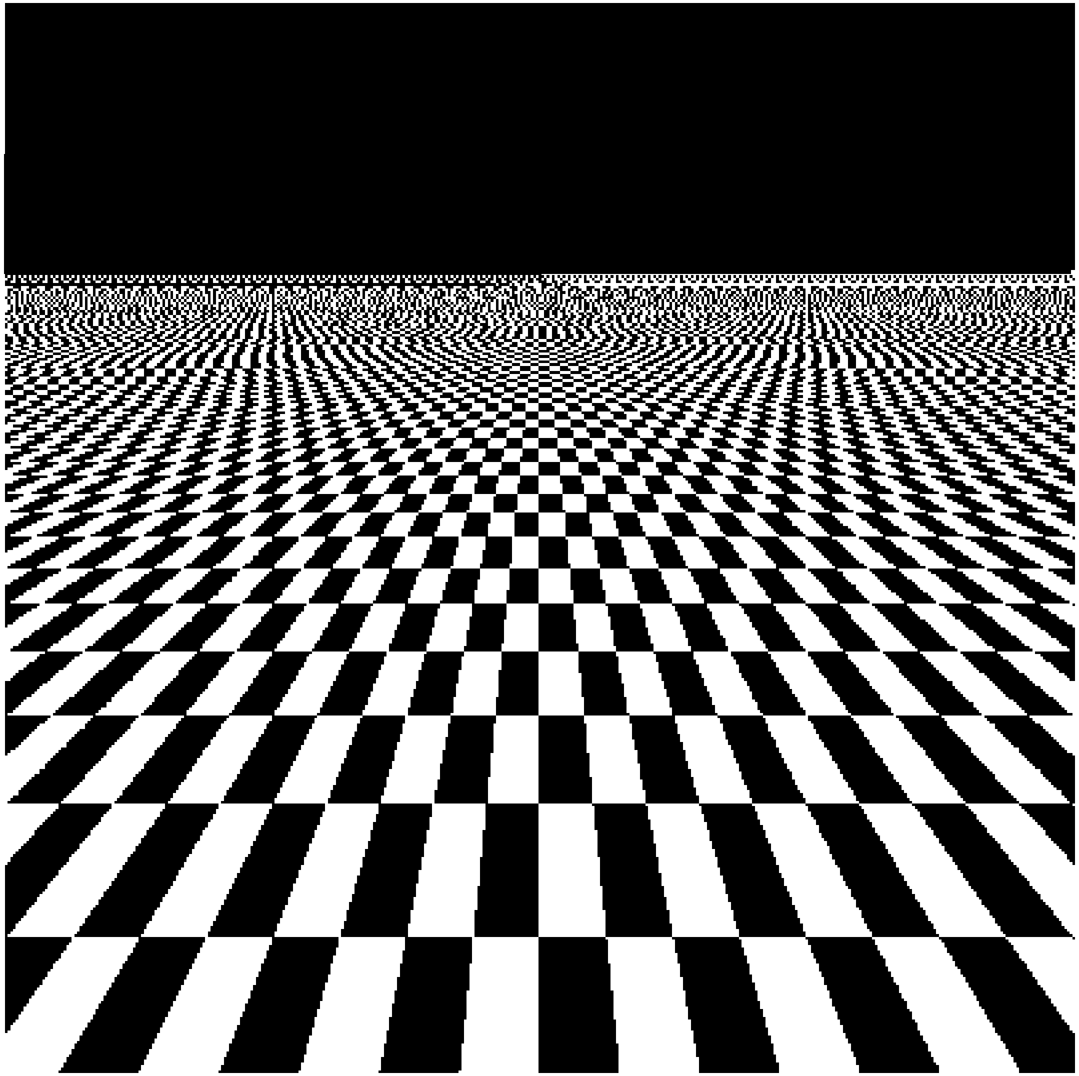
Questions?

Supersampling

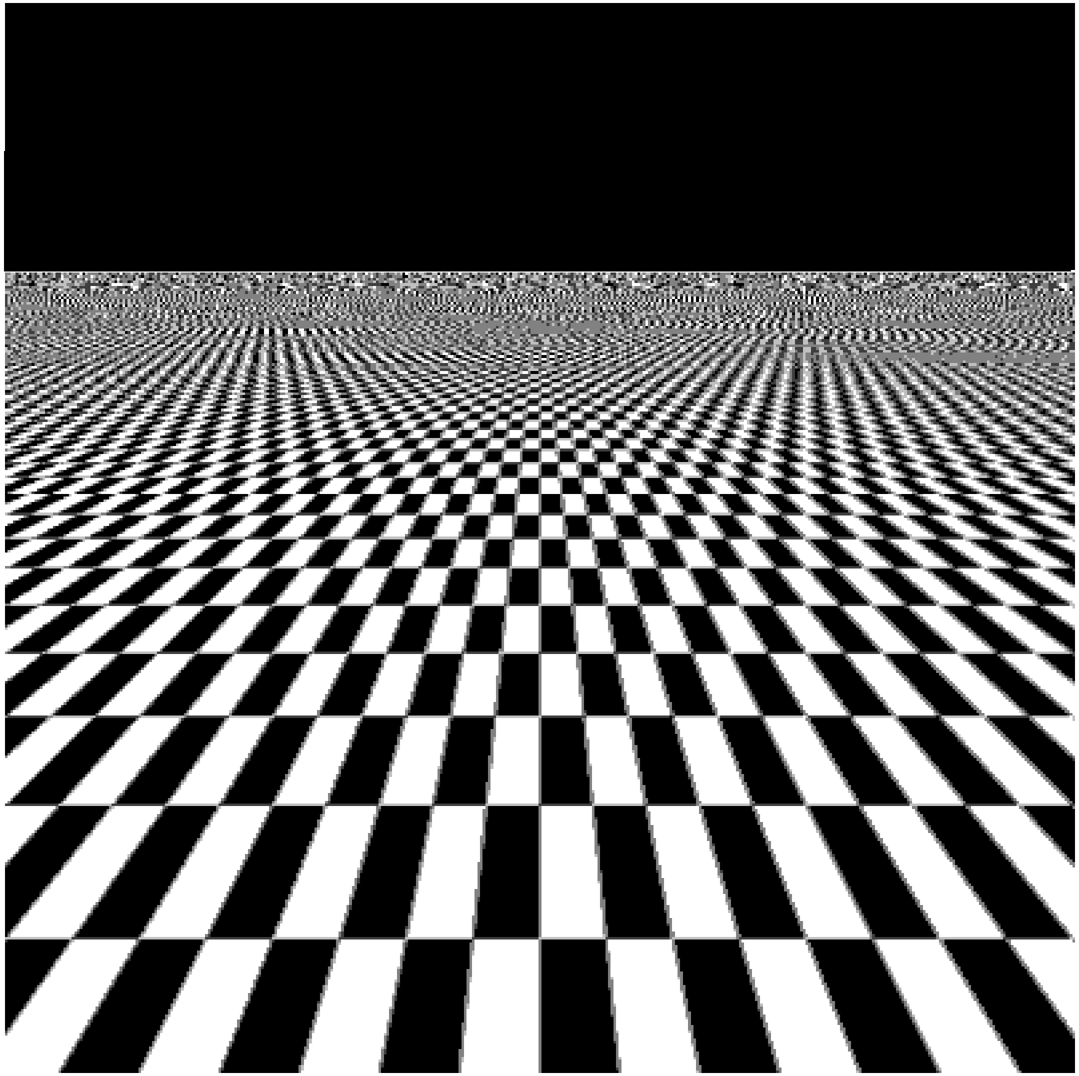
- Trivial to do with rasterization as well
- Often rates of 2x to 8x
- Requires to compute per-pixel average at the end
- Most effective against edge jaggies
- Usually with jittered sampling
  - pre-computed pattern for a big block of pixels



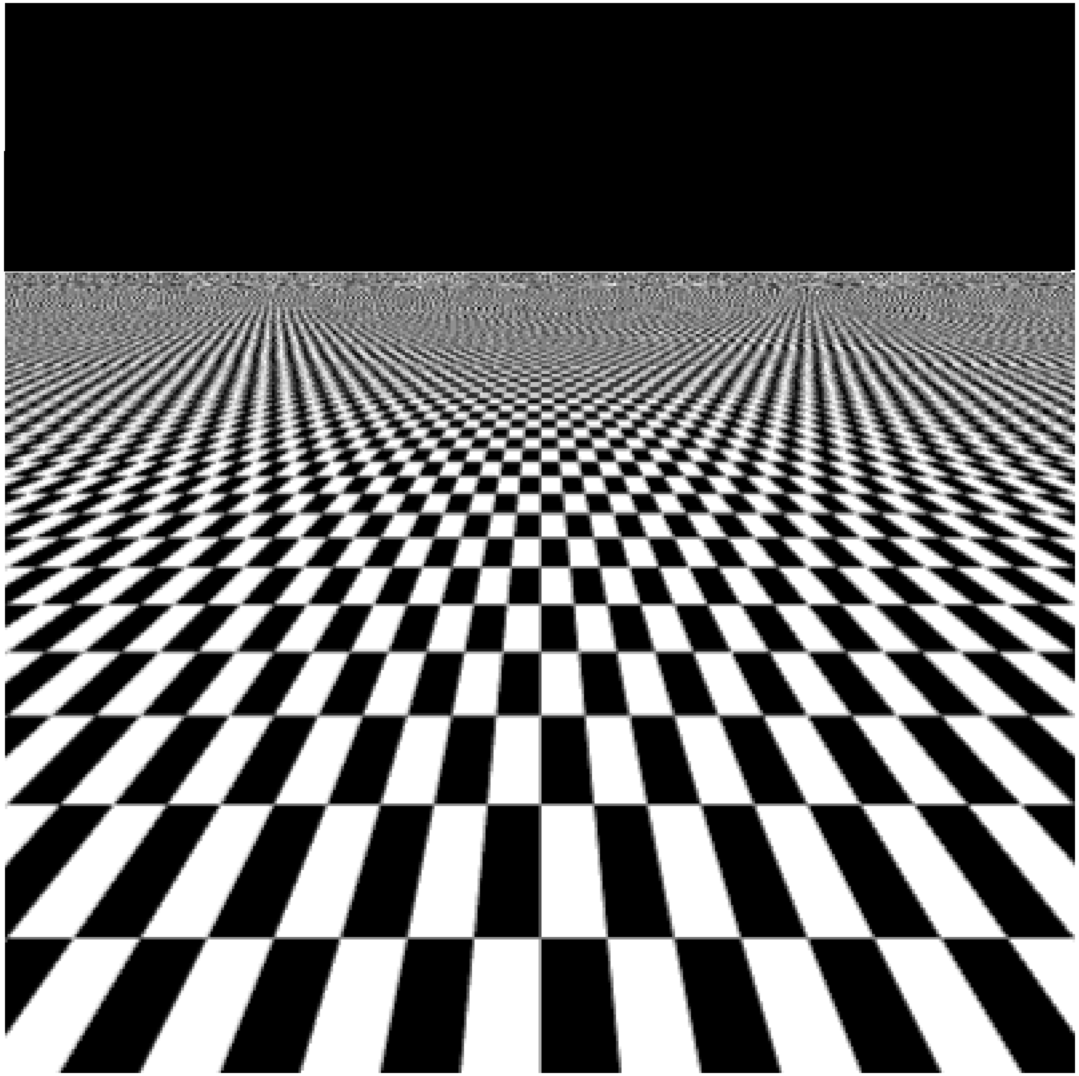
1 Sample / Pixel



4 Samples / Pixel



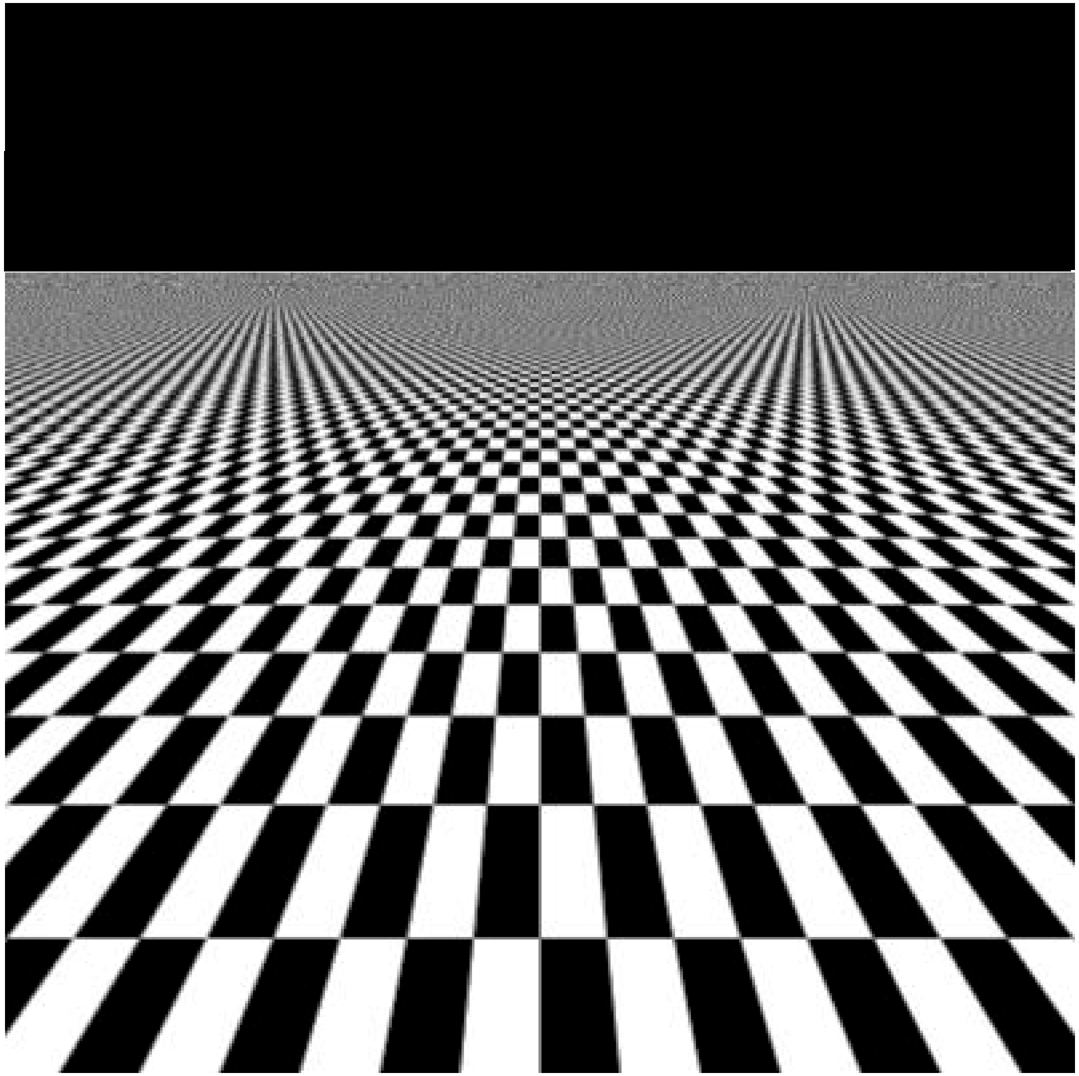
16 Samples / Pixel



100 Samples / Pixel

Even this sampling rate cannot get rid of all aliasing artifacts!

We are really only pushing the problem farther.

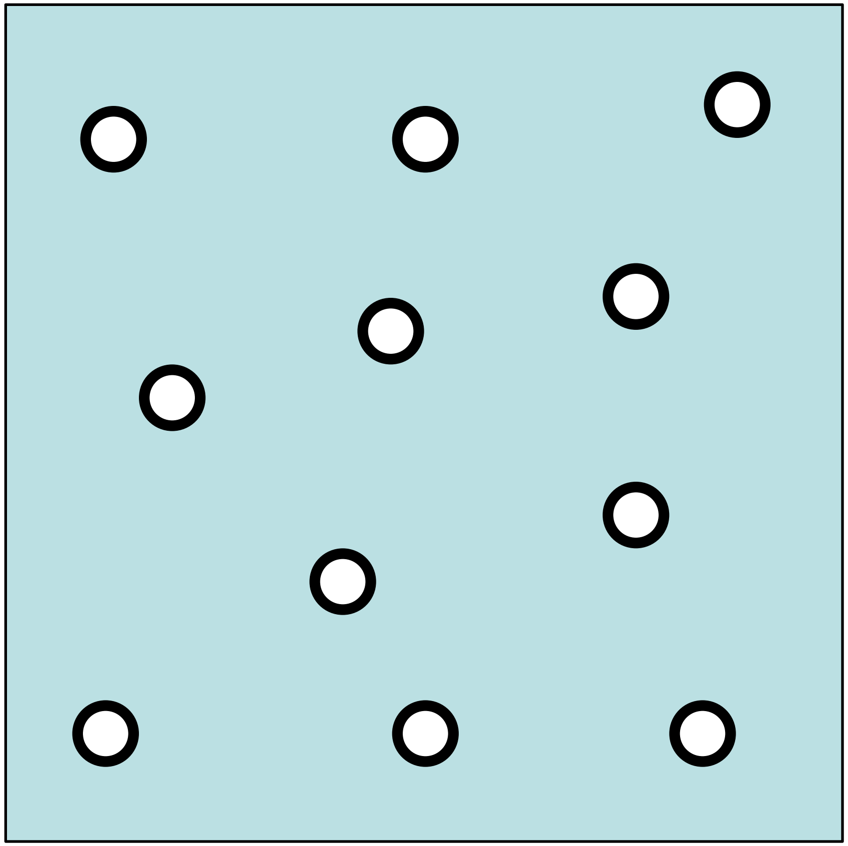


Related Idea: Multisampling

- Problem
  - Shading is very expensive today (complicated shaders)
  - Full supersampling has linear cost in #samples (k*k)
- Goal: High-quality edge antialiasing at lower cost
- Solution
  - Compute shading only once per pixel for each primitive, but resolve visibility at “sub-pixel” level
    - Store (k*width, k*height) frame and z buffers, but share shading results between sub-pixels within a real pixel
  - When visibility samples within a pixel hit different primitives, we get an average of their colors
    - Edges get antialiased without large shading cost

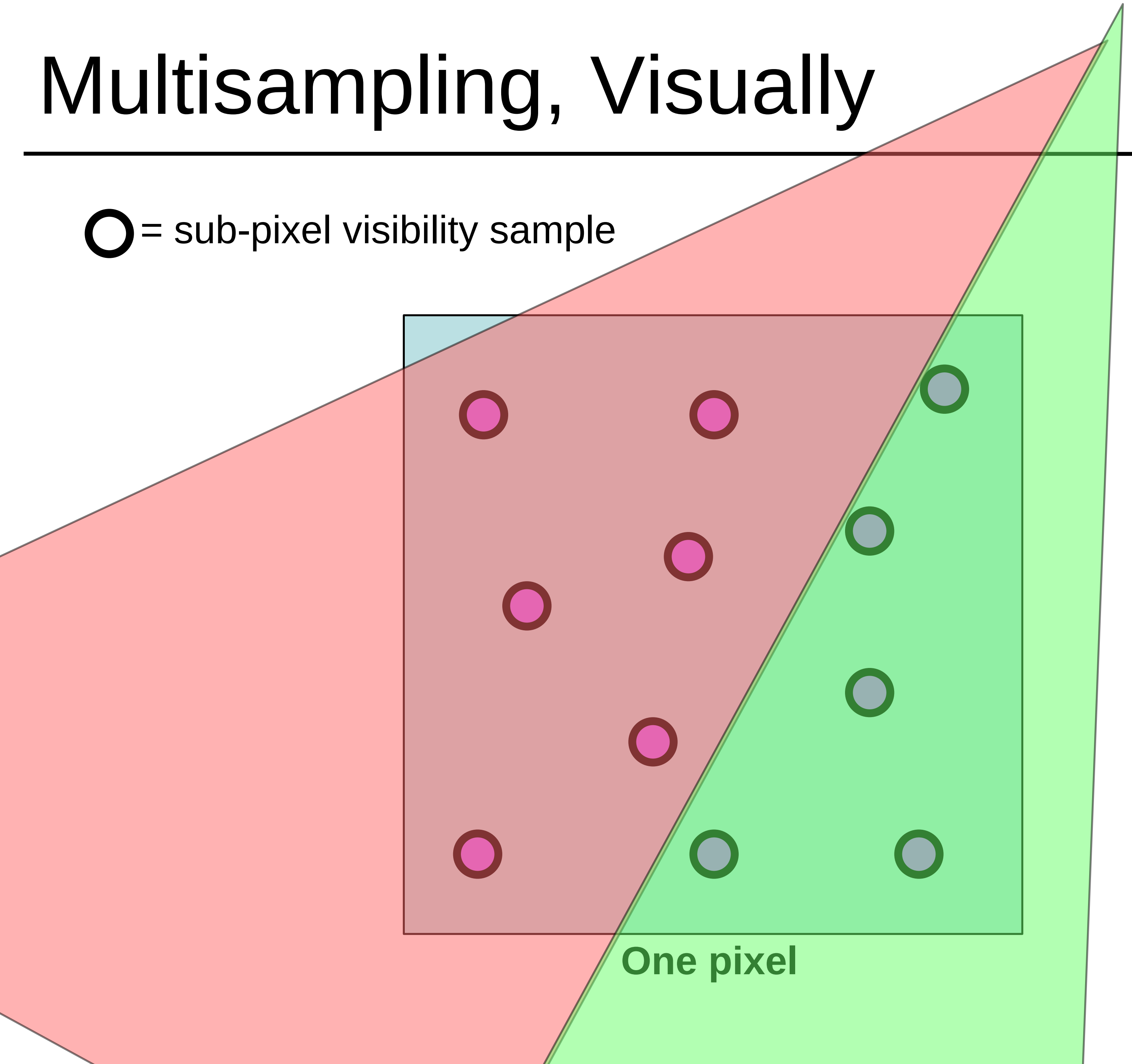
Multisampling, Visually

○ = sub-pixel visibility sample



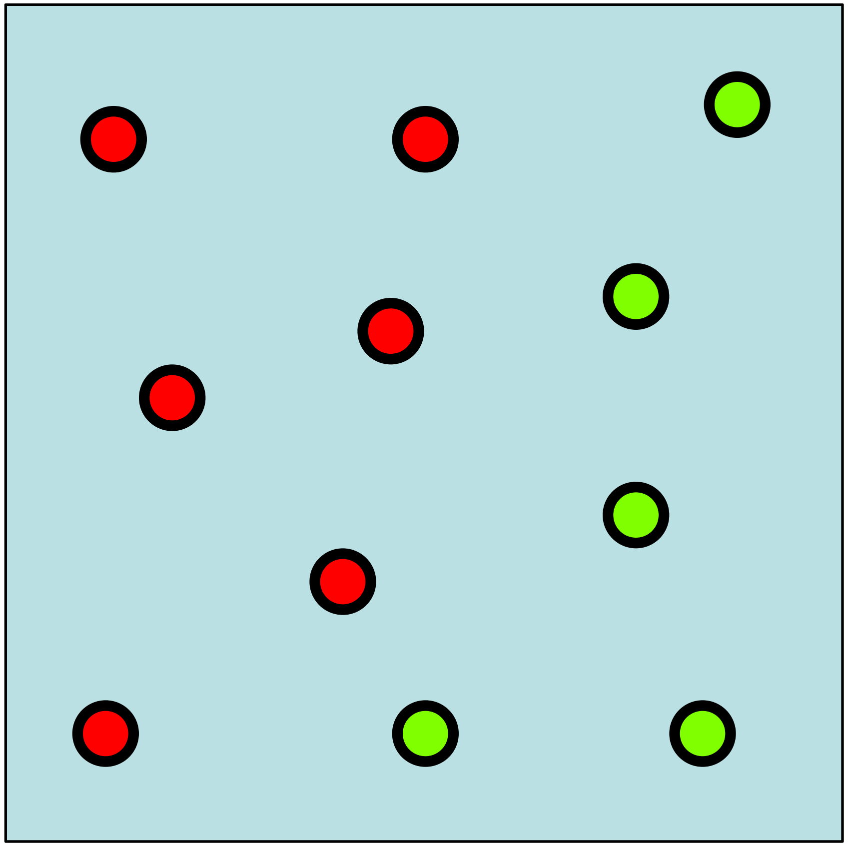
One pixel

Multisampling, Visually



Multisampling, Visually

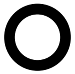
○ = sub-pixel visibility sample

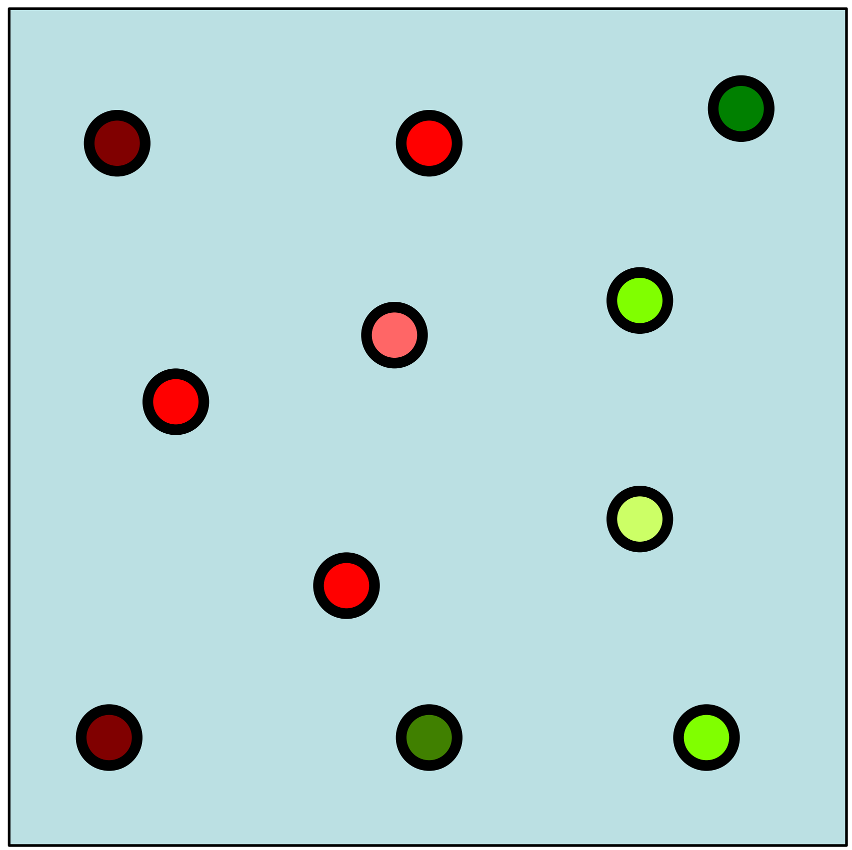


One pixel

The color is only computed **once per pixel per triangle** and reused for all the visibility samples that are covered by the triangle.

Supersampling, Visually

 = sub-pixel visibility sample



One pixel

When supersampling, we compute colors independently for all the visibility samples.

Multisampling Pseudocode

```
For each triangle
  For each pixel
    if pixel overlaps triangle
      color=shade() // only once per pixel!
      for each sub-pixel sample
        compute edge equations & z
        if subsample passes edge equations
           && z < zbuffer[subsample]
          zbuffer[subsample]=z
          framebuffer[subsample]=color
```

Multisampling Pseudocode

```
For each triangle
  For each pixel
    if pixel overlaps triangle
      color=shade() // only once per pixel!
      for each sub-pixel sample
        compute edge equations & z
        if subsample passes edge equations
           && z < zbuffer[subsample]
          zbuffer[subsample]=z
          framebuffer[subsample]=color
At display time: //this is called “resolving”
  For each pixel
    color = average of subsamples
```

Multisampling vs. Supersampling

- Supersampling
  - Compute an entire image at a higher resolution, then downsample (blur + resample at lower res)
- Multisampling
  - Supersample visibility, compute expensive shading only once per pixel, reuse shading across visibility samples
- But Why?
  - Visibility edges are where supersampling really works
  - Shading can be prefiltered more easily than visibility
- This is how GPUs perform antialiasing these days

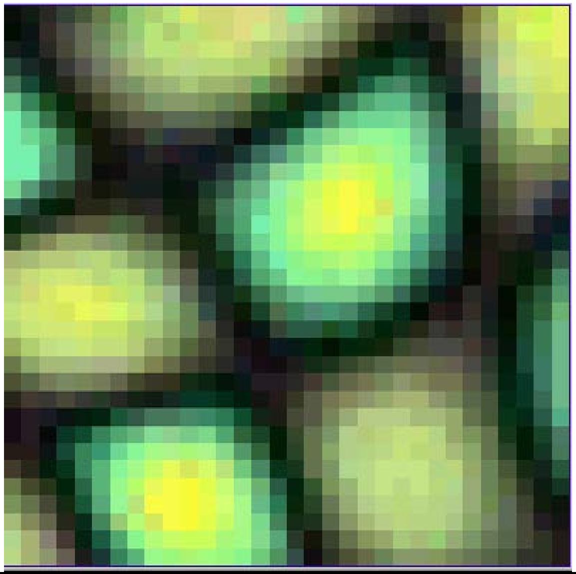
Multisampling vs. Supersampling

- Supersampling
  - Compute an entire image at a higher resolution, then downsample (blur + resample at lower res)
- Multisampling
  - Supersample visibility, compute expensive shading only once per pixel, reuse shading across visibility samples
- But Why?
  - Visibility edges are where supersampling really works
  - Shading can be prefiltered more easily than visibility
- This is how GPUs perform antialiasing these days

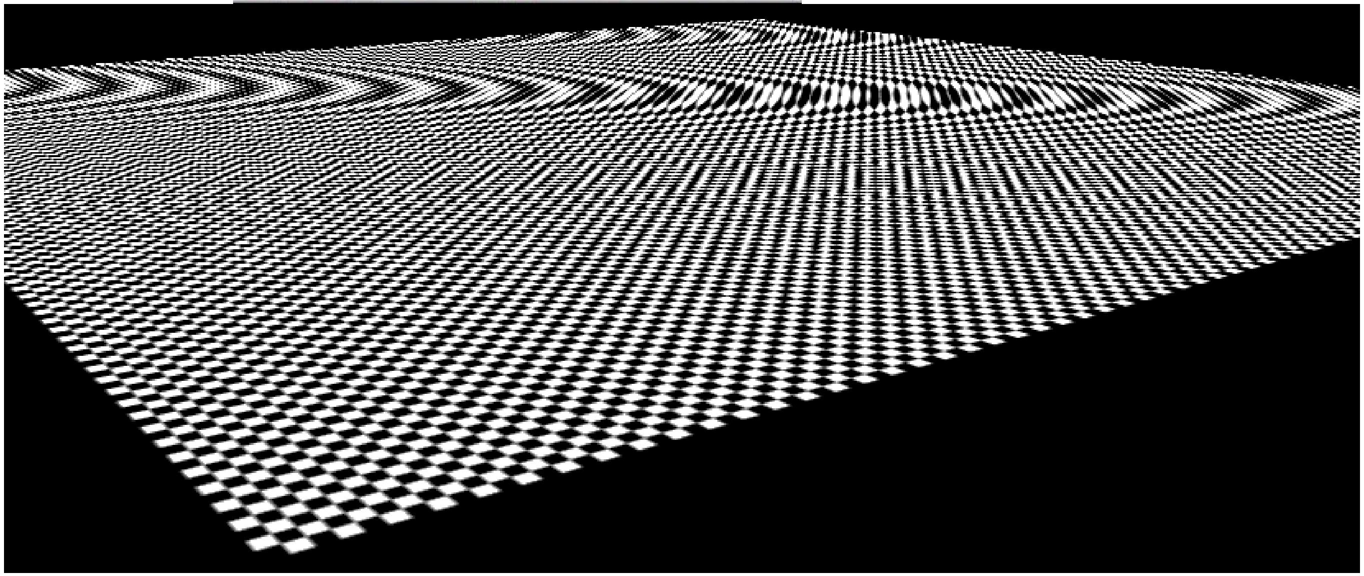
Questions?

Examples of Texture Aliasing

Magnification

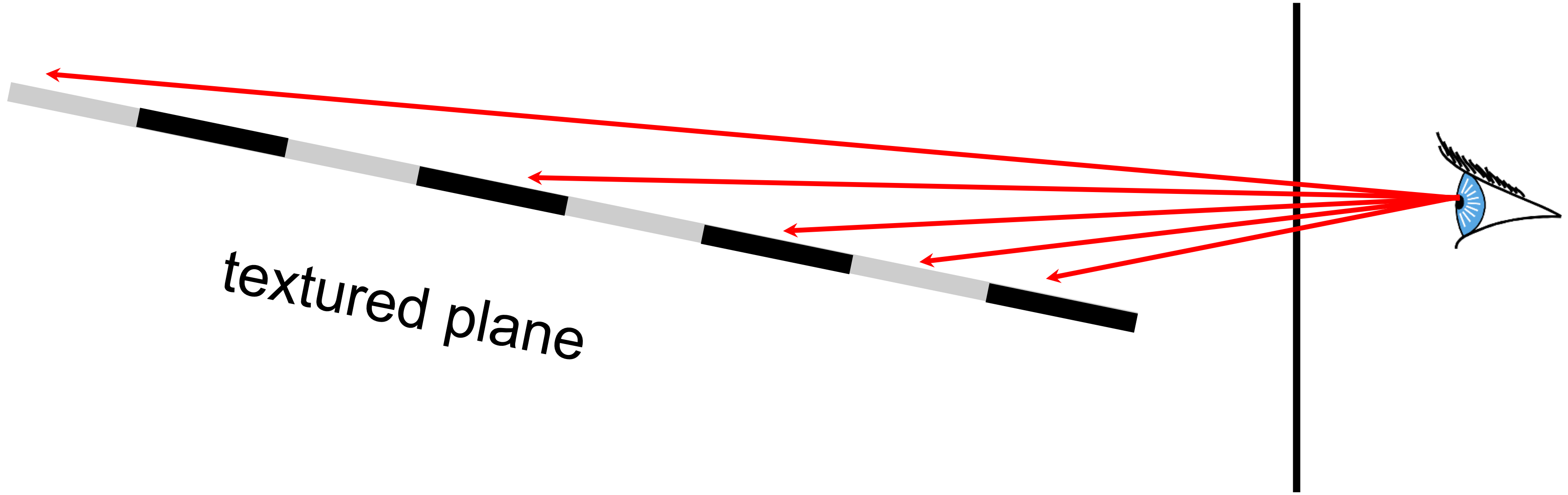


Minification



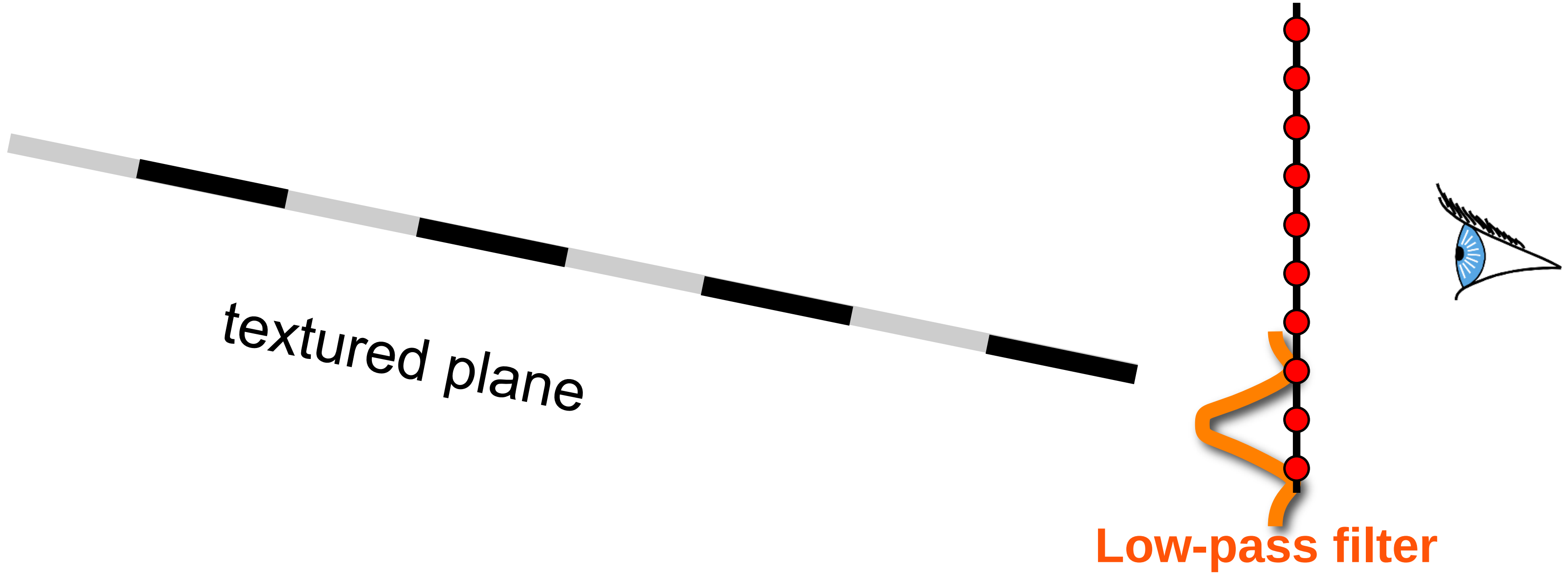
Texture Filtering

- Problem: Prefiltering is impossible when you can only take point samples
  - This is why visibility (edges) need supersampling
- Texture mapping is simpler
  - Imagine again we are looking at an infinite textured plane



Texture Filtering

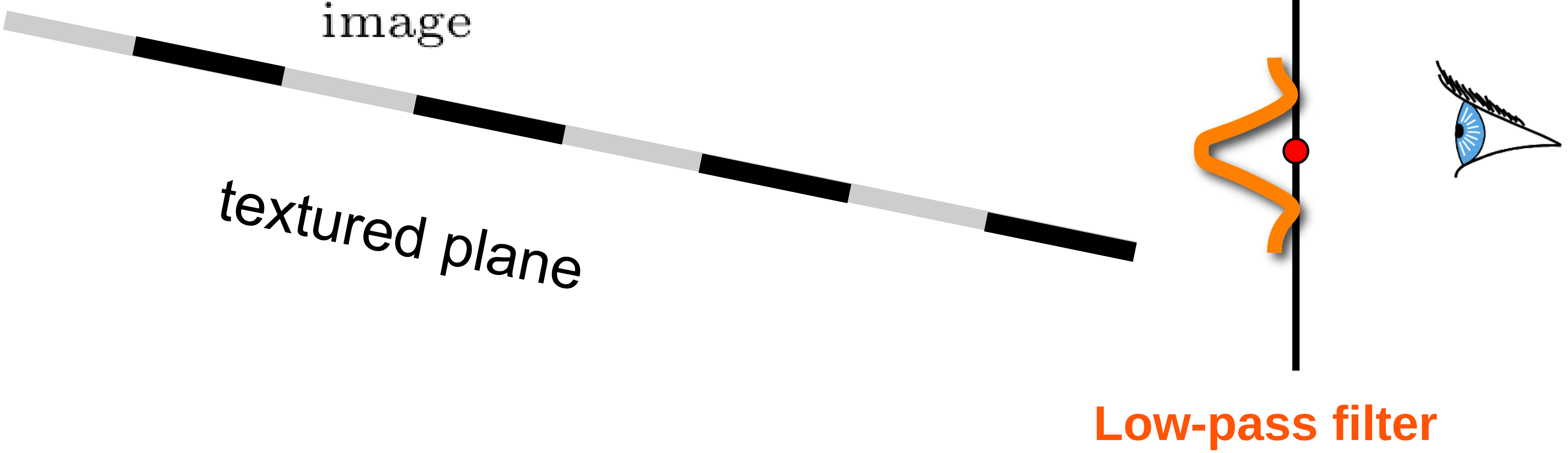
- We should pre-filter image function *before sampling*
  - That means blurring the image function with a low-pass filter (convolution of image function and filter)



Texture Filtering

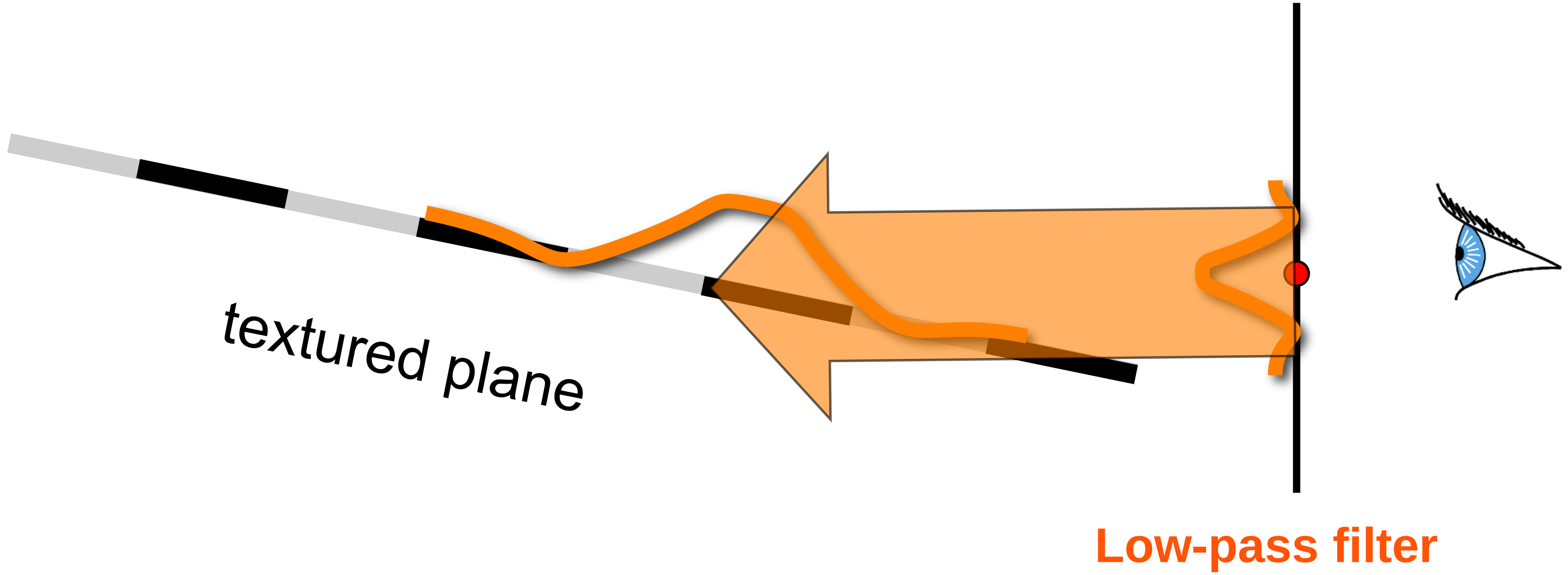
- We can combine low-pass and sampling
  - The value of a sample is the integral of the product of the image *f* and the filter *h* centered at the sample location
    - "A local average of the image *f* weighted by the filter *h*"

$$\hat{f}_i = \int_{\text{image}} f(x)\, h(x)\, \mathrm{d}x$$



Texture Filtering

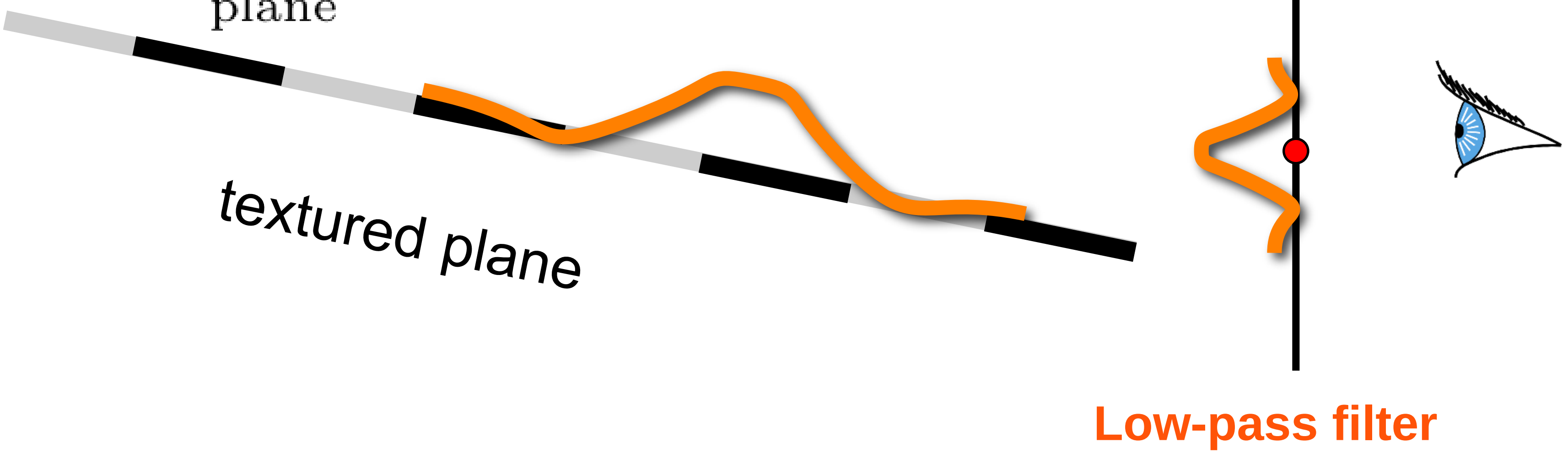
- Well, we can just as well change variables and compute this integral *on the textured plane instead*
  - In effect, we are projecting the pre-filter onto the plane



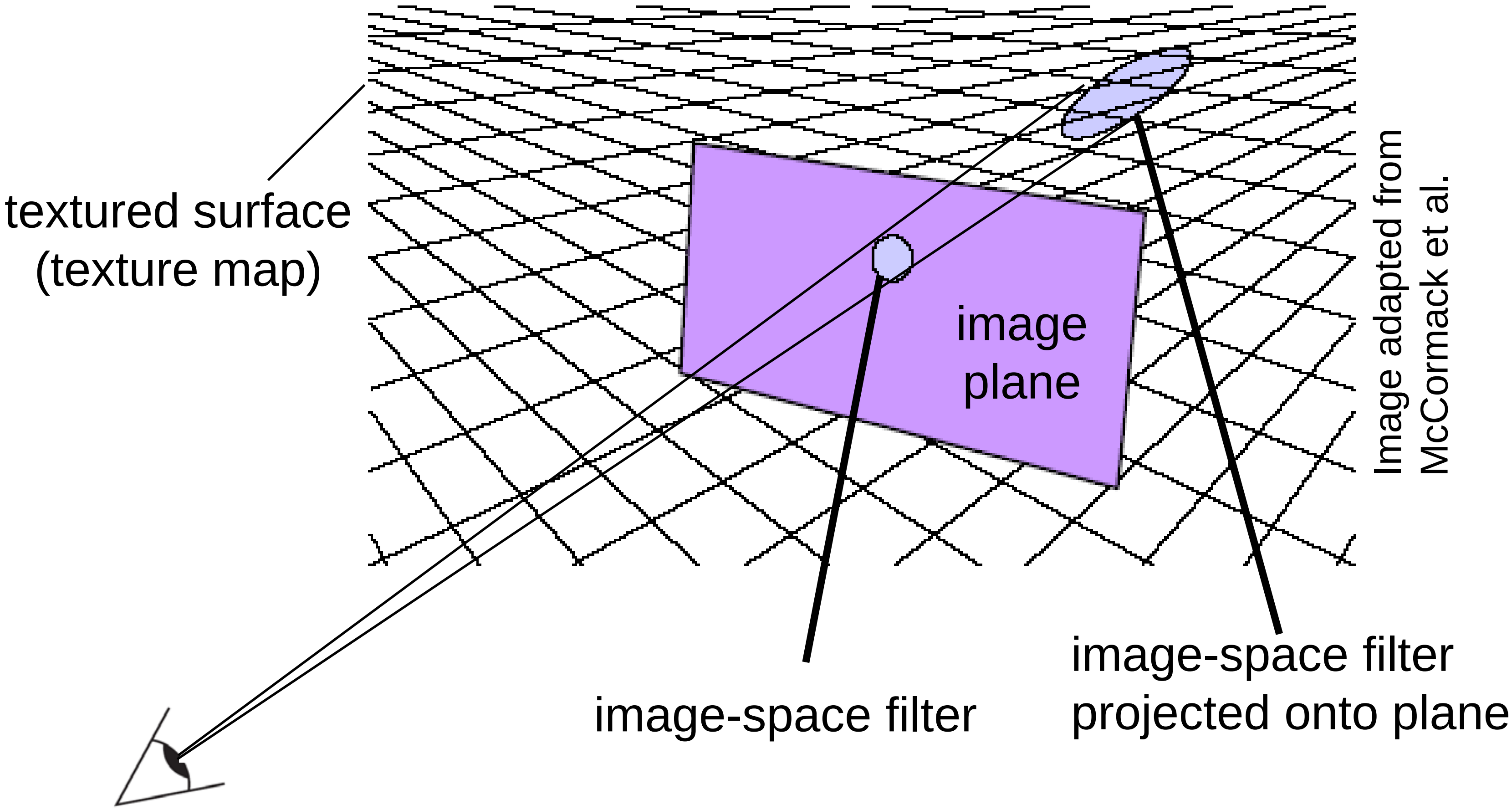
Texture Filtering

- Well, we can just as well change variables and compute this integral *on the textured plane instead*
  - In effect, we are projecting the pre-filter onto the plane
  - It's still a weighted average of the texture under filter

$$\hat{f}_i = \int_{\text{plane}} f(x')\, h(x')\, \left|J(x, x')\right| \mathrm{d}x'$$



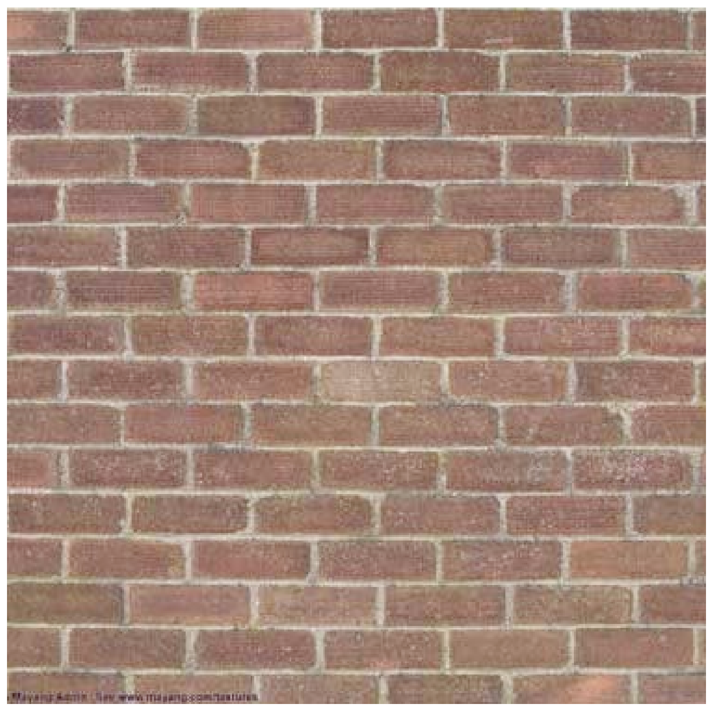
Texture Pre-Filtering, Visually



- Must still integrate product of projected filter and texture

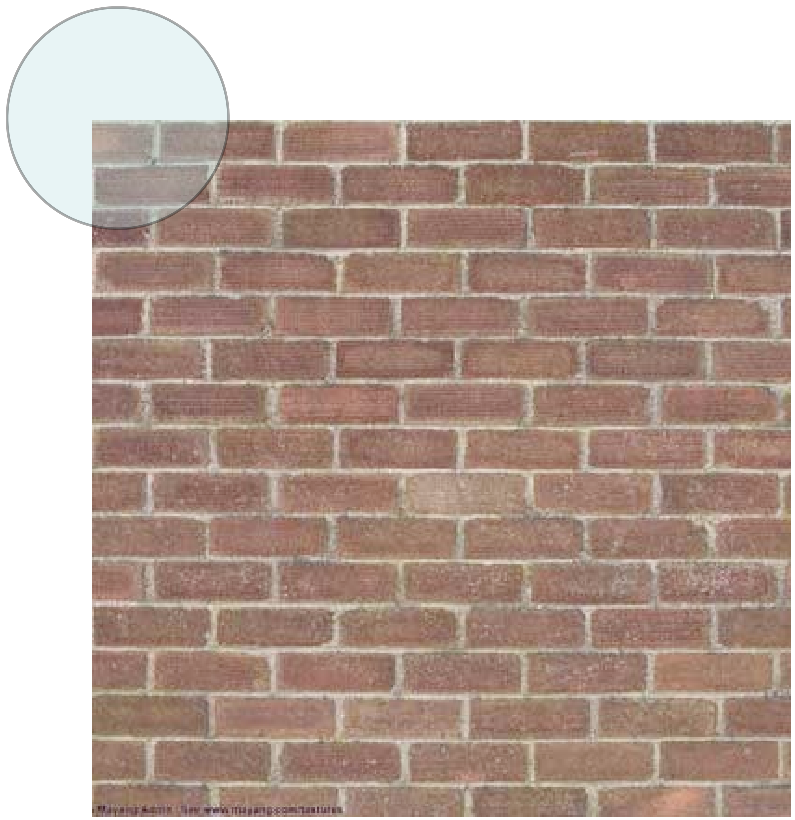
Solution: Precomputation

- We'll precompute and store a set of prefiltered results from each texture with different sizes of prefilters



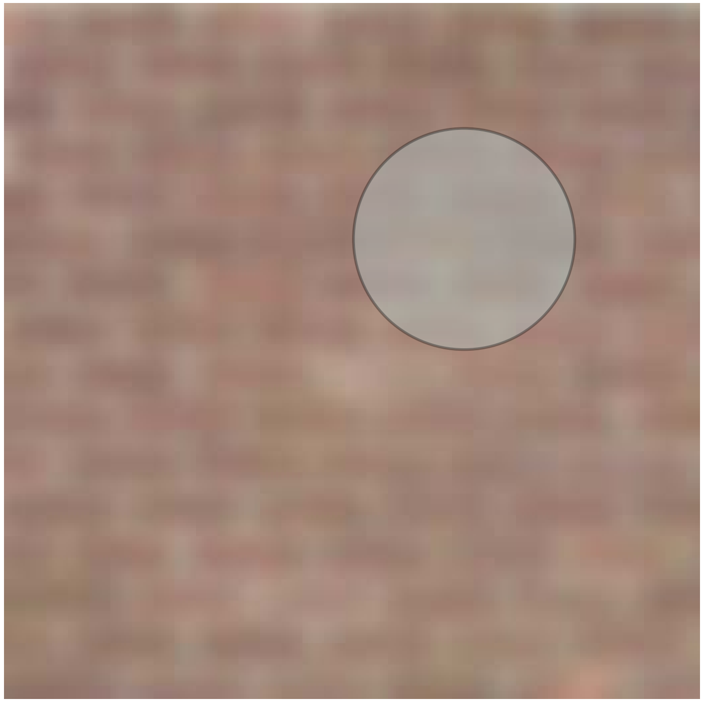
Solution: Precomputation

- We'll precompute and store a set of prefiltered results from each texture with different sizes of prefilters



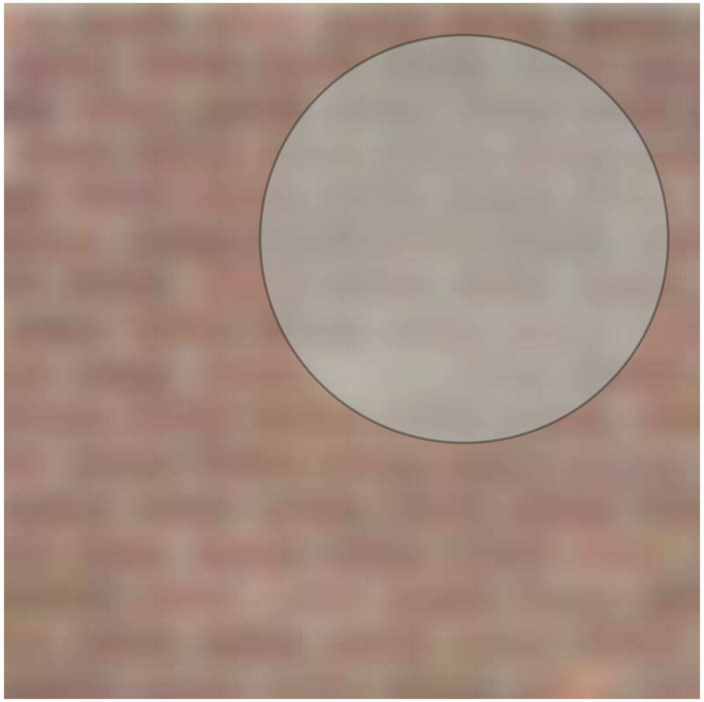
Solution: Precomputation

- We'll precompute and store a set of prefiltered results from each texture with different sizes of prefilters
  - Because it's low-passed, we can also subsample



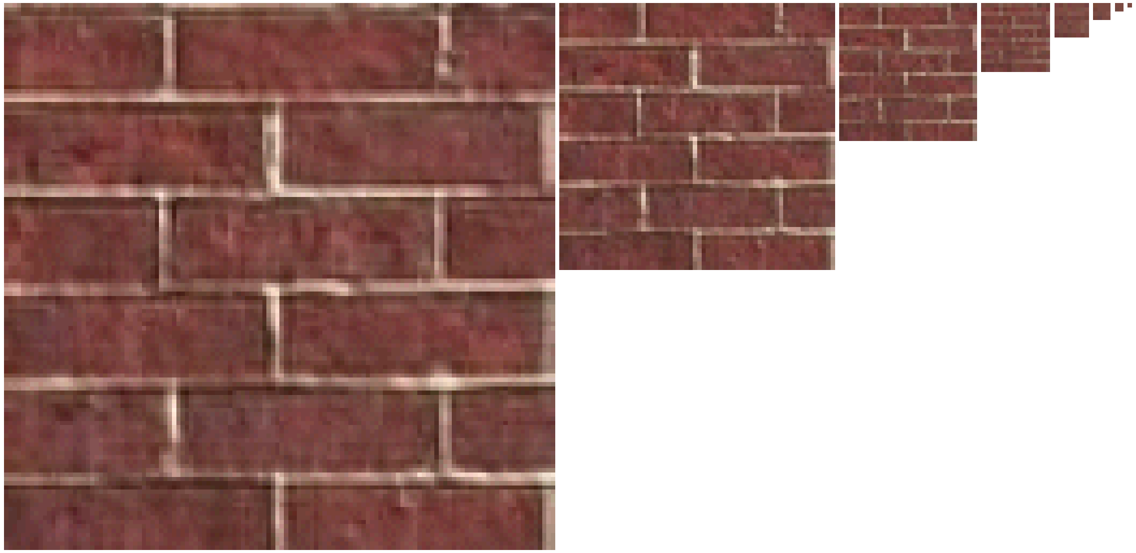
Solution: Precomputation

- We’ll precompute and store a set of prefiltered results from each texture with different sizes of prefilters
  - Because it’s low-passed, we can also subsample



This is Called “MIP-Mapping”

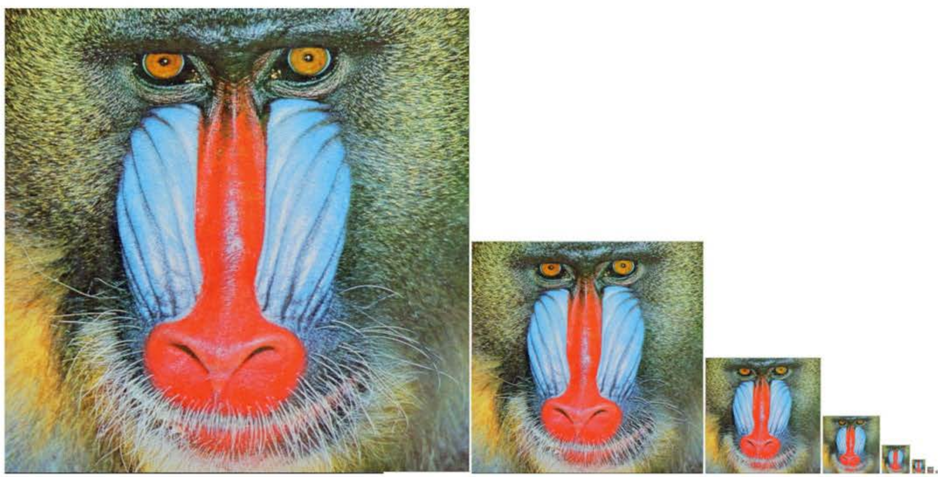
- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling

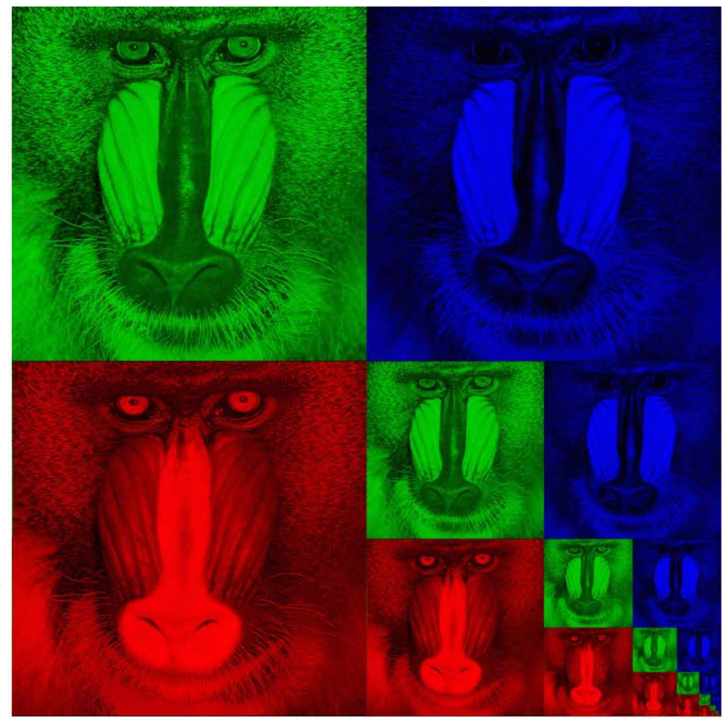


- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

Storing MIP Maps

- Can be stored compactly: Only 1/3 more space!

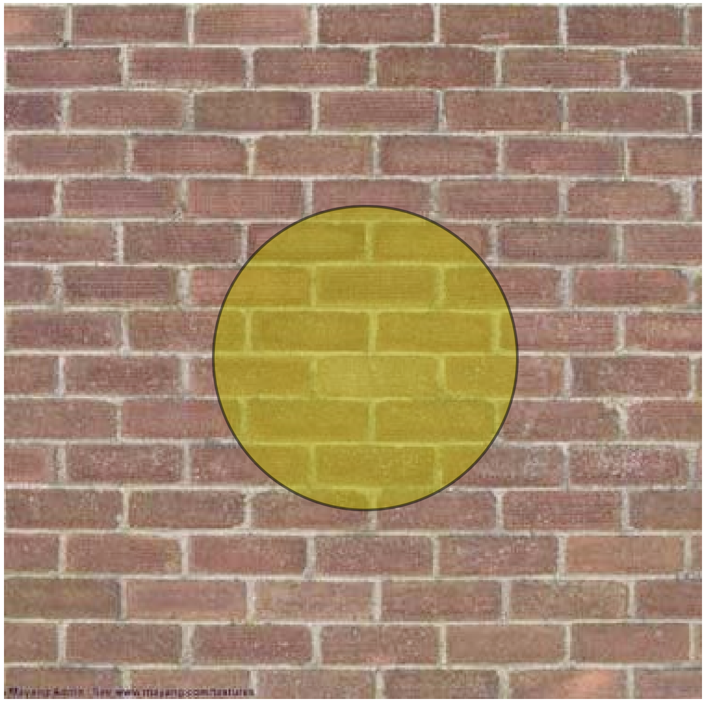




MIP-Mapping

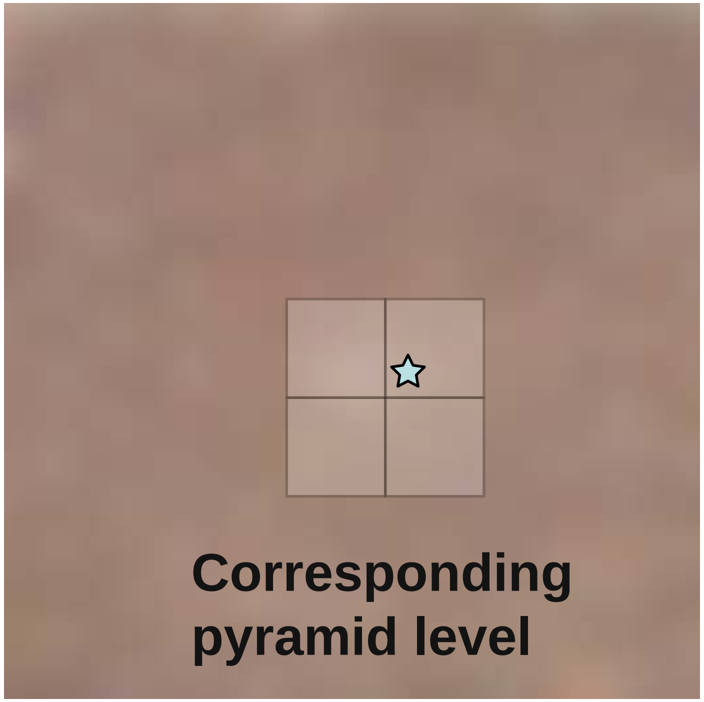
- When a pixel wants an integral of the pre-filtered texture, we must find the “closest” results from the precomputed MIP-map pyramid
  - Must compute the “size” of the projected pre-filter in the texture UV domain

Projected pre-filter

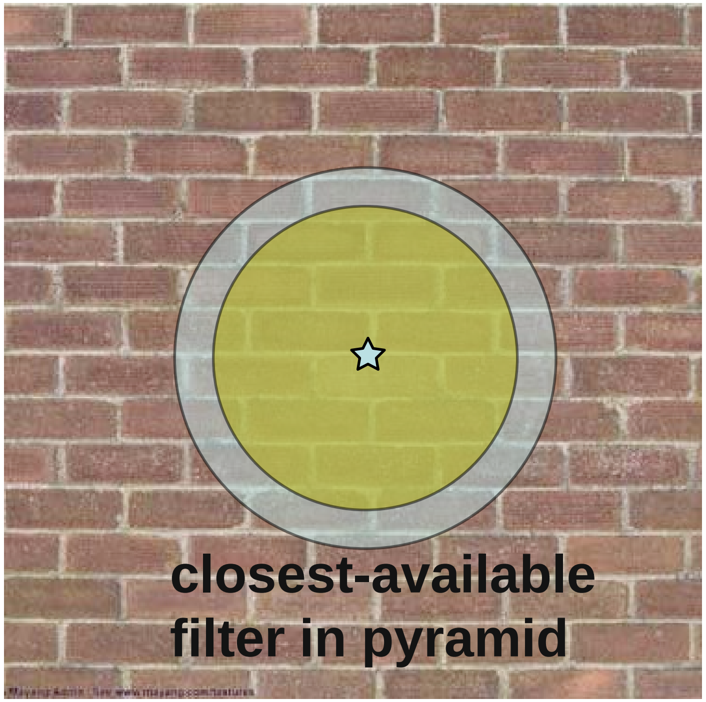


MIP-Mapping

- Simplest method: Pick the scale closest, then do usual reconstruction on that level (e.g. bilinear between 4 closest texture pixels)



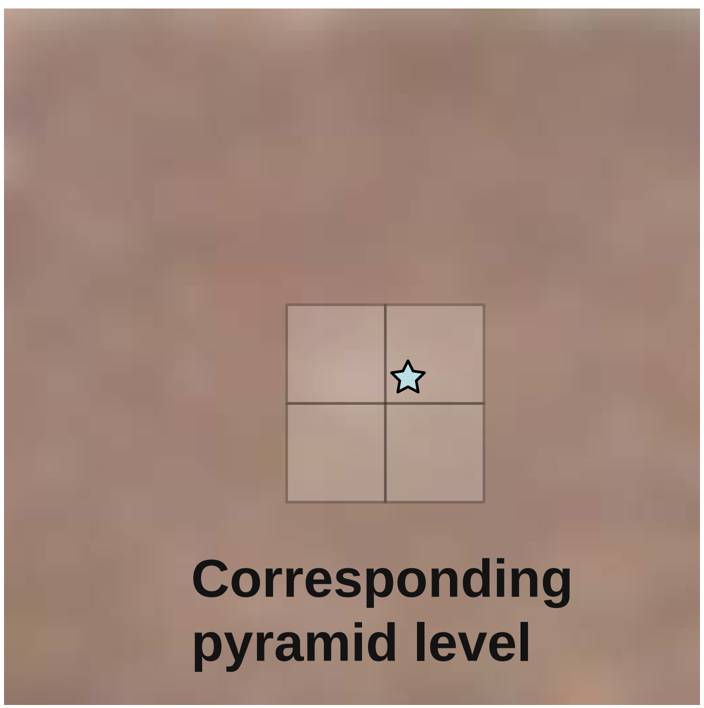
Projected pre-filter

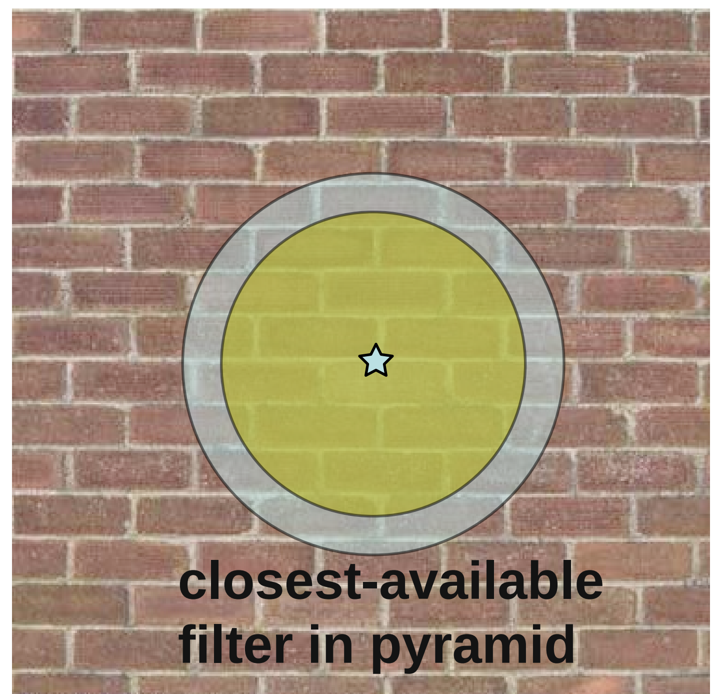


MIP-Mapping

- Simplest method: Pick the scale closest, then do usual reconstruction on that level (e.g. bilinear between 4 closest texture pixels)
- Problem: discontinuity when switching scale

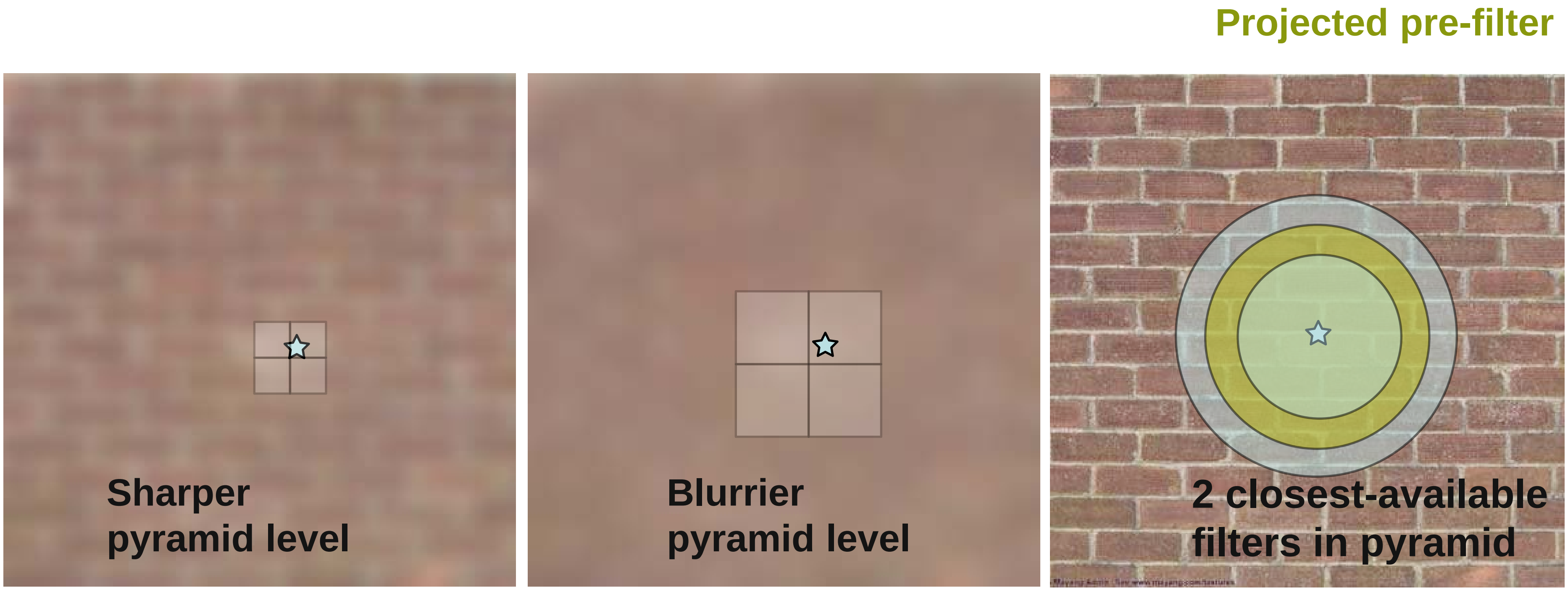
Projected pre-filter





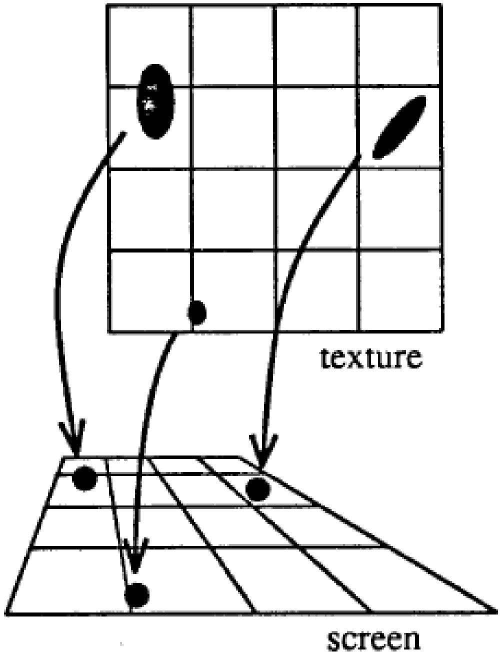
Tri-Linear MIP-Mapping

- Use **two** closest scales,
  compute reconstruction results from both,
  and linearly interpolate between them

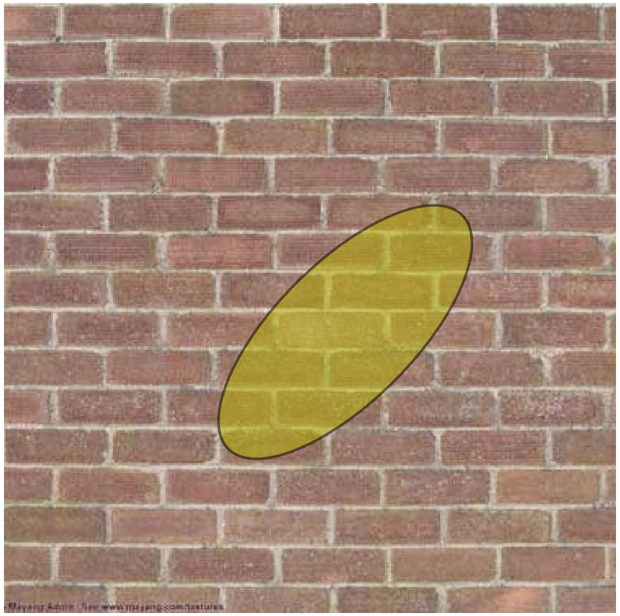


Tri-Linear MIP-Mapping

- Use **two** closest scales,
  compute reconstruction results from both,
  and linearly interpolate between them
- Problem: our filter might not be circular, because of foreshortening



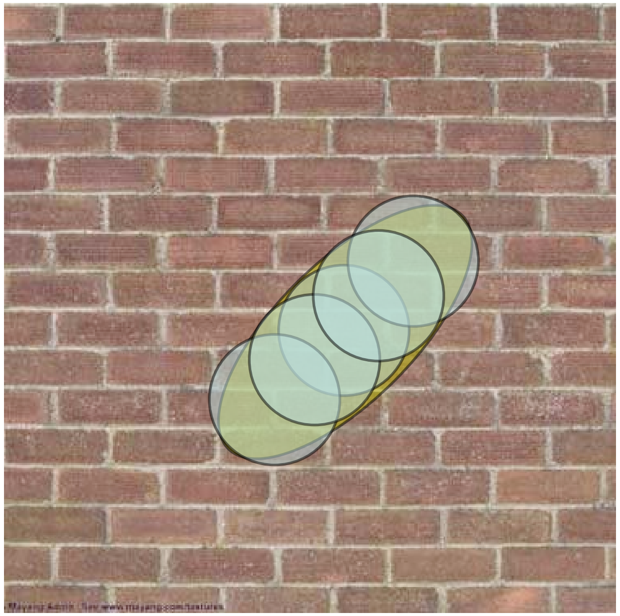
Projected pre-filter



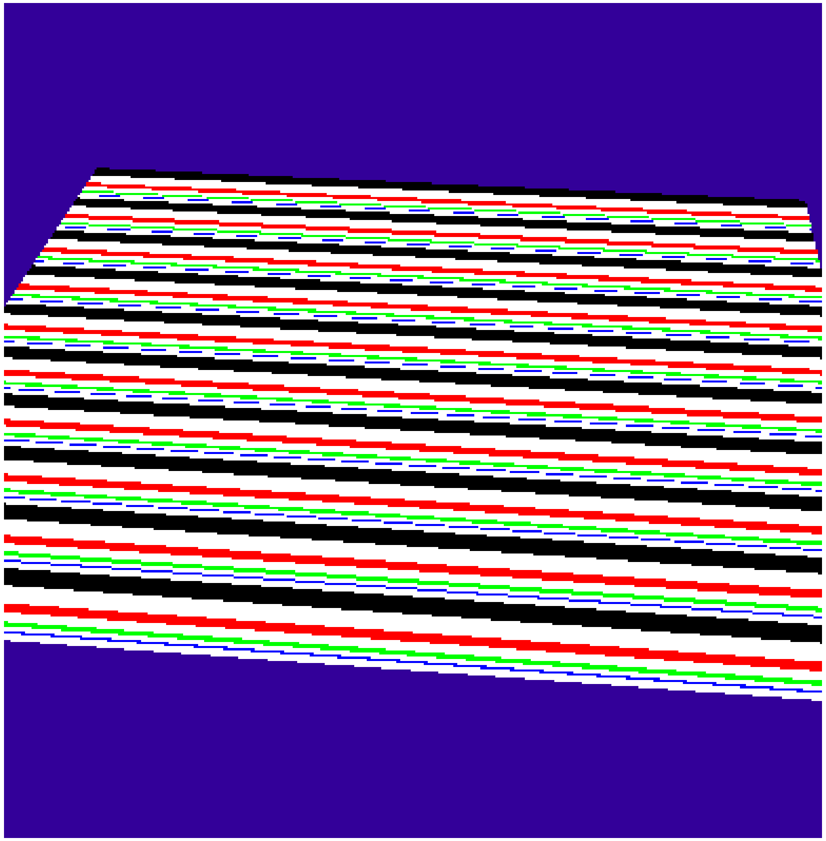
Anisotropic filtering

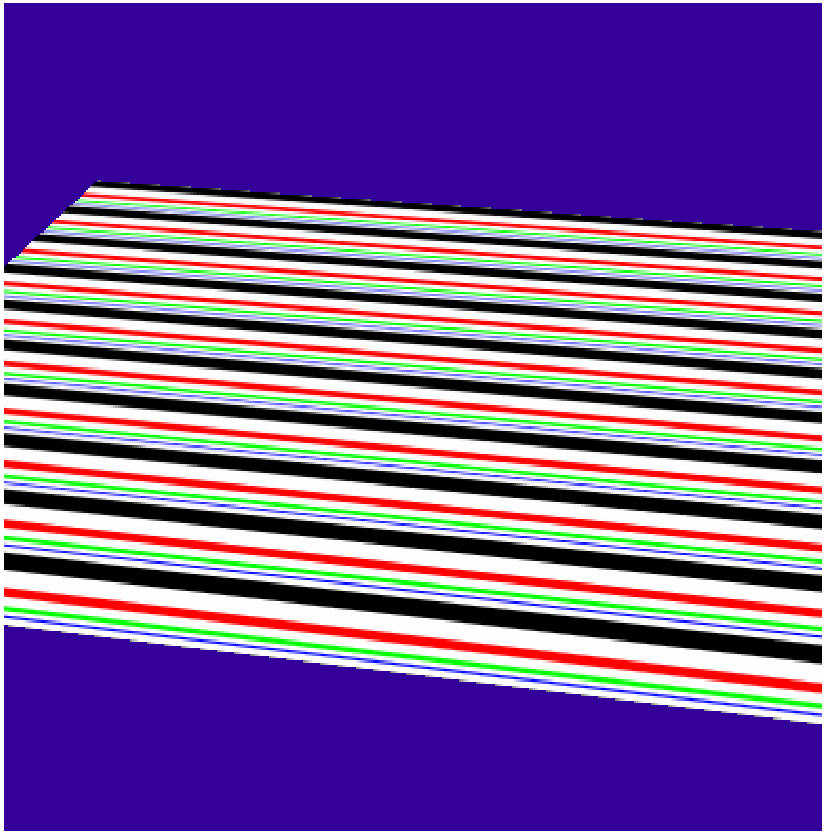
- Approximate Elliptical filter with multiple circular ones (usually 5)
- Perform trilinear lookup at each one
- i.e. consider five times eight values
  - fair amount of computation
  - this is why graphics hardware has dedicated units to compute trilinear mipmap reconstruction

Projected pre-filter

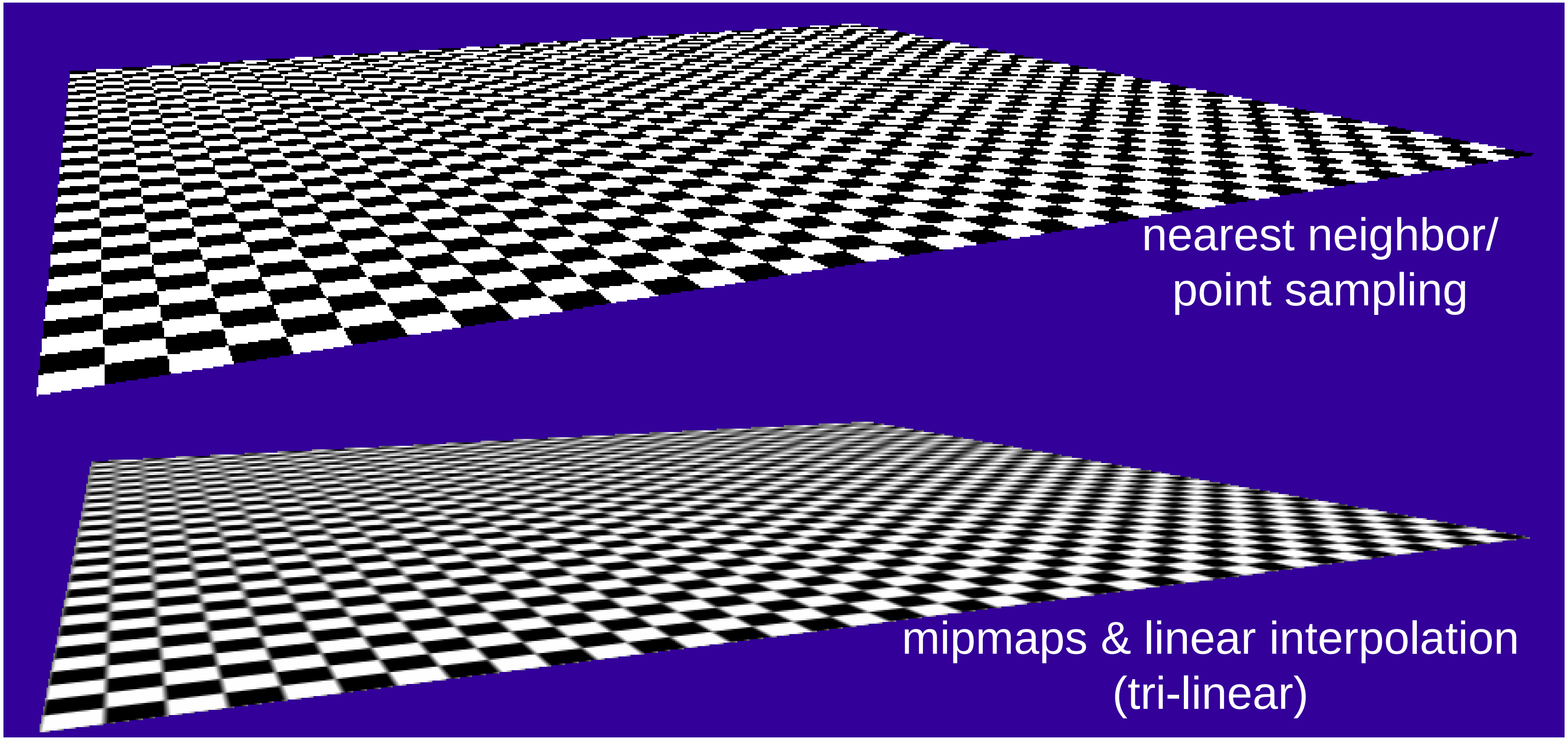


MIP Mapping Example

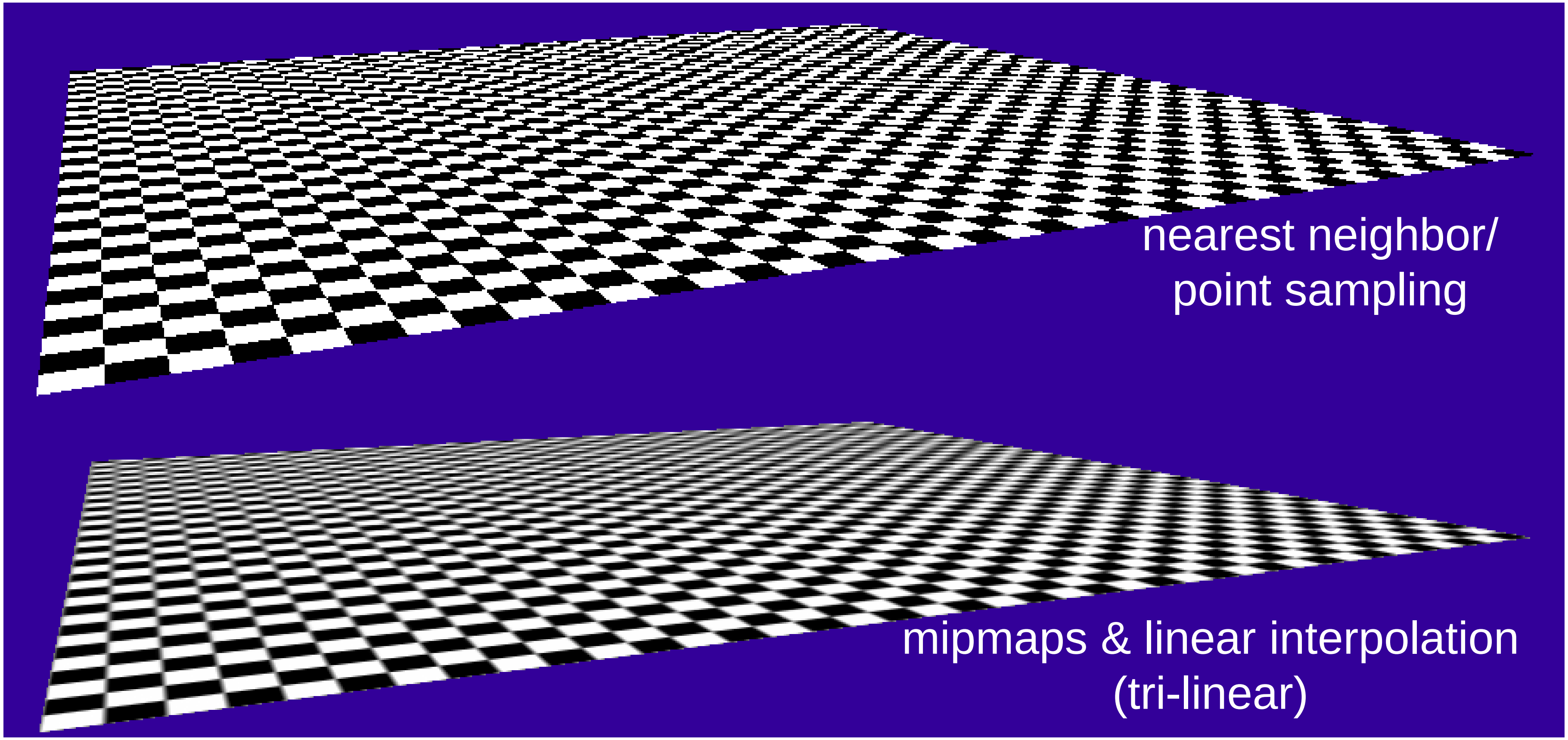

Nearest Neighbor


MIP Mapped (Tri-Linear)

MIP Mapping Example



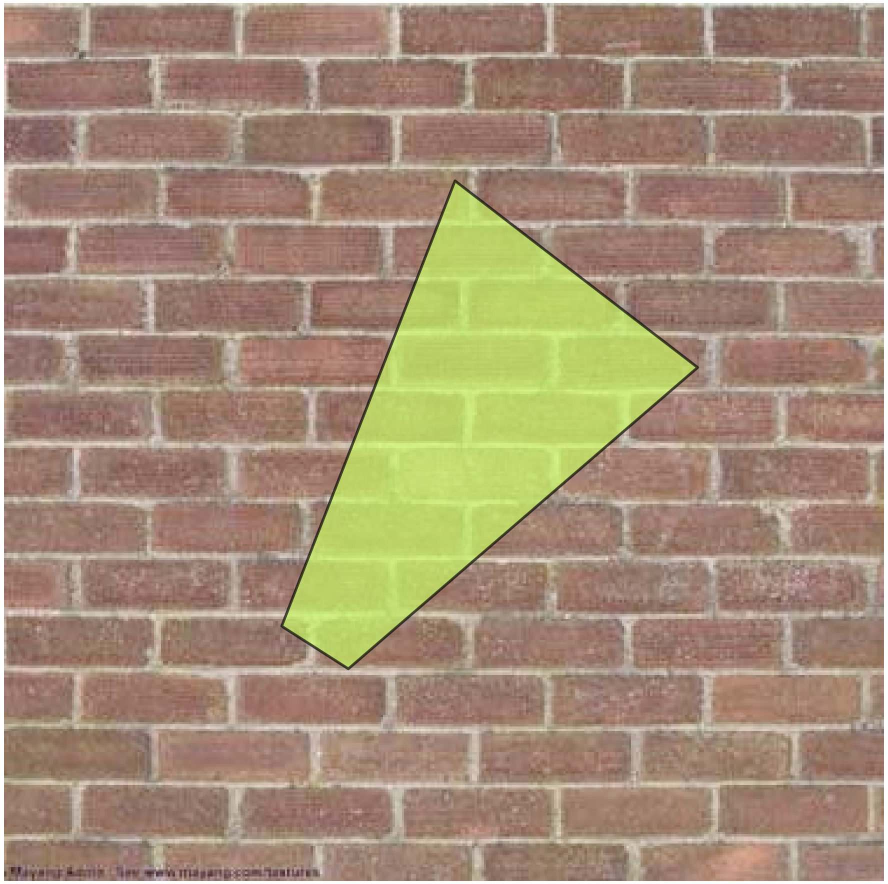
MIP Mapping Example Questions



Finding the MIP Level

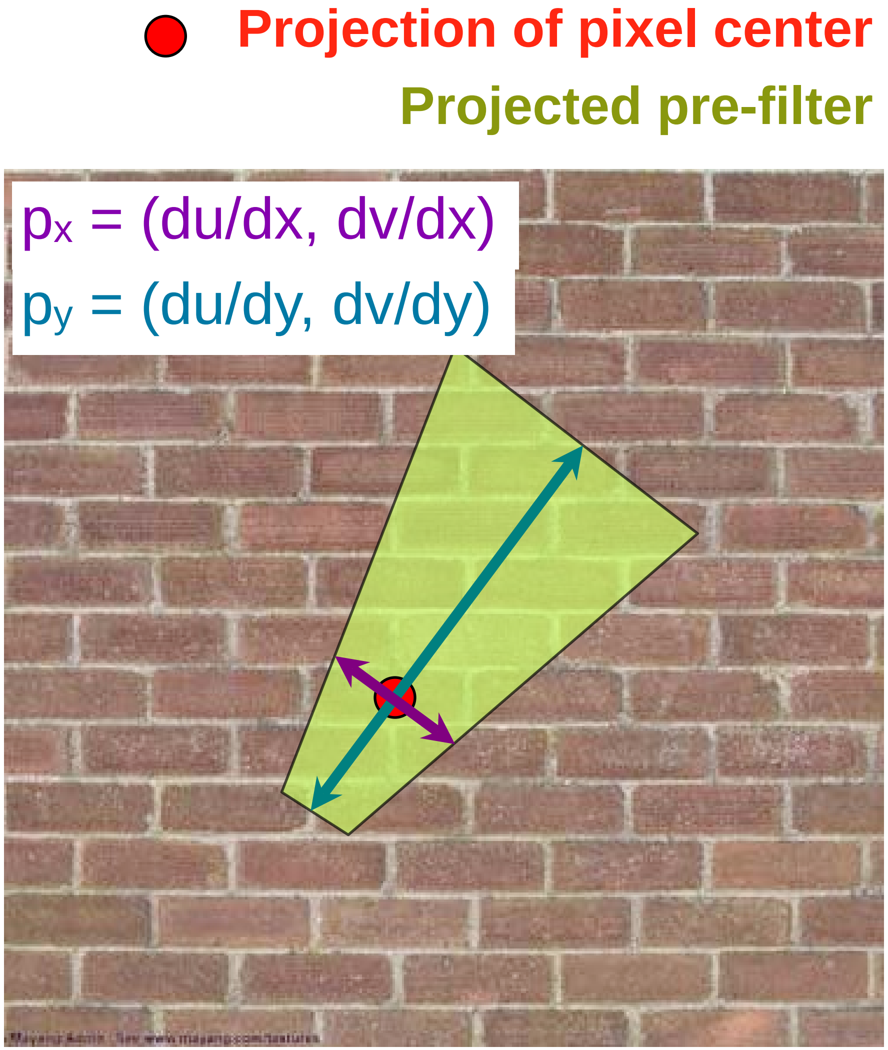
- Often we think of the pre-filter as a box
  - What is the projection of the square pixel “window” in texture space?

Projected pre-filter



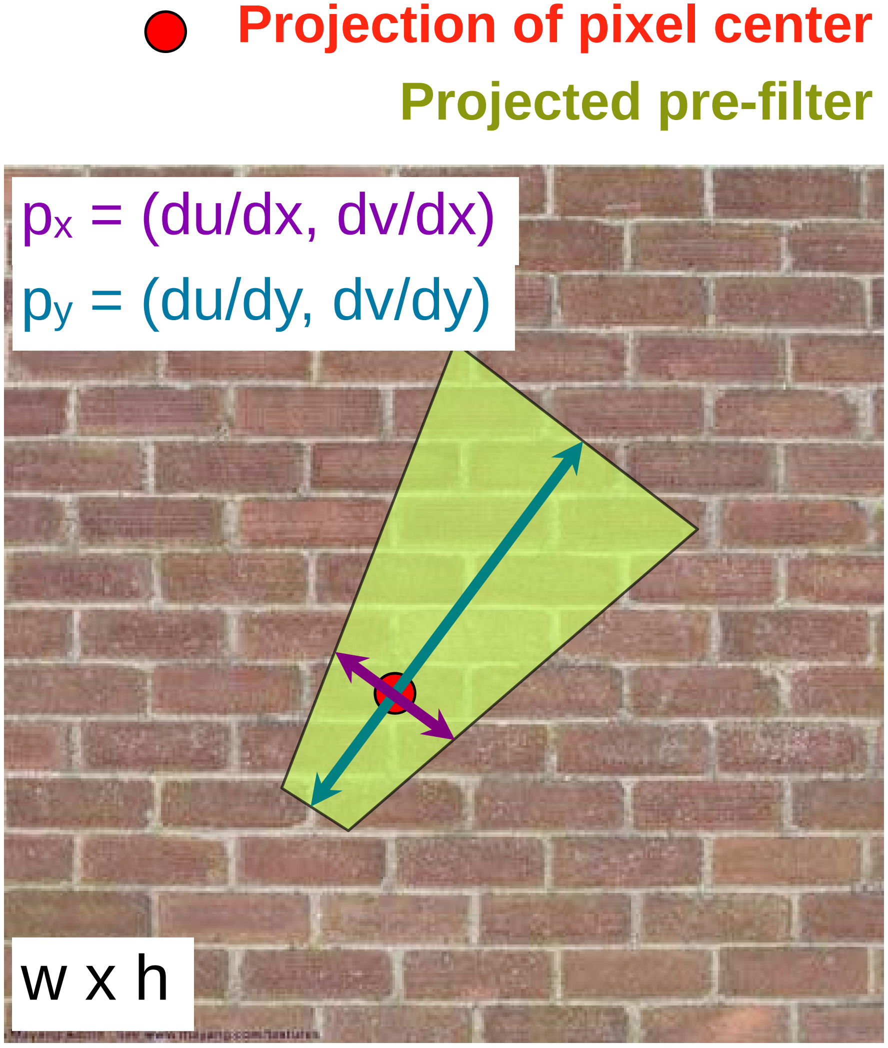
Finding the MIP Level

- Often we think of the pre-filter as a box
  - What is the projection of the square pixel “window” in texture space?
  - Answer is in the partial derivatives p_x and p_y of (u,v) w.r.t. screen (x,y)



For isotropic trilinear mipmapping

- No right answer, circular approximation
- Two most common approaches are
  - Pick level according to the length (in texels) of the longer partial
    $$\log_2 \max\{w|p_x|, h|p_y|\}$$
  - Pick level according to the length of their sum
    $$\log_2 \sqrt{(w|p_x|)^2 + (h|p_y|)^2}$$

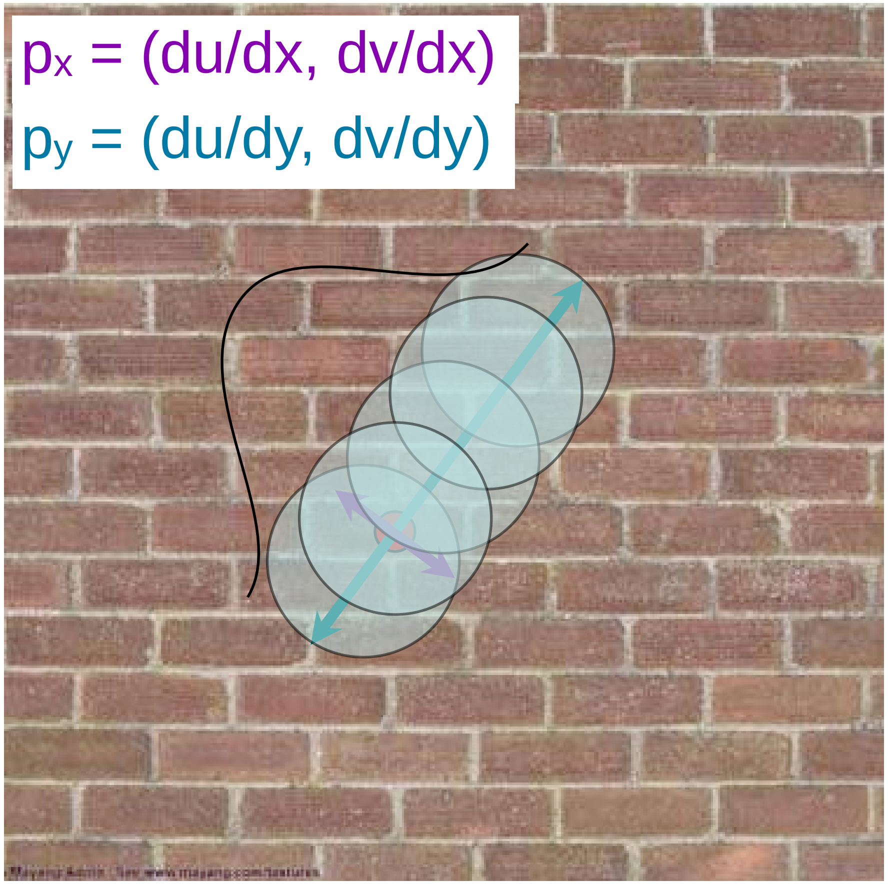


Anisotropic filtering

- Pick levels according to smallest partial
  - well, actually max of the smallest and the largest/5
- Distribute circular "probes" along longest one
- Weight them by a Gaussian

Projection of pixel center

Projected pre-filter



How Are Partials Computed?

- You can derive closed form formulas based on the *uv* and *xyw* coordinates of the vertices...
  - This is what used to be done
- ..but shaders may compute texture coordinates programmatically, not necessarily interpolated
  - No way of getting analytic derivatives!
- **In practice, use finite differences**
  - GPUs process pixels in blocks of (at least) 4 anyway
    - These 2x2 blocks are called *quads*

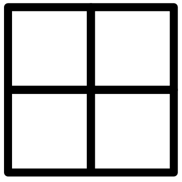
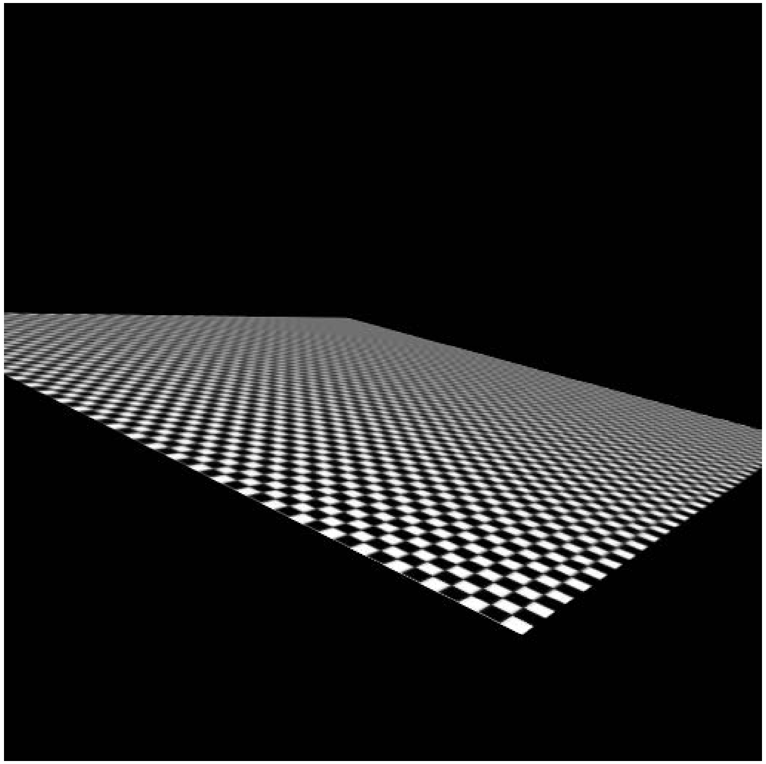
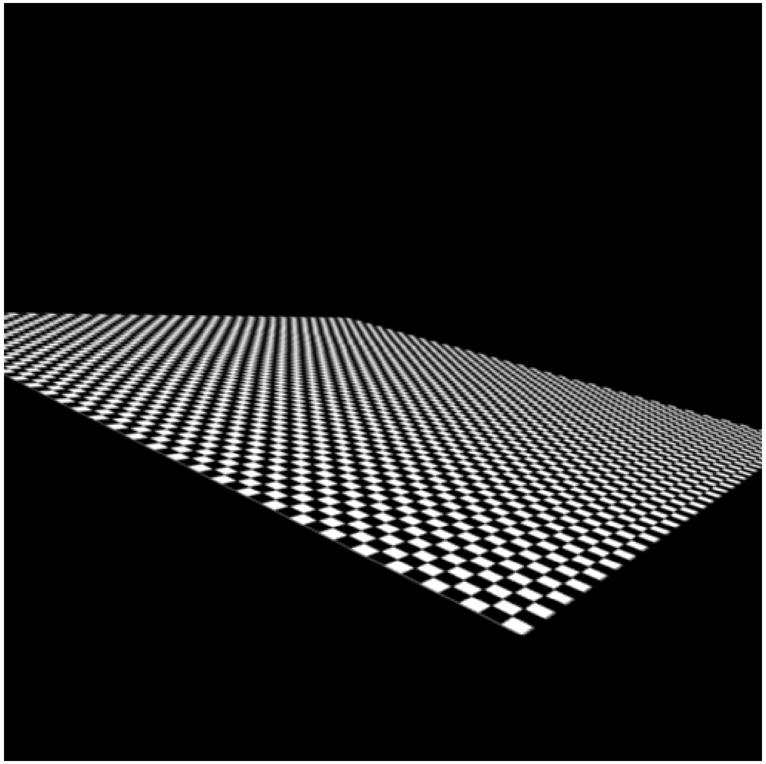


Image Quality Comparison



trilinear mipmapping
(excessive blurring)



anisotropic filtering

Further Reading

- Paul Heckbert published seminal work on texture mapping and filtering in his master's thesis (!)
  - Including EWA
  - Highly recommended reading!
  - See http://www.cs.cmu.edu/~ph/texfund/texfund.pdf
- More reading
  - [Feline: Fast Elliptical Lines for Anisotropic Texture Mapping, McCormack, Perry, Farkas, Jouppi SIGGRAPH 1999]()
  - [Texram: A Smart Memory for Texturing Schilling, Knittel, Strasser,. IEEE CG&A, 16(3): 32-41]()

Arf!

Further Reading Questions?

- Paul Heckbert published seminal work on texture mapping and filtering in his master's thesis (!)
  - Including EWA
  - Highly recommended reading!
  - See http://www.cs.cmu.edu/~ph/texfund/texfund.pdf
- More reading
  - Feline: Fast Elliptical Lines for Anisotropic Texture Mapping, McCormack, Perry, Farkas, Jouppi SIGGRAPH 1999
  - Texram: A Smart Memory for Texturing Schilling, Knittel, Strasser,. IEEE CG&A, 16(3): 32-41

Arf!

Ray Casting vs. Rasterization

Ray Casting

```
For each pixel
    For each object
```

- Ray-centric
- Needs to store scene in memory
- (Mostly) Random access to scene

Rasterization

```
For each triangle
For each pixel
```

- Triangle centric
- Needs to store image (and depth) into memory
- (Mostly) random access to frame buffer

Which is smaller? Scene or Frame?
  Frame
Which is easiest to access randomly?
  Frame because regular sampling

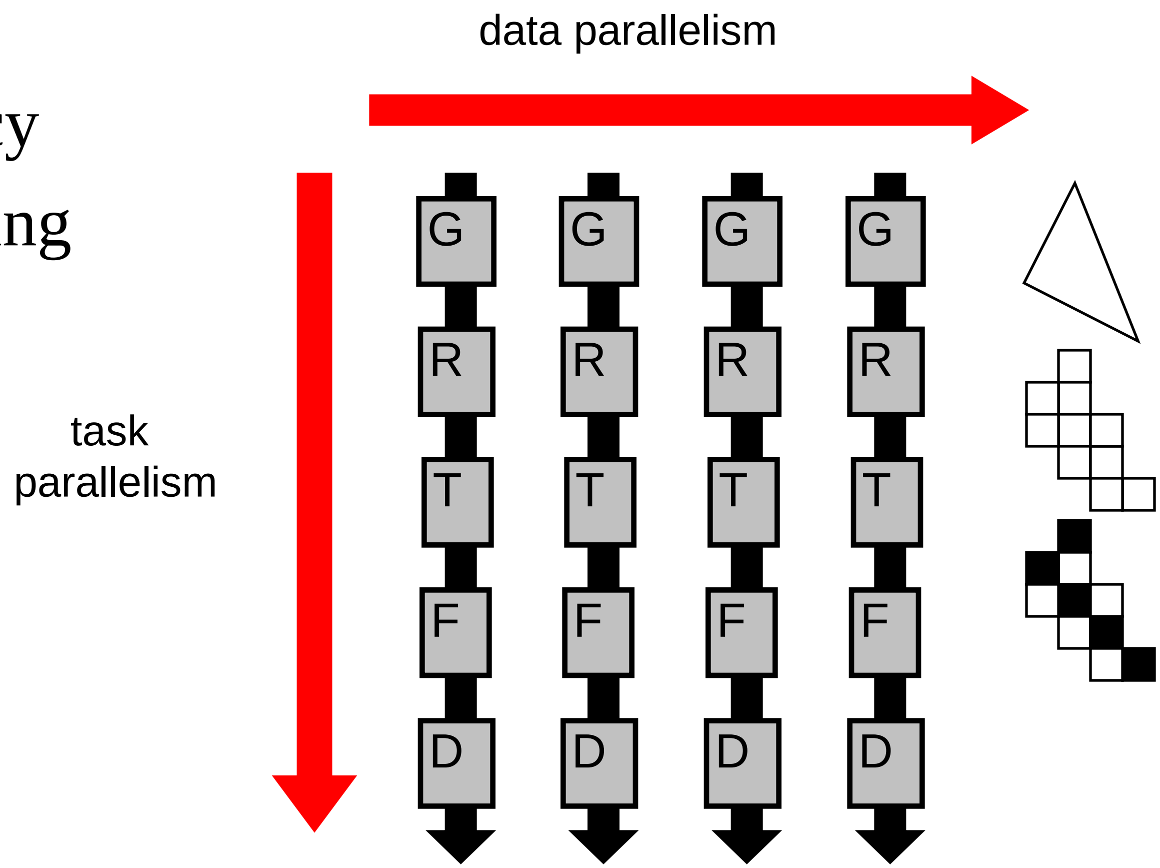
Good References

- http://www.tomshardware.com/reviews/ray-tracing-rasterization,2351.html
- http://c0de517e.blogspot.com/2011/09/raytracing-myths.html
- http://people.csail.mit.edu/fredo/tmp/rendering.pdf

Graphics Hardware

- High performance through
  - Parallelism
  - Specialization
  - No data dependency
  - Efficient pre-fetching

- More next week

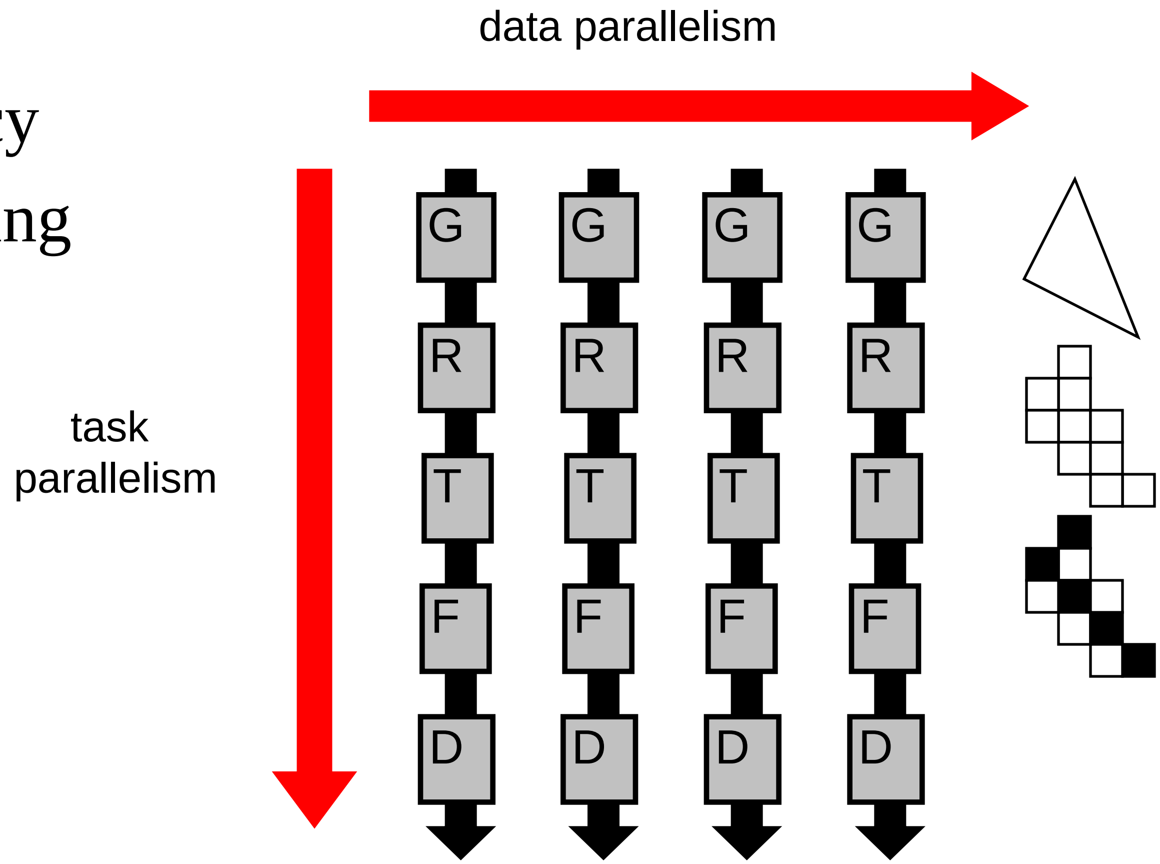


Graphics Hardware

Questions?

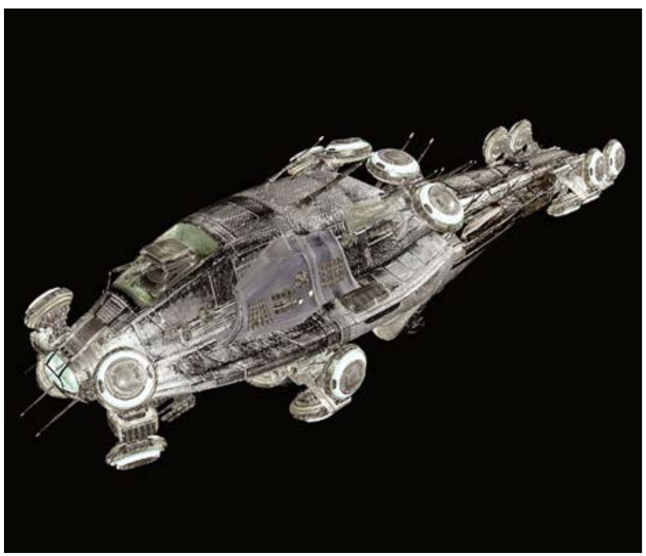
- High performance through
  - Parallelism
  - Specialization
  - No data dependency
  - Efficient pre-fetching

- More next week



Movies

both rasterization and ray tracing



Games

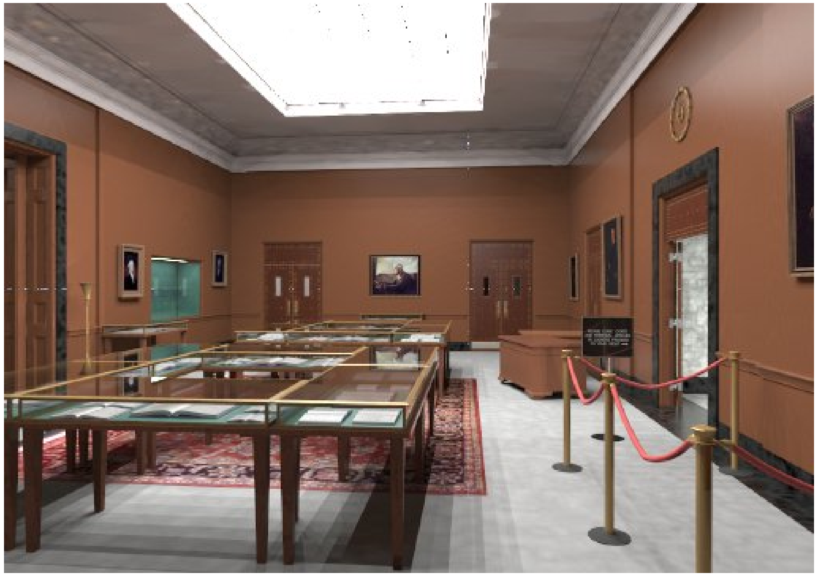
rasterization

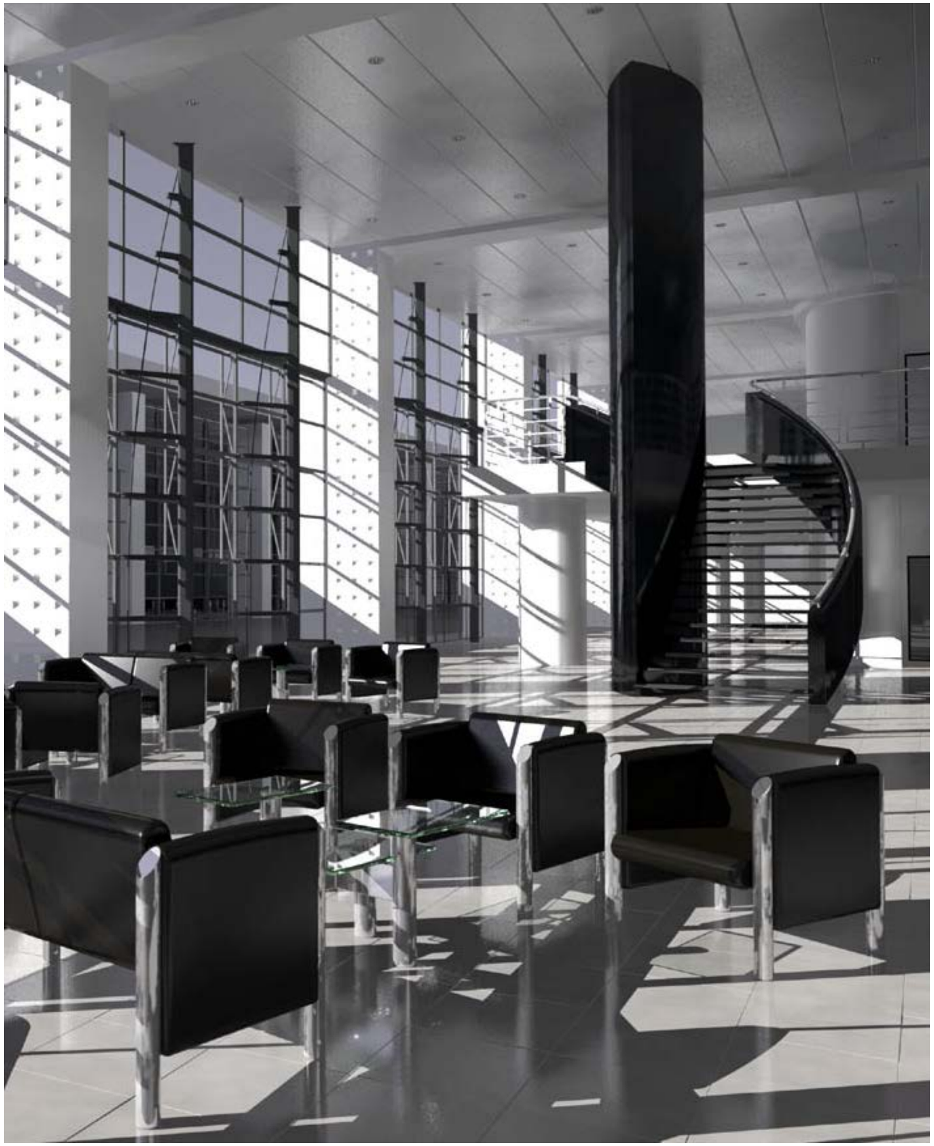
CAD-CAM & Design

rasterization for GUI,
anything for final image

Architecture

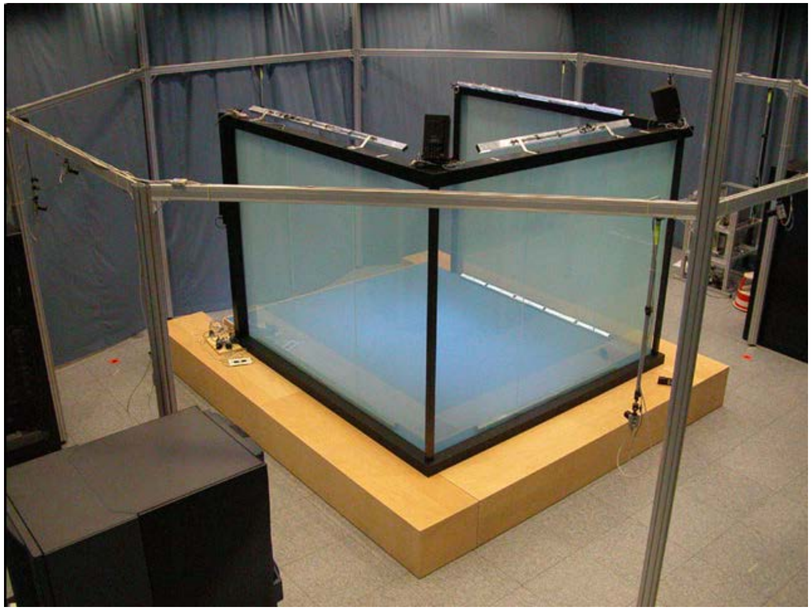
ray-tracing, rasterization with preprocessing for complex lighting

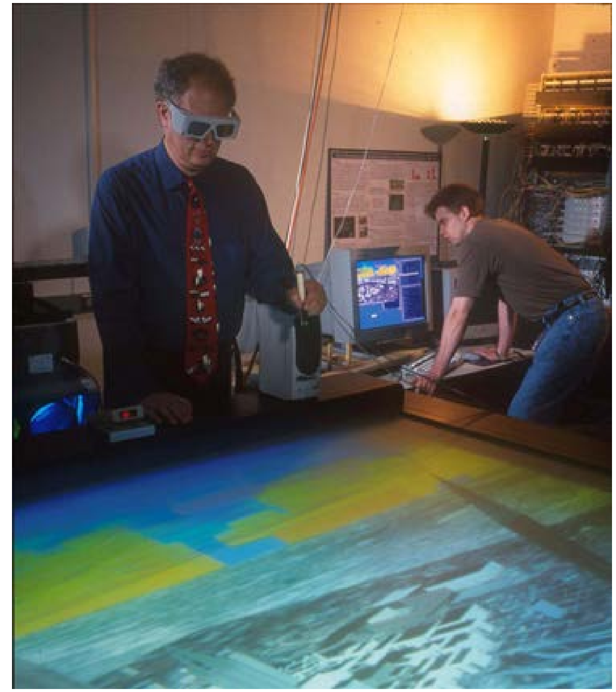




Virtual Reality

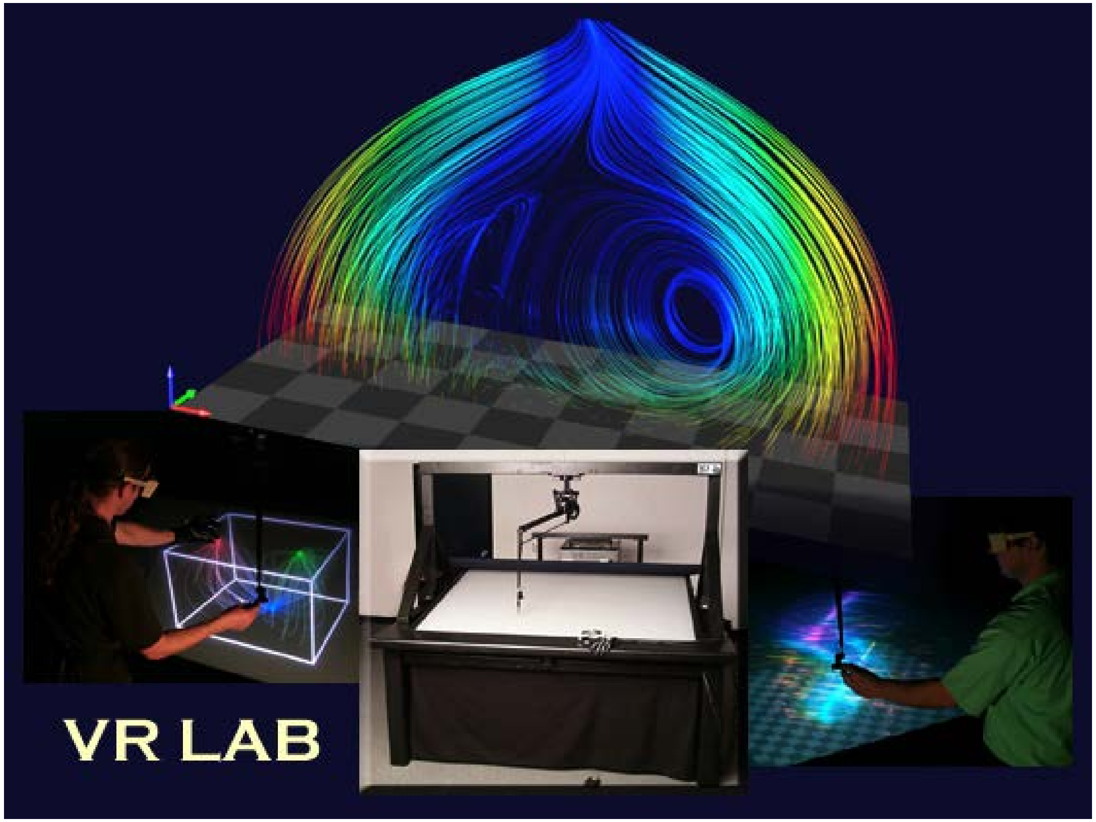
rasterization

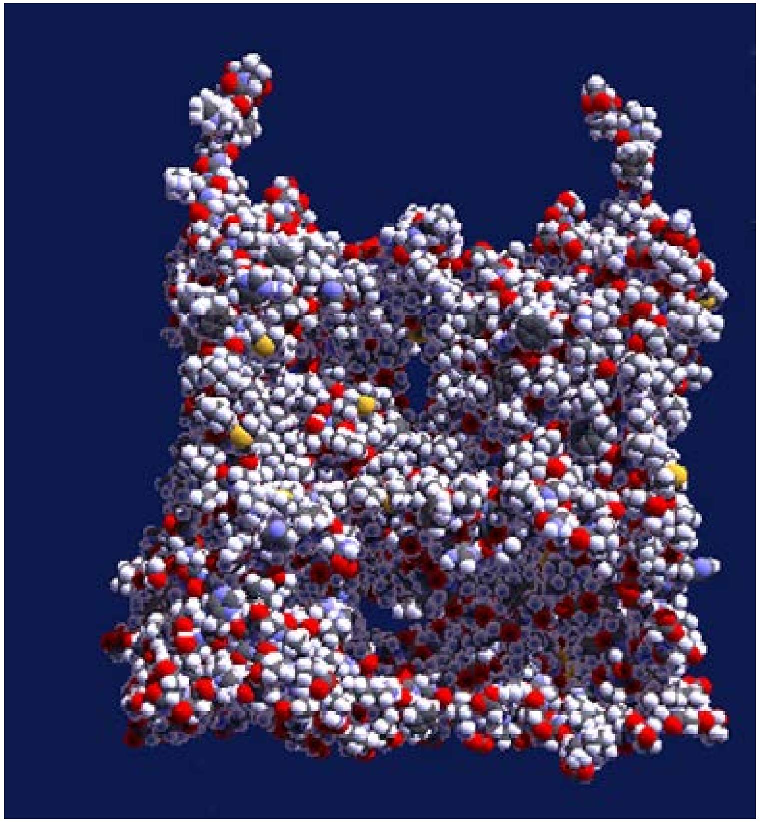




Visualization

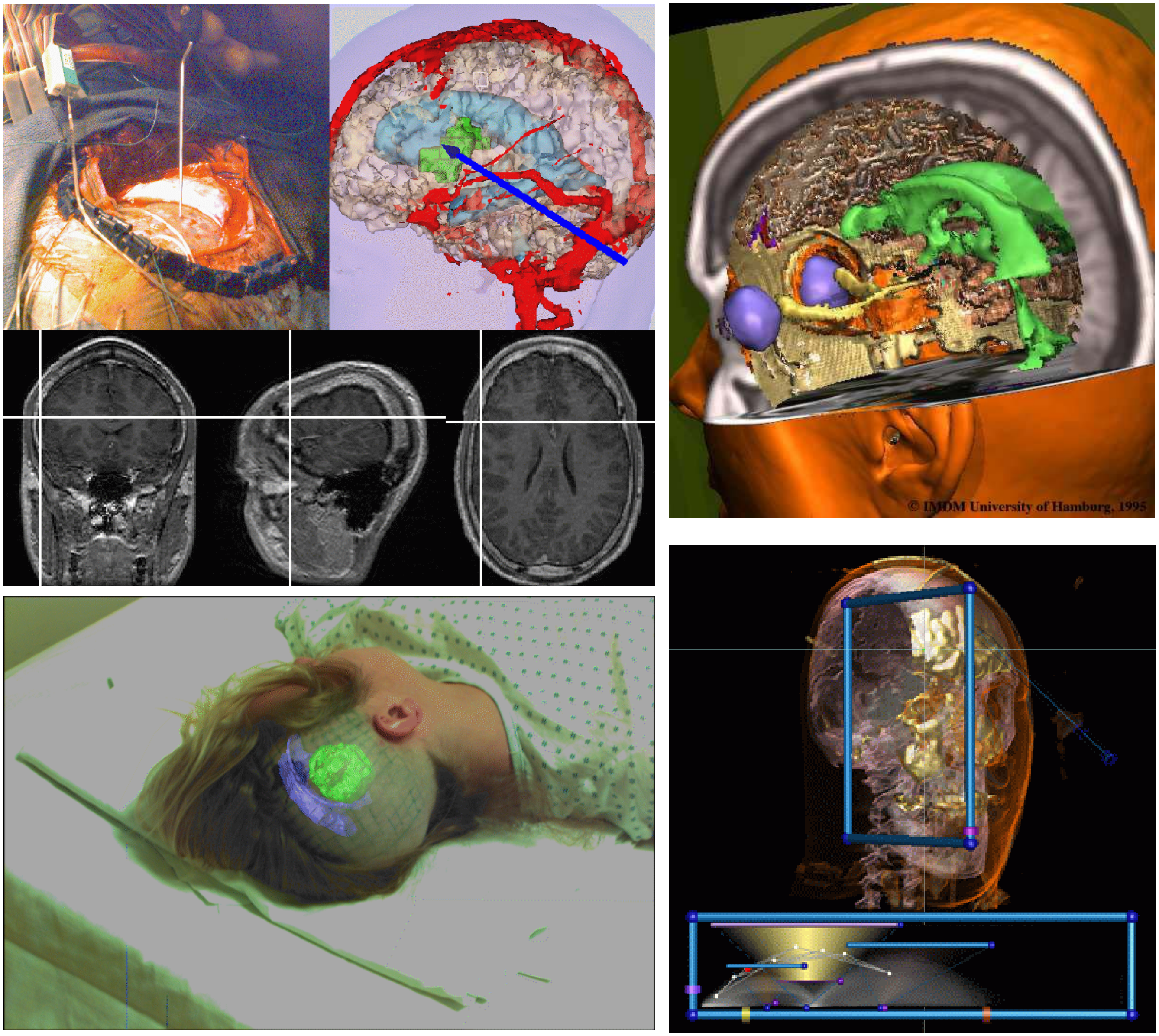
mostly rasterization,
interactive ray-tracing is starting





Medical Imaging

same as visualization



Questions?