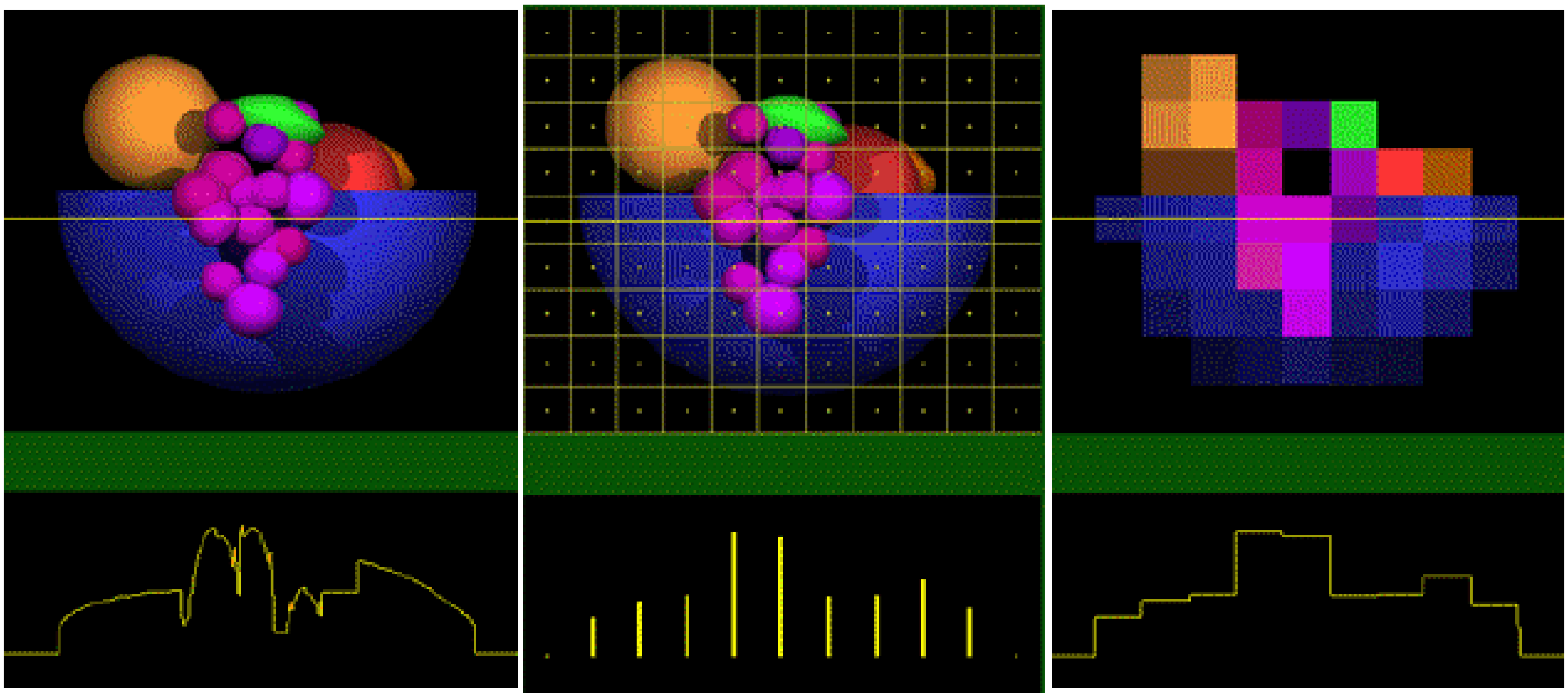
Sampling, Aliasing, & Mipmaps

MIT EECS 6.837 Computer Graphics
Wojciech Matusik, MIT EECS

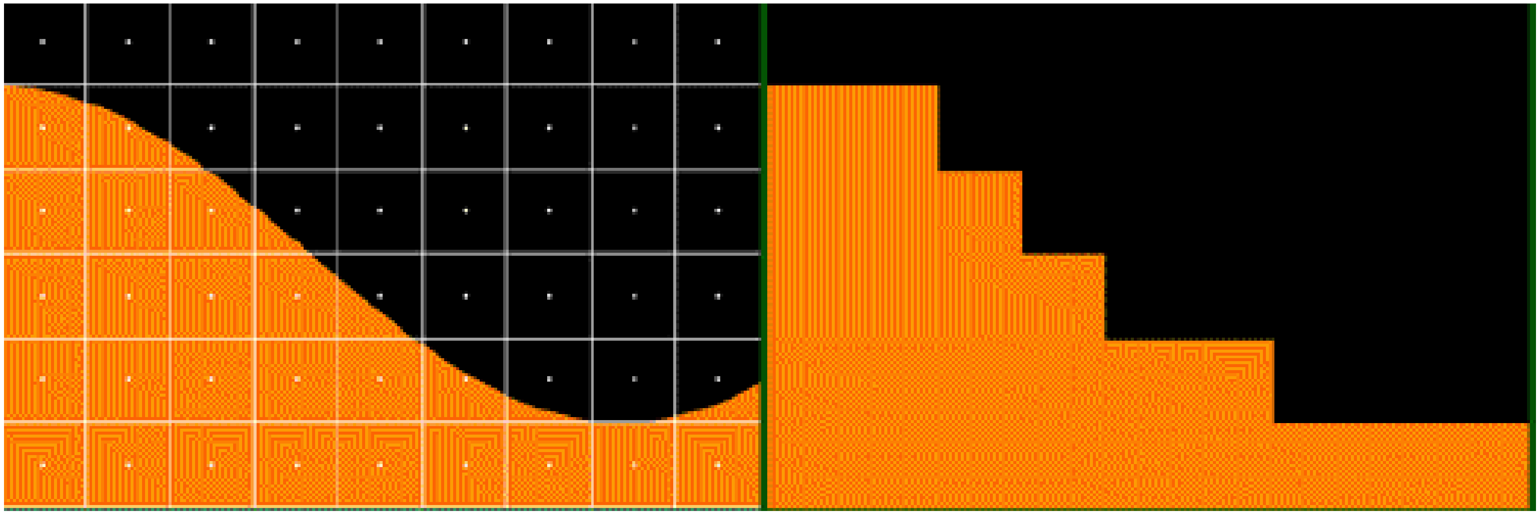
Examples of Aliasing



Original Image Samples Reconstruction

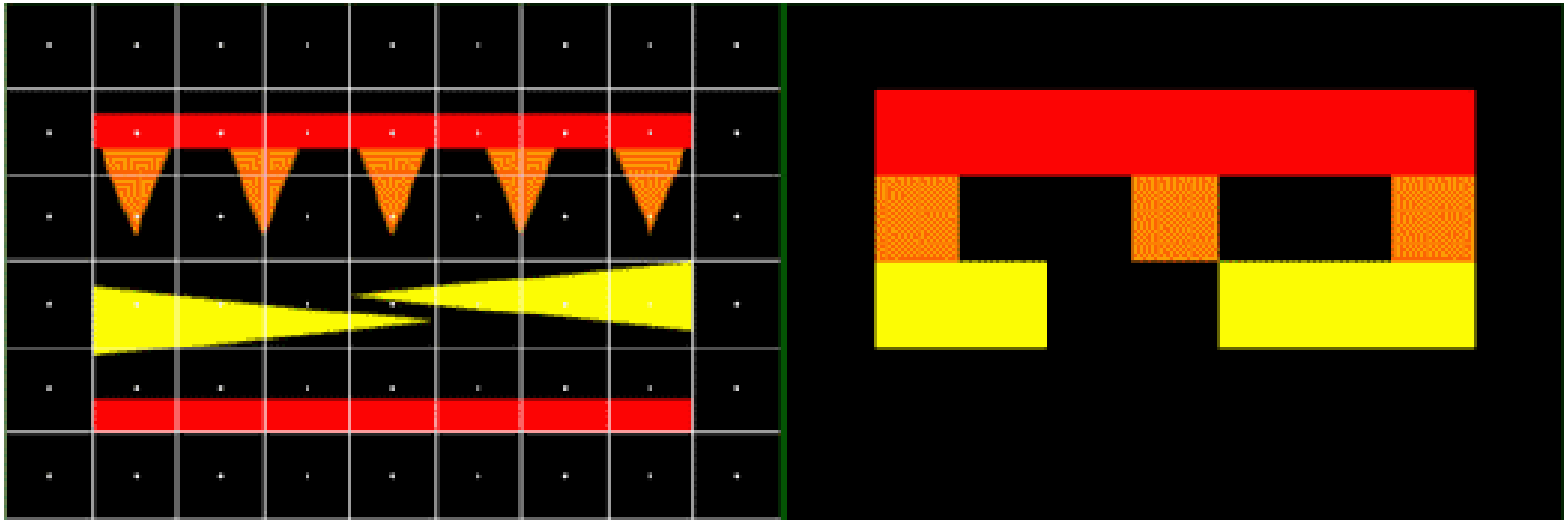
Examples of Aliasing

Jagged boundaries



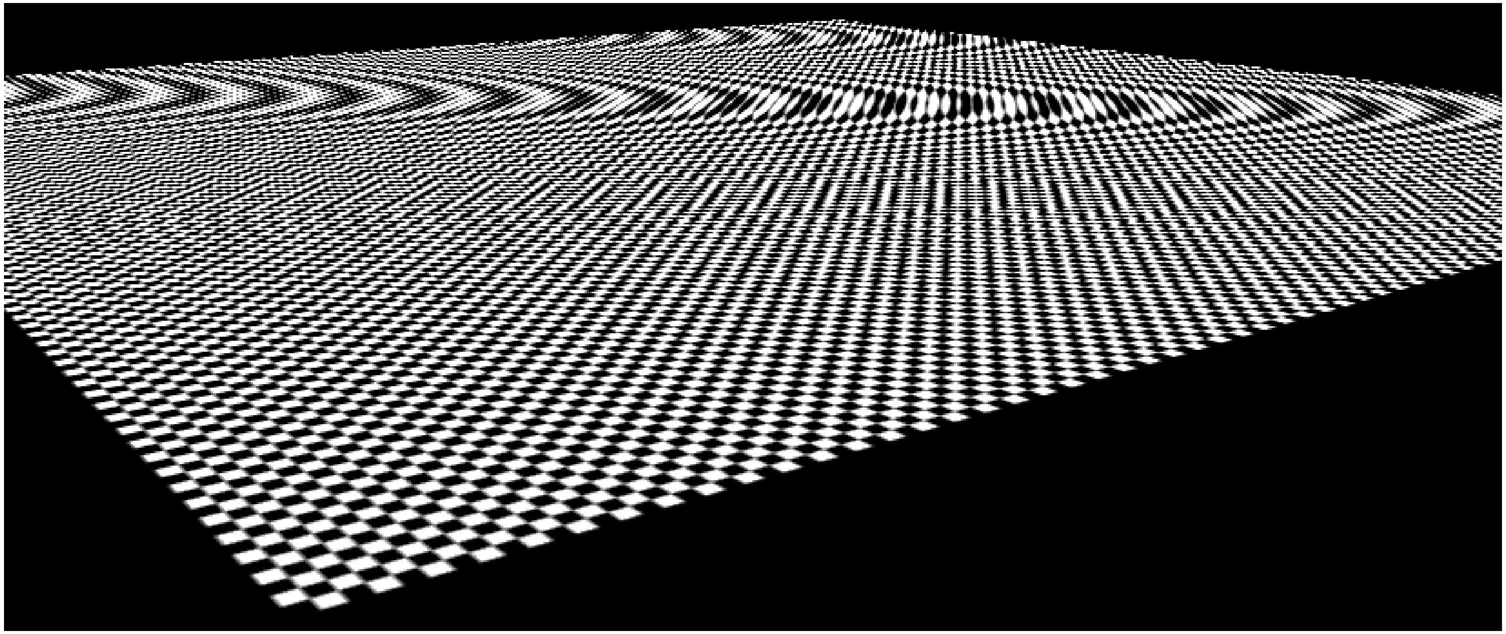
Examples of Aliasing

Improperly rendered detail



Examples of Aliasing

Texture Errors



In photos too

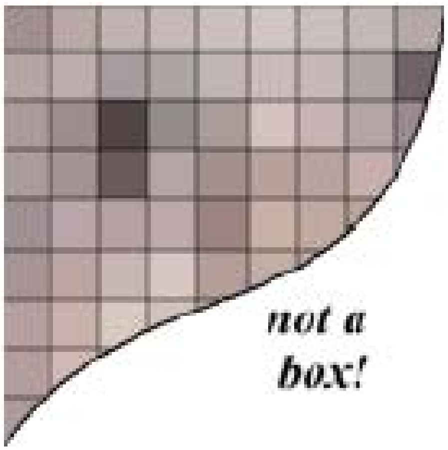
See also http://vimeo.com/26299355

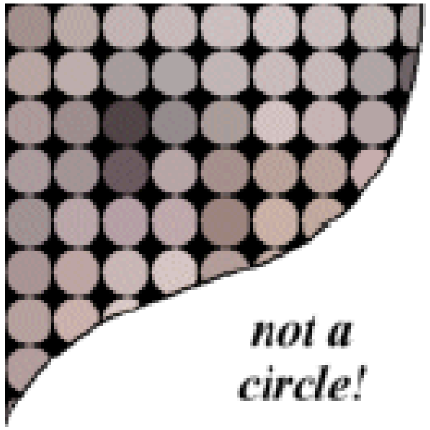
Philosophical perspective

- The physical world is continuous, inside the computer things need to be discrete
- Lots of computer graphics is about translating continuous problems into discrete solutions
  - e.g. ODEs for physically-based animation, global illumination, meshes to represent smooth surfaces, rasterization, antialiasing
- Careful mathematical understanding helps do the right thing

What is a Pixel?

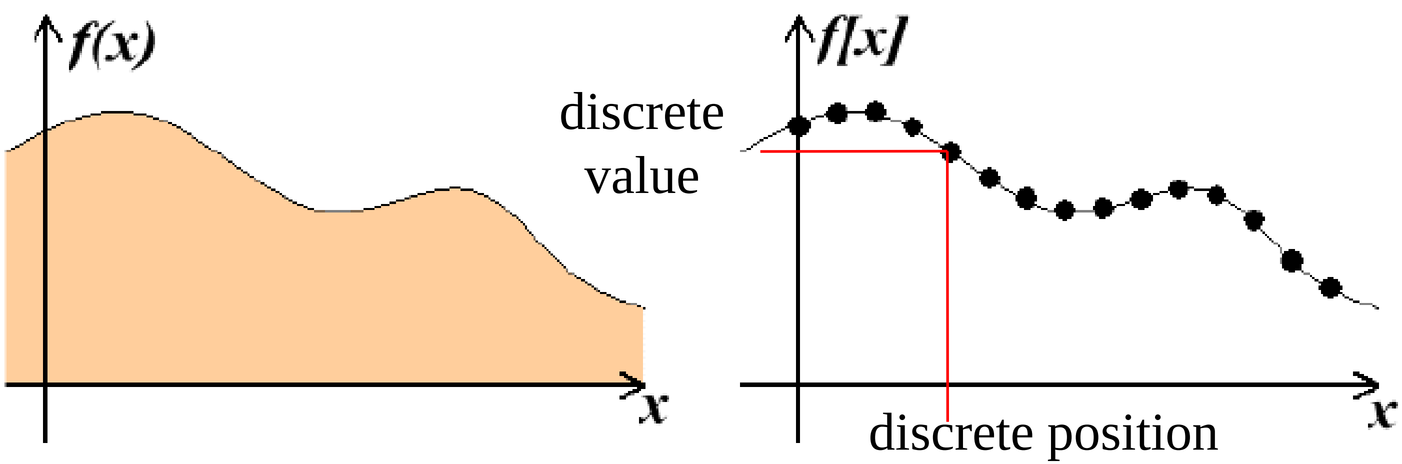
- A pixel is not:
  - a box
  - a disk
  - a teeny tiny little light
- A pixel "looks different" on different display devices
- A pixel is a sample
  - it has no dimension
  - it occupies no area
  - it cannot be seen
  - it has a coordinate
  - it has a value





More on Samples

- In signal processing, the process of mapping a continuous function to a discrete one is called *sampling*
- The process of mapping a continuous variable to a discrete one is called *quantization*
  - Gamma helps quantization
- To represent or render an image using a computer, we must both sample and quantize
  - Today we focus on the effects of sampling and how to fight them



Sampling & reconstruction

The visual array of light is a continuous function

1/ we sample it

- with a digital camera, or with our ray tracer
- This gives us a finite set of numbers,
  not really something we can see
- We are now inside the discrete computer world

2/ we need to get this back to the physical world:
we reconstruct a continuous function

- for example, the point spread of a pixel on a CRT or LCD

- Both steps can create problems
  - pre-aliasing caused by sampling
  - post-aliasing caused by reconstruction
  - We focus on the former

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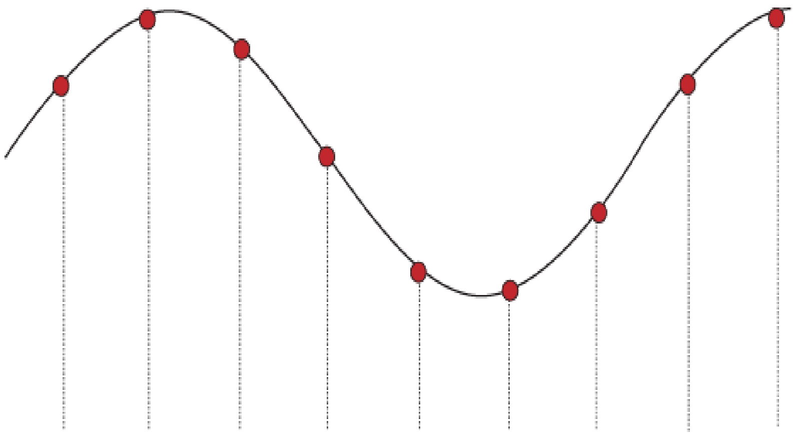
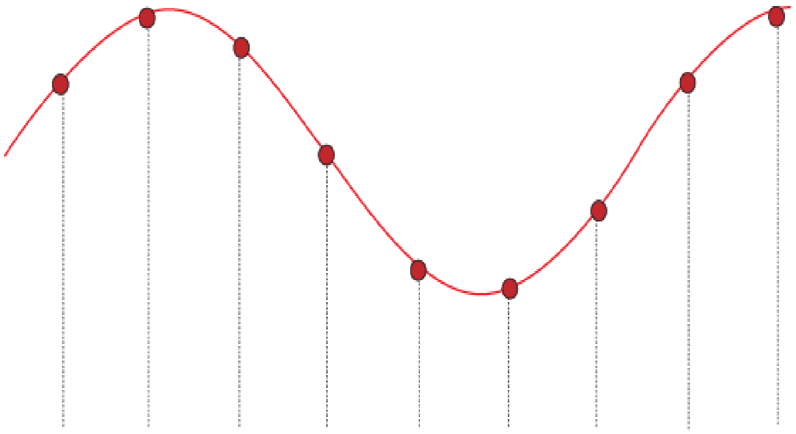
Questions?

Sampling Density

- If we're lucky, sampling density is enough



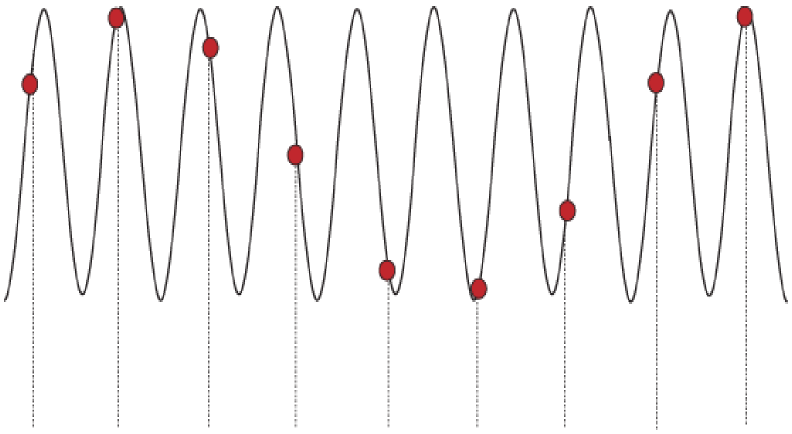
Input

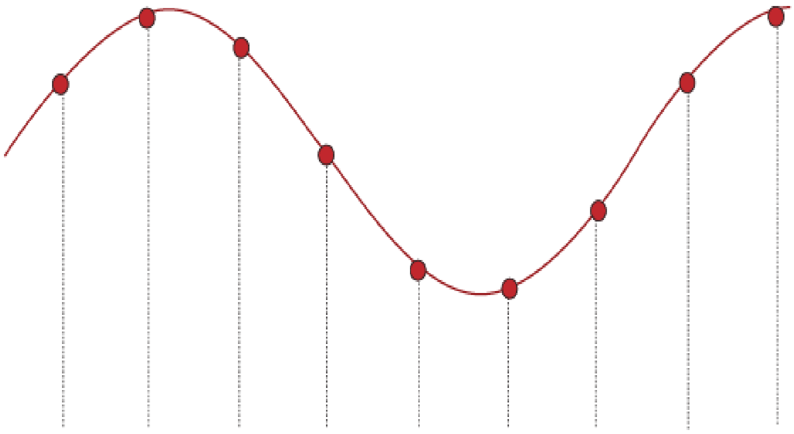


Reconstructed

Sampling Density

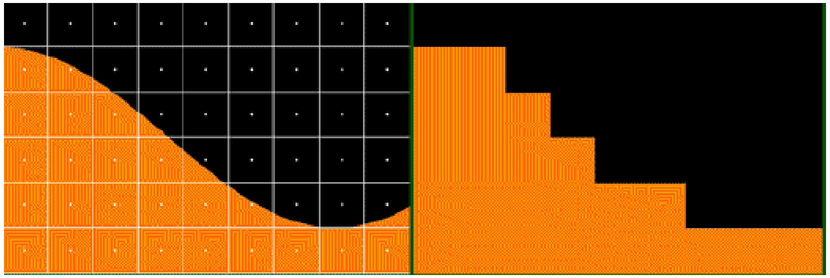
- If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)
- This is why it’s called aliasing: the new low-frequency sine wave is an alias/ghost of the high-frequency one

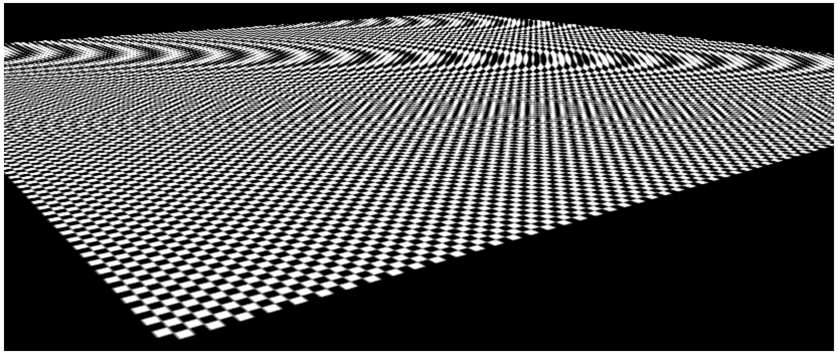




Discussion

- Types of aliasing
  - Edges
    - mostly directional aliasing (vertical and horizontal edges rather than actual slope)
  - Repetitive textures
    - Paradigm of aliasing
    - Harder to solve right
    - Motivates fun mathematics





Solution?

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- We blur!
  - In the case of audio, people first include an analog low-pass filter before sampling
  - For ray tracing/rasterization: compute at higher resolution, blur, resample at lower resolution
  - For textures, we can also blur the texture image before doing the lookup
- To understand what really happens, we need serious math

Solution? Questions?

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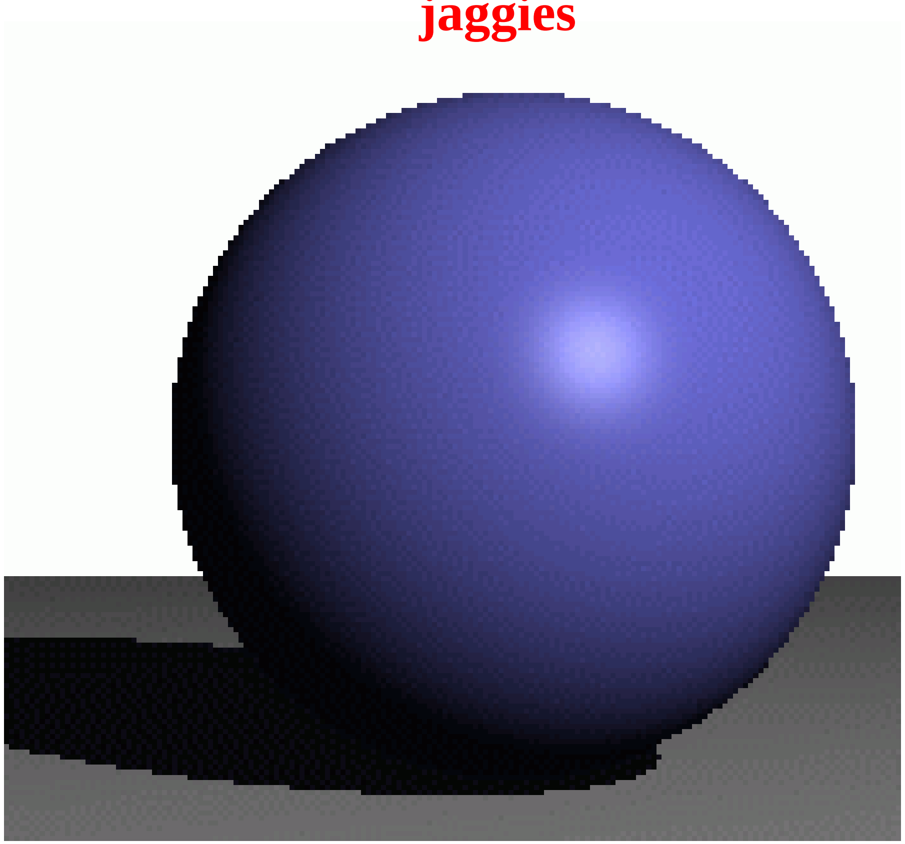
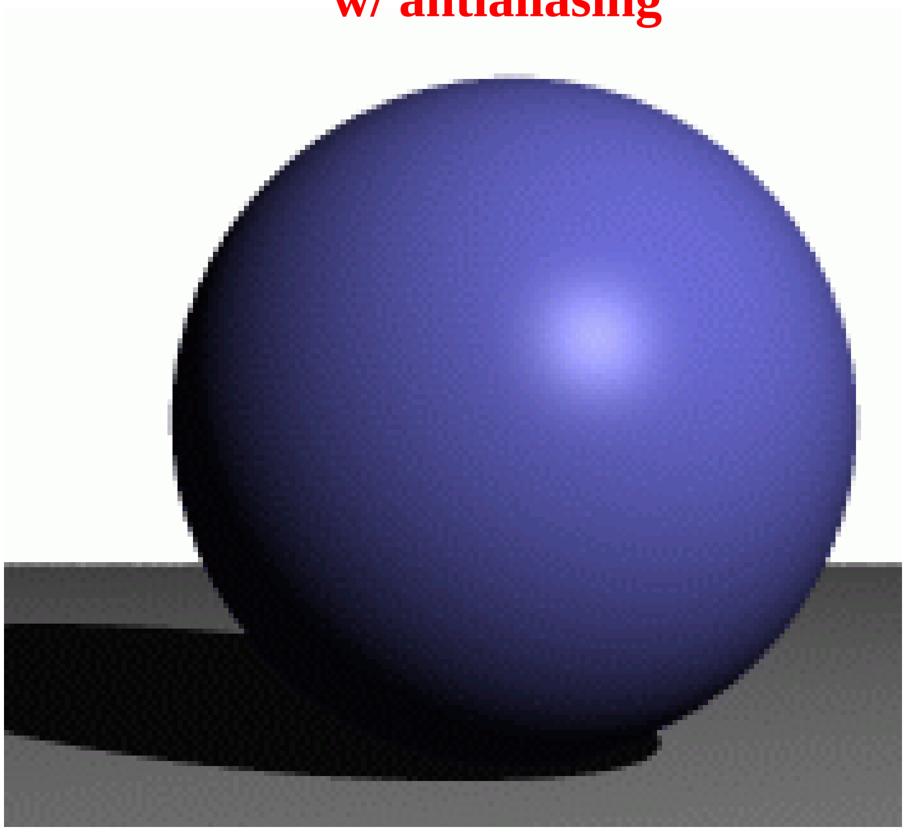
In practice: Supersampling

- Your intuitive solution is to compute multiple color values per pixel and average them

jaggies

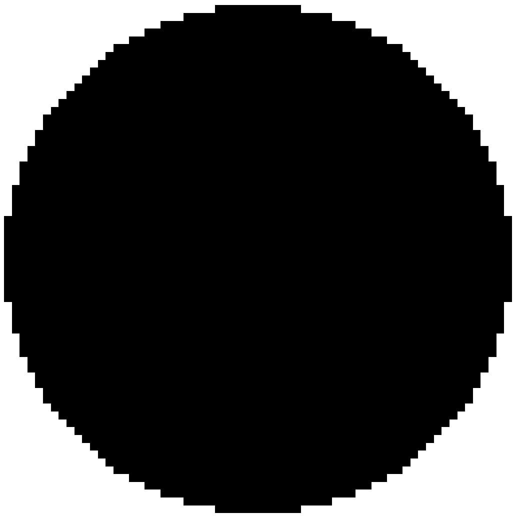


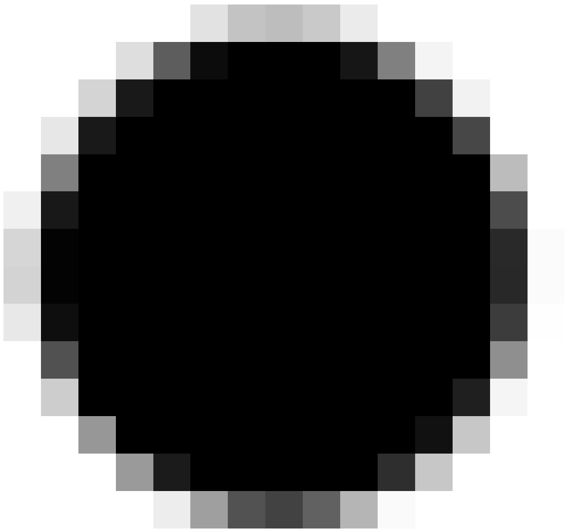
w/ antialiasing



Uniform supersampling

- Compute image at resolution k*width, k*height
- Downsample using low-pass filter
  (e.g. Gaussian, sinc, bicubic)

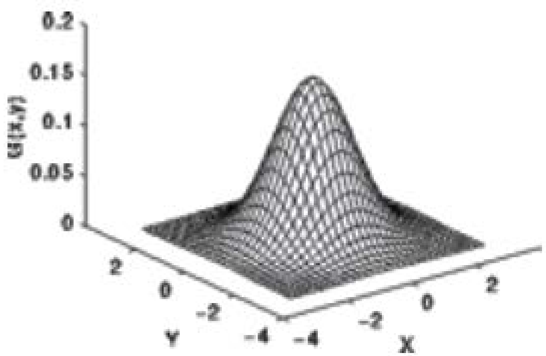




Low pass / convolution

- Each output (low-res) pixel is a weighted average of input subsamples
- Weight depends on relative spatial position
- For example:
  - Gaussian as a function of distance
  - 1 inside a square, zero outside (box)

$$\frac{1}{273}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

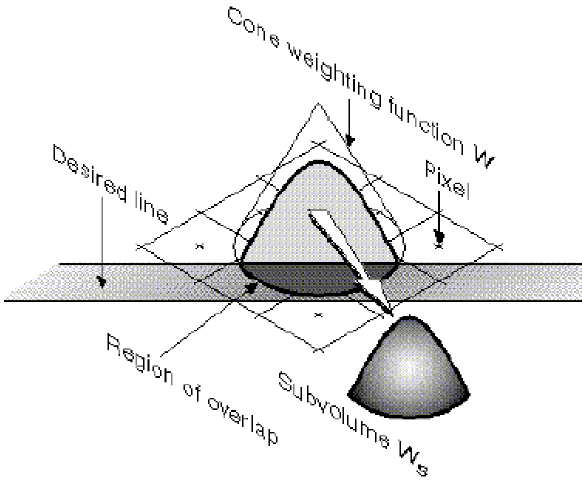


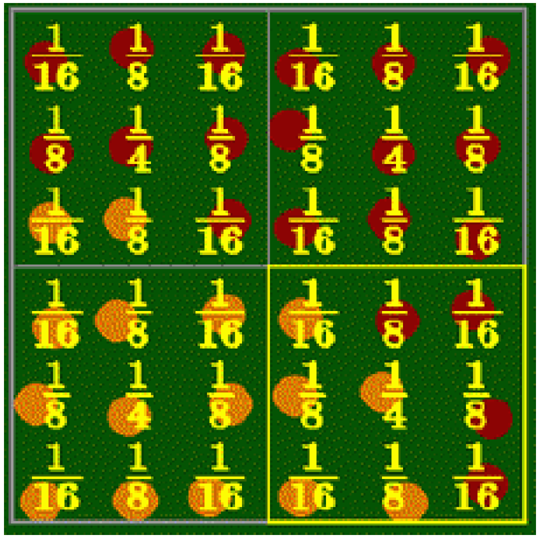
Gaussian

http://homepages.inf.ed.ac.uk/rbf/HIPR2/gsmooth.htm

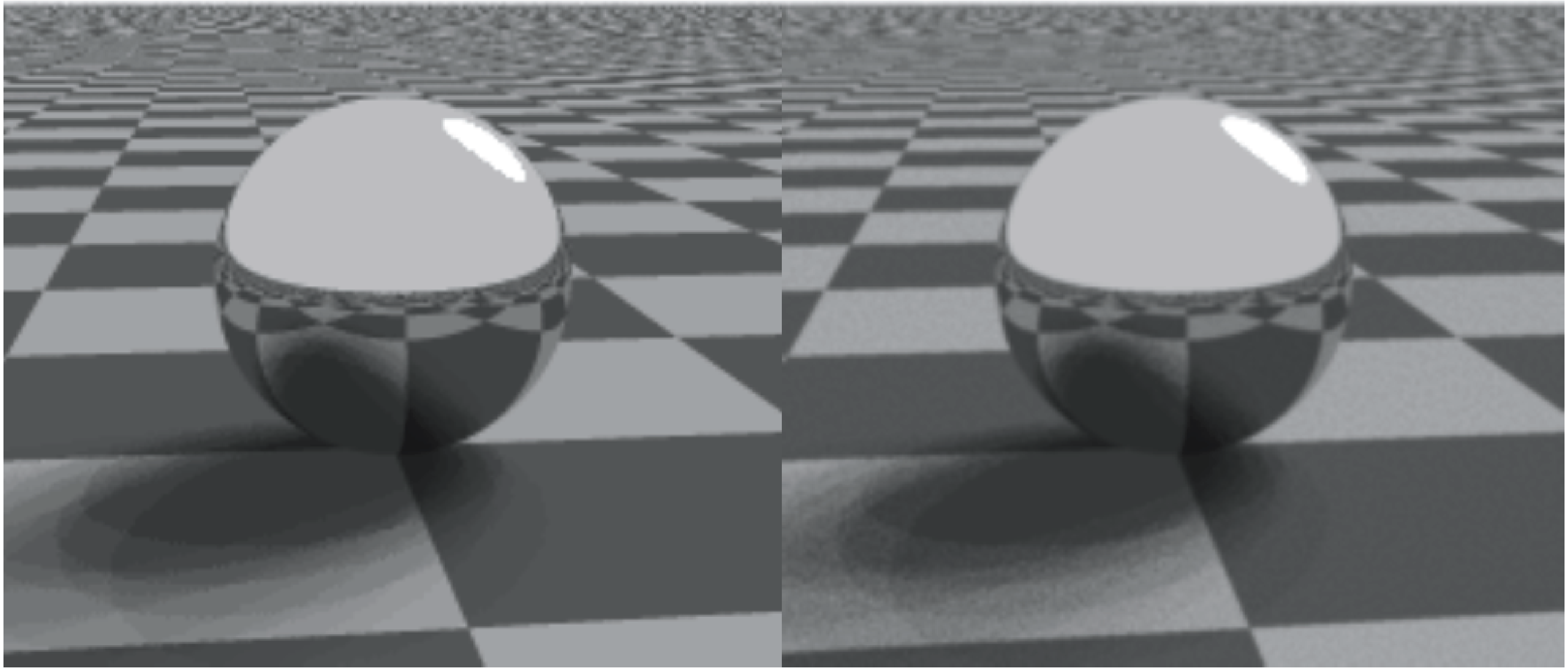
In practice: Supersampling

- Your intuitive solution is to compute multiple color values per pixel and average them
- A better interpretation of the same idea is that
  - You first create a higher resolution image
  - You blur it (low pass, prefilter)
  - You resample it at a lower resolution





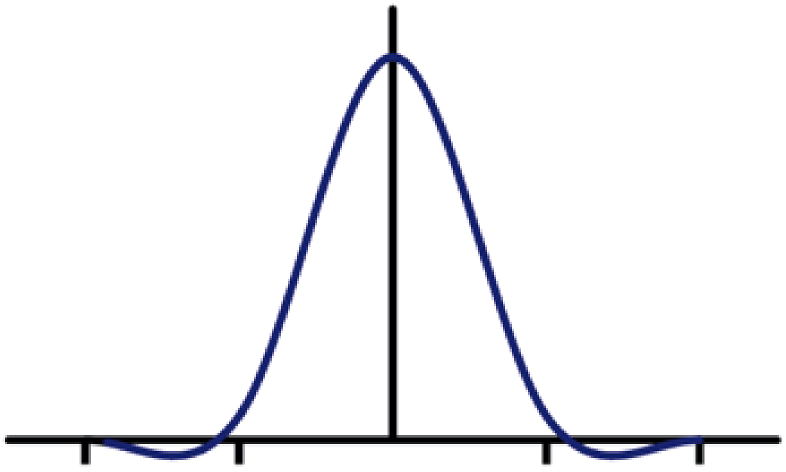
Box Gaussian



http://www.cs.unc.edu/~narain/courses/comp870/a1-sampling/

Recommended filter

- Bicubic
  - http://www.mentallandscape.com/Papers_siggraph88.pdf
- Good tradeoff between sharpness and aliasing



http://de.wikipedia.org/wiki/Datei:Mitchell_Filter.svg

Piecewise-cubic

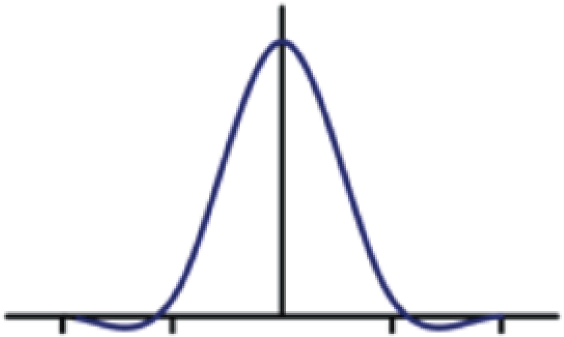
- General formula

$$k(x) = \begin{cases} P|x|^3 + Q|x|^2 + R|x| + S & \text{if } |x| < 1 \\ T|x|^3 + U|x|^2 + V|x| + W & \text{if } 1 \le |x| < 2 \\ 0 & \textit{otherwise} \end{cases}$$

where P, Q, R, S, T, U, V, W are parameters

- But we want the derivatives to be zero at the boundary and constant signals to be well reconstructed. Reduces to 2 parameters

$$k(x) = \frac{1}{6}\begin{cases} (12-9B-6C)|x|^3 + & \text{if } |x| < 1 \\ \quad (-18+12B+6C)|x|^2 + (6-2B) & \\ (-B-6C)|x|^3 + (6B+30C)|x|^2 + & \text{if } 1 \le |x| < 2 \\ \quad (-12B-48C)|x| + (8B+24C) & \\ 0 & \textit{otherwise} \end{cases}$$

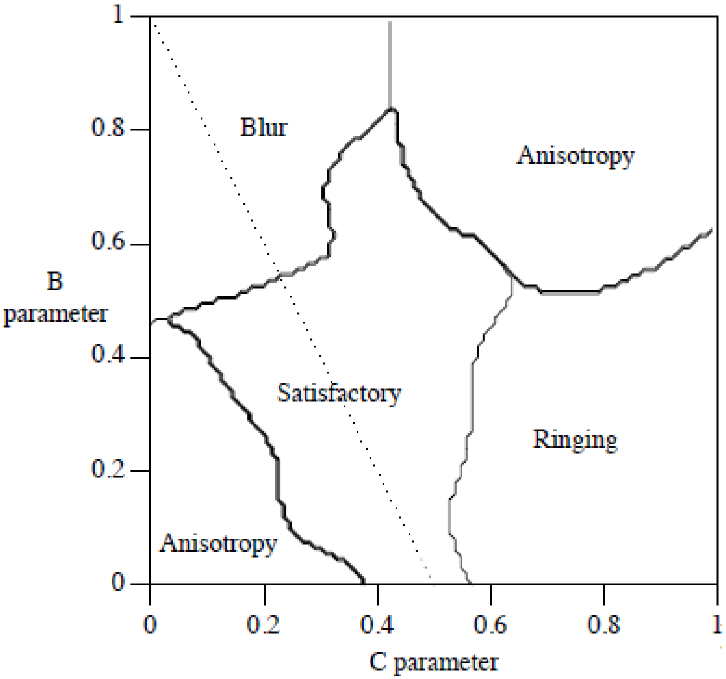


Choosing the parameters

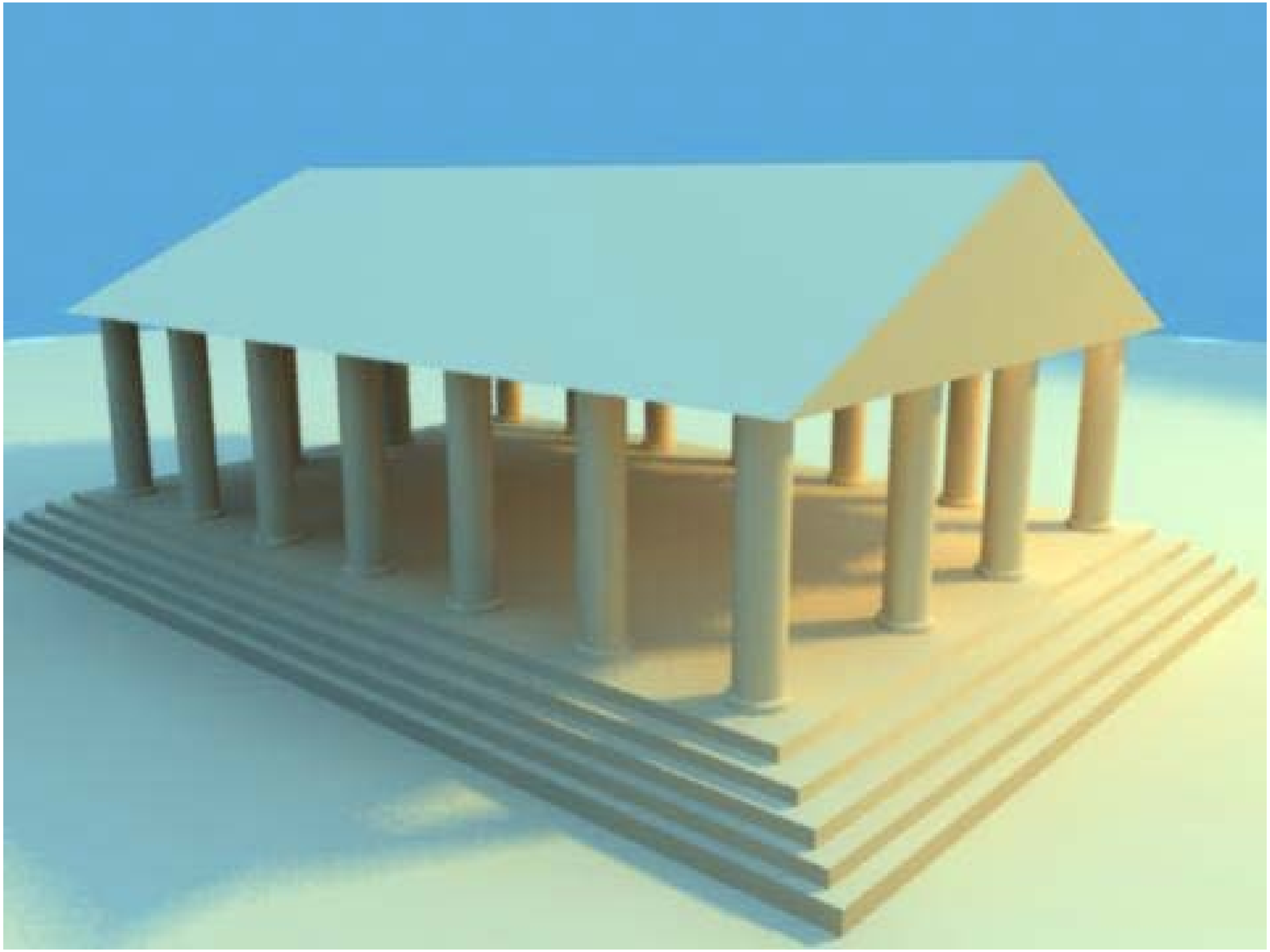
- Empirical tests determined usable parameters
  - Mitchell, Don and Arun Netravali, "Reconstruction Filters in Computer Graphics", SIGGRAPH 88.

    http://www.mentallandscape.com/Papers_siggraph88.pdf

    http://dl.acm.org/citation.cfm?id=378514

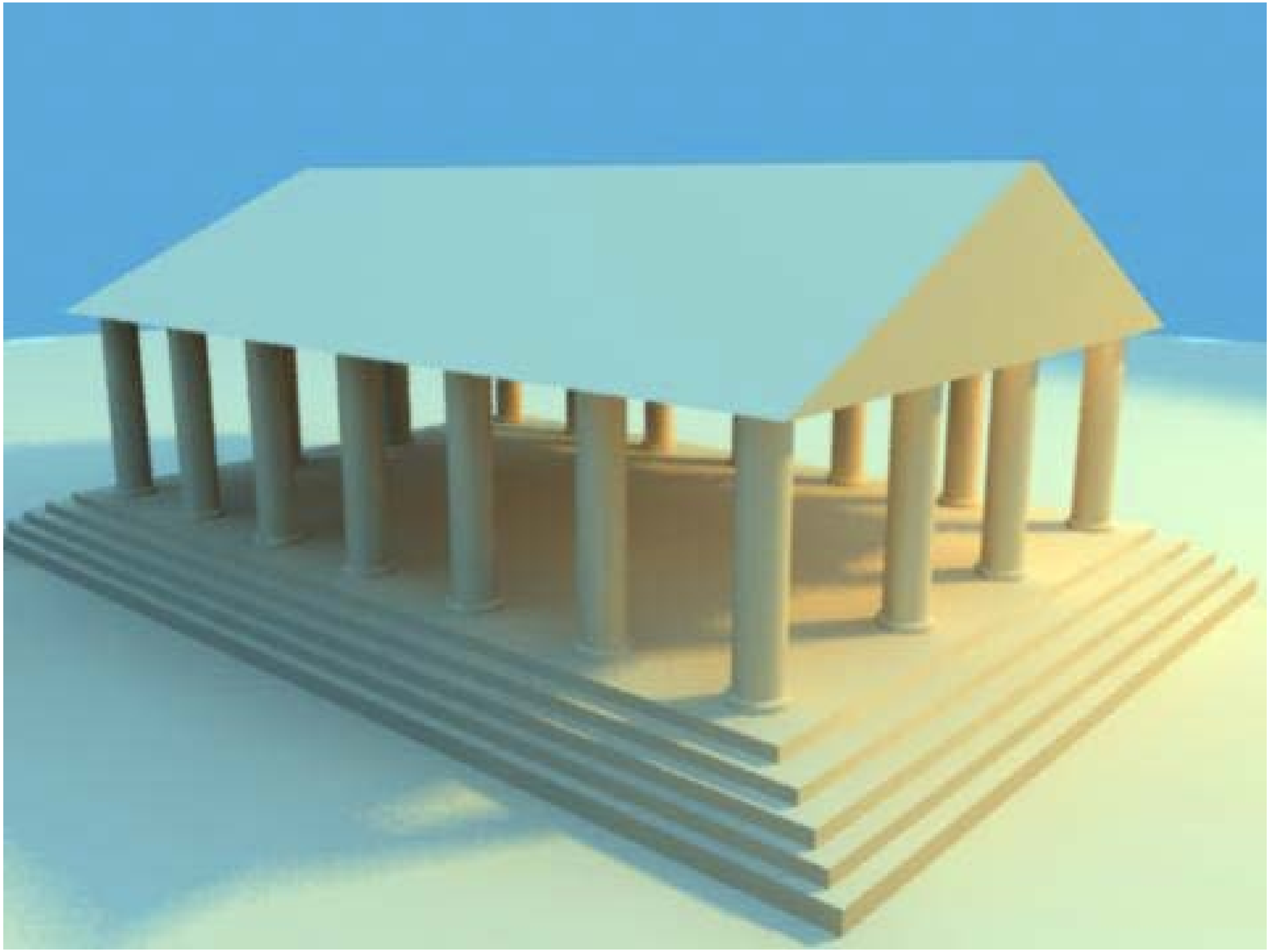


Box



http://www.willsmith.org/maya_gi_tutorial_1/Wills_Maya_GI_Tweaking_Guide_1.html

Gauss



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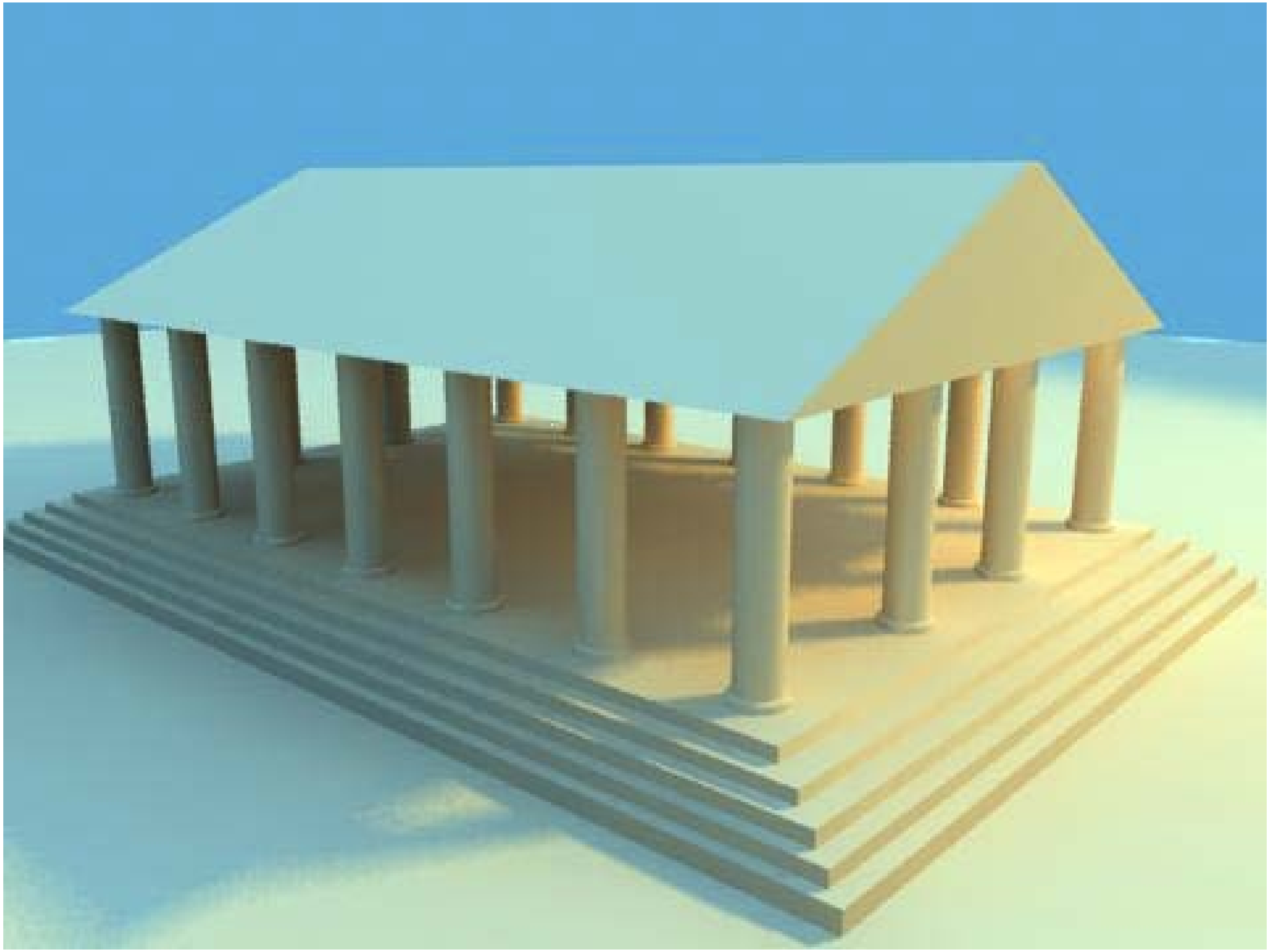
Mitchell bicubic



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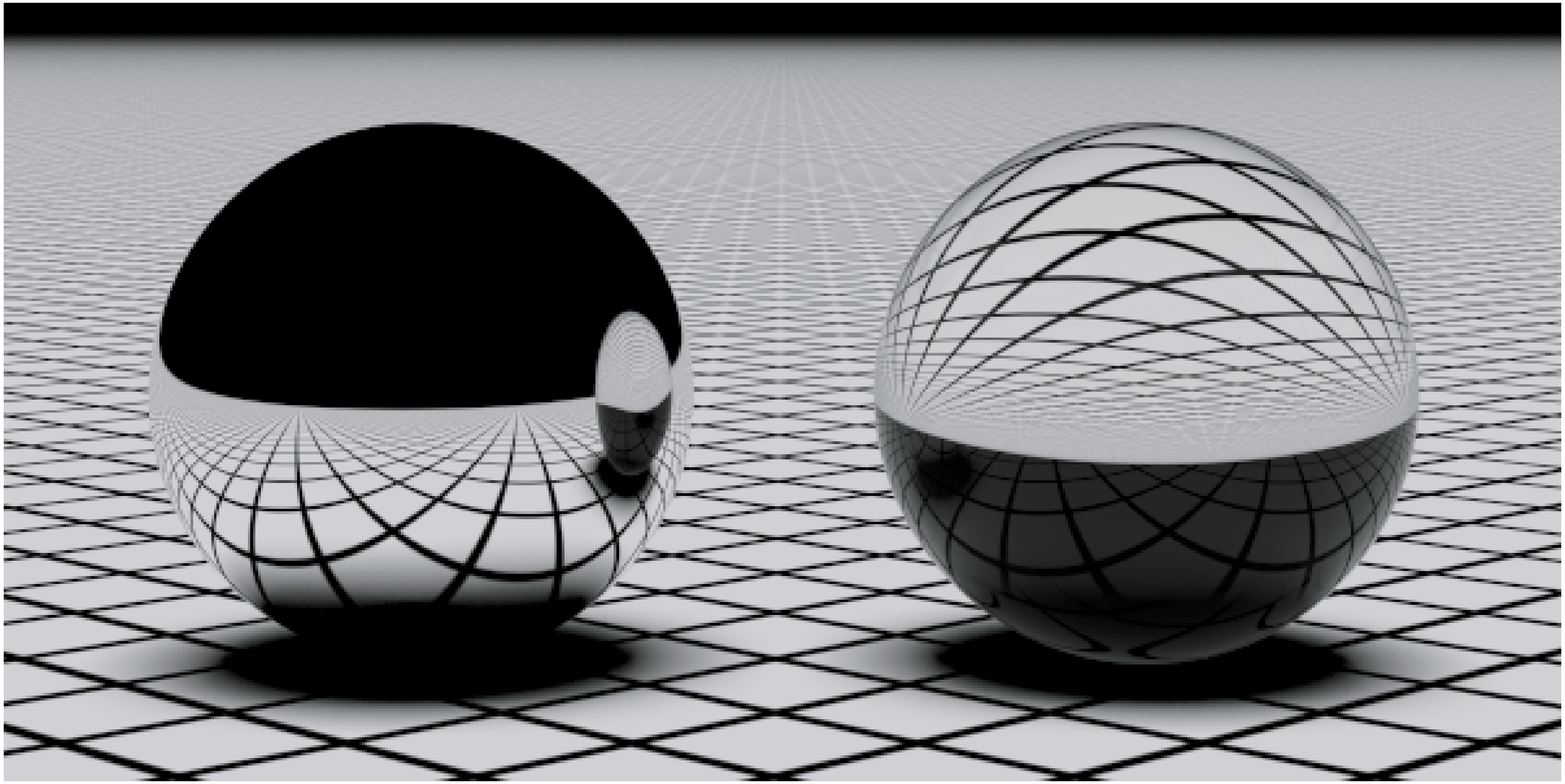
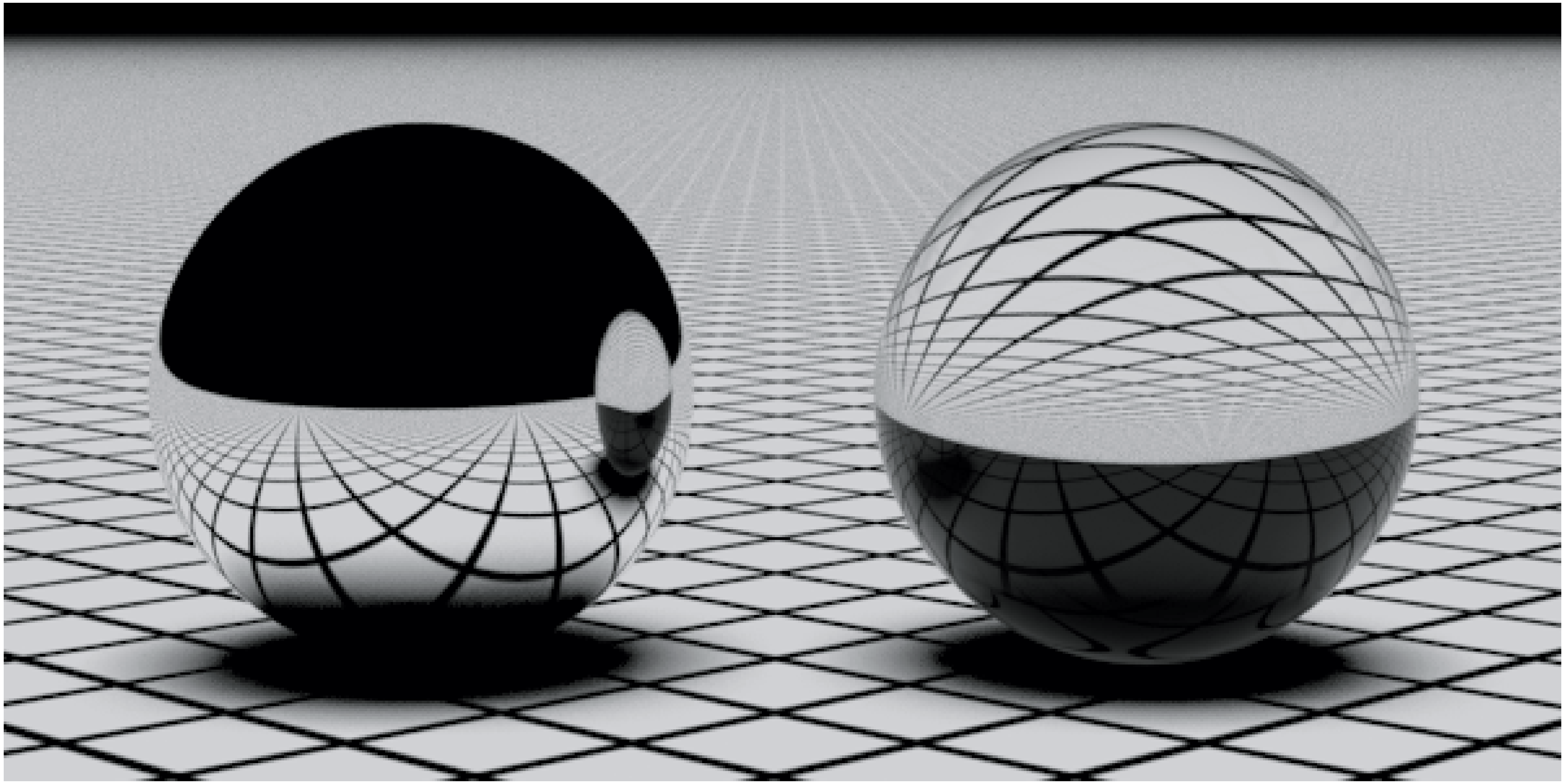
Box



http://rise.sourceforge.net/cgi-bin/makepage.cgi?Filtering

Mitchell-Netravali cubic (1/3. 1/3)



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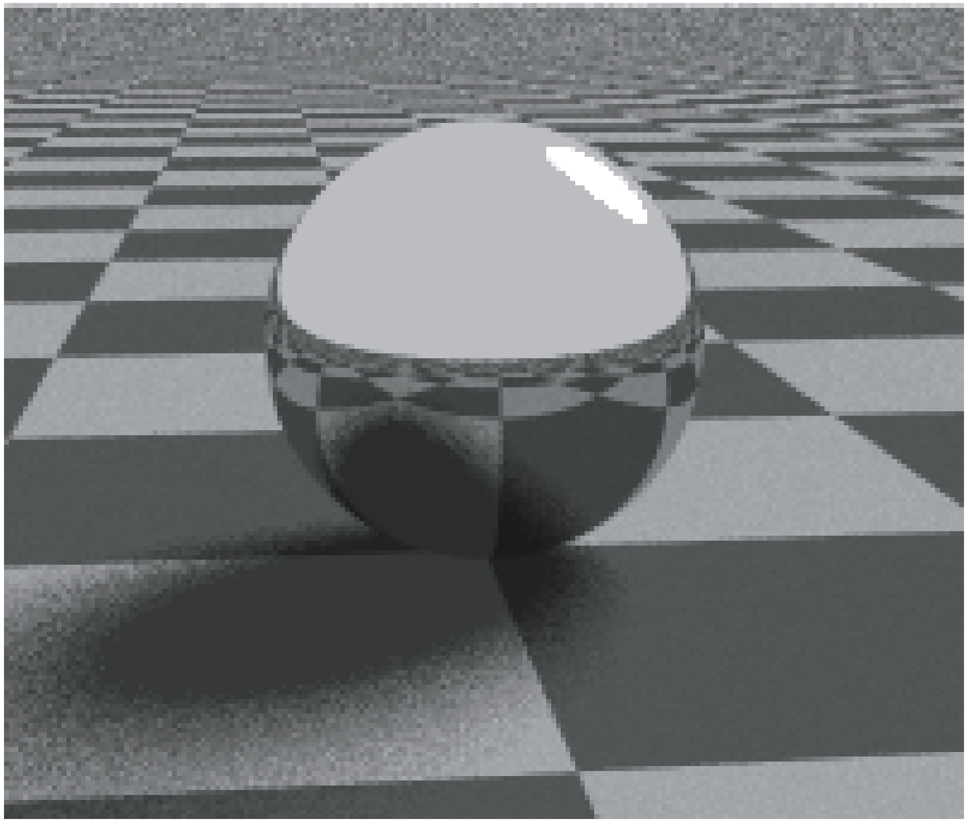

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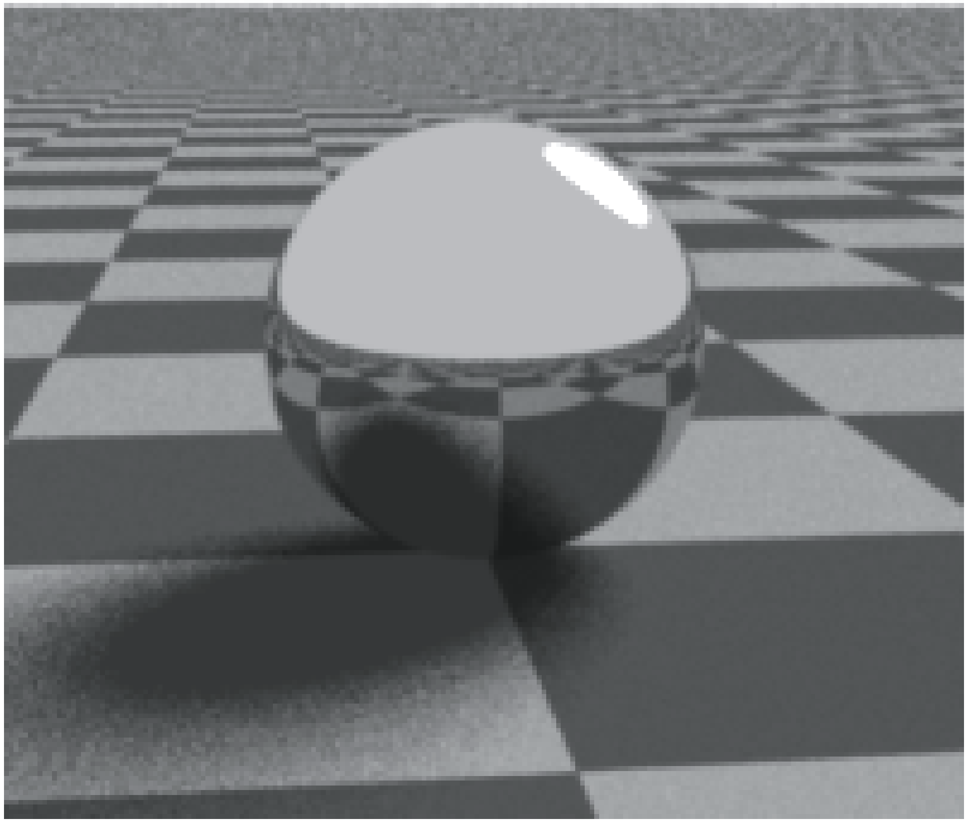

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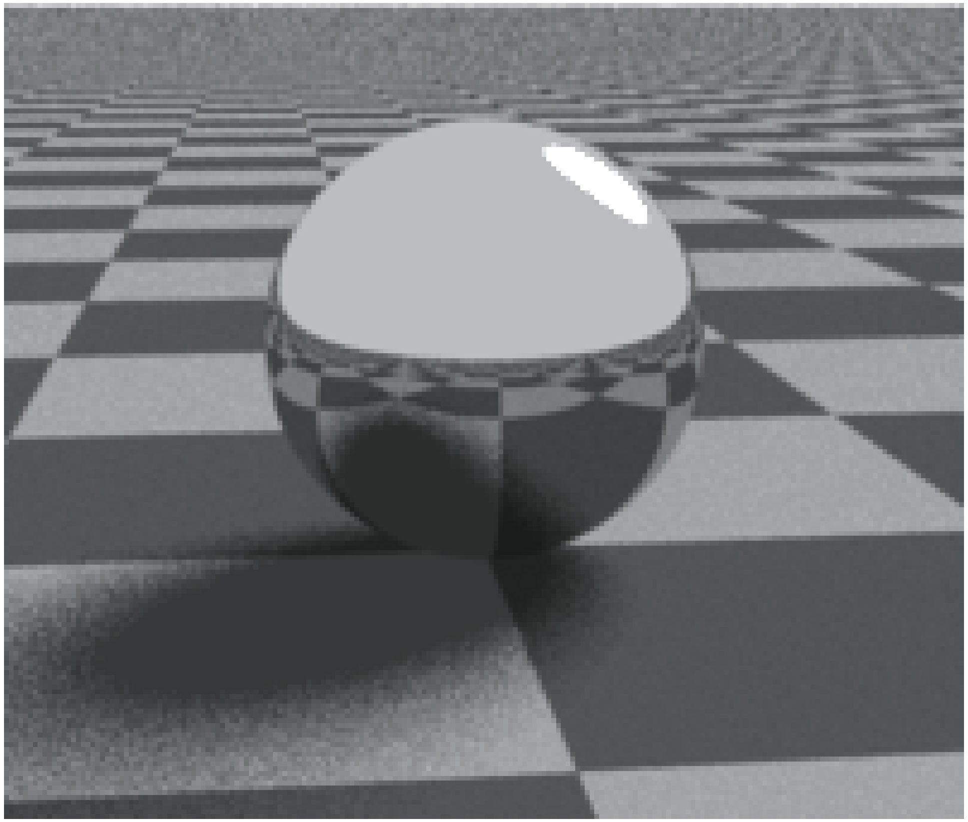

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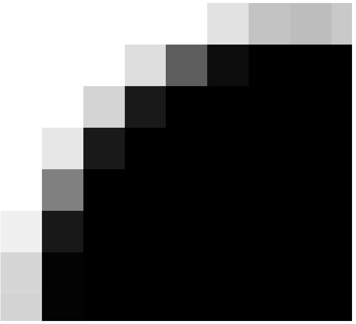
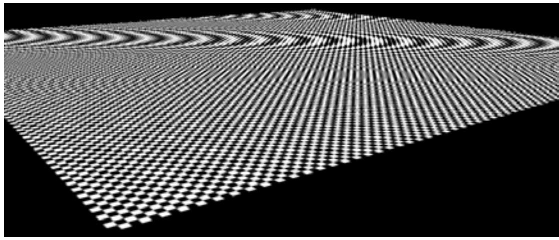
Uniform supersampling

- Advantage:
  - The first (super)sampling captures more high frequencies that are not aliased
  - Downsampling can use a good filter
- Issues
  - Frequencies above the (super)sampling limit are still aliased
- Works well for edges, since spectrum replication is less an issue
- Not as well for repetitive textures
  - But solution soon





Uniform supersampling Questions?

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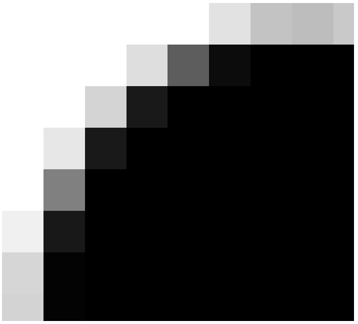
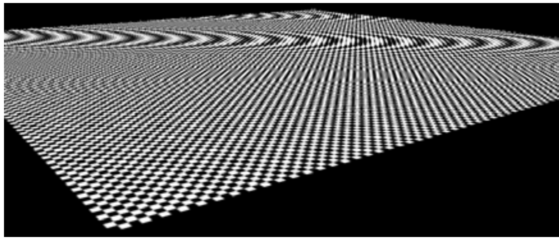
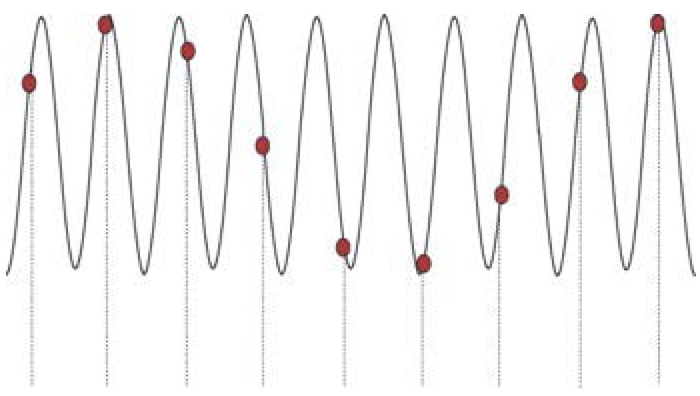




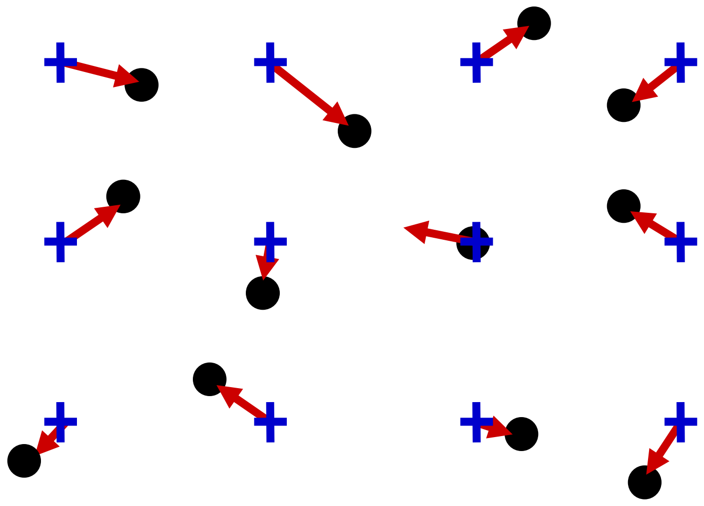
Uniform supersampling

- Problem: supersampling only pushes the problem further: The signal is still not bandlimited
- Aliasing happens
- Especially if the signal and the sampling are regular



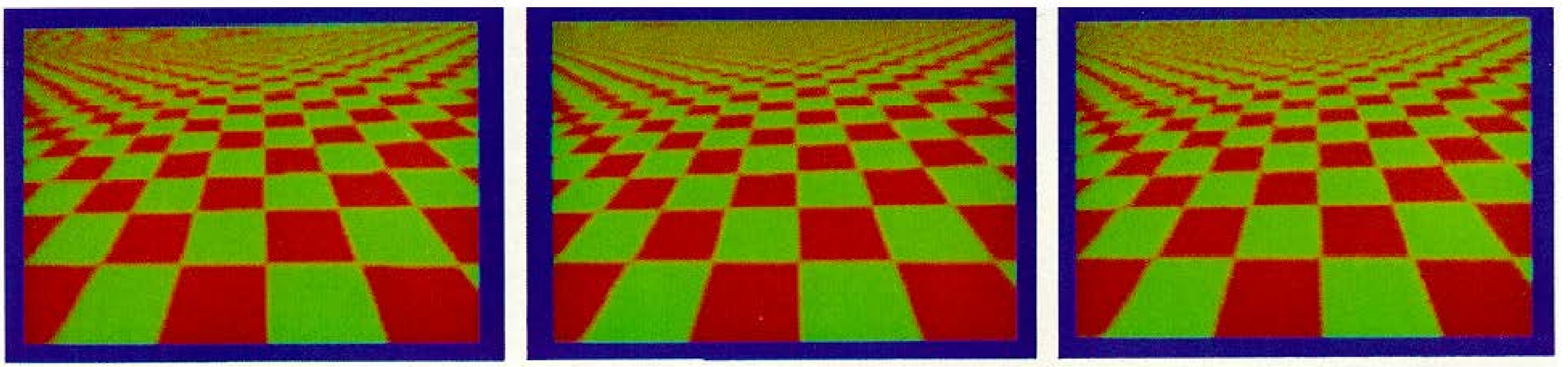
Jittering

- Uniform sample + random perturbation
- Sampling is now non-uniform
- Signal processing gets more complex
- In practice, adds noise to image
- But noise is better than aliasing Moiré patterns



Jittered supersampling

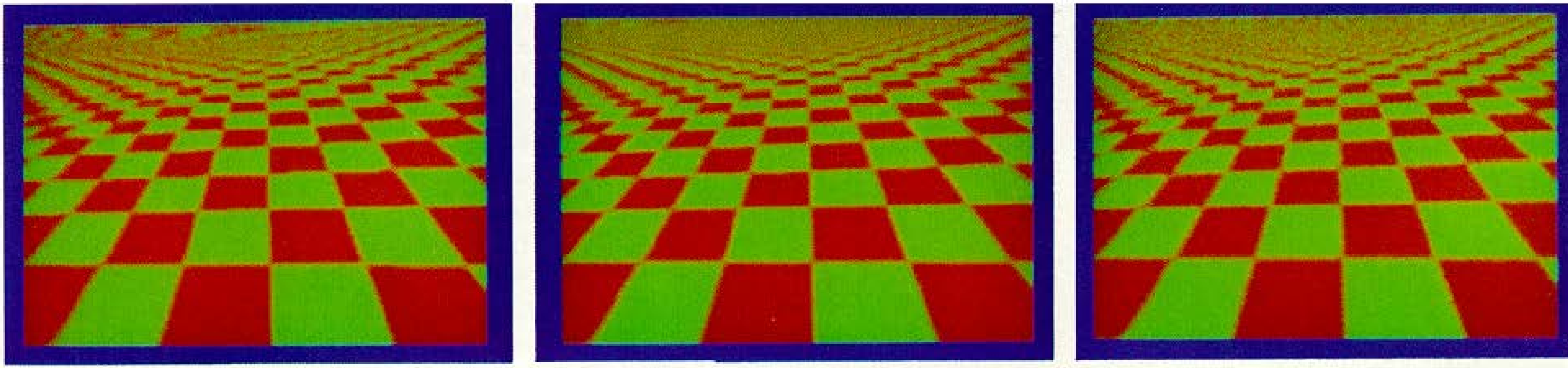
1 sample / pixel



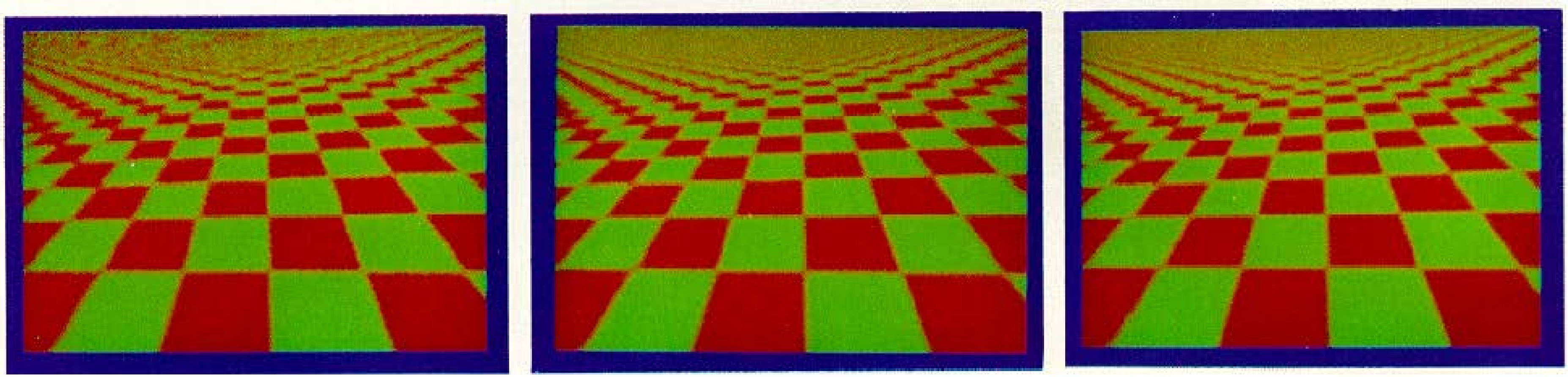
0 jittering   jittering by 0.5   jittering by 1

Jittered supersampling

1 sample / pixel



2 sample / pixel



0 jittering jittering by 0.5 jittering by 1

Jittering

- Displaced by a vector a fraction of the size of the subpixel distance
- Low-frequency Moire (aliasing) pattern replaced by noise
- Extremely effective
- Patented by Pixar!
- When jittering amount is 1, equivalent to stratified sampling (cf. later)

Recap: image antialiasing

- Render multiple samples per pixel
- Jitter the sample locations
- Use appropriate filter to reconstruct final image
  - Bicubic for example

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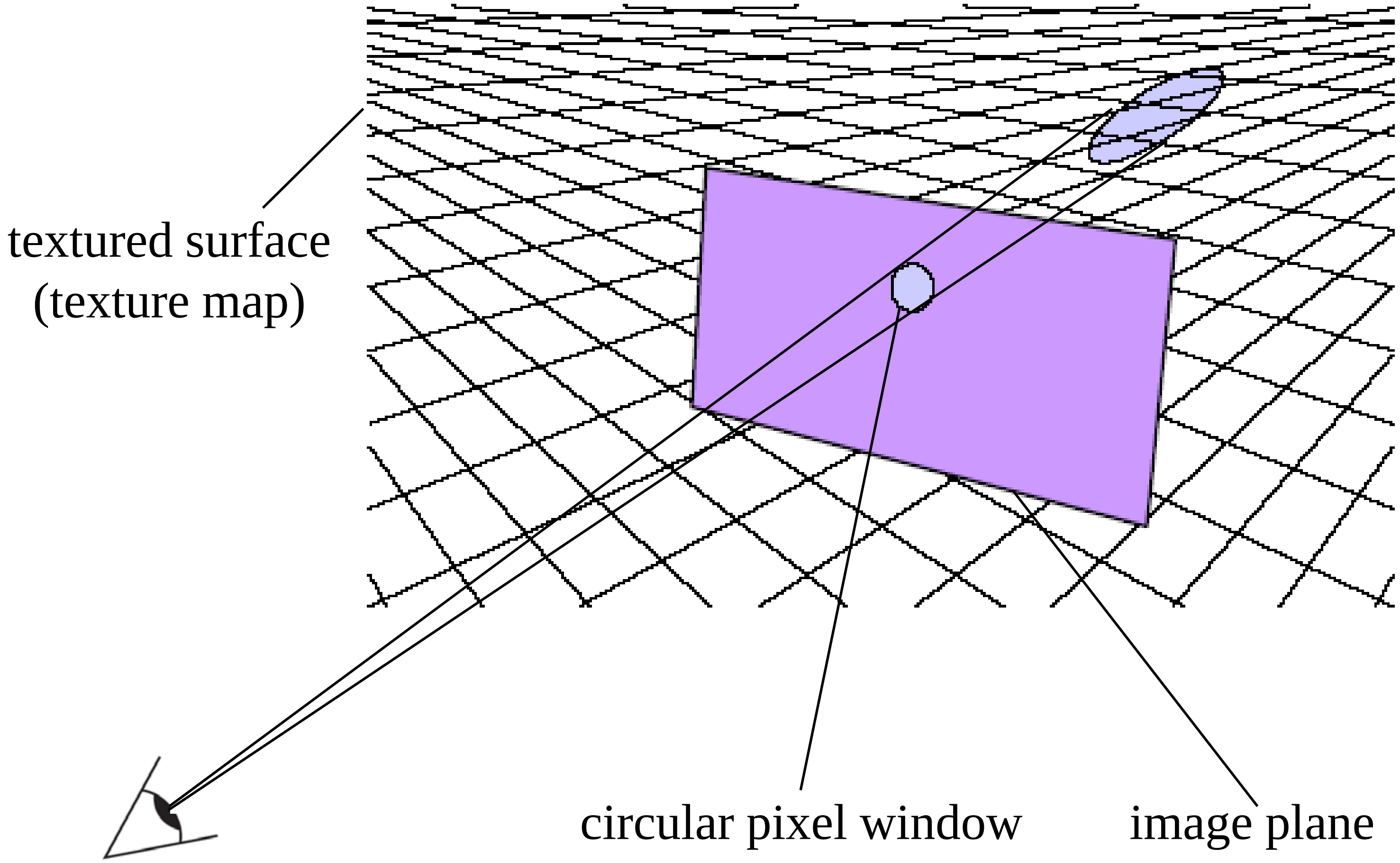
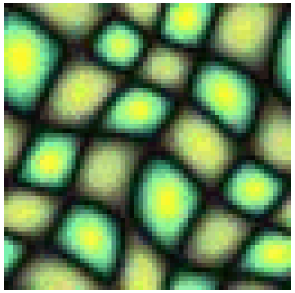
Sampling Texture Maps



- How to map the texture area seen through the pixel window to a single pixel value?

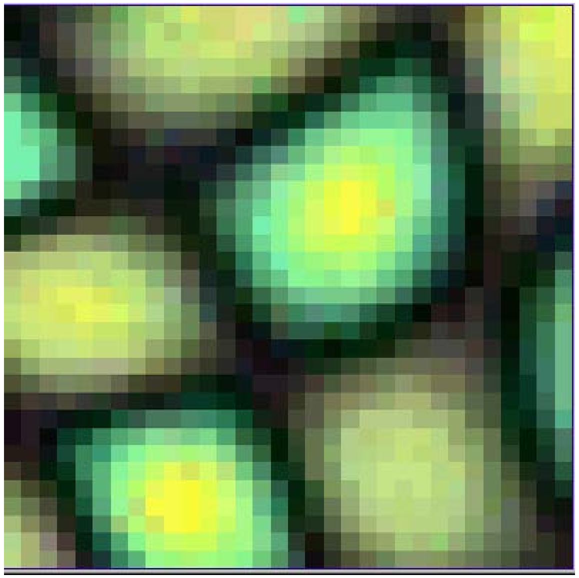
Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.

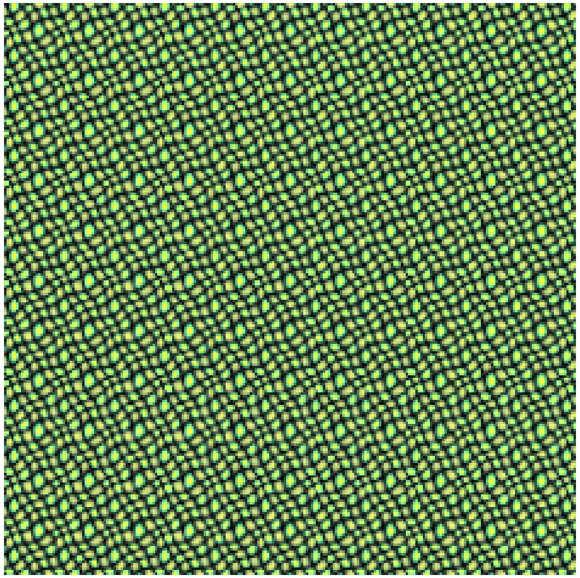


64x64 pixels

Original Texture



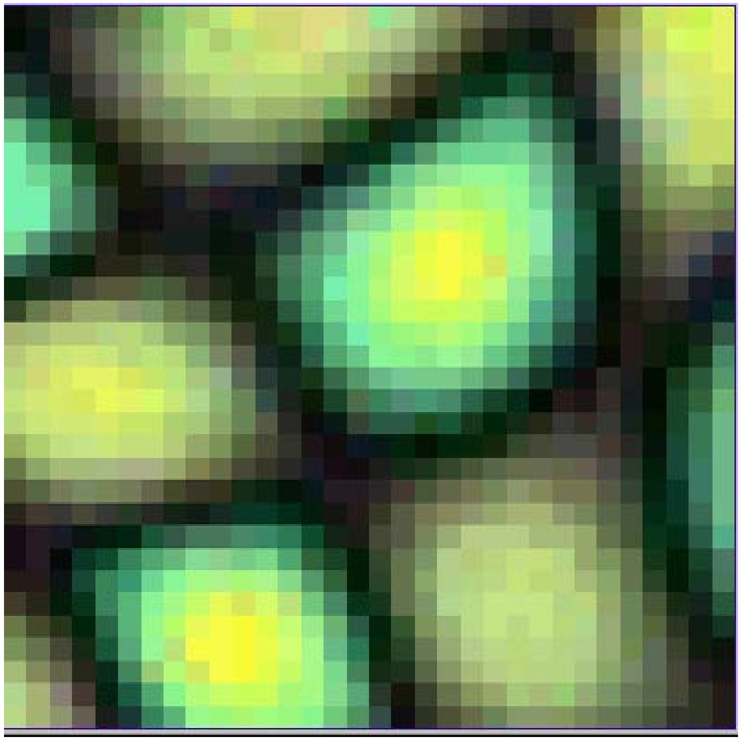
Magnification for Display

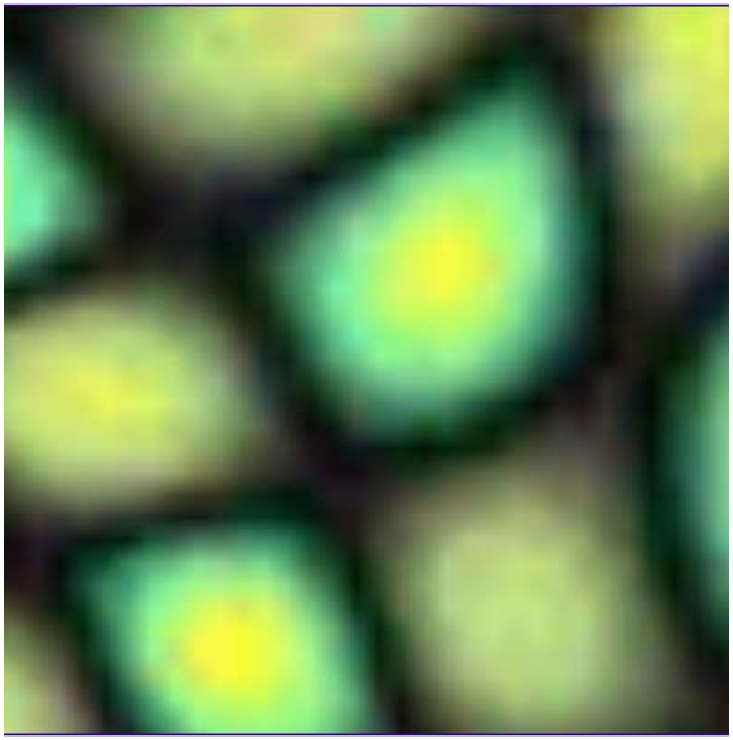


Minification for Display

Linear Interpolation

- Tell OpenGL to use a tent filter instead of a box filter.
- Magnification looks better, but blurry
  - (texture is under-sampled for this resolution)
  - Oh well.





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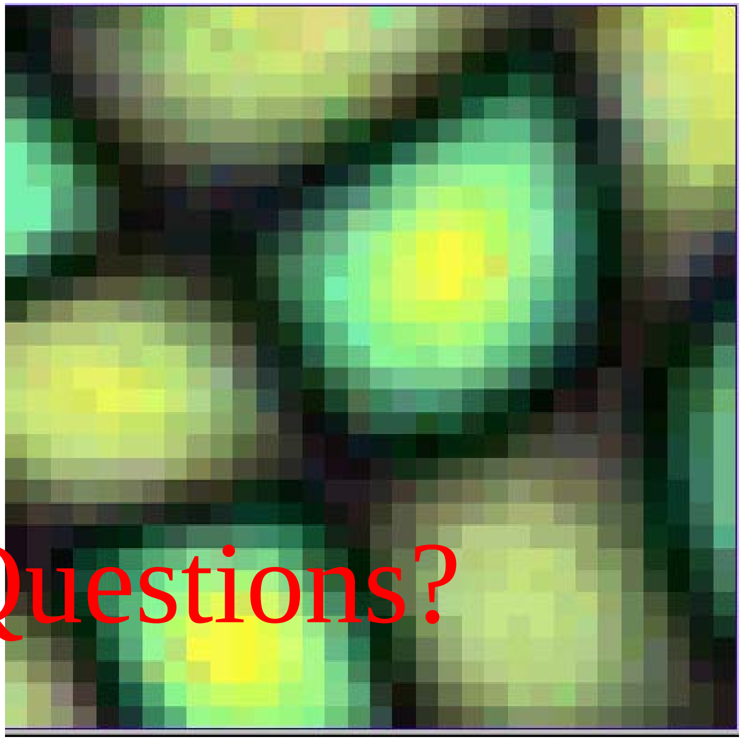
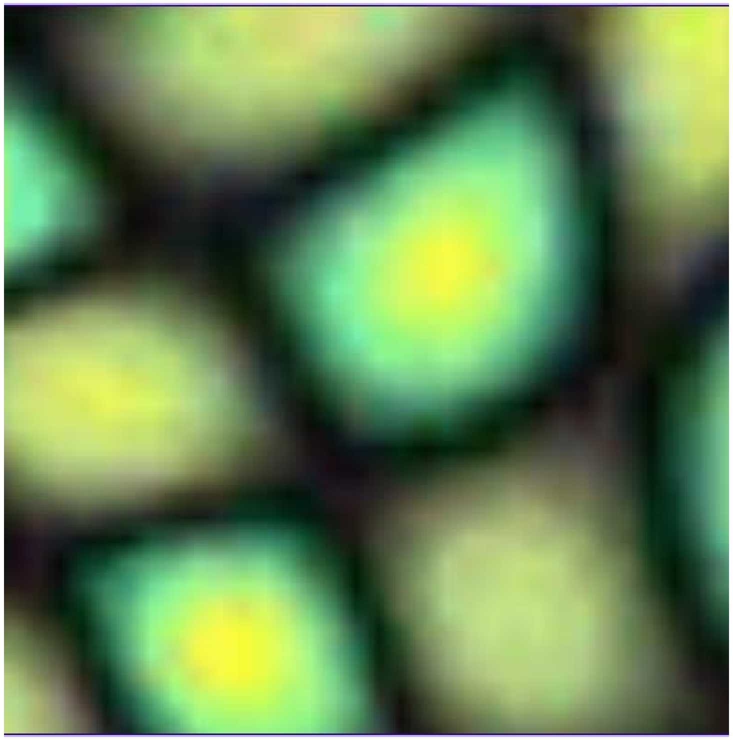
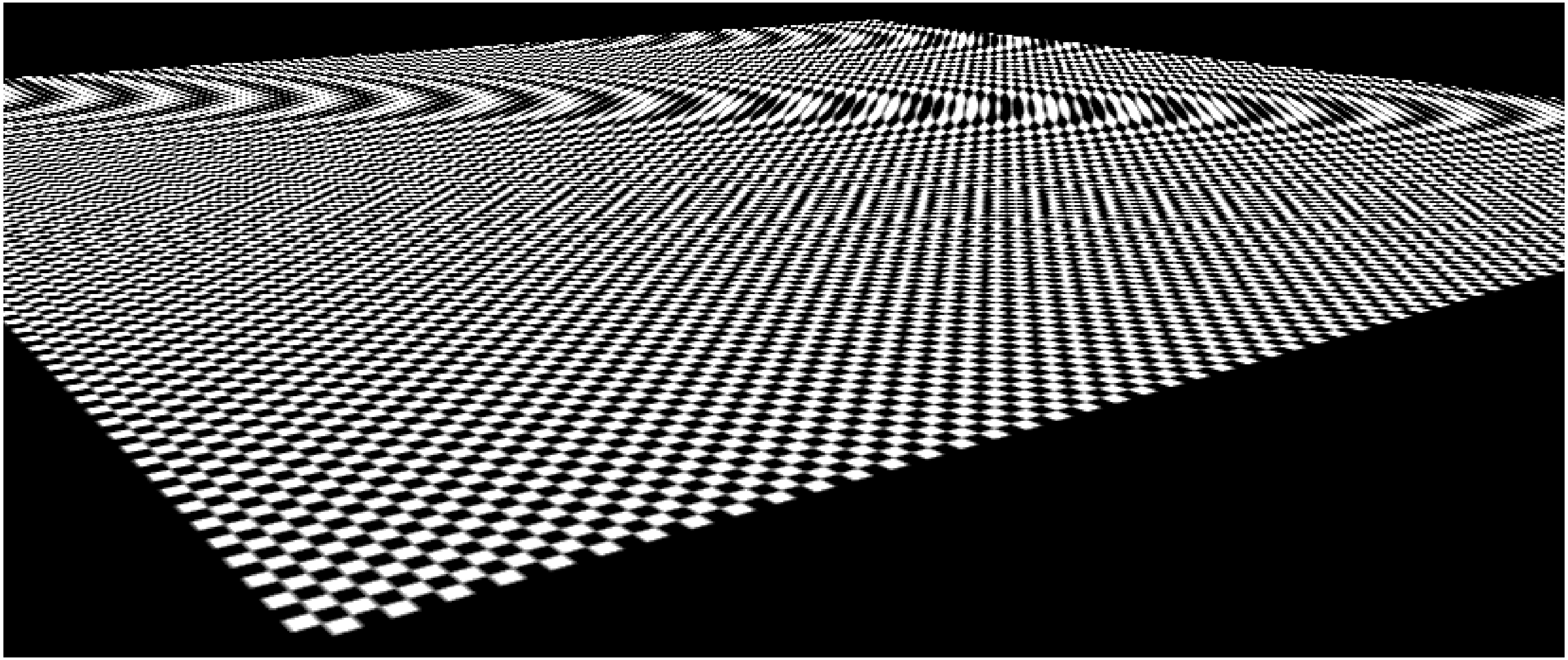




Questions?

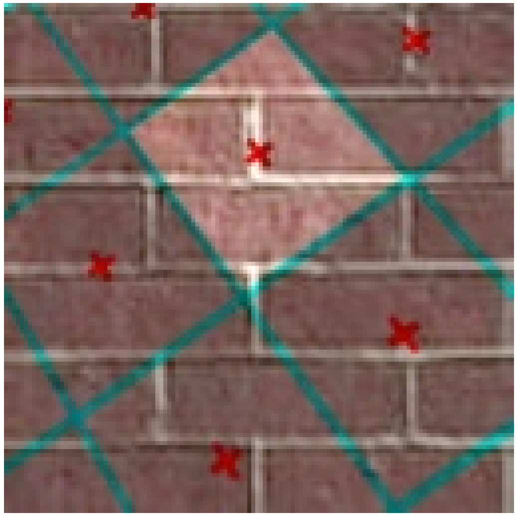
Minification: Examples of Aliasing



Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed

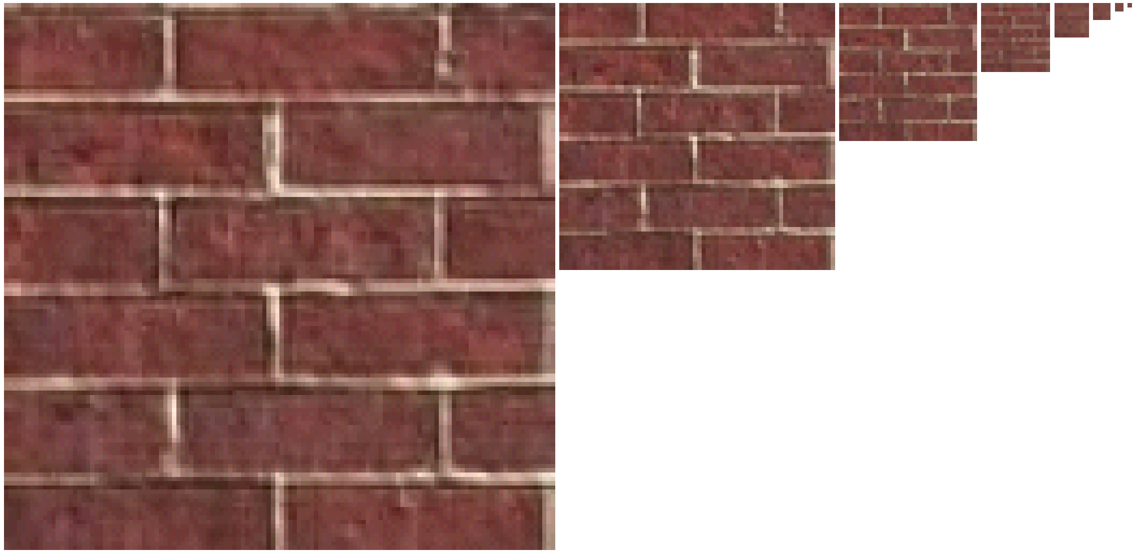
projected texture in image plane



pixels projected in texture plane

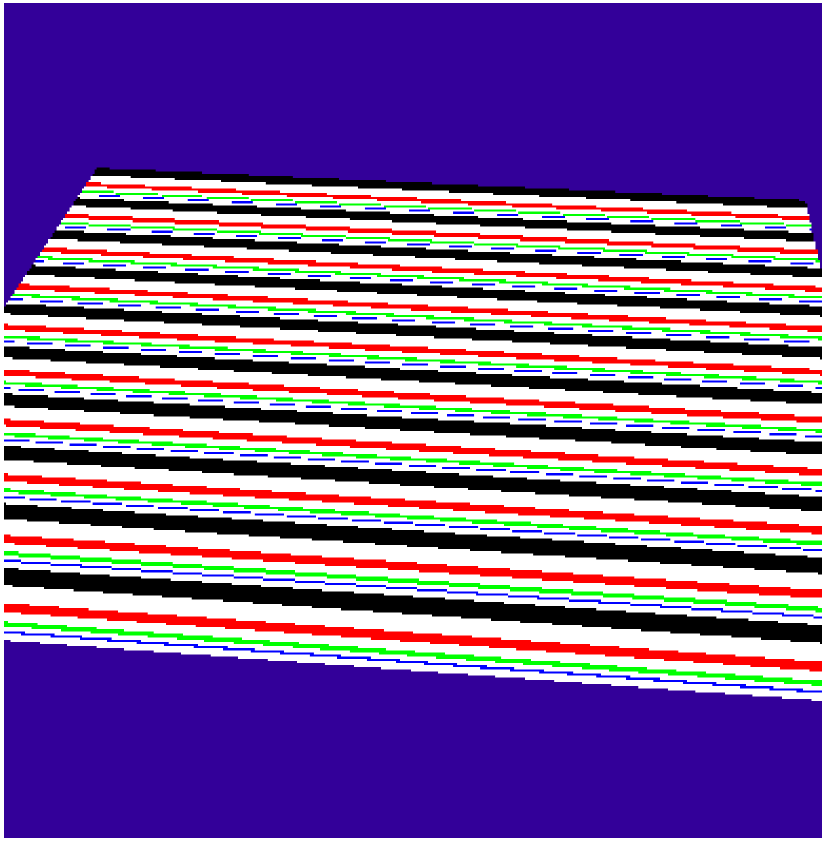
MIP Mapping

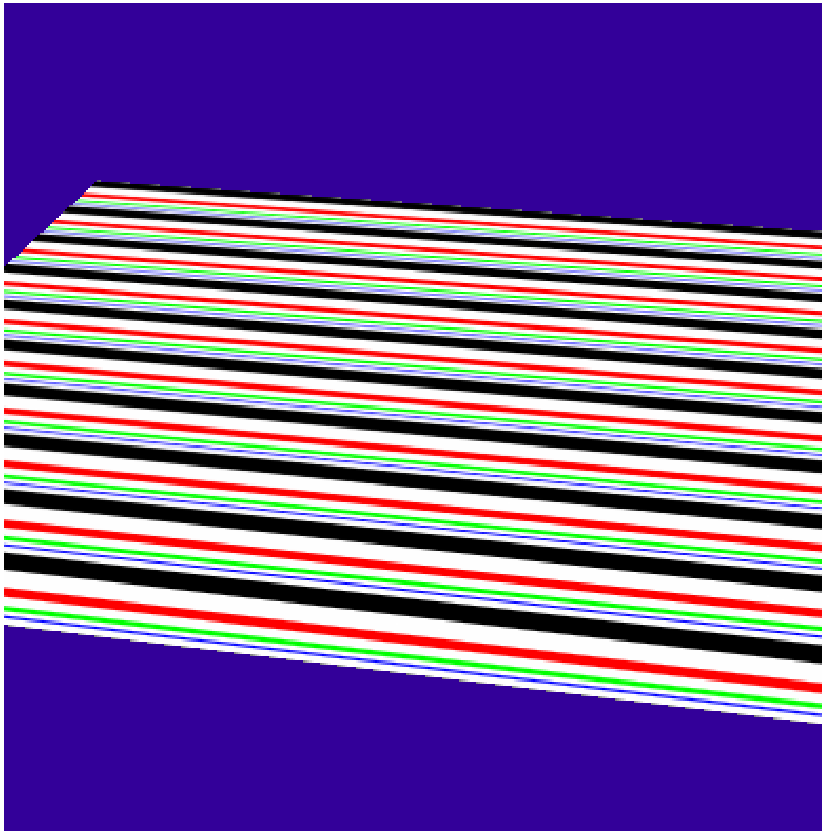
- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling



- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

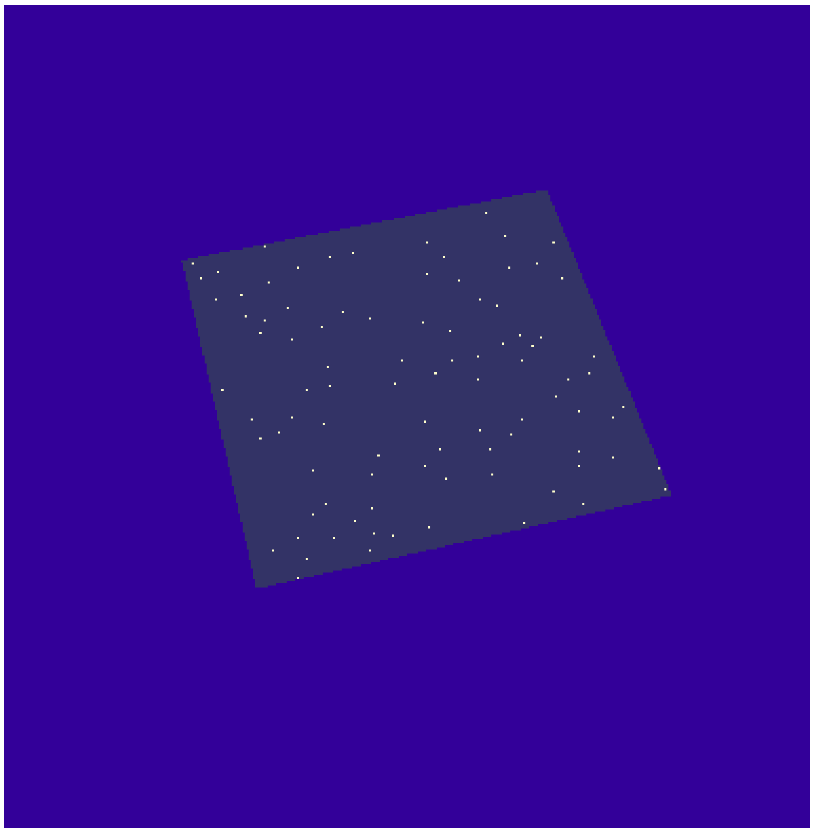
MIP Mapping Example


Nearest Neighbor

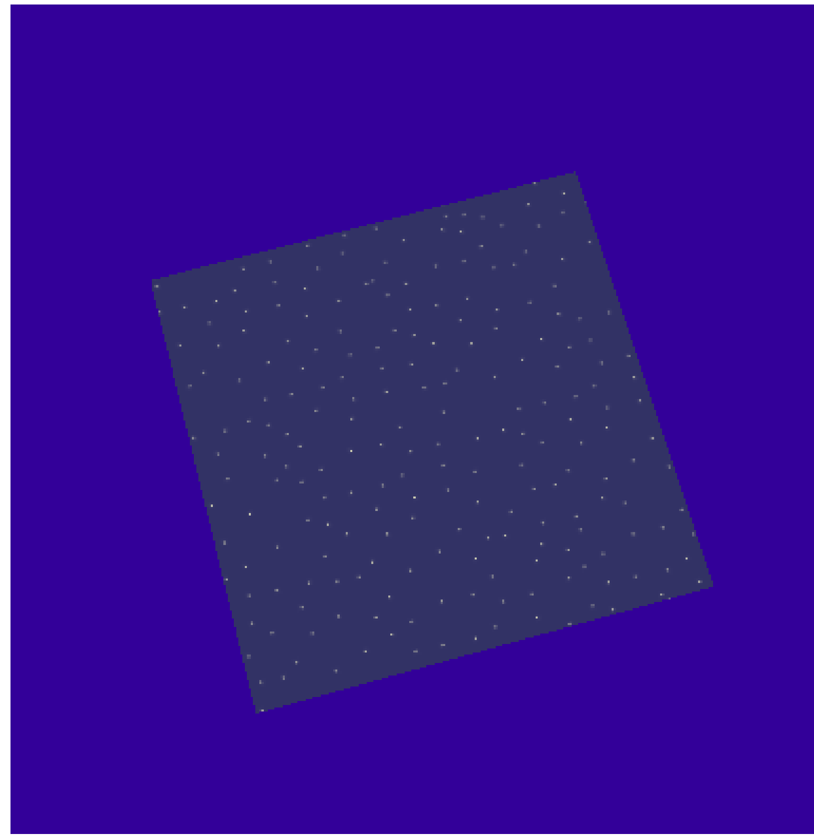

MIP Mapped (Bi-Linear)

MIP Mapping Example

- Small details may "pop" in and out of view



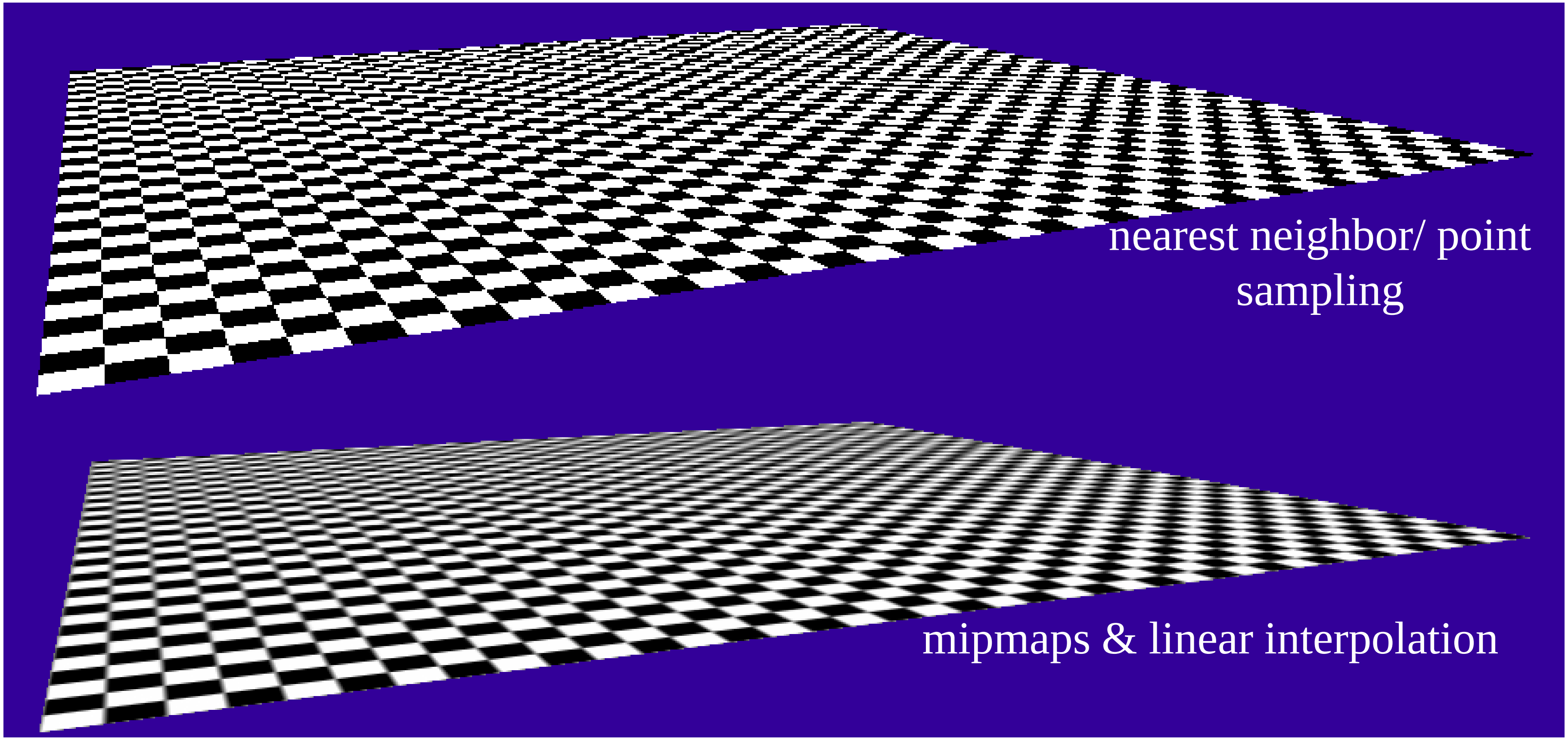
Nearest Neighbor



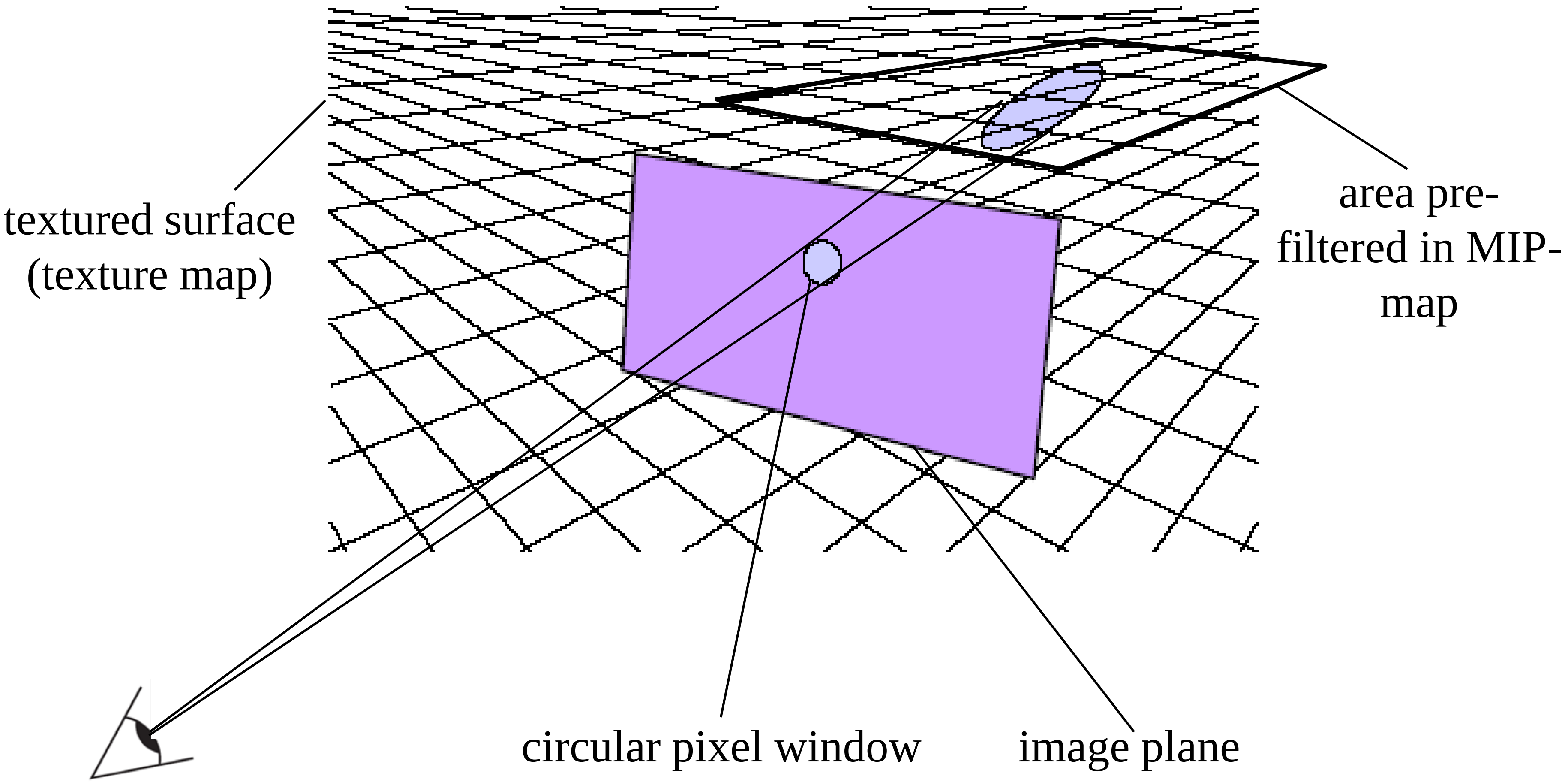
MIP Mapped (Bi-Linear)

Examples of Aliasing

Texture Errors



Finding the mip level



- Square MIP-map area is a bad approximation

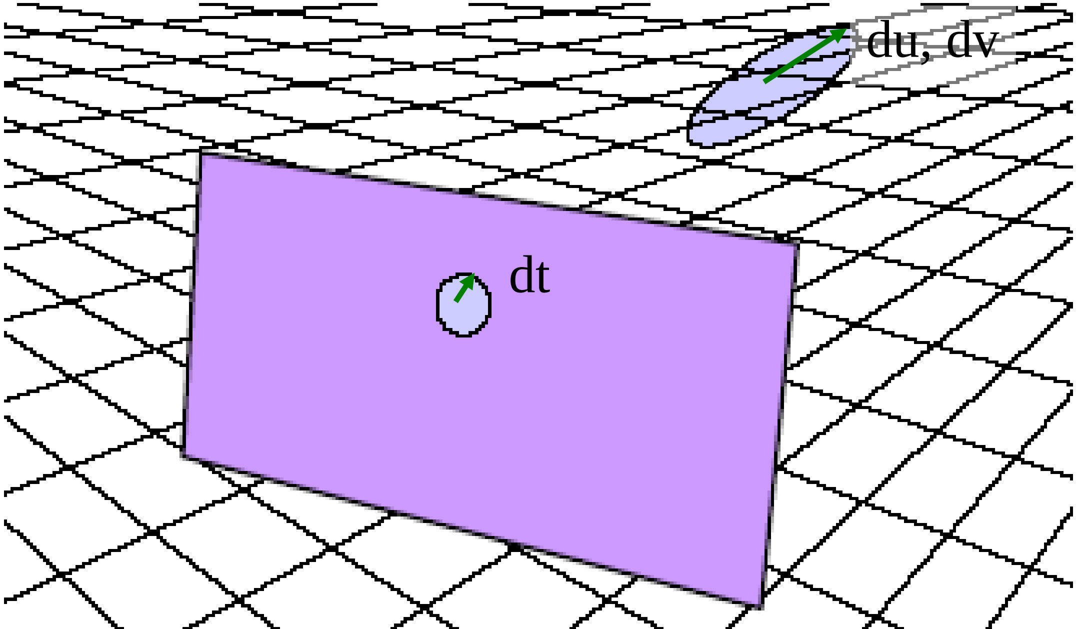
Finding the MIP level

How does a screen-space change dt relates to a texture-space change du,dv.

=> derivatives, (du/dt, dv/dt).

e.g. computed by hardware during rasterization

often: finite difference (pixels are handled by quads)



MIP Indices

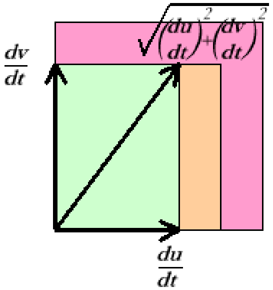
Actually, you have a choice of ways to translate this **derivative value** into a MIP level.

Because we have two derivatives, for u and for v (anisotropy)

This also brings up one of the shortcomings of MIP mapping. MIP mapping assumes that both the u and v components of the texture index are undergoing a uniform scaling, while in fact the terms du/dt and dv/dt are relatively independent. Thus, we must make some sort of compromise. Two of the most common approaches are given below:

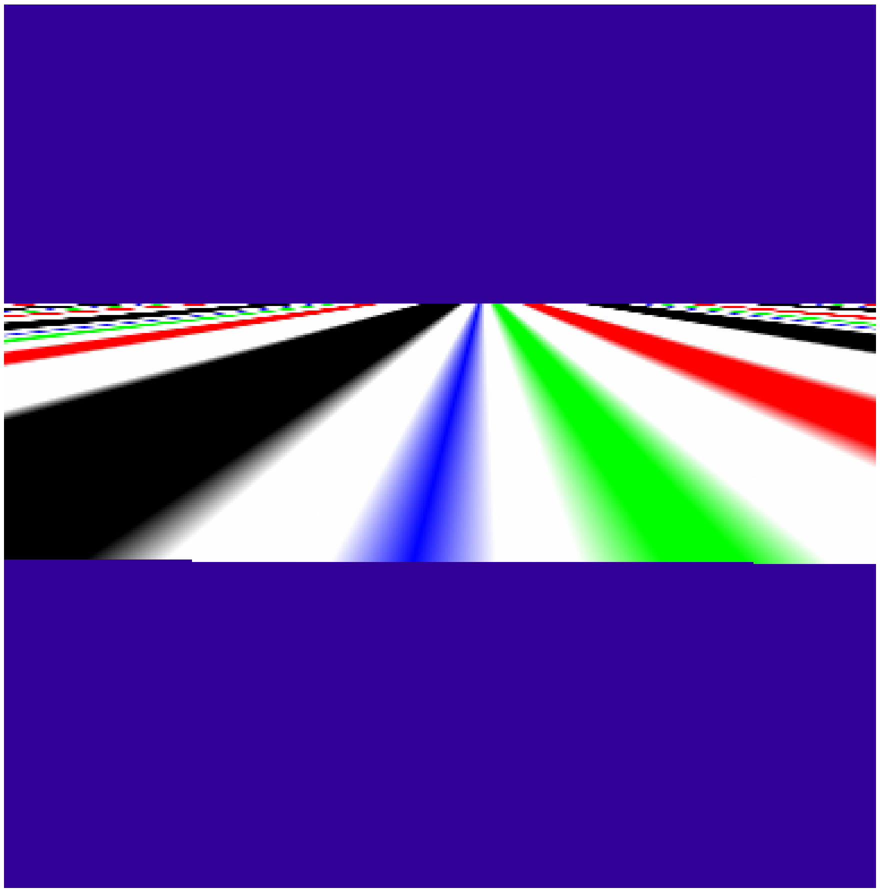
$$level = \log_2\left(\sqrt{\left(\frac{du}{dt}\right)^2 + \left(\frac{dv}{dt}\right)^2}\right)$$

$$level = \log_2\left(Max\left(\left|\frac{du}{dt}\right|, \left|\frac{dv}{dt}\right|\right)\right)$$

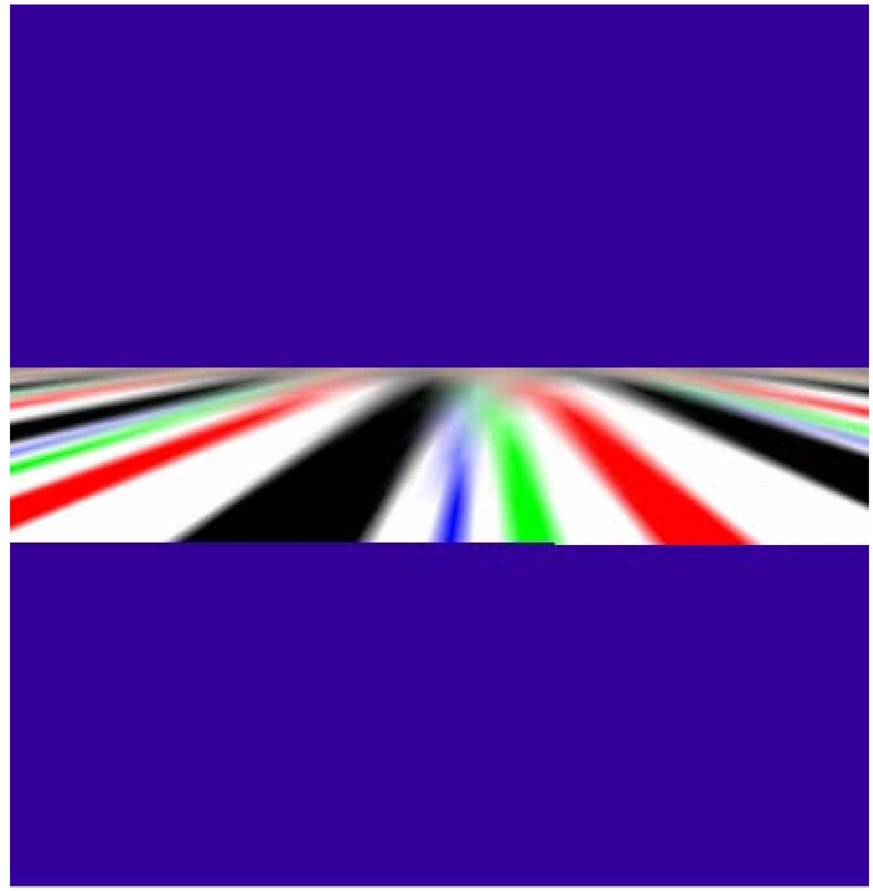


Anisotropy & MIP-Mapping

- What happens when the surface is tilted?



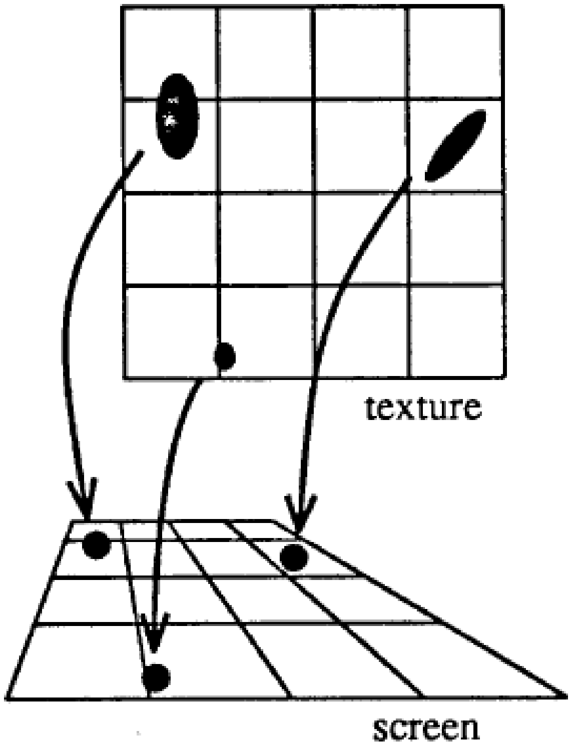
Nearest Neighbor



MIP Mapped (Bi-Linear)

Elliptical weighted average

- Isotropic filter wrt screen space
- Becomes anisotropic in texture space
- e.g. use anisotropic Gaussian
- Called Elliptical Weighted Average (EWA)



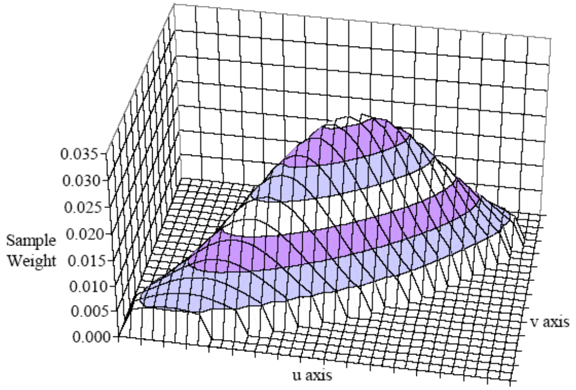


Figure 3: A perspective projection of a Gaussian filter into texture space.

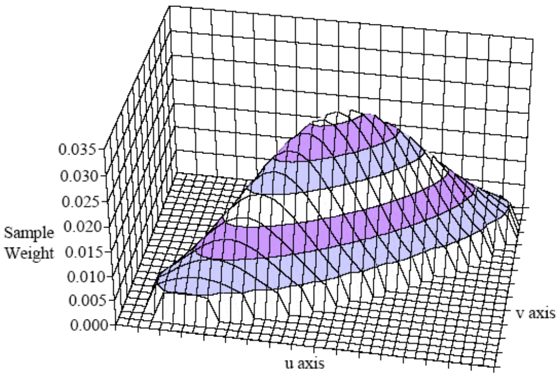
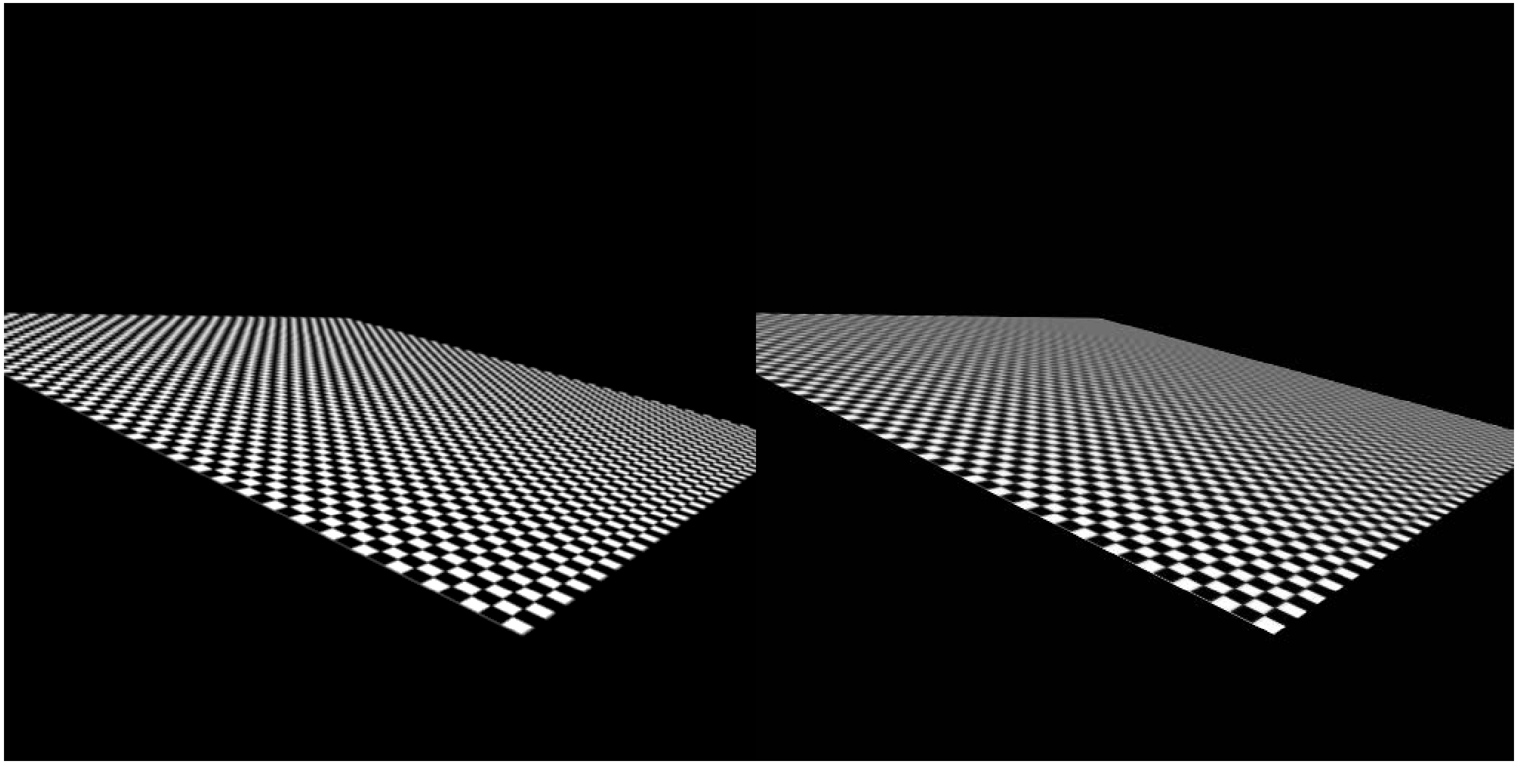


Figure 4: An affine projection of a Gaussian filter into texture space.

Image Quality Comparison

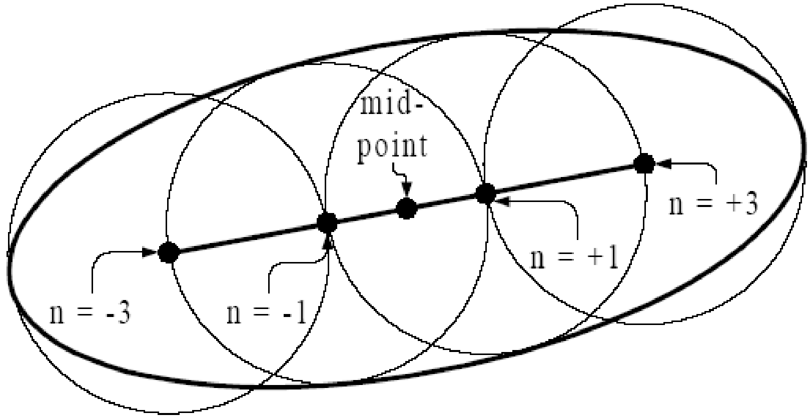
- Trilinear mipmapping



EWA trilinear mipmapping

Approximation of anisotropic

- Feline: Fast Elliptical Lines for Anisotropic Texture Mapping Joel McCormack, Ronald Perry, Keith I. Farkas, and Norman P. Jouppi SIGGRAPH 1999
- Andreas Schilling, Gunter Knittel & Wolfgang Strasser. Texram: A Smart Memory for Texturing. IEEE Computer Graphics and Applications, 16(3): 32-41, May 1996.
- Aproximate Anisotropic Gaussian by a set of isotropic "probes"



FELINE results

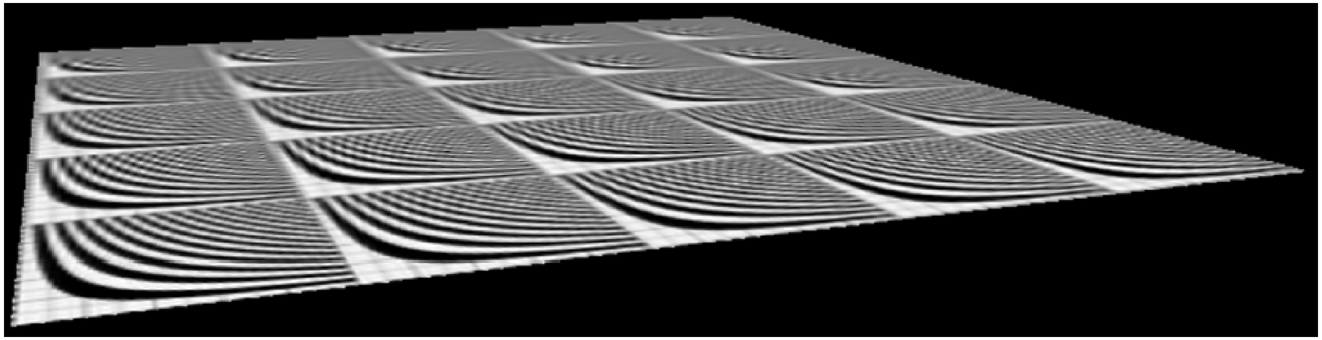


Figure 10: Trilinear paints curved lines with blurring.

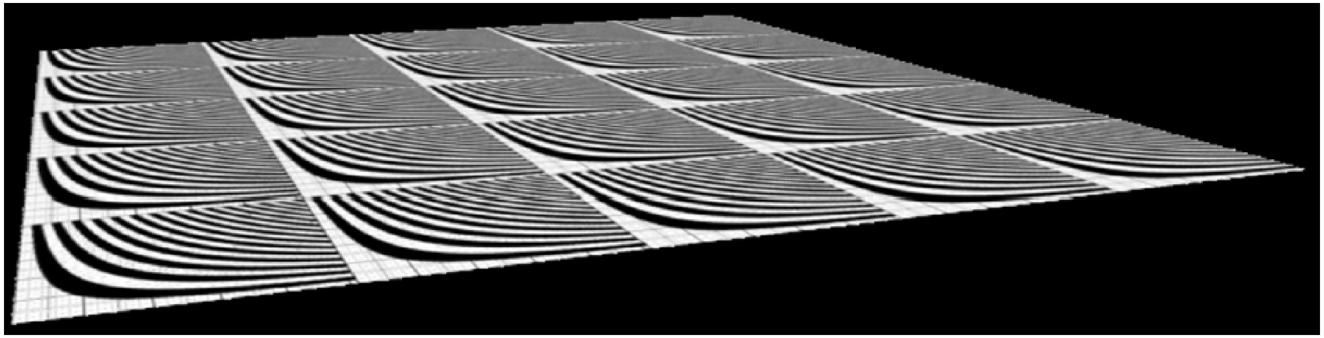


Figure 13: “High-quality” Simple Feline paints curved lines with few artifacts.

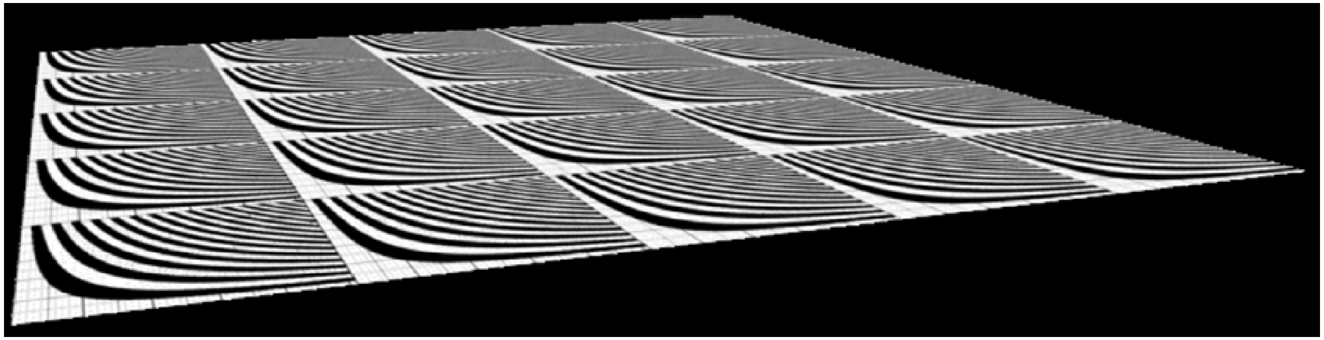


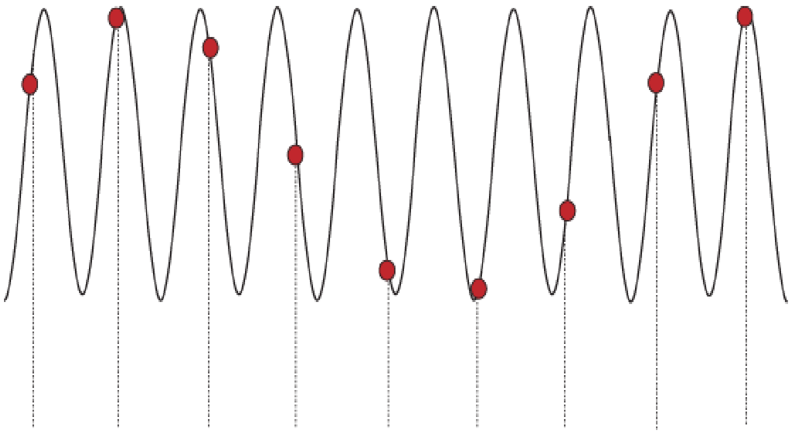
Figure 14: Mip-mapped EWA paints curved lines with few artifacts.

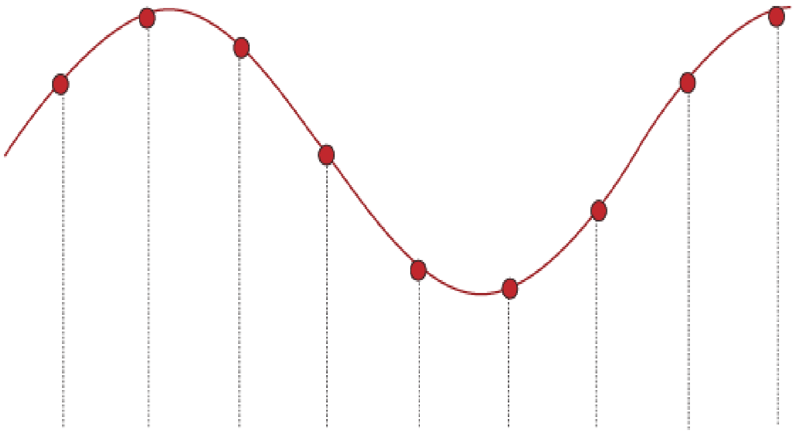
Questions?

- What follows is useful mathematical background that was not covered in class

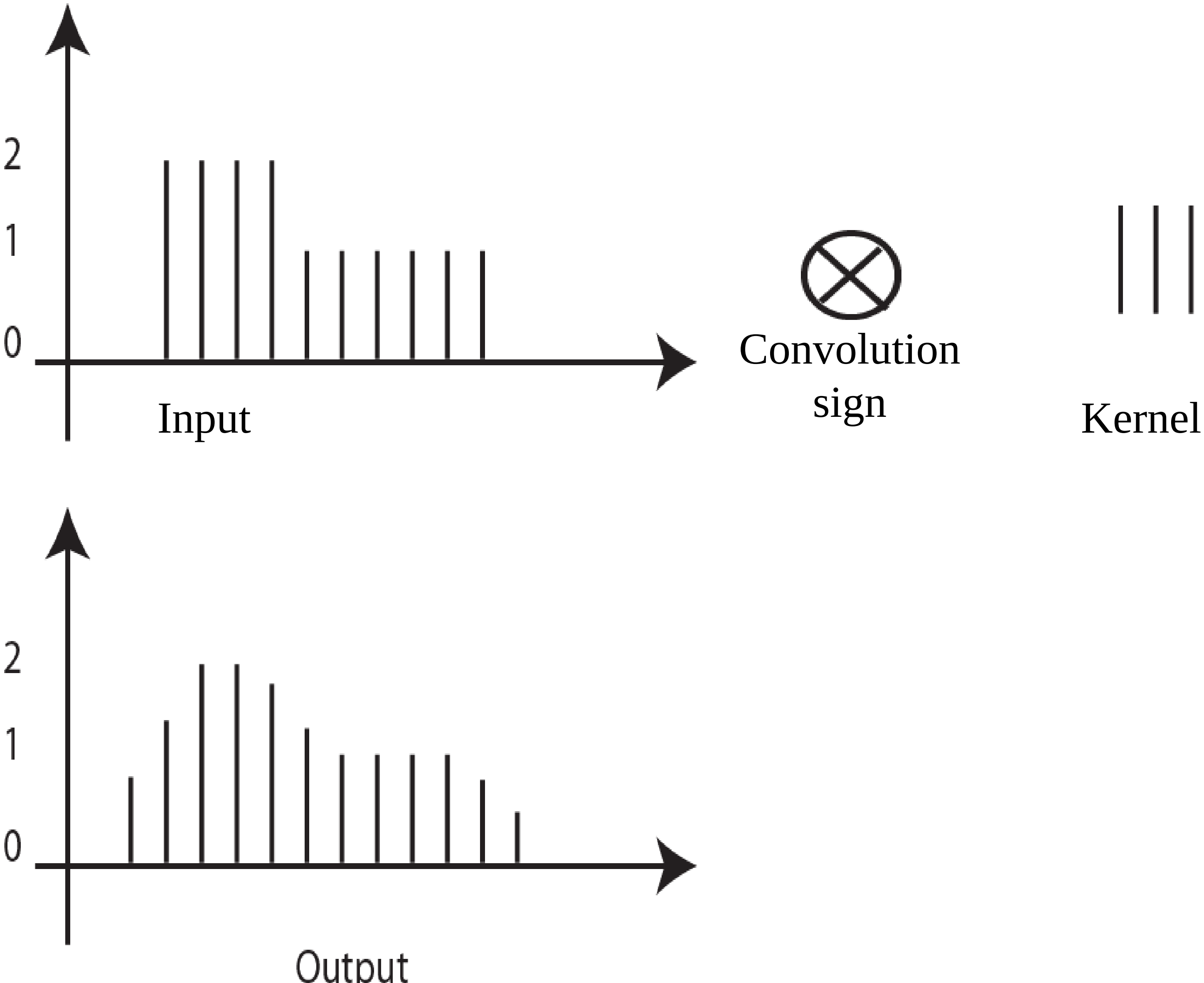
Signal processing 101

- Sampling and filtering are best understood in terms of Fourier analysis
- We already saw that sine waves generate aliasing: a high frequency sine wave turns into a low frequency one when undersampled

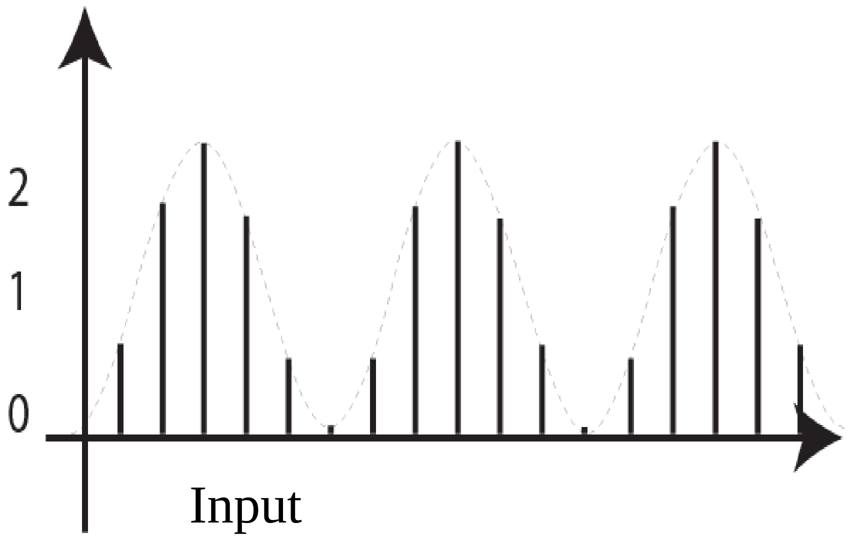




Blurring: convolution



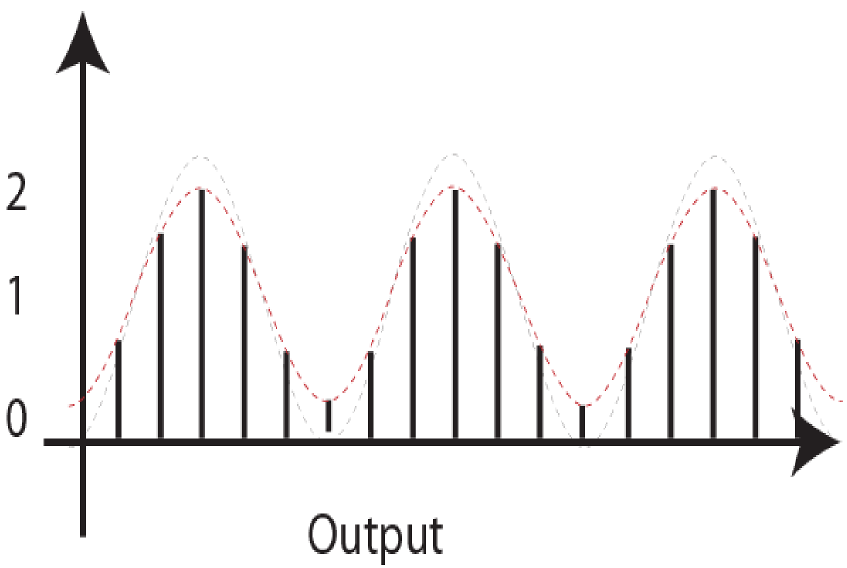
Blurring: convolution



Input

Convolution sign

Kernel



Same shape, just reduced contrast!!!

This is an eigenvector
(output is the input multiplied by a constant)

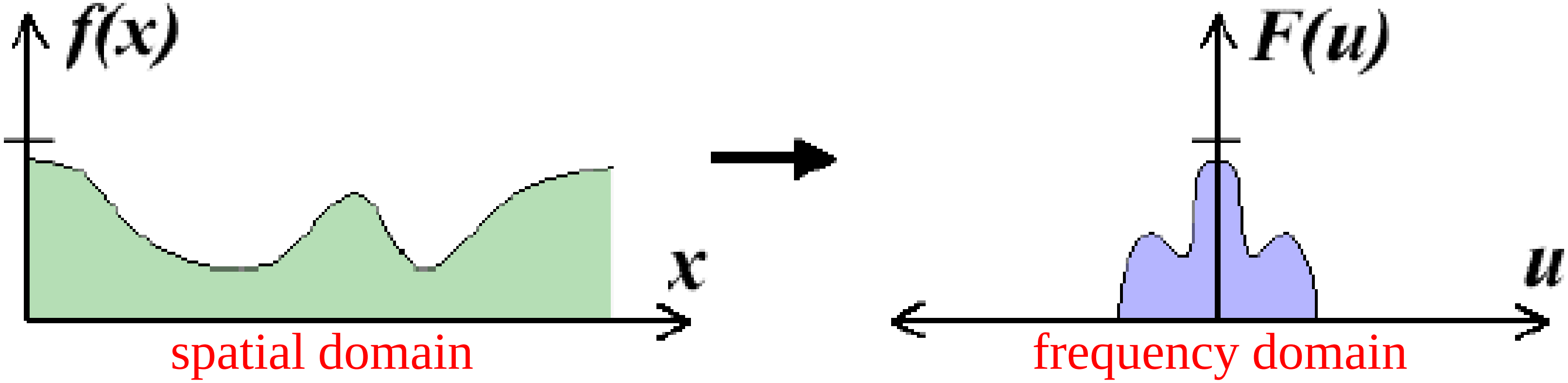
Motivation for Fourier analysis

- Sampling/reconstruction is a linear process
- The sampling grid is a periodic structure
  - Fourier is pretty good at handling periodic stuff
- Plus we saw that a sine wave has serious problems with sampling
- The solution is about blurring
  - We have seen that sine wave are simple wrt blur
- In general, the Fourier transform is just a change of basis
  - In that basis, aliasing & blurring are easier to understand
  - become diagonal (we can consider frequencies individually)

Questions?

Remember Fourier Analysis?

- Every periodic signal in the *spatial domain* has a dual in the *frequency domain* (change of basis)



- This particular signal is *band-limited*, meaning it has no frequencies above some threshold

Remember Fourier Analysis?

- We can transform from one domain to the other using the Fourier Transform
(Just a change of basis)

frequency domain

spatial domain

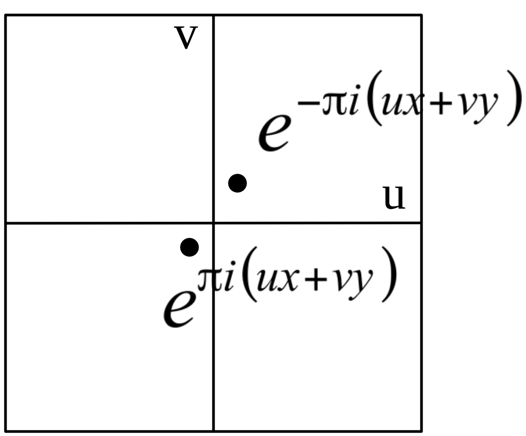
Fourier Transform

$$F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-i2\pi(ux+vy)}\,dxdy$$

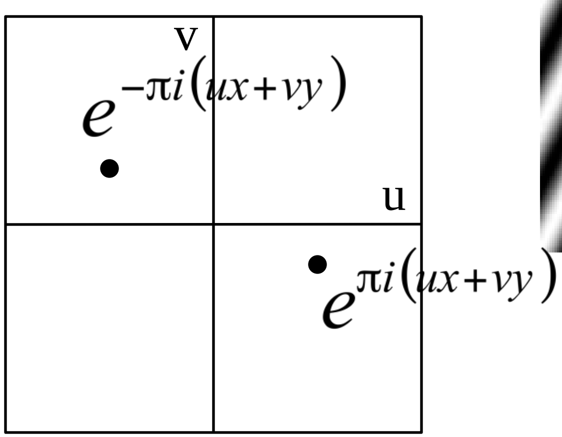
Inverse Fourier Transform

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{\;i2\pi(ux+vy)}\,dudv$$

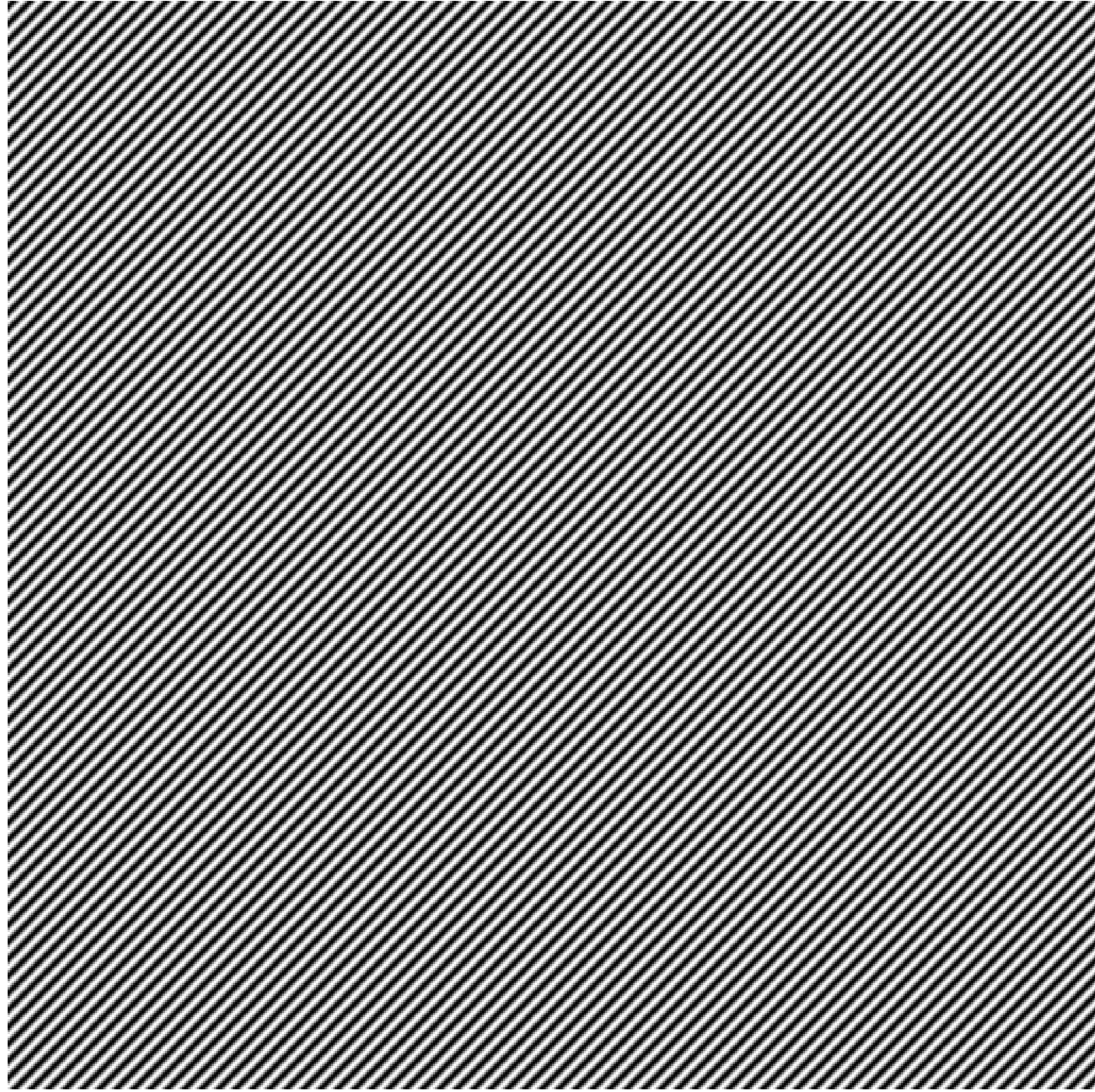
To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.

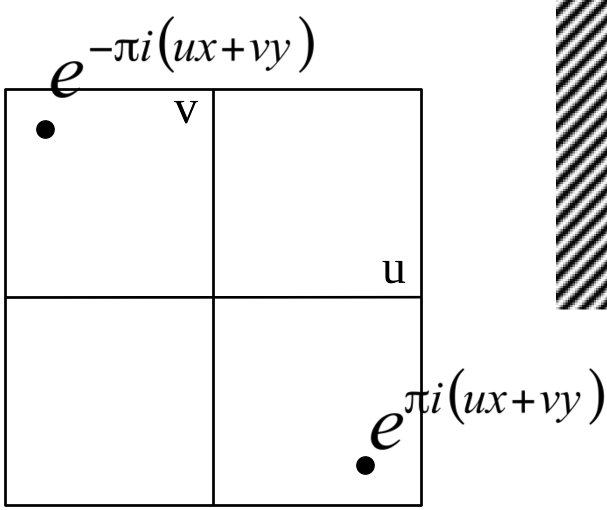


Here u and v are
larger than in
the previous
slide.

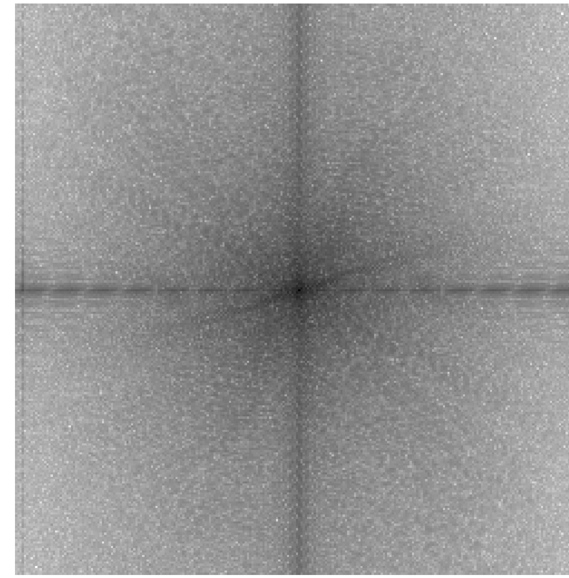


And larger still...

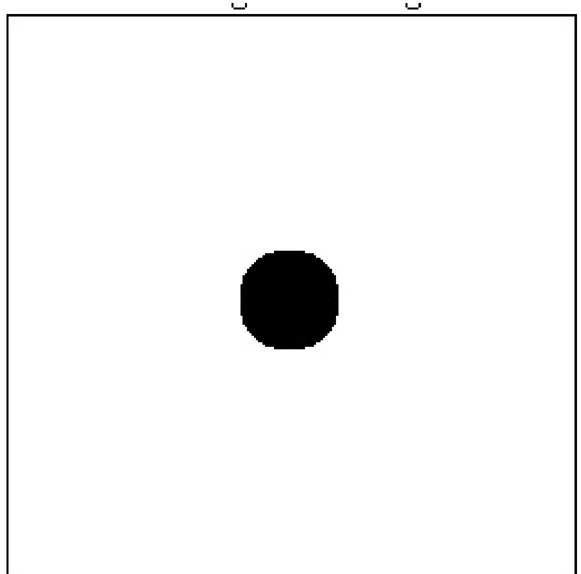




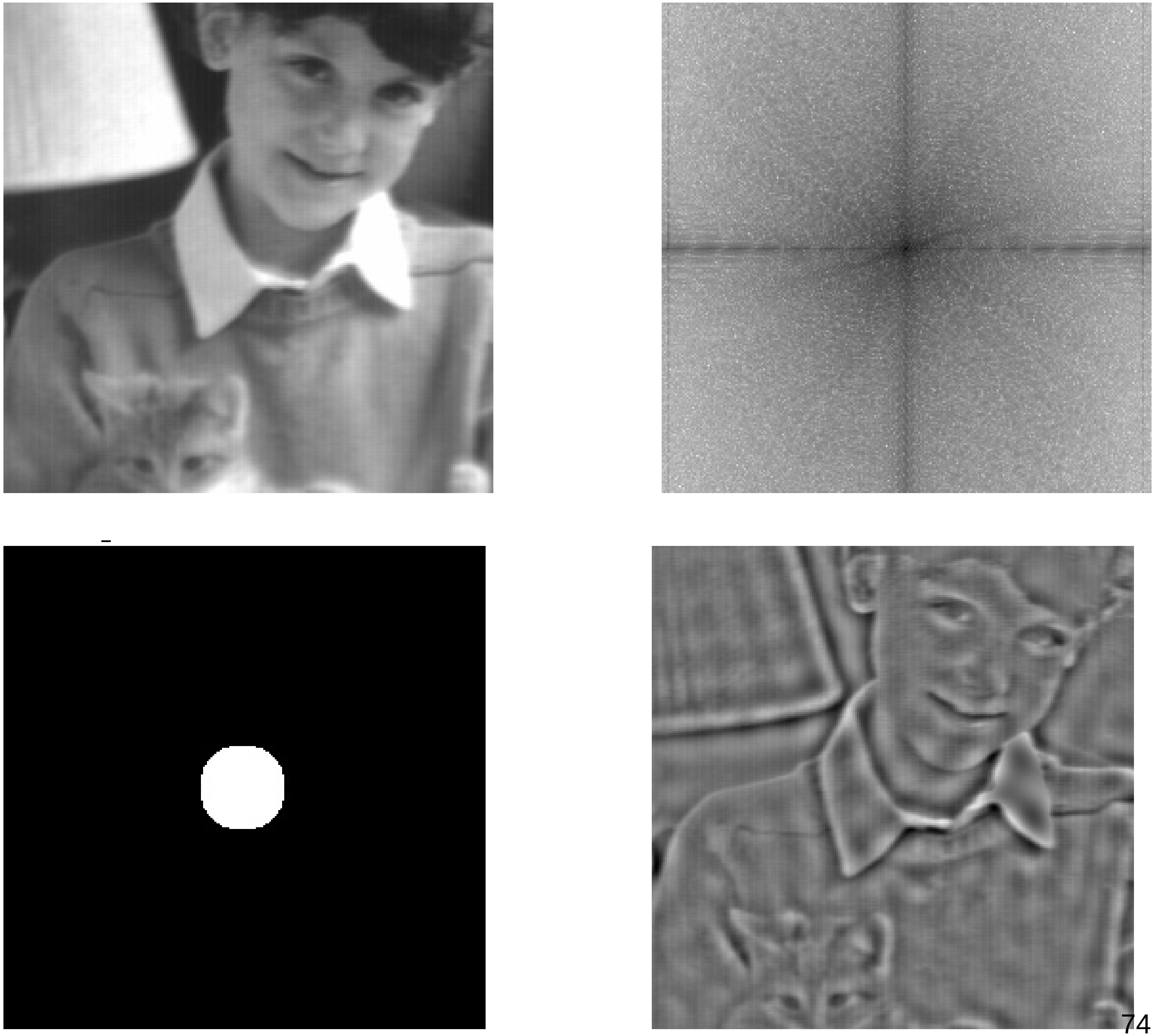
Low pass http://www.reindeergraphics.com



black means 1,
white means 0



High pass http://www.reindeergraphics.com



Questions?

Remember Convolution?

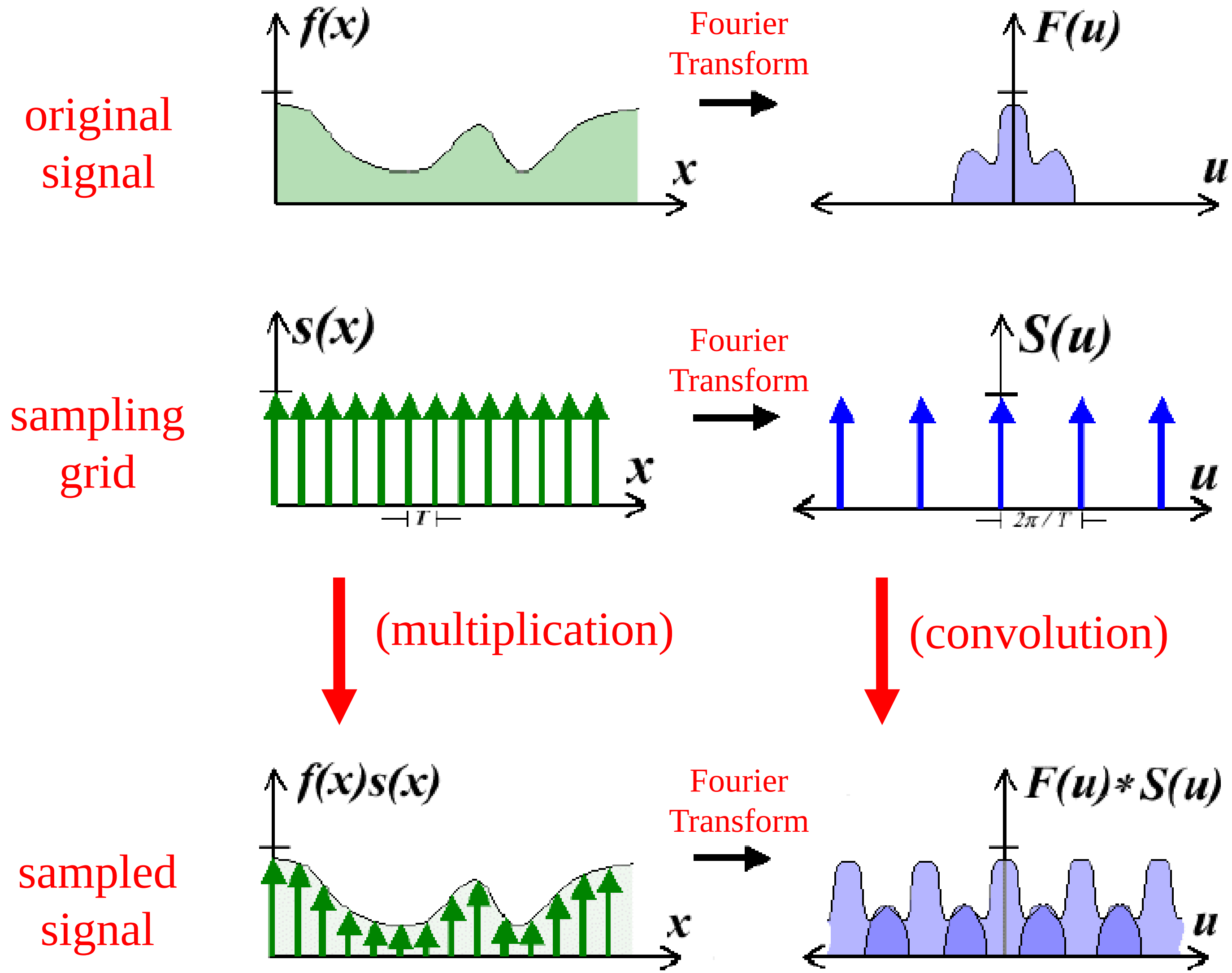
- Some operations that are difficult in the spatial domain are simpler in the frequency domain.
- For example, convolution in the spatial domain is the same as multiplication in the frequency domain.
  - Because sine waves are eigen functions

$$f(x) * h(x) \rightarrow F(u)H(u)$$

- And, convolution in the frequency domain is the same as multiplication in the spatial domain

$$F(u) * H(u) \rightarrow f(x)h(x)$$

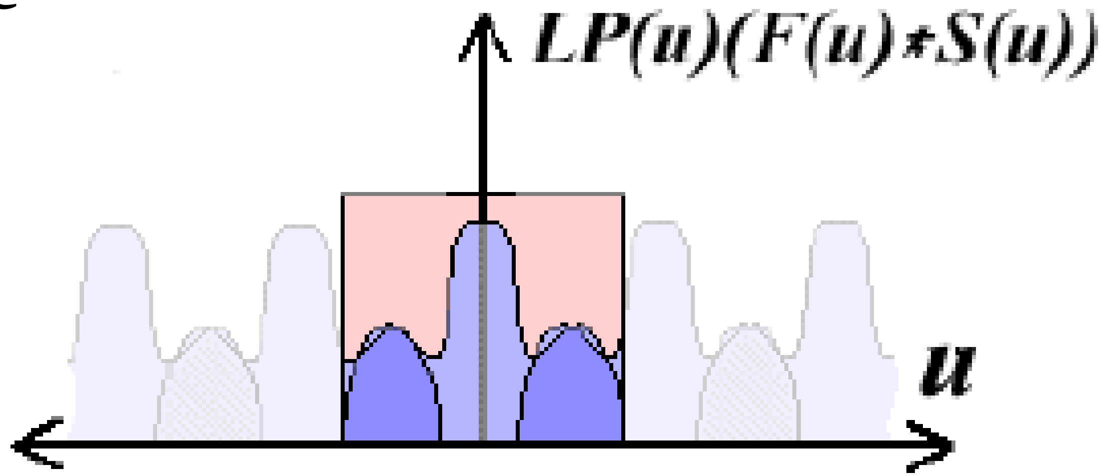
Sampling in the Frequency Domain



Reconstruction

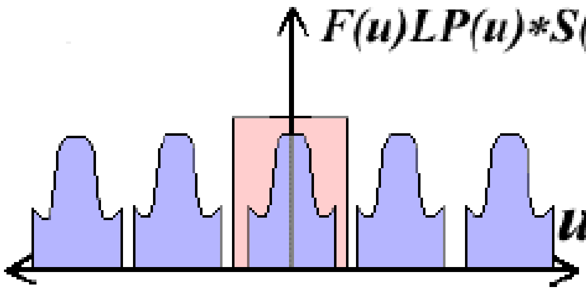
- If we can extract a copy of the original signal from the frequency domain of the sampled signal, we can reconstruct the original signal!

- But there may be overlap between the copies.

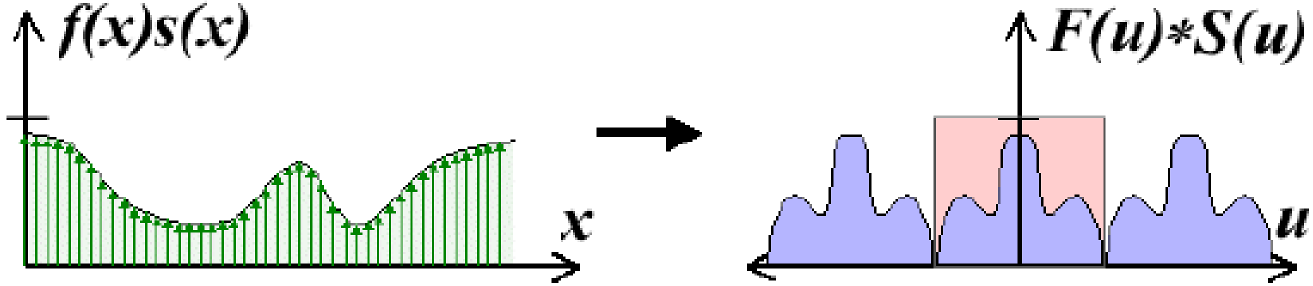


Guaranteeing Proper Reconstruction

- Separate by removing high frequencies from the original signal (low pass pre-filtering)



- Separate by increasing the sampling density



- If we can't separate the copies, we will have overlapping frequency spectrum during reconstruction → *aliasing*.

Recap: sampling/reconstruction

- 1/ Prefilter signal
  - blur with given filter
- 2/ Sample
  - creates replicas
- 3/ Reconstruct
  - Another blur

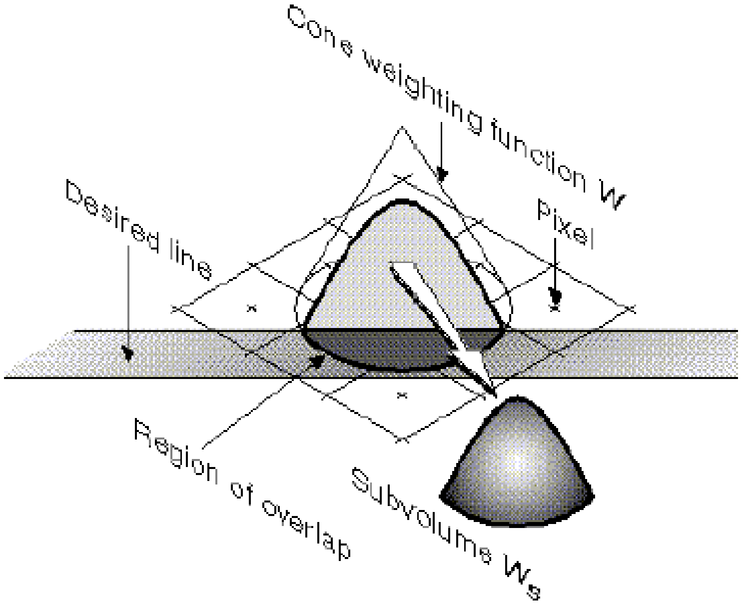
Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be *greater than twice* the highest frequency of the input signal in order to be able to reconstruct the original perfectly from the sampled version
(Shannon, Nyquist, Whittaker, Kotelnikov)

Questions?

Filters (a.k.a convolution kernel)

- Weighting function or a convolution kernel
- Area of influence often bigger than "pixel"
- Sum of weights = 1
  - Each sample contributes the same total to image
  - Constant brightness as object moves across the screen.
- No negative weights/colors (optional)

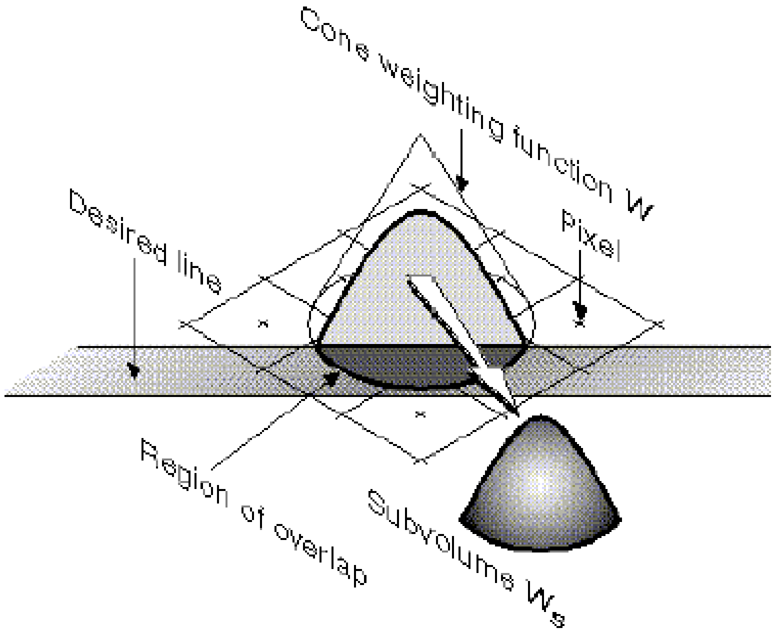


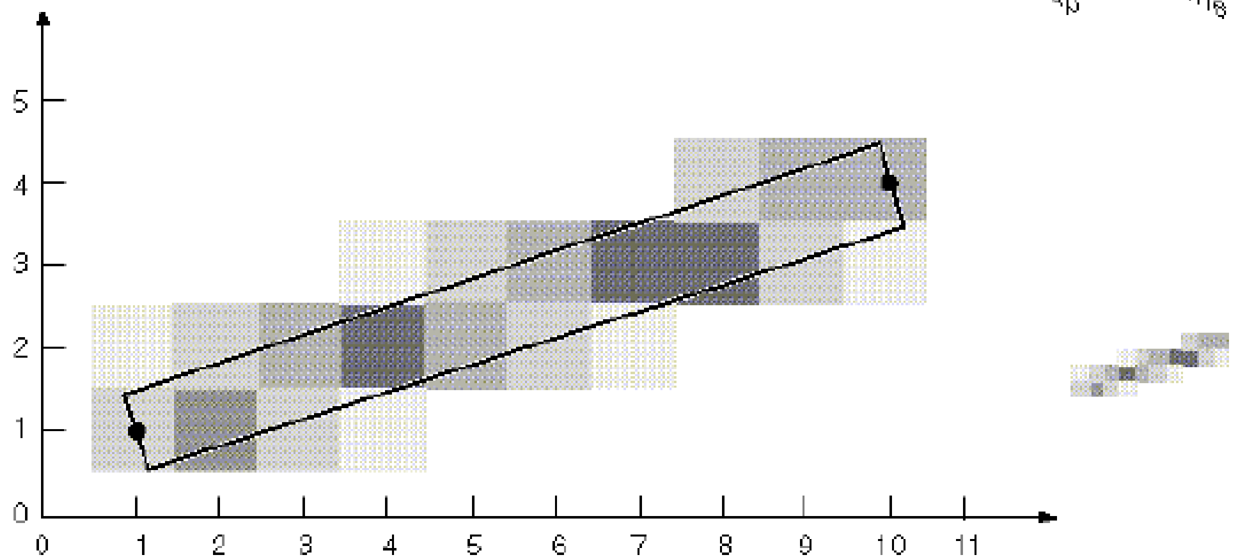
Filters

- Filters are used to
  - reconstruct a continuous signal from a sampled signal (reconstruction filters)
  - band-limit continuous signals to avoid aliasing during sampling (low-pass filters)
- Desired frequency domain properties are the same for both types of filters
- Often, the same filters are used as reconstruction and low-pass filters

Pre-Filtering

- Filter continuous primitives
- Treat a pixel as an area
- Compute weighted amount of object overlap
- What weighting function should we use?

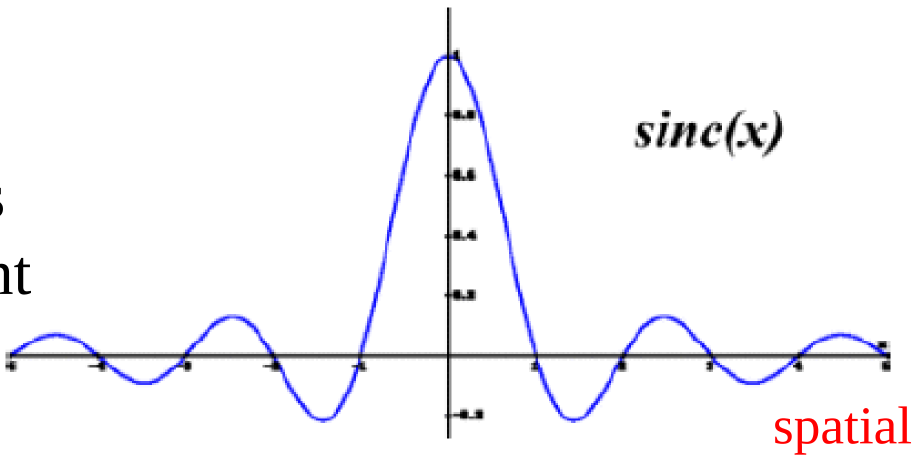


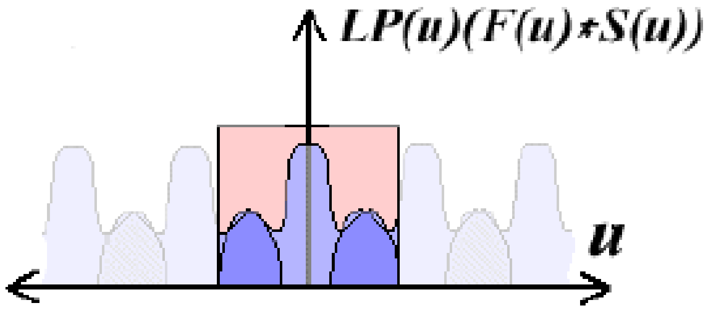


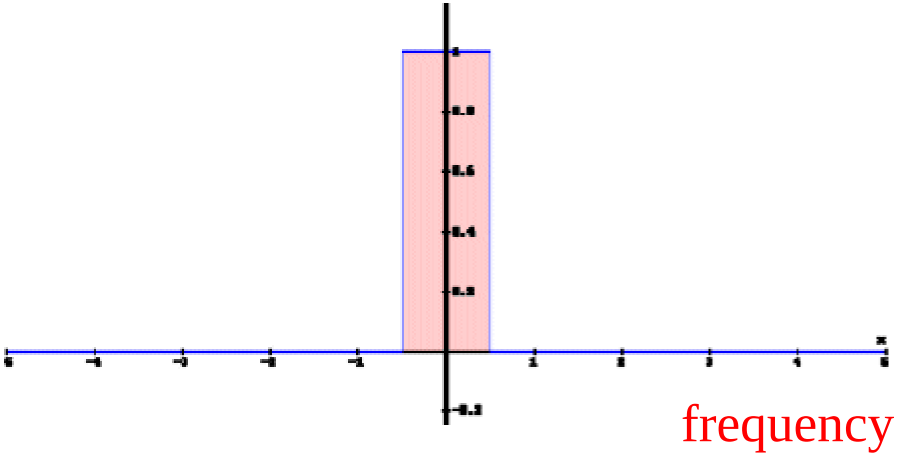
Source: Foley, VanDam, Feiner, Hughes - Computer Graphics, Second Edition, Addison Wesley

The Ideal Filter

- Unfortunately it has *infinite* spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute

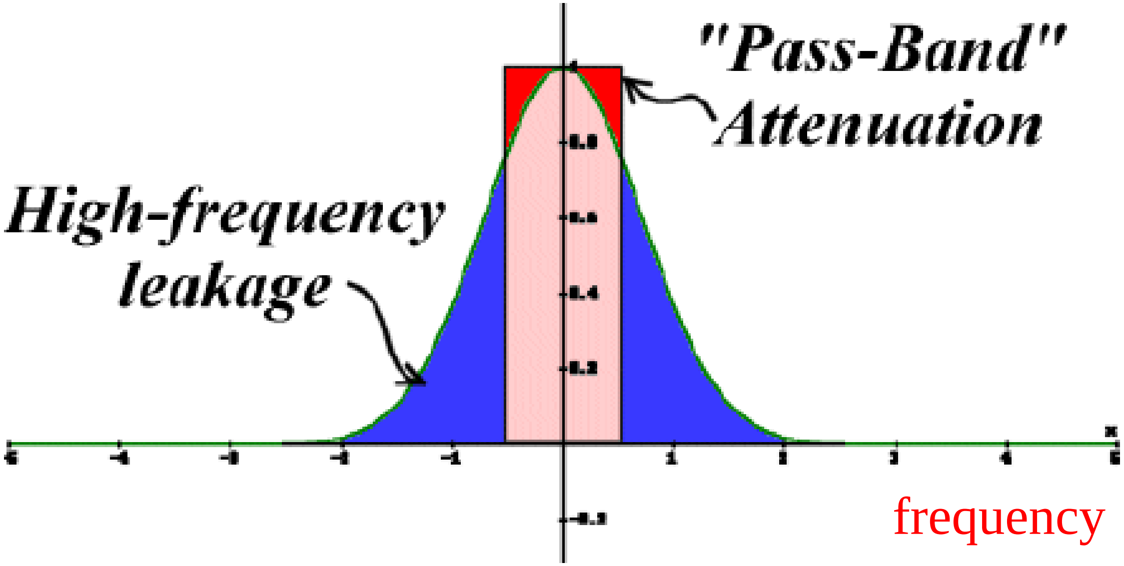






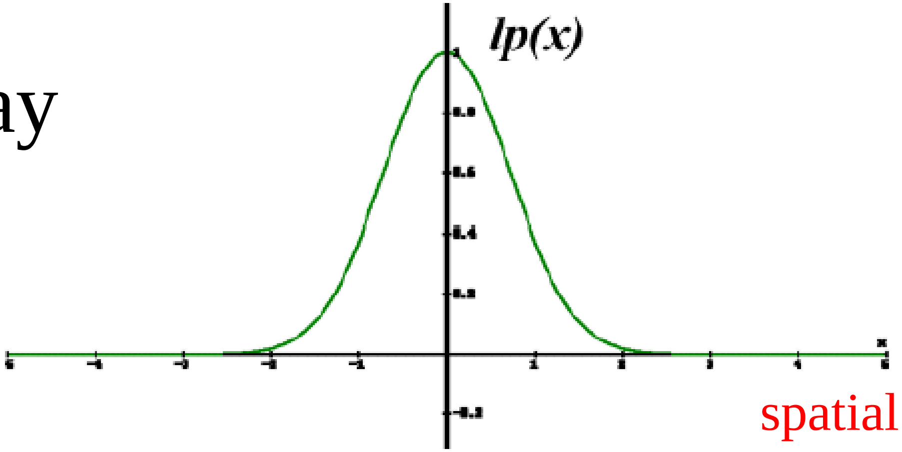
Problems with Practical Filters

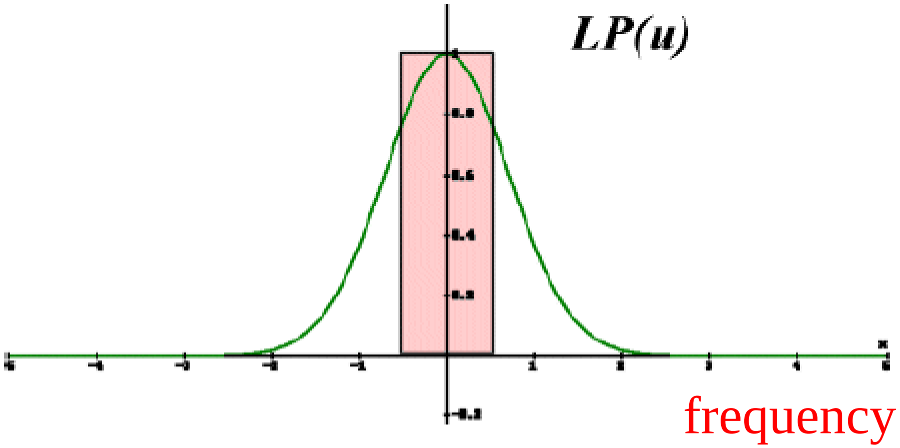
- Many visible artifacts in re-sampled images are caused by poor reconstruction filters
- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage causes "ringing" and can accentuate th sampling grid (anisotropy)



Gaussian Filter

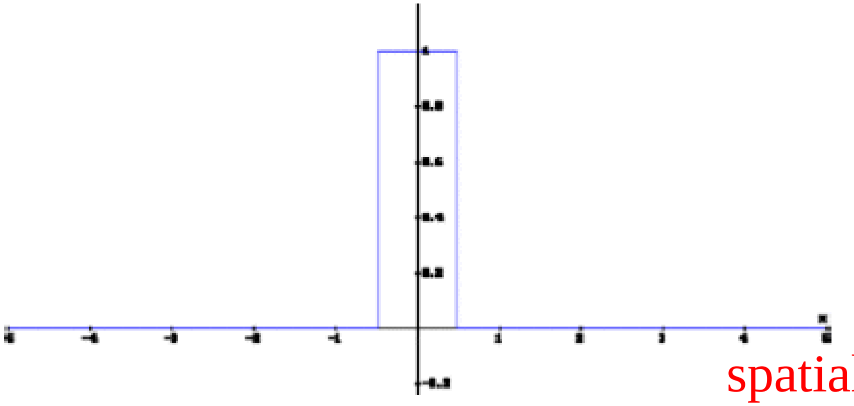
- This is more or less what a physical display does for free!

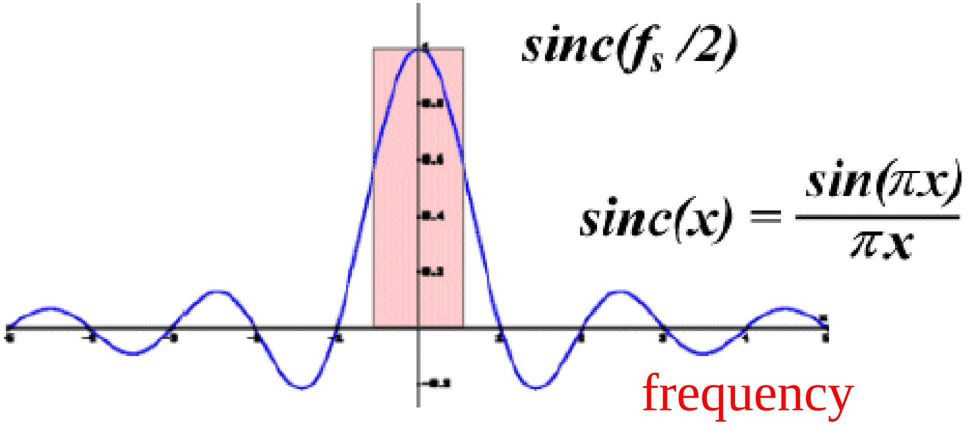




Box Filter / Nearest Neighbor

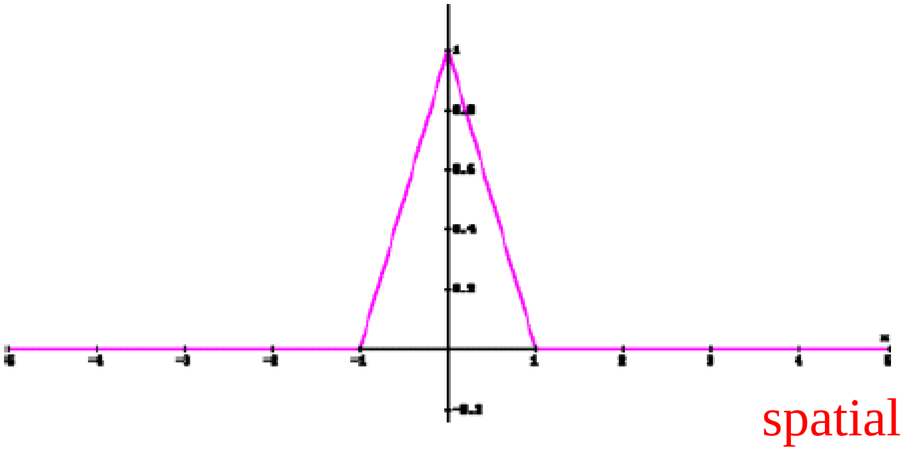
- Pretending pixels are little squares.

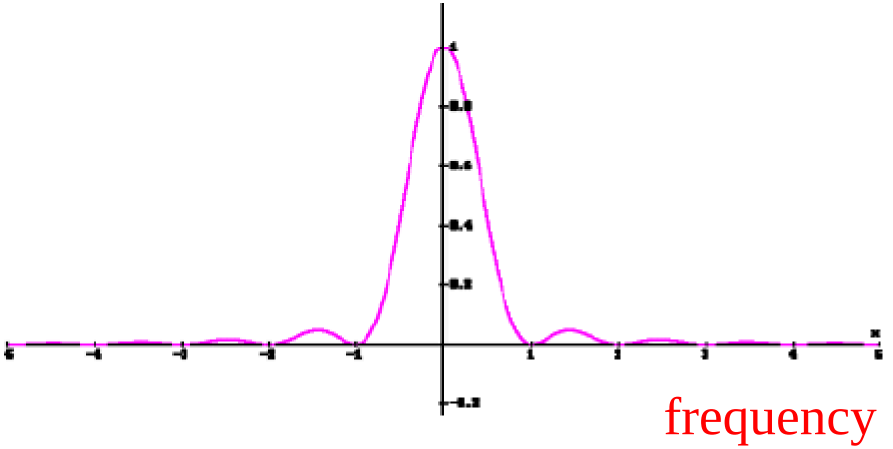




Tent Filter / Bi-Linear Interpolation

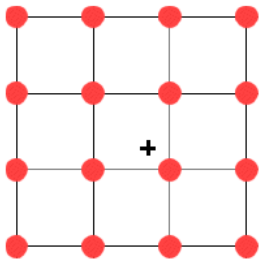
- Simple to implement
- Reasonably smooth

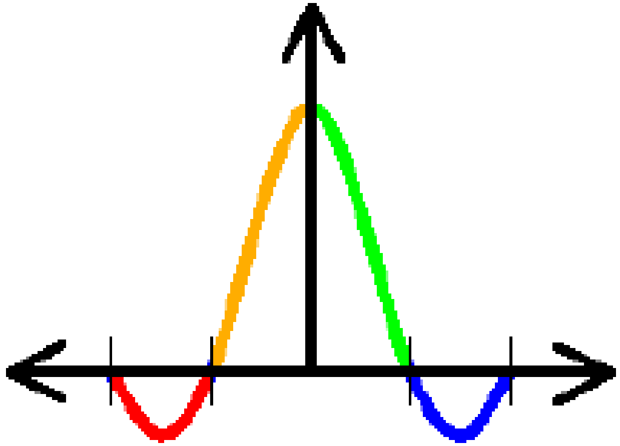


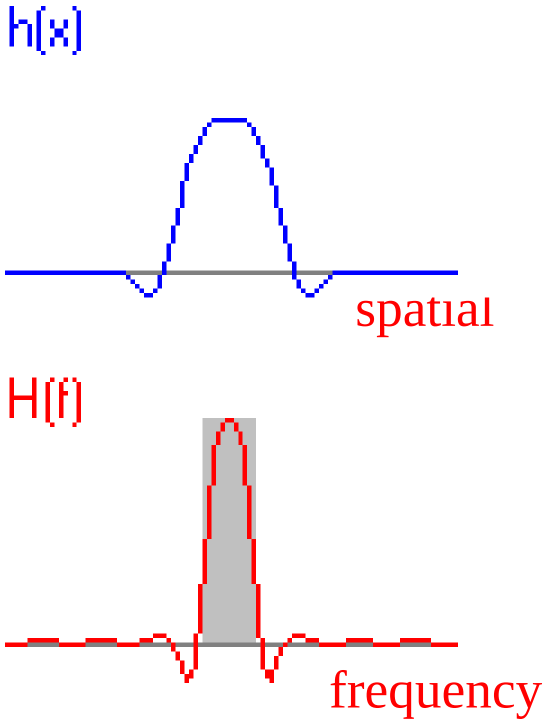


Bi-Cubic Interpolation

- Begins to approximate the ideal spatial filter, the sinc function

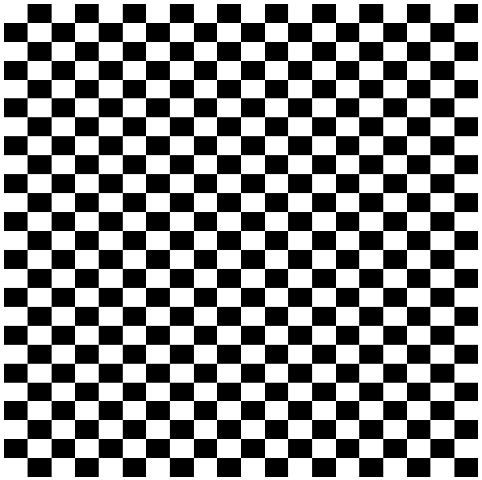




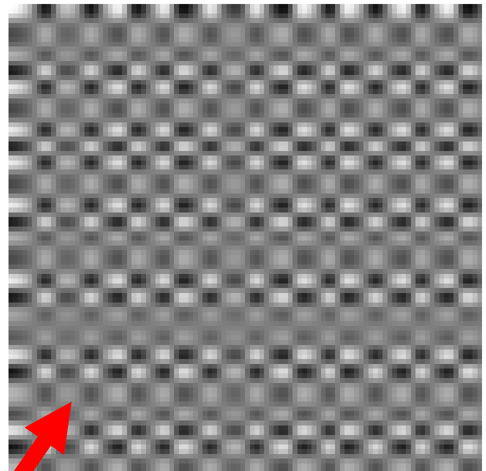


Why is the Box filter bad?

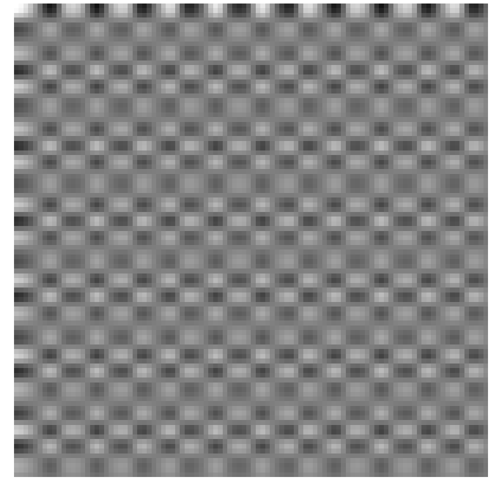
- (Why is it bad to think of pixels as squares)



Original high-resolution image

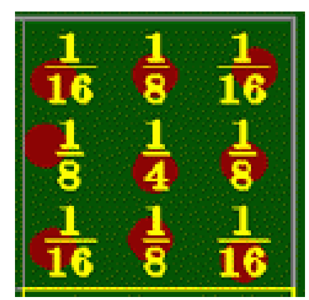


Down-sampled with a 5x5 box filter (uniform weights)



Down-sampled with a 5x5 Gaussian filter (non-uniform weights)

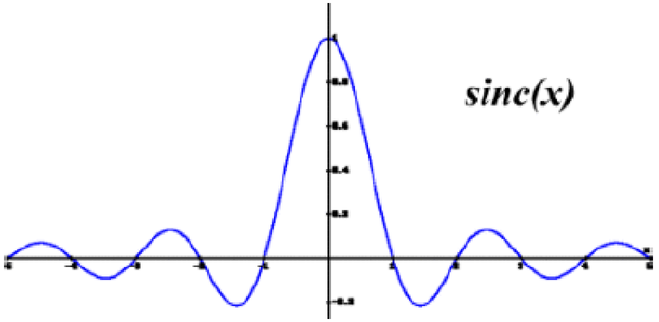
notice the ugly horizontal banding

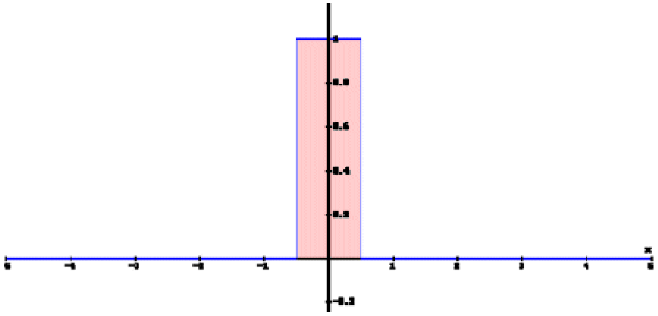


Questions?

Recap: Ideal sampling/reconstruction

- Pre-filter with a perfect low-pass filter
  - Box in frequency
  - Sinc in time
- Sample at Nyquist limit
  - Twice the frequency cutoff
- Reconstruct with perfect filter
  - Box in frequency, sinc in time
- And everything is great!



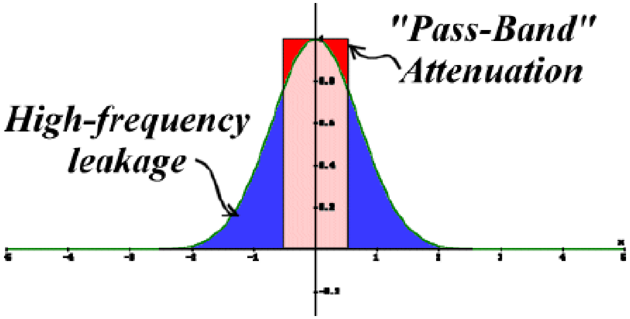


Difficulties with perfect sampling

- Hard to prefilter
- Perfect filter has infinite support
  - Fourier analysis assumes infinite signal and complete knowledge
  - Not enough focus on local effects
- And negative lobes
  - Emphasizes the two problems above
  - Negative light is bad
  - Ringing artifacts if prefiltering or supports are not perfect

At the end of the day

- Fourier analysis is great to understand aliasing
- But practical problems kick in
- As a result there is no perfect solution
- Compromises between
  - Finite support
  - Avoid negative lobes
  - Avoid high-frequency leakage
  - Avoid low-frequency attenuation



- Everyone has their favorite cookbook recipe
  - Gaussian, tent, Mitchell bicubic