Convergence by transforms: Central linit theorem Recall: convergence in distribution: Xn dix $F_X(x) \rightarrow F(x)$ for all continuity points of F_X Example: (X_n) , $X_n \in Bin(n, \frac{\lambda}{n})$, $\lambda > 0$, $\Psi_{\chi_n(6)} = (q + p e^{it})^n = (1 - \frac{\lambda}{n} + \frac{\lambda}{n} e^{it})^n$ $\frac{n-\infty}{n - \infty}$ Enler ideality $e^{\lambda (e^{i+1})} = (p_{0}(x))$ Enler: $\left(1+\frac{4}{n}\right)^n = 2$ This snggests that

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Xn -> X with X \in Po (x).

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2.

Thm (Continuity theorem) X and X, X2,

random variables and suppose that

 $\Psi_{\chi_n}(b) \longrightarrow \Psi_{\chi}(b) \quad \forall b \in \mathbb{R}$

Then $\chi_n \to \chi$ as $n \to \infty$

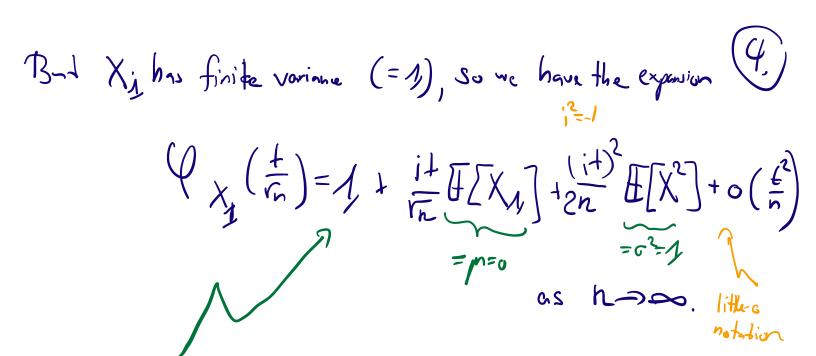
Remark: If $(\chi_n(i)) \rightarrow (f(i))$ with (f(i)) some function continuous at t=0,

than then is a random variable X such that

 $\times n \xrightarrow{d} \times and \varphi(i) = \varphi_{\chi(i)}$

The converse is also true: H Xn X, the (x10 > (x11) V6CR.

Thm (Central limit thm). (Xx) iid randon variables (3) with [[[X]= M and Var Xk=02co. $\sqrt{n}\left(\frac{1}{n}\sum_{k=1}^{n}\chi_{k}-\mu\right)\frac{d}{n}\mathcal{N}(0,0^{2}),$ or equivalently $\frac{1}{\sqrt{n}}\left(\sum_{k=1}^{n}X_{k}-n_{k}\right)\xrightarrow{d}\mathcal{N}(0,1)$ Proof: Let $Z_n := \left(\frac{2}{2} \times_{k-1}^n \right)$. It suffices to prove by the continuity theorem that $(2(1) \rightarrow (20)$ Con assum that p=0 and o=1 (why?) with ZeW1011). (1) = If [etim Z X] = If [etim X] identically distributed $= \left(\varphi_{X_{1}} \left(\frac{t}{r_{n}} \right) \right)^{n}.$



See notes about characteristic function.

Used that
$$y = e^{\log y}$$
 (here $\log = \ln n = \log (1 - \frac{\xi^2}{2n} + o(\frac{\xi^2}{n}))$).

So $\left(\varphi_{X_{1}}\left(\frac{6}{5}\right)^{n}=e^{n\left(\sqrt{1-\frac{4^{2}}{2n}}+o\left(\frac{6^{2}}{2}\right)\right)}$.

Taylor exponsion
$$\log(1+2) = 2 + o(3)$$
 as $|3| \rightarrow 0$
 $= e^{in(-\frac{\xi^2}{2n} + o(\frac{\xi^2}{n}))}$
 $= e^{-\frac{\xi^2}{2} + no(\frac{\xi^2}{n})}$.

but $n \cdot o\left(\frac{n^2}{n}\right) \rightarrow 0$ as $n \rightarrow \infty$ for any fixed b. Moreover, how e = qx(b), XeW(o,)

Exercise. Prove the following veck law of large numbers:

(Xk) iid r.v. with $\exists X_k = \mu$.

Show this iid $x = \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} = \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} = \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} = \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} \times \frac{1}{n} = \frac{1}{n} \times \frac{1}{n$