KTH Matematik

Homework 2
Mathematical Systems Theory, SF2832
Fall 2023
Passing grade: 12p

1. Consider the pair (c, A), where

$$A = \begin{bmatrix} 1 & a \\ -1 & -2 \end{bmatrix}$$
$$c = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where a is a real constant.

(a) Solve the Lyapunov equation $A^T P + PA + c^T c = 0$. .... (2p)

(b) Determine for what a the solution P is positive definite and positive semi-definite respectively. .... (2p)

2. You will in this problem derive and investigate a number of realizations for the transfer function

$$R(s) = \begin{bmatrix} \dfrac{s+2}{s+1} \\ \\ \dfrac{1}{s+3} \end{bmatrix}$$

(a) Determine the standard reachable realization of $R(s)$. .... (1p)

(b) Is the realization in (a) observable? .... (1p)

(c) Determine the standard observable realization of $R(s)$. .... (2p)

3. Two state space representations (A, B, C) and $\bar{A}, \bar{B}, \bar{C})$ are said to be equivalent if there exists a nonsingular matrix T such that

$$(\bar{A}, \bar{B}, \bar{C}) = (TAT^{-1}, TB, CT^{-1})$$

Use whatever method you can to verify if the following pairs of realizations are equivalent or not:

(a)

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \end{bmatrix} \right)$$

.... (2p)

(*b*)

$$(A, B, C) = \left(\begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \end{bmatrix} \right)$$
$$(\bar{A}, \bar{B}, \bar{C}) = \left(\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \end{bmatrix} \right)$$

. . . . (2p)

4. Consider a minimal SISO system

$$\dot{x} = Ax + bu$$
$$y = cx, \ \ x \in R^n.$$

Find further conditions on matrices A, b, c such that for any $u = kx$, the pair $(c, A + bk)$ is always observable. . . . . (3p)

5. Suppose the following is a realization of a given $R(s)$:

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \right)$$

(*a*) Find a feedback control $u = Kx$ that assigns poles to $\{-1, -2, -3, -4\}$. . (2p)

(*b*) Is the realization minimal? If not, use Kalman decomposition to find a minimal realization. . . . . (3p)