SF2940: Distributions and their characteristic functions

Distribution	Notation	Probability function	$\varphi_X(t)$
One point	$\delta(a)$, $a \in \mathbb{R}$	$p(a) = 1$	e^{ita}
Bernoulli	$\mathrm{Be}(p)$; $0 \le p \le 1$	$p(0) = q$, $p(1) = p$; $q = 1 - p$	$q + p\mathrm{e}^{\mathrm{i}t}$
Binomial	$\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$; $0 \le p \le 1$	$p(k) = \binom{n}{k} p^k q^{n-k}$, $k = 0, 1, 2, \ldots, n$; $q = 1 - p$	$(q + p\mathrm{e}^{\mathrm{i}t})^n$
Geometric	$\mathrm{Ge}(p)$, $0 \le p \le 1$	$p(k) = pq^k$, $k = 0, 1, 2, \ldots, n$; $q = 1 - p$	$\frac{p}{1-q\mathrm{e}^{\mathrm{i}t}}$
Poisson	$\mathrm{Po}(\lambda)$, $\lambda > 0$	$p(k) = \mathrm{e}^{-\lambda}\frac{\lambda^k}{k!}$, $k = 0, 1, 2, \ldots$	$\mathrm{e}^{\lambda(\mathrm{e}^{\mathrm{i}t}-1)}$

Table 1. Discrete distributions

Distribution	Notation	Density	$\varphi_X(t)$
Uniform	$\mathrm{U}(a, b)$, $a < b$	$f(x) = \frac{1}{b-a}$, $a < x < b$	$\frac{\mathrm{e}^{\mathrm{i}tb} - \mathrm{e}^{\mathrm{i}ta}}{\mathrm{i}t(b-a)}$
Exponential	$\mathrm{Exp}(\lambda)$, $\lambda > 0$	$f(x) = \frac{1}{\lambda}\mathrm{e}^{-x/\lambda}$, $x > 0$	$\frac{1}{1-\lambda\mathrm{i}t}$
Laplace	$\mathrm{L}(\lambda)$, $\lambda > 0$	$f(x) = \frac{1}{2\lambda}\mathrm{e}^{-\lvert x\rvert/\lambda}$, $x \in \mathbb{R}$	$\frac{1}{1+\lambda^2 t^2}$
Beta	$\beta(r, s)$, $r, s > 0$	$f(x) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1}(1-x)^{s-1}$, $0 < x < 1$	$*$
Normal	$\mathrm{N}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{1}{2}(x-\mu)^2/\sigma^2}$, $x \in \mathbb{R}$	$\mathrm{e}^{\mathrm{i}\mu t - \frac{1}{2}\sigma^2 t^2}$
Standard normal	$\mathrm{N}(0, 1)$	$f(x) = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}x^2}$, $x \in \mathbb{R}$	$\mathrm{e}^{-\frac{1}{2}t^2}$
Log-normal	$\mathrm{LN}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$	$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{1}{2}(\log x - \mu)^2/\sigma^2}$, $x > 0$	$*$
Cauchy	$\mathrm{C}(m, \eta)$, $\eta > 0$, $m \in \mathbb{R}$	$f(x) = \frac{1}{\pi}\frac{\eta}{(x-m)^2+\eta^2}$, $x \in \mathbb{R}$	$\mathrm{e}^{\mathrm{i}mt - \eta\lvert t\rvert}$

Table 2. Continuous distributions

$*$ indicates either a too complicated expression or no explicit expression exists.
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