Dag 10

Undersök om funktionen

$$f(x, y) = x^4 + y^4 - 2(x - y)^2$$

har några stationära punkter och om dessa i så fall är lokala extrempunkter.

$f'_x = 4x^3 - 4(x-y)$

$f'_y = 4y^3 - 4(x-y)(-1) = 4y^3 - 4(y-x)$

$$\begin{cases} x^3 - (x-y) = 0 \\ y^3 - (y-x) = 0 \end{cases}$$

addera (1) och (2)

$x^3 + y^3 = 0 \Leftrightarrow x = -y$

$x^3 - 2x = 0 \Leftrightarrow x(x^2 - 2) = 0$

$x = 0, \; x = \sqrt{2}, \; x = -\sqrt{2}$

$(0,0), \; (\sqrt{2}, -\sqrt{2}), \; (-\sqrt{2}, \sqrt{2})$

$Q = f''_{xx}(a,b)h^2 + 2f''_{xy}(a,b)hk + f''_{yy}(a,b)k^2$

$f''_{xx} = 12x^2 - 4, \quad f''_{yy} = 12y^2 - 4$

$f''_{xy} = 4$

$(\sqrt{2}, -\sqrt{2})$ $\quad Q = 20h^2 + 8hk + 20k^2$

$20(h^2 + \frac{2}{5}hk + k^2) =$

$20\left((h + \frac{1}{5}k)^2 + \frac{24}{25}k^2\right)$

positiva koeff. $\Rightarrow$ pos. def.

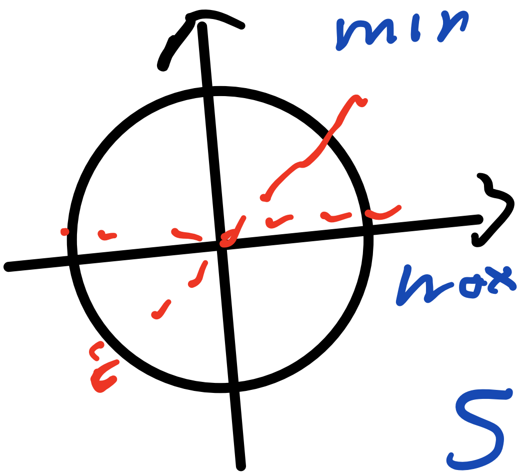
$\Rightarrow (\sqrt{2}, -\sqrt{2})$ min punkt.

$(-\sqrt{2}, \sqrt{2})$ $\quad Q = 20h^2 + 8hk + 20k^2$

pos. def. min punkt.

$(0,0)$ $\quad Q = -4h^2 + 8hk - 4h^2$

$= -4(h-k)^2$ neg sem. def.

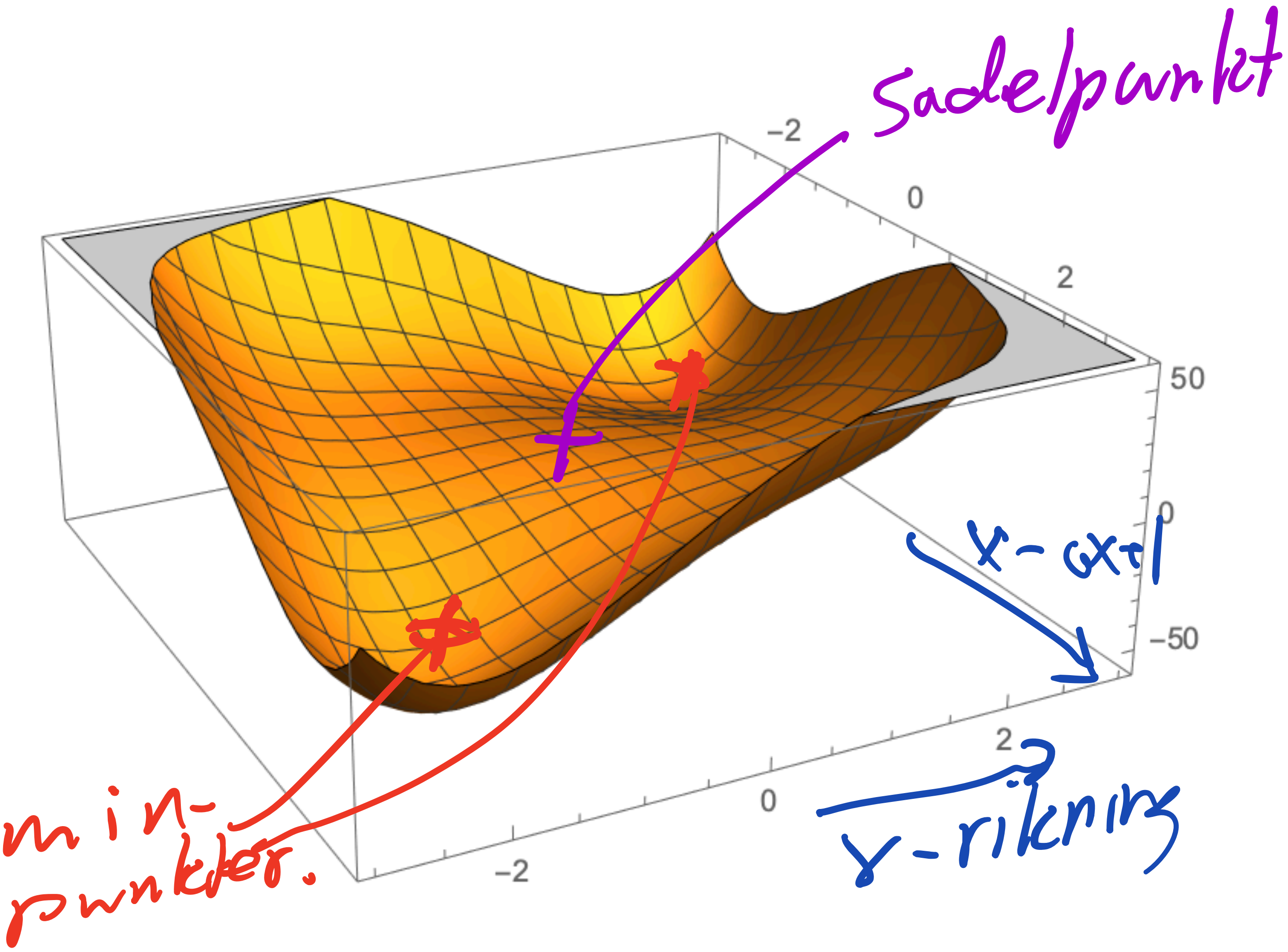


$f(t,t) = 2t^4 \geq 0$

$f(t,0) = t^4 - 2t^2$

max i origo.

Sadelpunkt.



Bestäm karaktären hos följande kvadratiska former:

$$Q_1 = 2h^2 + 9k^2 + 5l^2 - 4hk + 2kl + 4hl,$$

$$Q_2 = k^2 - l^2 + 2hk - 3kl + 4hl.$$

$$Q_1 = 2(h^2 - 2hk + 2hl) + 9k^2 + 5l^2 + 2kl$$

$$= 2(h - k + l)^2 + 7k^2 + 3l^2 + 6kl$$

$$= 2(h-k+l)^2 + 7\left(k^2 + \tfrac{3}{7}l^2 + \tfrac{6}{7}kl\right)$$

$$= 2(h-k+l)^2 + 7\left(\left(k + \tfrac{3}{7}l\right)\right)^2 + \tfrac{12}{9}l^2$$

all koeff positive

pos. def. form

Q_2 k^2 och l^2 har olika tecken.

$Q_2(0,k,0) = k^2$

$Q_2(0,0,l) = -l^2$

Q är indefinit

$\geq 0 \quad > 0$

Bestäm alla stationära punkter till funktionen $f(x, y, z) = (x^2 + y^2 + z^2)e^{-\frac{2}{3}(x+y+z)}$.

$$f_x' = 2xe^{-\frac{2}{3}(x+y+z)} - \frac{2}{3}(x^2+y^2+z^2)e^{-\frac{2}{3}(x+y+z)}$$

$$= \left(2x - \frac{2}{3}(x^2+y^2+z^2)\right)e^{-\frac{2}{3}(x+y+z)}$$

$$f_y' = \left(2y - \frac{2}{3}(x^2+y^2+z^2)\right)e^{-\frac{2}{3}(x+y+z)}$$

$$f_z' = \left(2z - \frac{2}{3}(x^2+y^2+z^2)\right)e^{-\frac{2}{3}(x+y+z)}$$

$$\begin{cases} 2x - \frac{2}{3}(x^2+y^2+z^2) = 0 \\ 2y - \frac{2}{3}(x^2+y^2+z^2) = 0 \\ 2z - \frac{2}{3}(x^2+y^2+z^2) = 0 \end{cases}$$

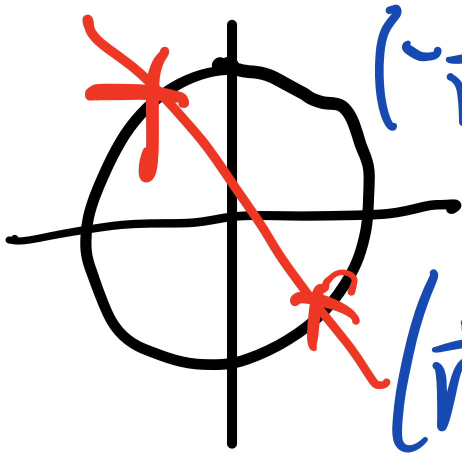
$$x = y = z$$

$$2x - \frac{2}{3}(x^2+x^2+x^2) = 0$$

$$2x - 2x^2 = 0 \qquad x = 0, \quad x = 1$$

$$(0,0,0) \qquad (1,1,1)$$

$$\begin{cases} x^2 + y^2 = 1 \\ x + y = 0 \end{cases}$$



$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Vad har $(0,0,0)$ för karaktär.

$f \geq 0$ överallt! $f(0,0,0) = 0$

$(0,0,0)$ är en global minpunkt.

Alltså speciellt en lokal minpunkt.