

Assignment TentaAnalys_210518 due 05/18/2021 at 02:00pm CEST

(a) Beräkna gränsvärdet (2 poäng):

$$\lim_{n \rightarrow \infty} (\sqrt{3n^2 + n} - \sqrt{3n^2 - 4n}).$$

Gränsvärdet är: ____.

(b) Beräkna gränsvärdet (3 poäng):

$$\lim_{x \rightarrow 0} \frac{\arctan(9x^2) - (\arctan(3x))^2}{x^4}.$$

Gränsvärdet är: ____.

Answer(s) submitted:

- 0
- 54

(score 0.5)

Undersök lokala och globala extremvärden, konvexitetsegenskaper och asymptoter till funktionen $f(x) = -|x| + 2\arctan(1/x)$, samt skissera grafen. Bestäm även funktionens värdemängd. (5 poäng):

Funktionen har ett lokalt extremvärde i punkten $x =$ ____.

Asymptoten då $x \rightarrow \infty$ är $y =$ ____.

Asymptoten då $x \rightarrow -\infty$ är $y =$ ____.

Answer(s) submitted:

- -1
- -x
- x

(correct)

Bestäm största och minsta värde till funktionen $f(x, y) = x^2 + 2y^2 + 4y$ i området $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 8\}$. Ange även i vilka punkter respektive extremvärde antas. (5 poäng):

Minsta värdet är: ____, största värdet är: ____.

Answer(s) submitted:

- -2
- $16 + 4\sqrt{8}$

(correct)

Beräkna dubbelintegralen $\iint_D e^{3-y^2} dx dy$, där $D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq y \leq 2 \text{ och } y \geq |x|\}$. (5 poäng)

Dubbelintegralens värde är: ____.

Answer(s) submitted:

- $e^2 - e^{-1}$

(correct)

(a) Beräkna volymen av den rotations kropp som erhålls om det ändliga område som begränsas av koordinataxlarna, kurvan $y = \frac{1}{\sqrt{1+x^2/2}}$ och linjen $x = \sqrt{2}$ roterar kring x-axeln. (3 poäng):

Volymen är: ____.

(b) Beräkna volymen av den rotations kropp som fås om området i (a) istället roterar kring y-axeln. (2 poäng):

Volymen är: ____.

Answer(s) submitted:

- $\sqrt{2} \pi \arctan(1)$
- $4\pi(\sqrt{2} - 1)$

(correct)

(a) Bestäm den lösning till differentialekvationen $y' + xy = x^3$ som uppfyller villkoret $y(0) = 1$ (2.5 poäng).

Lösningen till begynnelsevärdesproblemet är $y(x) =$ ____.

(b) Bestäm den lösning till differentialekvationen $y' - xy + xy^2 = 0$ som uppfyller villkoret $y(0) = 3$ (2.5 poäng).

Lösningen till begynnelsevärdesproblemet är $y(x) =$ ____.

Answer(s) submitted:

- $x^2 - 2 + 3e^{-(x^2/2)}$
- $(3e^{(x^2/2)}) / (3e^{(x^2/2)} - 2)$

(correct)

Fragebogennummer: 403-0028-KDM

$$\textcircled{1} \lim_{n \rightarrow \infty} (\sqrt{3n^2+n} - \sqrt{3n^2-4n}) = \lim_{n \rightarrow \infty} n \left(\sqrt{3+\frac{1}{n}} - \sqrt{3-\frac{4}{n}} \right) =$$

$$\approx \lim_{n \rightarrow \infty} n(\sqrt{3} - \sqrt{3}) = \lim_{n \rightarrow \infty} n \cdot 0 = \underline{\underline{0}}$$

$$\textcircled{b} \lim_{x \rightarrow 0} \frac{\arctan(9x^2) - (\arctan(3x))^2}{x^4}$$

$x \rightarrow 0 \Leftrightarrow$ Maclaurin-Entwicklung $f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 \dots$

$$\arctan t = t - \frac{t^3}{3} + \frac{t^5}{5} + O(t^7) \Leftrightarrow \arctan(9x^2) = 9x^2 - \frac{9^3 x^6}{3} + O(x^{10})$$

$$\frac{1-r^n}{1-r} = \sum_{k=0}^{n-1} r^k \Leftrightarrow \frac{1}{1-r} = \frac{r^n}{1-r} + \sum_{k=0}^{n-1} r^k \quad r = -t^2 \quad (\arctan(3x))^2 = \left(3x - \frac{3^3 x^3}{3} + O(x^5)\right)^2 =$$

$$= 9x^2 - 2 \cdot 3^3 x^4 + 3^4 x^6 + O(x^{10})$$

$$\frac{1}{1+t^2} = 1 - t^2 + t^4 - \dots$$

$$\lim_{x \rightarrow 0} \frac{\arctan(9x^2) - (\arctan(3x))^2}{x^4} = \lim_{x \rightarrow 0} \frac{9x^2 - \frac{9^3 x^6}{3} + O(x^{10}) - (9x^2 - 2 \cdot 3^3 x^4 + 3^4 x^6 + O(x^{10}))}{x^4} =$$

$$= \lim_{x \rightarrow 0} \left(\frac{x^6 \left(-\frac{9^3}{3} - 9^2\right) + 2 \cdot 3^3 x^4 + O(x^{10})}{x^4} \right) = \lim_{x \rightarrow 0} \left(\frac{x^2 \left(-\frac{9^3}{3} - 9^2\right) + 2 \cdot 3^3 + O(x^6)}{1} \right) =$$

$$= 2 \cdot 3^3 = 2 \cdot 27 = 54$$

$$\textcircled{2} f(x) = -|x| + 2 \arctan(1/x) \quad f'(x) \text{ undefiniert nat } x=0$$

$$x > 0: f(x) = -x + 2 \arctan(1/x)$$

$$x < 0: f(x) = x + 2 \arctan(1/x)$$

$$f'(x) = -1 - \frac{2}{x^2+1}$$

$$f'(x) = 1 - \frac{2}{x^2+1}$$

$$f''(x) = \frac{4x}{(x^2+1)^2} = 0, x=0 \text{ def.}$$

$$f''(x) = \frac{4x}{(x^2+1)^2} = 0, x=0 \text{ def.}$$

$$f'(x) = 0 \Leftrightarrow -(x^2+1) = 2 \Leftrightarrow -x^2-1-2=0 \Leftrightarrow$$

$$f'(x) = 0 \Leftrightarrow x^2+1 = 2 \Leftrightarrow x^2-1=0$$

$$\Leftrightarrow x^2+3=0 \text{ keine reellen Lsg.}$$

$$x = \pm 1, -1$$

$$f'(x) < 0$$

$$\lim_{x \rightarrow \infty} (-x + 2 \arctan(1/x)) =$$

$$= \lim_{x \rightarrow \infty} (-x + 0)$$

$$\lim_{x \rightarrow -\infty} (x + 2 \arctan(1/x)) =$$

$$= \lim_{x \rightarrow -\infty} (x + 0)$$

Asymptot

$$\lim_{x \rightarrow 0^+} (-x + 2 \arctan(1/x))$$

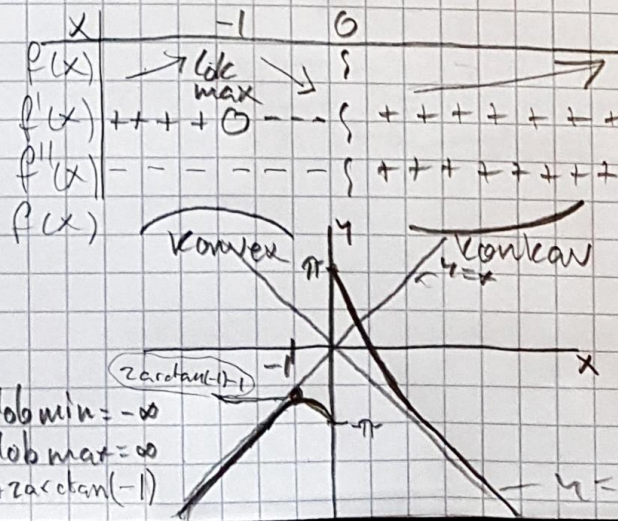
$$= 0 + 2 \cdot \frac{\pi}{2} = \pi$$

$$f(x) = 0 \tan x/2 + 1/x = 0$$

$$\lim_{x \rightarrow 0} (x + 2 \arctan(1/x)) =$$

$$= 0 - 2 \cdot \frac{\pi}{2} = -\pi$$

Glob. min = $-\infty$
 Glob. max = ∞
 Lok. max = $-1 + 2 \arctan(-1)$



③ $f(x, y) = x^2 + 2y^2 + 4y$

403-0028-KDM - Circlekriter!
 $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 8\} \quad r = \sqrt{8}$

- i) Randpunkte
~~ii) Punkte der Parabel~~
 iii) interne Extrempunkte
 d.h. $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$

iii) $\frac{\partial f}{\partial x} = 2x = 0, x = 0$

$\frac{\partial f}{\partial y} = 4y + 4 = 0, y = -1$

$f(0, -1) = 0 + 2 - 4 = \underline{\underline{-2}}$
min

i) $\begin{cases} x = \sqrt{8} \cos t \\ y = \sqrt{8} \sin t \end{cases}$

$h(t) = f(\sqrt{8} \cos t, \sqrt{8} \sin t) =$
 $= 8 \cos^2 t + 2 \cdot 8 \sin^2 t + 4 \cdot \sqrt{8} \sin t =$
 $= 8 + 8 \sin^2 t + 4 \cdot \sqrt{8} \sin t$

$4\sqrt{8} = \frac{\sqrt{2}}{2} \cdot 4\sqrt{8} = \frac{4 \cdot 4}{\sqrt{2}} = \frac{16}{\sqrt{2}} = 8\sqrt{2}$

$h'(t) = 16 \sin t \cos t + 4\sqrt{8} \cos t = 0 \Leftrightarrow$
 $\Leftrightarrow \cos t = 0, t = \pi/2 + \pi \cdot n \quad n \in \mathbb{R}$
 $\frac{16 \sin t \cos t}{4\sqrt{8} \cos t} = -1$

$\Leftrightarrow \frac{4}{\sqrt{8}} \sin t = -1 \Leftrightarrow \sin t = -\frac{\sqrt{8}}{4}$

$-\frac{\sqrt{8}}{4} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{6}}{4} = -\frac{1}{\sqrt{2}} \quad t = 5\pi/4, 7\pi/4 + 2\pi \cdot n$

$h(5\pi/4) = 8 + 8 \sin^2(5\pi/4) + 4\sqrt{8} \sin(5\pi/4) =$
 $= 8 + 8 \cdot (-\frac{1}{\sqrt{2}})^2 - 8\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 8 + 4 - 8 = 4$

$h(7\pi/4) = h(5\pi/4) = 4$

$h(\pi/2) = 8 + 8 \sin^2(\pi/2) + 4\sqrt{8} \sin(\pi/2) =$
 $= 8 + 8 + 4\sqrt{8} = \underline{\underline{16 + 4\sqrt{8}}}$

$h(3\pi/2) = 8 + 8 - 4\sqrt{8}$ max

(x, y)	$(0, -1)$	$(0, \sqrt{8})$
$f(x, y)$	-2 min	$16 + 4\sqrt{8}$ max

④ $\iint_D e^{3-y^2} dx dy$

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 $D = \{(x,y) \in \mathbb{R}^2 \mid 1 \leq y \leq 2, y \geq |x|\}$

$$\begin{aligned} \int_{-1}^1 \int_1^2 (e^{3-y^2}) dx dy &= \int_1^2 (e^{3-y^2} \int_{-y}^y dx) dy = \int_1^2 (e^{3-y^2} [x]_{-y}^y) dy = \\ &= \int_1^2 (e^{3-y^2} (2y)) dy = - \int_1^2 (-2y e^{3-y^2}) dy = - [e^{3-y^2}]_1^2 = \\ &= - (e^{3-4} - e^{3-1}) = - (e^{-1} - e^2) = \underline{\underline{e^2 - e^{-1}}} \end{aligned}$$

⑤ $y = \frac{1}{\sqrt{1+x^2/2}}$

$x = \sqrt{2}$

a) Rot. um x -achse: $V = \pi \int_0^{\sqrt{2}} \left(\left(\frac{1}{\sqrt{1+x^2/2}} \right)^2 \right) dx = \pi \int_0^{\sqrt{2}} \left(\frac{1}{1+x^2/2} \right) dx =$
 $= \left[t = \frac{x}{\sqrt{2}} \right] = \sqrt{2} \pi \int_0^1 \left(\frac{1}{1+t^2} \right) dt = \sqrt{2} \pi [\arctan t]_0^1 =$
 $= \underline{\underline{\sqrt{2} \pi \arctan(1)}}$

b) Rot. um y -achse:

$V = \pi \int_{-1}^1 \left(\left(\frac{1}{\sqrt{1+x^2/2}} \right)^2 \right) dy = \pi \int_{-1}^1 \left(\frac{1}{1+x^2/2} \right) dy = \pi \int_{-1}^1 \left(\frac{1}{1+x^2/2} \right) dy =$
 $= \pi \left[-\frac{1}{1+x^2/2} \right]_{-1}^1 = \pi \left(-\frac{1}{1+1/2} - \left(-\frac{1}{1+1/2} \right) \right) =$

$dV = 2\pi x y dx = 2\pi x \cdot \frac{1}{\sqrt{1+x^2/2}} dx =$
 $= 2\pi \frac{x}{\sqrt{1+x^2/2}} dx$
 $V = 2\pi \int_0^{\sqrt{2}} \left(\frac{x}{\sqrt{1+x^2/2}} \right) dx = \left[t = \frac{x^2}{2} \right] = 2\pi \int_0^1 \left(\frac{1}{\sqrt{1+t}} \right) dt =$
 $= 2\pi \left[\frac{\sqrt{1+t}}{1/2} \right]_0^1 = \underline{\underline{4\pi(\sqrt{2}-1)}}$

⑥ a) $y' + \frac{y}{x} = x^3$ $y(0) = 1$ $\int P_5 = \int 5 - \int 1 \cdot 5$ 403-0028-KDM
 $G(x) = x^{3/2}$

IF = $e^{\int \frac{1}{x} dx} = e^{x^{3/2}}$
 $y = \frac{1}{e^{x^{3/2}}} \int e^{x^{3/2}} \cdot x^3 dx = \frac{1}{e^{x^{3/2}}} \left(e^{x^{3/2}} x^2 - \int e^{x^{3/2}} \cdot 2x dx \right) =$
 $= e^{-x^{3/2}} \left(e^{x^{3/2}} x^2 - 2(e^{x^{3/2}}) \right) + C = x^2 - 2 + C e^{-x^{3/2}}$

$y(0) = -2 + C = 1 \Rightarrow C = 3$, $y = x^2 - 2 + 3e^{-x^{3/2}}$

b) $y' - xy + xy^2 = 0$ $y(0) = 3$ separabel.

$\frac{dy}{dx} - xy + xy^2 = 0 \Leftrightarrow dy - x dx (y + y^2) = 0 \Leftrightarrow \frac{1}{y+y^2} dy = x dx$

$\int \frac{1}{y+y^2} dy = \int x dx$

$\int \frac{1}{y^2+y} = \frac{A}{y} + \frac{B}{y+1}$ $1 = A \quad B = -1$

$\frac{1}{y(y+1)} = \frac{1}{y} - \frac{1}{y+1}$
 $\int \frac{1}{y+y^2} dy = \int \left(\frac{1}{y} - \frac{1}{y+1} \right) dy = \ln y - \ln(y+1) + C$
 $= \ln \left(\frac{y}{y+1} \right) + C$

$\int x dx = \frac{x^2}{2}$ $\frac{y}{y+1} = e^{\frac{x^2}{2} + C} = C_2 e^{\frac{x^2}{2}}$ $y = (y+1) C_2 e^{\frac{x^2}{2}}$

$y(1 - C_2 e^{\frac{x^2}{2}}) = C_2 e^{\frac{x^2}{2}}$ $y = \frac{C_2 e^{\frac{x^2}{2}}}{1 - C_2 e^{\frac{x^2}{2}}}$ $y(0) = \frac{C_2}{1 - C_2} = 3$ $C_2 = 3 - 3C_2$

$y = \frac{3/2 e^{\frac{x^2}{2}}}{1 - 3/2 e^{\frac{x^2}{2}}}$

$= \frac{-3}{2} \frac{e^{\frac{x^2}{2}}}{1 - 3/2 e^{\frac{x^2}{2}}} = \frac{3e^{\frac{x^2}{2}}}{3e^{\frac{x^2}{2}} - 2}$

$-2C_2 = 3$
 $C_2 = -3/2$

①

lim
n→∞

$$\left(\sqrt{3n^2 + n} + \sqrt{3n^2 - 4n} \right)^2 = 3n^2 + n - 2 \cdot \sqrt{(3n^2 + n) \cdot (3n^2 - 4n)} =$$

$$= 6n^2 - 3n - 2 \sqrt{9n^4 - 12n^3 + 3n^2 - 5n^2} =$$

$$= 6n^2 - 3n - 2 \sqrt{9n^2 - 9n - 5} = 6n^2 - 3n - 6n \sqrt{n^2 - n - 5/9} =$$

=