Anonymiseringskod: 403-0028-KDM

1) a) $\lim_{n\to\infty}\left(\sqrt{3n^2+n}-\sqrt{3n^2-4n}\right)=\lim_{n\to\infty} n\left(\sqrt{3+\underbrace{1/n}_{\to 0}}-\sqrt{3-\underbrace{4/n}_{\to 0}}\right)=$

$=\lim_{n\to\infty} n(\sqrt{3}-\sqrt{3})=\lim_{n\to\infty} n\cdot 0=\underline{\underline{0}}$

b) $\lim_{x\to 0}\dfrac{\arctan(9x^2)-(\arctan(3x))^2}{x^4}$

$x\to 0 \Leftrightarrow$ Maclaurinutveckling $f(0)+f'(0)(x)+f''(0)(x)^2\ldots$

$\arctan t = t-\frac{t^3}{3}+\frac{t^5}{5}+O(t^7) \Leftrightarrow \arctan(9x^2)=9x^2-\frac{9^3x^6}{3}+O(x^{10})$

$\frac{1-r^n}{1-r}=\sum_{k=0}^{n-1}(r^k) \Leftrightarrow \frac{1}{1-r}=\frac{r^n}{1-r}+\sum_{k=0}^{n-1}(r^k) \quad r=-t^2$

$\frac{1}{1+t^2}=1-t^2+t^4\ldots$

$\arctan t = t-\frac{t^3}{3}+\frac{t^5}{5}+O(t^7)$

$(\arctan(3x))^2=\left(3x-\frac{3^3x^3}{3}+O(x^5)\right)^2=$

$=9x^2-2\cdot 3^3x^4+3^4x^6+O(x^{10})$

$\lim_{x\to 0}\dfrac{\arctan(9x^2)-(\arctan(3x))^2}{x^4}=\lim_{x\to 0}\left(\dfrac{\cancel{9x^2}-\frac{9^3x^6}{3}+O(x^{10})-(\cancel{9x^2}-2\cdot 3^3x^4+3^4x^6+O(x^{10}))}{x^4}\right)=$

$=\lim_{x\to 0}\left(\dfrac{x^6(-\frac{9^3}{3}-9^2)+2\cdot 3^3x^4+O(x^{10})}{x^4}\right)=\lim_{x\to 0}\left(\dfrac{\overbrace{x^2(-\frac{9^3}{3}-9^2)}^{\to 0}+2\cdot 3^3+\overbrace{O(x^6)}^{\to 0}}{1}\right)=$

$=2\cdot 3^3=2\cdot 27=54$

2) $f(x)=-|x|+2\arctan(1/x)$ $\quad f'(x)$ odefinierad när $x=0$

$x>0: f(x)=-x+2\arctan(1/x)$

$f'(x)=-1-\frac{2}{x^2+1}$

$f''(x)=\frac{4x}{(x^2+1)^2}=0, x=0$ odef.

$f'(x)=0 \Leftrightarrow -(x^2+1)=2 \Leftrightarrow -x^2-1-2=0 \Leftrightarrow$

$\Leftrightarrow x^2+3=0$ icke reella rötter

$f'(x)<0$

$x<0: f(x)=x+2\arctan(1/x)$

$f'(x)=1-\frac{2}{x^2+1}$

$f''(x)=\frac{4x}{(x^2+1)^2}=0, x=0$ odef.

$f'(x)=0 \Leftrightarrow x^2+1=2 \Leftrightarrow x^2-1=0$

$x=(1), -1$

| $x$ | | $-1$ | | $0$ | |
|---|---|---|---|---|---|
| $f(x)$ | $\nearrow$ | lok max | $\searrow$ | | $\nearrow$ |
| $f'(x)$ | $++++$ | $0$ | $---$ | | $+++++++$ |
| $f''(x)$ | $-----$ | | $-$ | | $+++++++$ |
| $f(x)$ | konvex | | | | konkav |

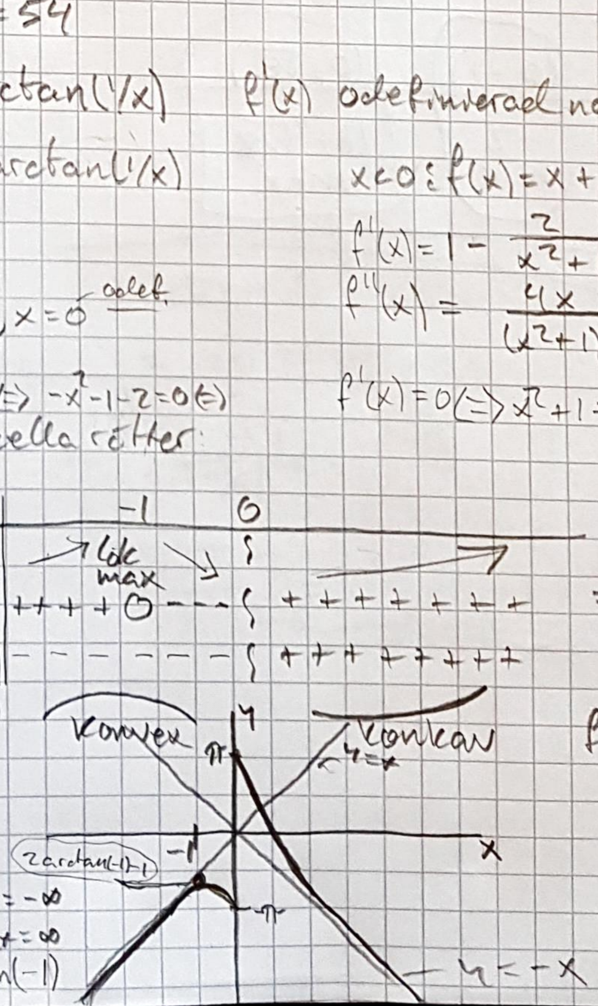
$\lim_{x\to\infty}(-x+2\arctan(1/x))=$
$=\lim_{x\to\infty}(-x+0)$
Asymptot

$\lim_{x\to -\infty}(x+2\arctan(1/x))=$
$=\lim_{x\to\infty}(x+0)$
Asymptot

$\lim_{x\to 0^+}(-x+2\arctan(1/x))$
$=0+2\cdot\pi/2=\pi$

$f(x)=0 \quad \tan x/2+1/x=0$

$\lim_{x\to 0^-}(x+2\arctan 1/x)=$
$=0-2\cdot\pi/2=\underline{-\pi}$




Glob min $=-\infty$
Glob max $=\infty$
Lokmax $=-1+2\arctan(-1)$

403-0028-KDM — Cirkelskiva!

③ $f(x,y) = x^2 + 2y^2 + 4y$

$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 8\}$ $r = \sqrt{8}$

i) Randpunkter
~~ii) punkter där f ej är deriverbar~~
iii) inre extrempunkter
i.e. $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$

i) $\begin{cases} x = \sqrt{8}\cos t \\ y = \sqrt{8}\sin t \end{cases}$

$h(t) = f(\sqrt{8}\cos t, \sqrt{8}\sin t) =$

$= 8\cos^2 t + 2 \cdot 8\sin^2 t + 4 \cdot \sqrt{8}\sin t =$

$= 8 + 8\sin^2 t + 4 \cdot \sqrt{8}\sin t$

$4\sqrt{8} = \frac{\sqrt{2}}{\sqrt{2}} 4\sqrt{8} = \frac{4 \cdot 4}{\sqrt{2}} = \frac{\sqrt{2} \cdot 16}{2} = 8\sqrt{2}$

$h'(t) = 16\sin t\cos t + 4\sqrt{8}\cos t = 0 \Leftrightarrow$

$\Leftrightarrow \cos t = 0$, $t = \pi/2 + \pi \cdot n$ $n \in \mathbb{R}$

, $\frac{16\sin t\cancel{\cos t}}{4\sqrt{8}\cancel{\cos t}} = -1$

$\Leftrightarrow \frac{4}{\sqrt{8}}\sin t = -1 \Leftrightarrow \sin t = -\frac{\sqrt{8}}{4}$

$-\frac{\sqrt{8}}{4} = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{16}}{4} = \frac{-1}{\sqrt{2}}$ $t = 5\pi/4, 7\pi/4$ $+2\pi \cdot n$

$h(5\pi/4) = 8 + 8\sin^2(5\pi/4) + 4\sqrt{8}\sin(5\pi/4) =$
$= 8 + 8 \cdot (-\frac{1}{\sqrt{2}})^2 - 8\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 8 + 4 - 8 = 4$

$h(7\pi/4) = h(5\pi/4) = 4.$

$h(\pi/2) = 8 + 8\sin^2(\pi/2) + 4\sqrt{8}\sin(\pi/2) =$
$= 8 + 8 + 4\sqrt{8} = \underline{\underline{16 + 4\sqrt{8}}}$
max

$h(3\pi/2) = 8 + 8 - 4\sqrt{8}$

iii) $\frac{\partial f}{\partial x} = 2x = 0$, $x = 0$

$\frac{\partial f}{\partial y} = 4y + 4 = 0$, $y = -1$

$f(0,-1) = 0 + 2 - 4 = \underline{\underline{-2}}$
min

| $(x,y)$ | $(0,-1)$ | $(0,\sqrt{8})$ |
|---|---|---|
| $f(x,y)$ | $-2$ min | $16+4\sqrt{8}$ max |

403-0028-KDM

④ $\iint_D e^{3-y^2}\,dx\,dy \qquad D=\{(x,y)\in\mathbb{R}^2 \mid 1\le y\le 2,\ y\ge |x|\}$

$|x|\le y \Leftrightarrow -y\le x\le y$

$$\int_1^2\int_{-y}^{y}\left(e^{3-y^2}\right)dx\,dy=\int_1^2\left(e^{3-y^2}\int_{-y}^{y}dx\right)dy=\int_1^2\left(e^{3-y^2}\Big[x\Big]_{-y}^{y}\right)dy=$$

$$=\int_1^2\left(e^{3-y^2}(2y)\right)dy=-\int_1^2\left(-2y\,e^{3-y^2}\right)dy=-\Big[e^{3-y^2}\Big]_1^2=$$

$$=-\left(e^{3-4}-e^{3-1}\right)=-\left(e^{-1}-e^{2}\right)=\underline{\underline{e^{2}-e^{-1}}}$$

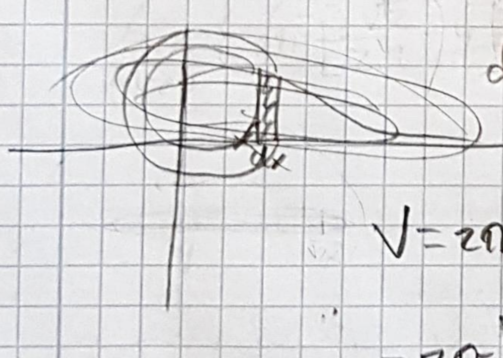
⑤ $y=\dfrac{1}{\sqrt{1+x^2/2}} \qquad x=\sqrt{2}$

a) Rot. kring x-axeln: $V=\pi\int_0^{\sqrt{2}}\left(\left(\frac{1}{\sqrt{1+\frac{x^2}{2}}}\right)^2\right)dx=\pi\int_0^{\sqrt{2}}\left(\frac{1}{1+\frac{x^2}{2}}\right)dx=$

$$=\begin{bmatrix}t=\frac{x}{\sqrt{2}}\\ dt=\frac{1}{\sqrt{2}}dx\end{bmatrix}=\sqrt{2}\pi\int_0^1\left(\frac{1}{1+t^2}\right)dt=\sqrt{2}\pi\Big[\arctan t\Big]_0^1=$$

$$=\underline{\underline{\sqrt{2}\,\pi\arctan(1)}}$$

b) Rot. kring y-axeln:



$$dV=2\pi x y\,dx=2\pi x\cdot\frac{1}{\sqrt{1+x^2/2}}\,dx=$$

$$=2\pi\frac{x}{\sqrt{1+x^2/2}}\,dx$$

$$V=2\pi\int_0^{\sqrt{2}}\left(\frac{x}{\sqrt{1+x^2/2}}\right)dx=\begin{bmatrix}t=\frac{x^2}{2}\\ dt=x\,dx\end{bmatrix}=2\pi\int_0^1\left(\frac{1}{\sqrt{1+t}}\right)dt=$$

$$=2\pi\left[\frac{\sqrt{1+t}}{1/2}\right]_0^1=\underline{\underline{4\pi\left(\sqrt{2}-1\right)}}$$

403-0028-KDM

(6) a) $\underset{g(x)}{y'+xy}=x^3 \quad y(0)=1 \qquad \int f'g = fg - \int fg'$

$G(x)=x^2/2$

$I.F = e^{x^2/2}$

$y = \frac{1}{e^{x^2/2}}\int e^{x^2/2}\cdot x^3 dx = e^{-x^2/2}\left(e^{x^2/2}x^2 - \int e^{x^2/2}\cdot 2x\,dx\right) =$

$= e^{-x^2/2}\left(e^{x^2/2}x^2 - 2\left(e^{x^2/2}\right) + C\right) = x^2 - 2 + Ce^{-x^2/2}$

$y(0) = -2 + C = 1 \Leftrightarrow C = 3, \quad \underline{\underline{y = x^2 - 2 + 3e^{-x^2/2}}}$

b) $y' - xy + xy^2 = 0 \qquad y(0) = 3 \qquad$ separabel.

$\frac{dy}{dx} - xy + xy^2 = 0 \Leftrightarrow dy - x\,dx\,(y+y^2) = 0 \Leftrightarrow \frac{1}{y+y^2}dy = x\,dx$

$\int \frac{1}{y+y^2}dy = \int x\,dx$

$\frac{1}{y^2+y} = \frac{A}{y} + \frac{B}{y+1} \qquad 1 = A \quad B = -1$

$\frac{1}{y(y+1)}$

$\int \frac{1}{y+y^2}dy = \int\left(\frac{1}{y} - \frac{1}{y+1}\right)dy = \ln y - \ln(y+1) + C$

$= \ln\left(\frac{y}{y+1}\right) + C$

$\int x\,dx = \frac{x^2}{2} \qquad \frac{y}{y+1} = e^{x^2/2 + C_1} = C_2 e^{x^2/2} \qquad y = (y+1)C_2 e^{x^2/2}$

$y - yC_2 e^{x^2/2} = C_2 e^{x^2/2}$

$y(1 - C_2 e^{x^2/2}) = C_2 e^{x^2/2} \quad y = \frac{C_2 e^{x^2/2}}{1 - C_2 e^{x^2/2}} \qquad y(0) = \frac{C_2}{1-C_2} = 3 \qquad C_2 = 3 - 3C_2$

$-2C_2 = 3$

$C_2 = -3/2$

$y = \frac{3/2\, e^{x^2/2}}{1 + 3/2\, e^{x^2/2}} =$

$= \frac{-3}{2}\,\frac{e^{x^2/2}}{1 + 3/2\, e^{x^2/2}} = \underline{\underline{\frac{3e^{x^2/2}}{3e^{x^2/2} - 2}}}$

① $\lim\limits_{n\to\infty} \left(\sqrt{3n^2+n} - \sqrt{3n^2-4n}\right)^2 = 3n^2+n-2\cdot\sqrt{(3n^2+n)\cdot(3n^2-4n)}+3n^2-4n =$

$= 6n^2-3n-2\sqrt{9n^4-12n^3+3n^3-5n^2} =$

$= 6n^2-3n-2n\sqrt{9n^2-9n-5} = 6n^2-3n-6n\sqrt{n^2-n-5/9} =$

$=$