Dag 6

Lämna gärna in Bonus något tidigare!

$f(x,y) = \frac{x^2}{1+x^2+y^2}$ $\lim_{x^2+y^2 \to \infty} \frac{x^2}{1+x^2+y^2}$

$f(0,t) = \frac{0^2}{1+0^2+t^2} = 0$ $\lim_{t\to\infty} f(0,t) = 0$

$f(t,t) = \frac{t^2}{1+t^2+t^2} \to \frac{1}{2}$ när $t \to \infty$

$\lim_{t\to\infty} f(0,t) \neq \lim_{t\to\infty} f(t,t)$ gränsvärde saknas

$\lim_{(x,y)\to\infty}$ $\lim_{x^2+y^2\to\infty}$

$$f(x,y) = \frac{x^3 + y^2}{x^4 + y^4}$$

$$\lim_{x^2+y^2\to\infty} \frac{x^3 + y^3}{x^4 + y^4} = 0 \qquad \begin{cases} x = r\cos t \\ y = r\sin\theta \end{cases}$$

$$\frac{x^3 + y^3}{x^4 + y^4} = \frac{(r\cos\theta)^3 + (r\sin\theta)^3}{(r\cos\theta)^4 + (r\sin\theta)^4} = \underbrace{\frac{1}{r}}_{\to 0} \underbrace{\frac{\cos^3\theta + \sin^3\theta}{\cos^4\theta + \sin^4\theta}}_{\text{begr.}}$$

$$|\cos^3\theta + \sin^3\theta| \le 1 + 1 = 2$$

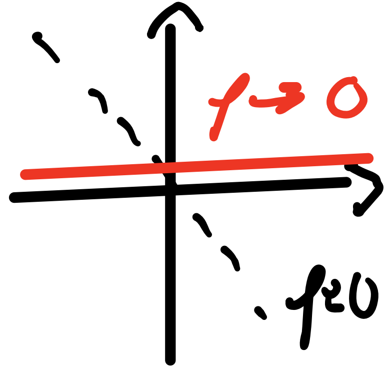
$$\cos^4\theta + \sin^4\theta \ge \frac{1}{4} \quad \left(\ge \frac{1}{2}\right)$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left|\frac{\cos^3\theta + \sin^3\theta}{\cos^4\theta + \sin^4\theta}\right| \le \frac{2}{\frac{1}{4}} = 8.$$

$$f(x,y) = \frac{\sin^2(x+y)}{\sin^2 x + \sin^2 y}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin^2(x+y)}{\sin^2 x + \sin^2 y}$$



$x+y=0$: $f = 0$

$$\begin{cases} x = t \\ y = -t \end{cases} \qquad \lim_{t\to 0} f(t,-t) = \underline{\underline{0}}$$

x-axeln

$$\begin{cases} x = t \\ y = 0 \end{cases} \qquad f(t,0) = \frac{\sin^2 t}{\sin^2 t} = 1 \qquad \lim_{t\to 0} f(t,0) = \underline{\underline{1}}$$

Limes saknas

$$f(x,y) = e^{-e^{xy}} \qquad \lim_{x^2+y^2\to\infty} e^{-e^{xy}}$$

$x = y$

$$\begin{cases} x = t \\ y = t \end{cases} \qquad f(t,t) = e^{-e^{t^2}} \to 0, \quad t \to \infty$$

$x = -y$

$$\begin{cases} x = t \\ y = -t \end{cases} \qquad f(t,-t) = e^{-\overset{\to 0}{e^{-t^2}}} \to 1$$

Limes saknas.

$$\lim_{x^2+y^2\to\infty} \frac{\ln(1+x^4-x^2y^2+y^4)}{\ln(1+x^4+x^2y^2+y^4)}$$

$$\lim_{x^2+y^2\to\infty} \frac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^4}$$ existerar inte.

$$\boxed{-\frac{1}{2}(x^4+y^4)\leq x^2y^2 \leq \frac{1}{2}(x^4+y^4)}$$

$$0\leq \frac{1}{2}(x^2+y^2)^2, \quad 0\leq \frac{1}{2}(x^2-y^2)^2$$

$$\frac{1}{2}(x^4+y^4) \leq x^4-x^2y^2+y^4$$

$$\Rightarrow \frac{1}{2}(1+x^4+y^4)\leq 1+x^4-x^2y^2+y^4$$

$$\underline{\ln\frac{1}{2}+\ln(1+x^4+y^4)\leq \ln(1+x^4-x^2y^2+y^4)}$$

$$x^4+x^2y^2+y^4\leq \frac{3}{2}(x^4+y^4)$$

$$\underline{\ln(1+x^4+x^2y^2+y^4)}\leq \underbrace{\ln\left(\frac{3}{2}(1+x^4+y^4)\right)}_{\underline{\ln\frac{3}{2}+\ln(1+x^4+y^4)}}$$

$$\frac{\ln\frac{1}{2}+\ln(1+x^4+y^4)}{\ln\frac{3}{2}+\ln(1+x^4+y^4)} < \frac{\ln(1+x^4-x^2y^2+y^4)}{\ln(1+x^4+x^2y^2+y^4)} \le 1$$

$\to 1$ $\to 1$

$\longrightarrow 1$ Instängningslagen

$$\frac{\frac{\ln\frac{1}{2}}{\ln(1+x^4+y^4)}+1}{\frac{\ln\frac{3}{2}}{\ln(1+x^4+y^4)}+1} \longrightarrow 1$$

$$\ln(1+x^4+y^4) \overset{?}{\longrightarrow} \infty$$

$$1+x^4+y^4 = 1+r^4(\underbrace{\cos^4\theta+\sin^4\theta}_{\ge 1/4})$$

$$\ge 1+\frac{1}{4}r^4 \longrightarrow \infty, \quad r\to\infty$$

Rolles sats

I P&B är Rolles sats en del av beviset av Medelvärdes-satsen

f kont på $[a,b]$, f deriverbar på $]a,b[$

$f(a)=f(b)$

Då finns $\xi \in]a,b[$ så att $f'(\xi)=0$

Bevis:

I. Om f är deriverbar i en inre extrempunkt så är derivatan lika med noll.

II Satsen om extremvärden.