Dag 9

$\frac{\partial f}{\partial x}$ f'_x

Munta: 27, 28, 29 oktober.

Sem 3: önskemål nästa recka.

Kedjeregeln i högre dimensioner.

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dt}(f(g(t), h(t))) = \frac{\partial f}{\partial s}(g(t), h(t)) \cdot g'(t) + \frac{\partial f}{\partial r}(g(t), h(t)) h'(t)$$

$$\mathbb{R}^m \xrightarrow{g} \mathbb{R}^n \xrightarrow{f} \mathbb{R}^p$$

$$g' = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_m} \\ \vdots & \ddots & \\ \frac{\partial g_n}{\partial x_1} & \cdots & \frac{\partial g_n}{\partial x_m} \end{pmatrix} \quad f' = \begin{pmatrix} \frac{\partial f_1}{\partial y_1} & \cdots & \frac{\partial f_1}{\partial y_n} \\ \vdots & & \vdots \\ \frac{\partial f_p}{\partial y_1} & \cdots & \frac{\partial f_p}{\partial y_n} \end{pmatrix}$$

Sats $f, g \in C^1$: $(f \circ g)' = f' g'$

$y_k = g_k$

$$\frac{\partial f_1}{\partial y_1}\frac{\partial g_1}{x_1} + \dots + \frac{\partial f_1}{\partial y_n}\frac{\partial g_n}{\partial x_1} = \frac{\partial}{\partial x_1}\left(f_1(g(\bar{x}))\right)$$

Differentialkalkylens universalprincip:

Lokalt är differentialkalkyl linjär algebra.

$$f \circ g \qquad \underset{a}{\mathbb{R}} \xrightarrow{g} \underset{b}{\mathbb{R}} \xrightarrow{f} \underset{c}{\mathbb{R}}$$

$$g(x) \approx g(a) + g'(a)(x-a)$$

$$f(y) \approx f(b) + f'(b)(y-b)$$

$$f \circ g(x) = f(g(a)) + \underbrace{f'(g(a))\,g'(a)}_{\text{detta är derivatan av } f\circ g \text{ i } a.}(x-a)$$

Taylors formel i en och flera variabler.

I P8B ser en- och två-
variabelfallen olika ut.

SATS. $f : \mathbb{R}^n \to \mathbb{R}$ tillhör klass C^{k+1}. Då kan vi skriva

$$f(\overline{x}) = \sum_{|\alpha| \le k} \frac{1}{\alpha!} D^\alpha f(\overline{a})(\overline{x} - \overline{a})^\alpha + O(|\overline{x}|^{k+1}).$$

vektorer

$\alpha = (\alpha_1, \dots \alpha_n)$ $\quad \alpha_k \ge 0$ heltal

multiindex $\quad n = 2 \quad \alpha = (1, 0)$

$= \frac{\partial f}{\partial x}$

$\alpha = (2, 3) \quad \frac{\partial^5 f}{\partial x^2 \partial y^3}$

$|\alpha| = \alpha_1 + \dots + \alpha_k$

$f(a+h, b+k) = f(a,b) + \frac{\partial f}{\partial x}(a,b)h + \frac{\partial f}{\partial y}(a,b)k + \dots$

$f(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) + \dots$

$h = x - a \quad k = y - b$

Taylors formel handlar om
"bästa möjliga" polynomapproximation

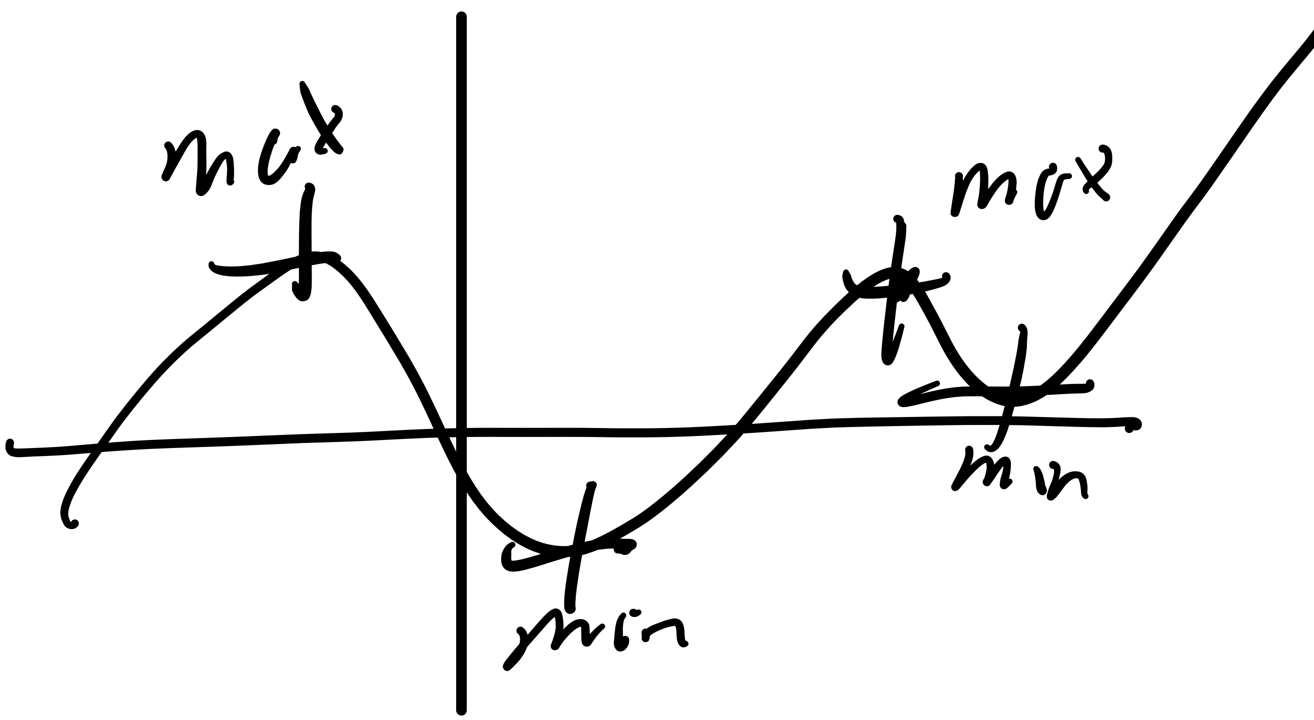
$n=1$. Gränsvärden

Exakta beräkningar med restterm

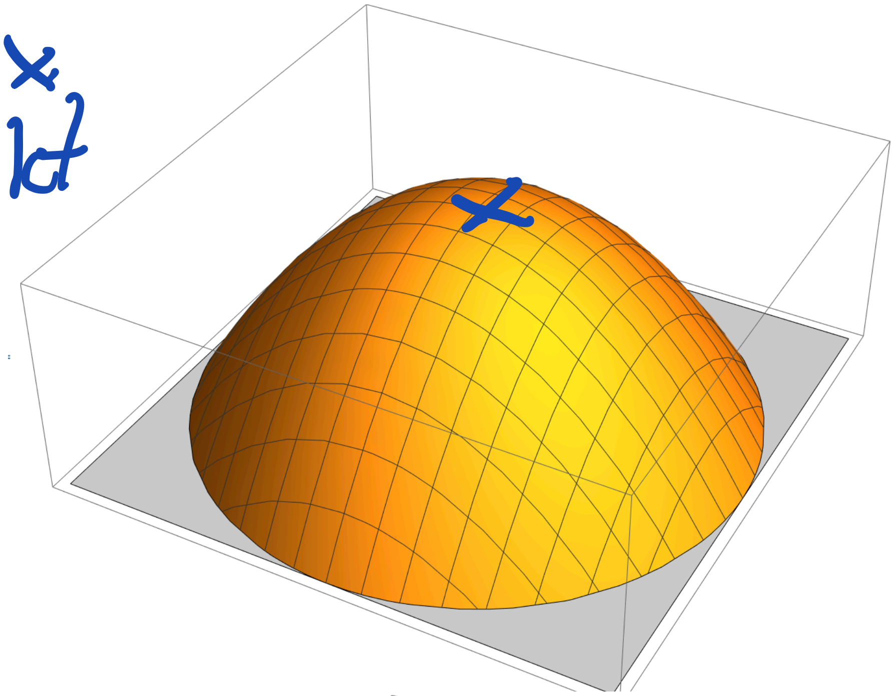
$n>1$ Avgöra karaktären av stationära punkter.

Stat punkt $(n=1)$: tangenten är parallell med x-axeln

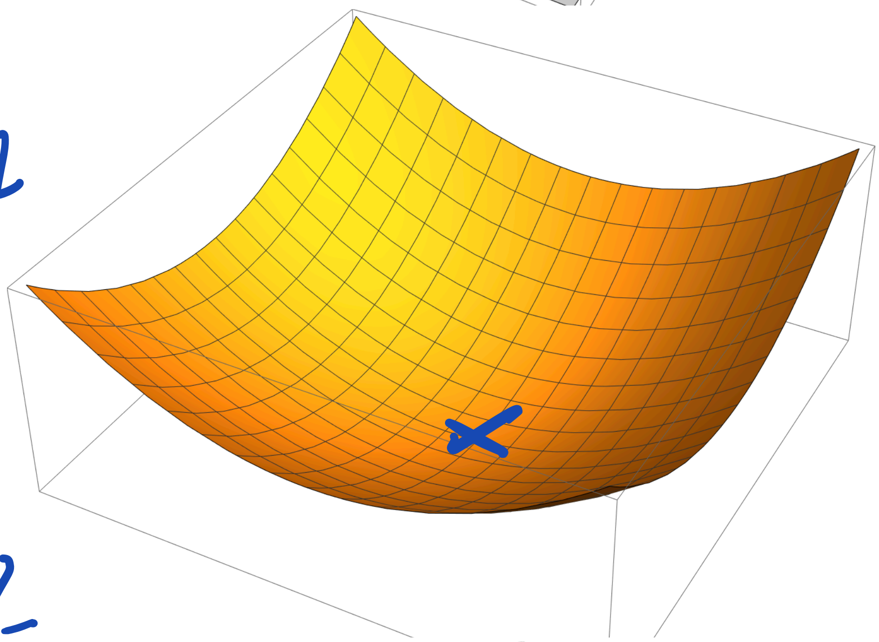
Stat punkter $(n=2)$: tangentplanet är parallellt med xy-planet.



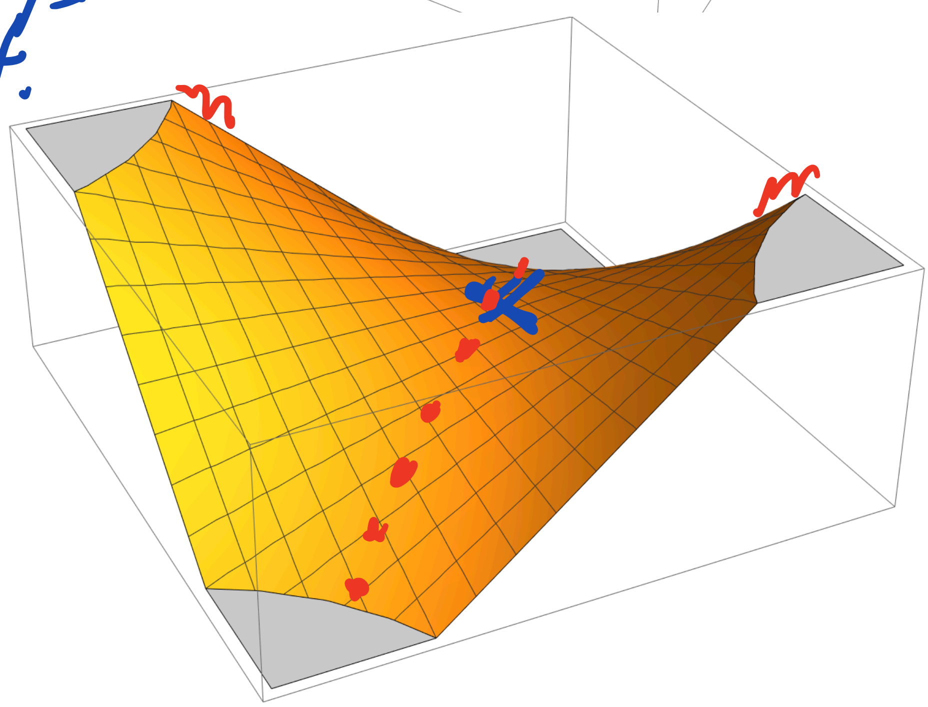
max. punkt

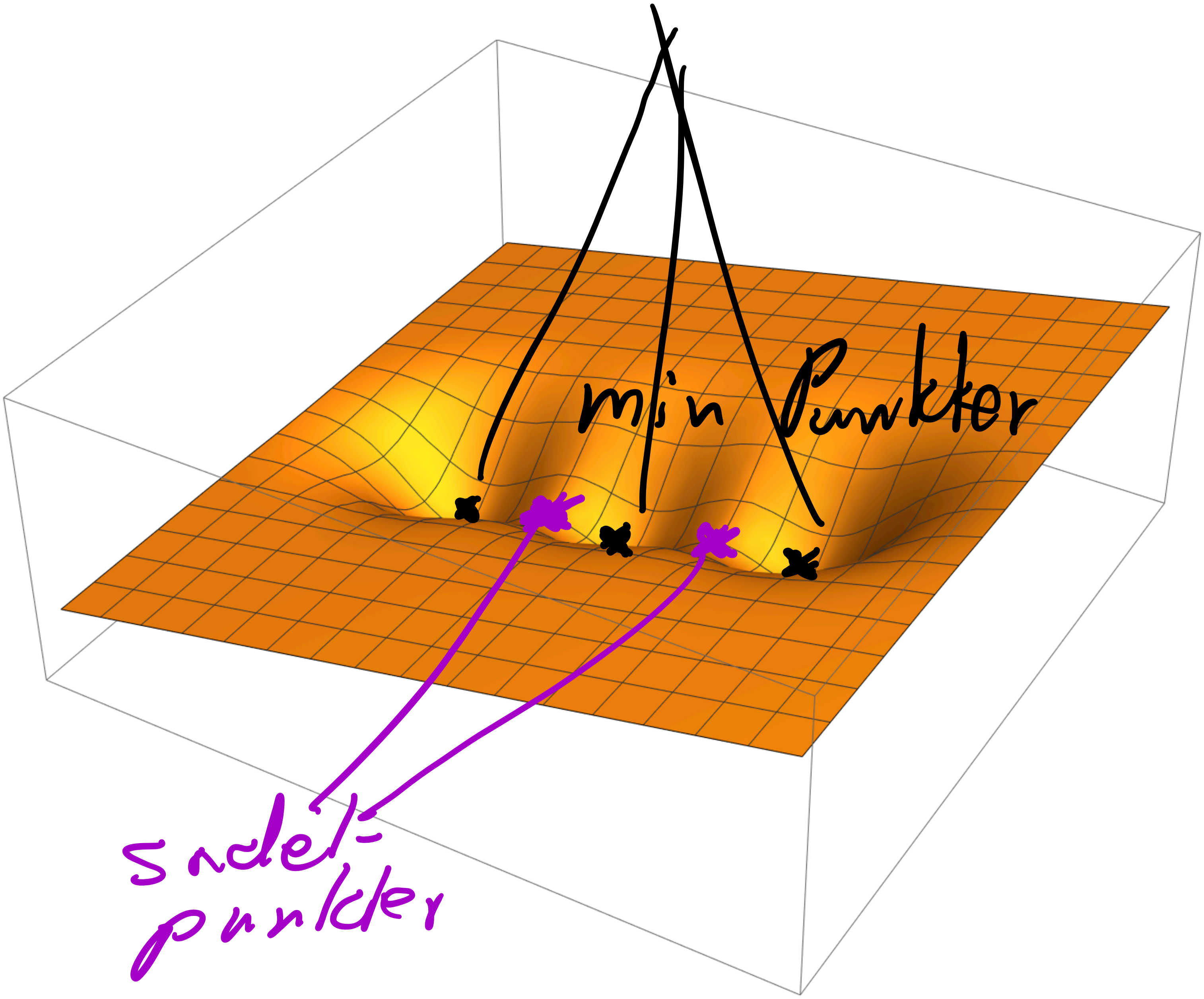


min punkt



Sadel-punkt.




min Punkter
sadel-
punkter

Två maxpunkter
Två minpunkter
Tre sadelpunkter