Lecture:
Face Recognition

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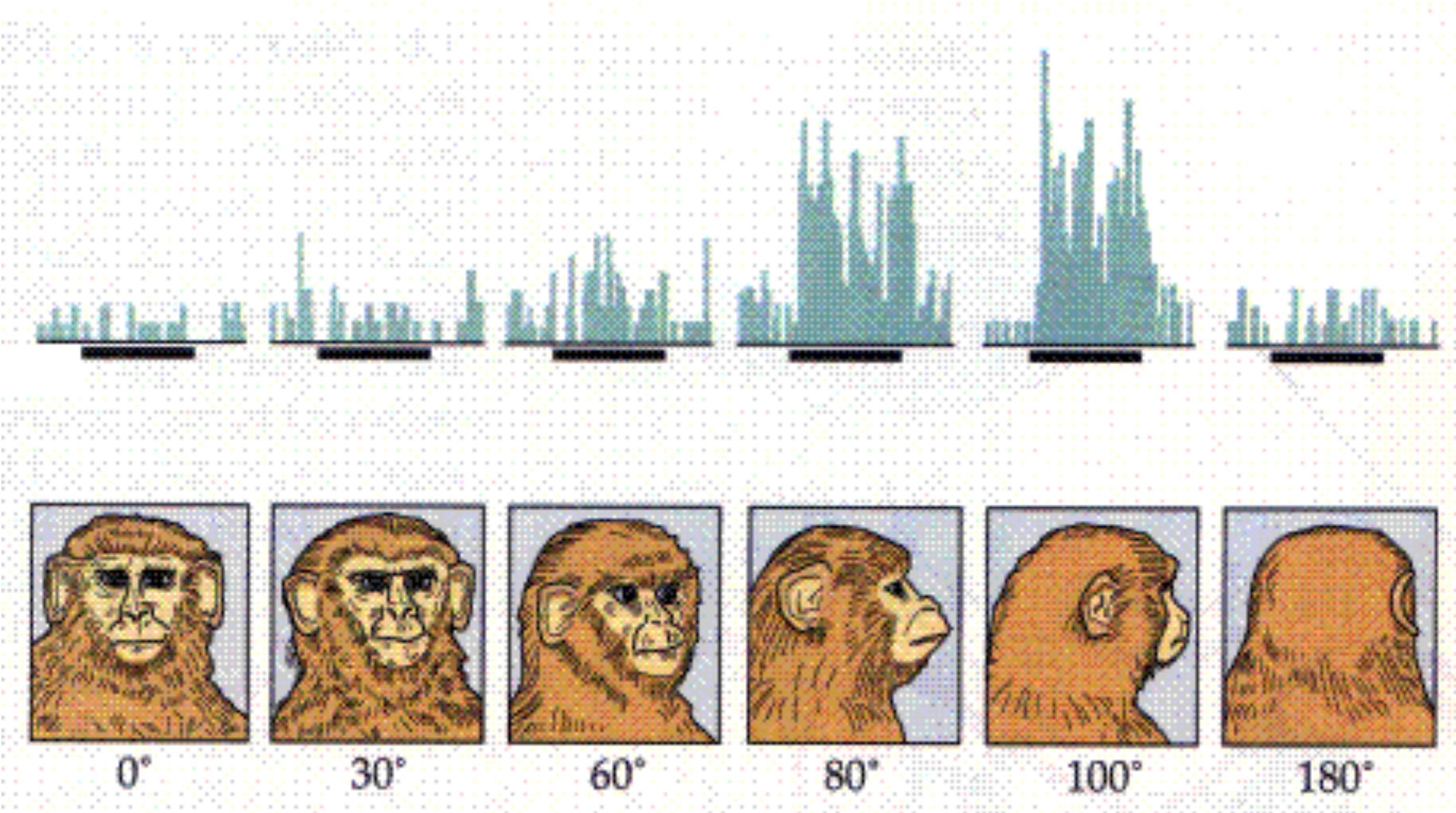
What we will learn today

- Introduction to face recognition
- The Eigenfaces Algorithm
- Linear Discriminant Analysis (LDA)

Turk and Pentland, Eigenfaces for Recognition, *Journal of Cognitive Neuroscience* **3** (1): 71–86.

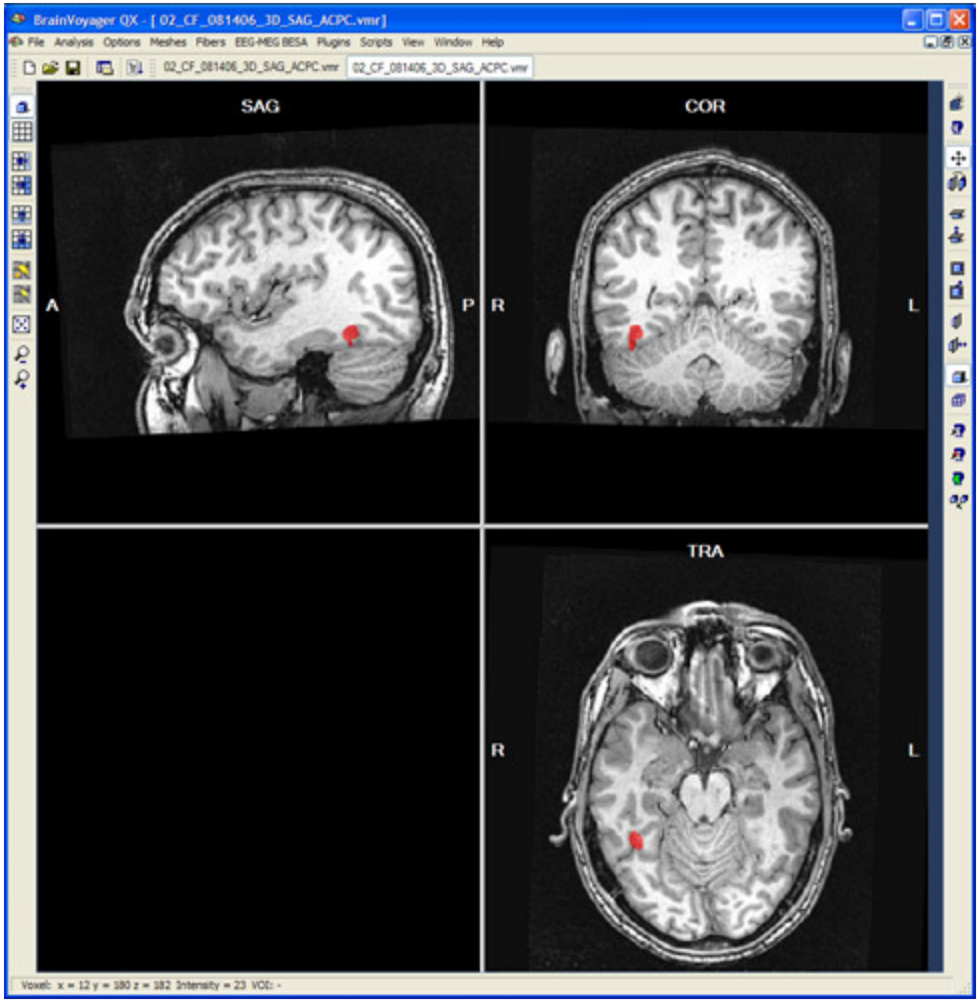
P. Belhumeur, J. Hespanha, and D. Kriegman. "Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection". *IEEE Transactions on pattern analysis and machine intelligence* **19** (7): 711. 1997.

“Faces” in the brain



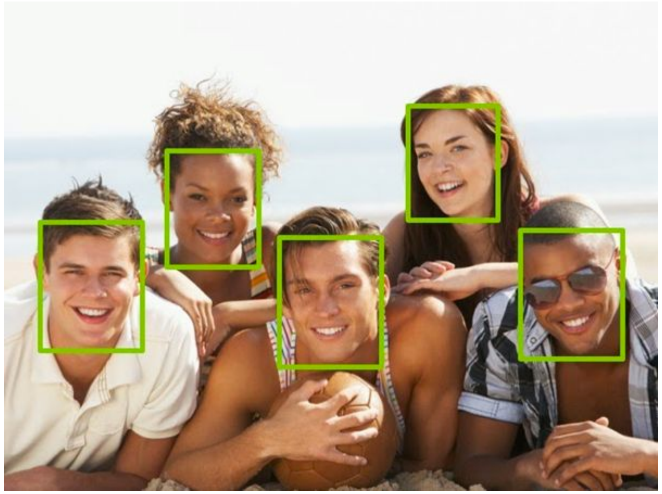
Courtesy of Johannes M. Zanker

"Faces" in the brain
fusiform **face area**

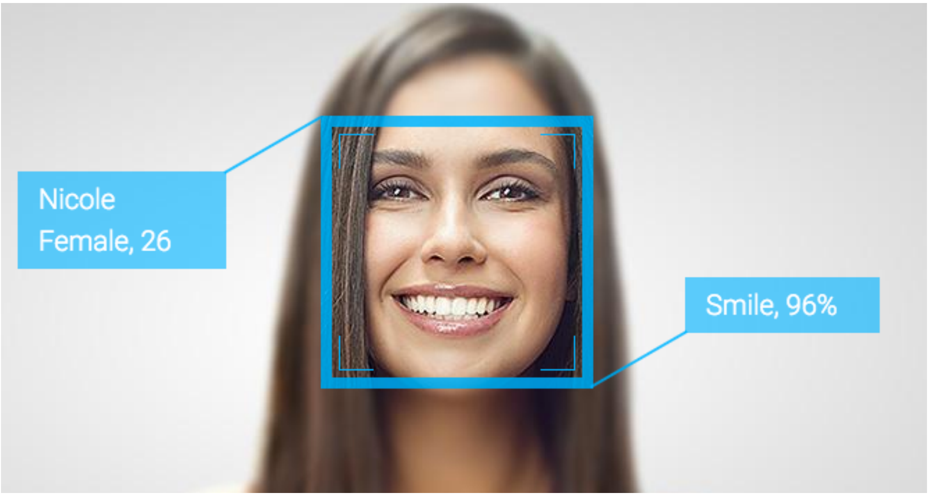


Kanwisher, et al. 1997

Detection versus Recognition



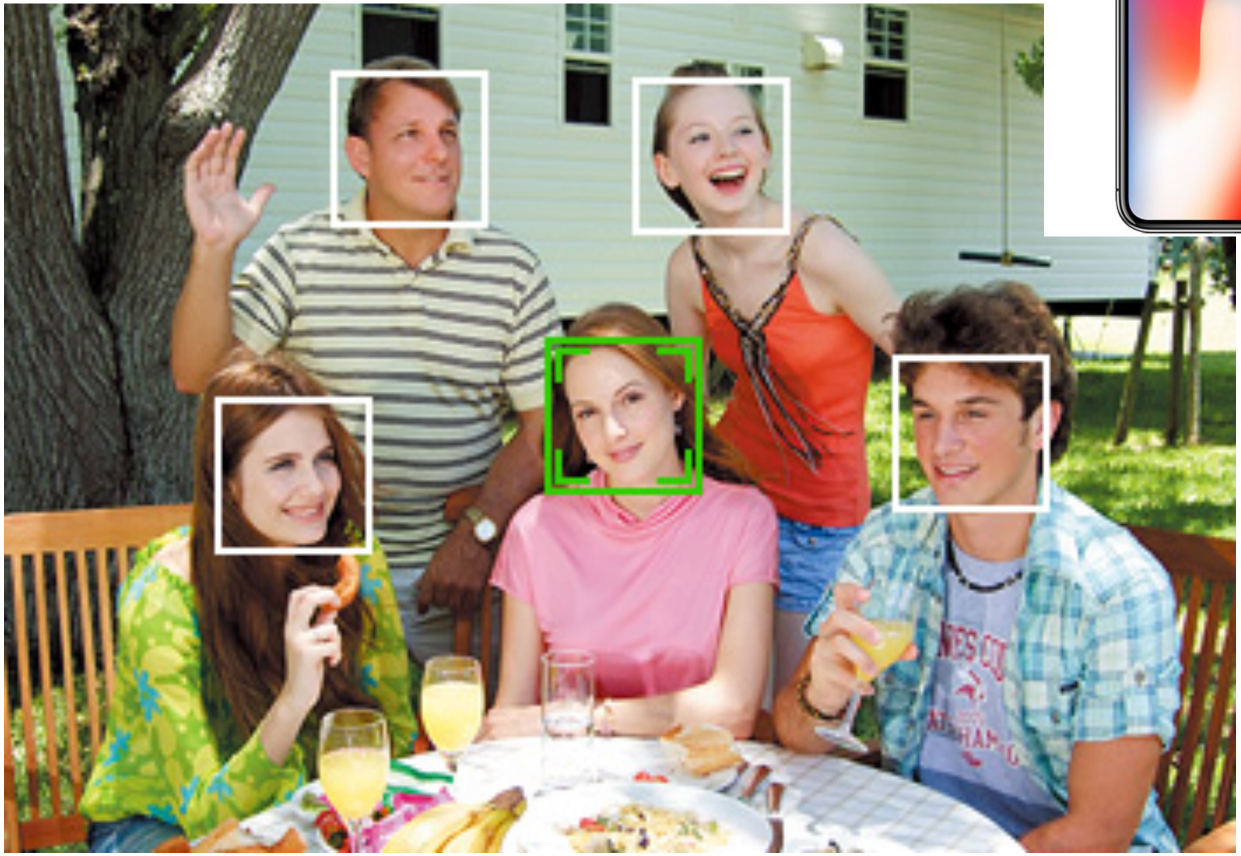
Detection finds the faces in images



Recognition recognizes WHO the person is

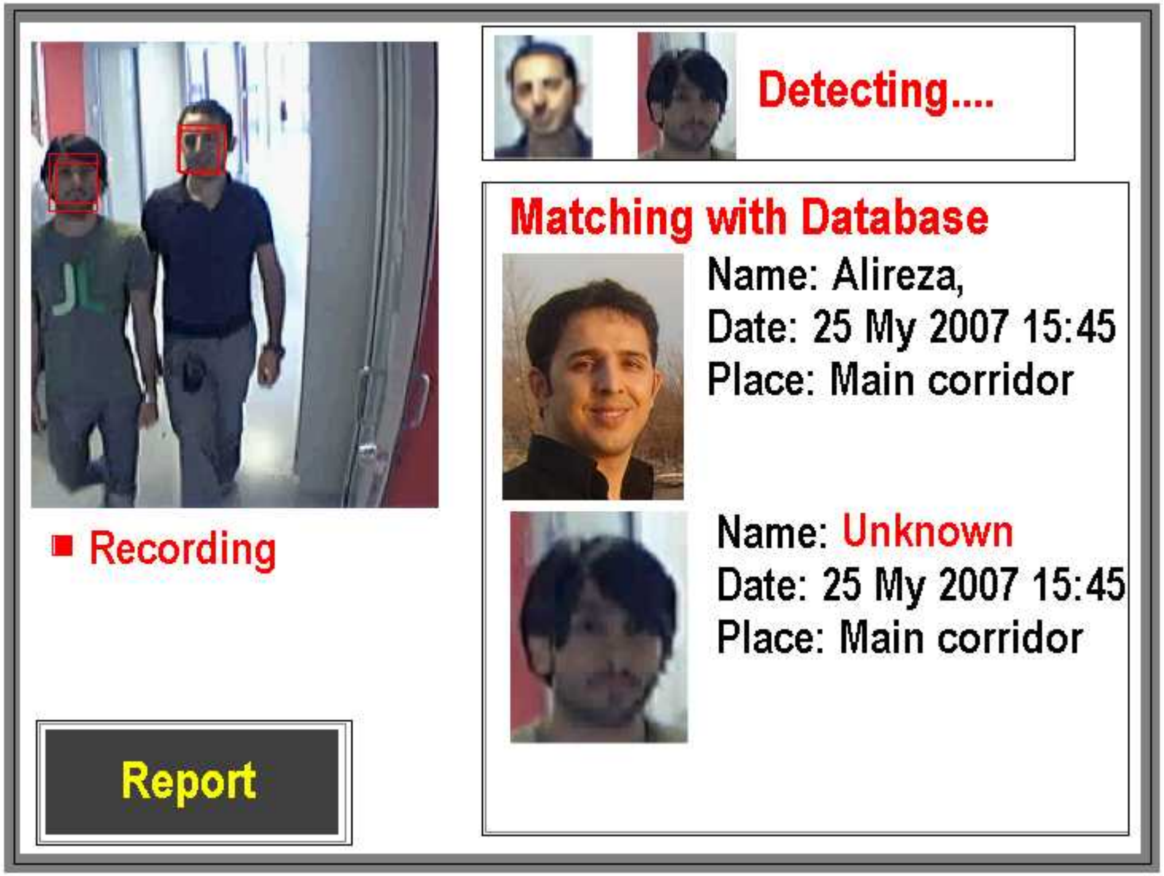
Face Recognition

- Digital photography



Face Recognition

- Digital photography
- Surveillance

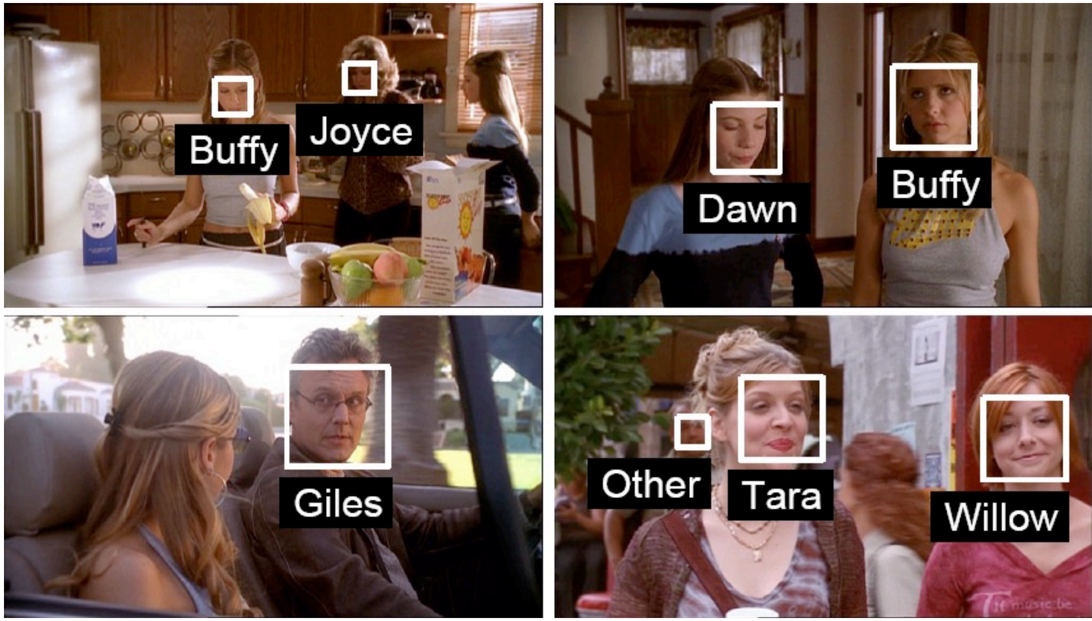


Face Recognition

- Digital photography
- Surveillance
- Album organization

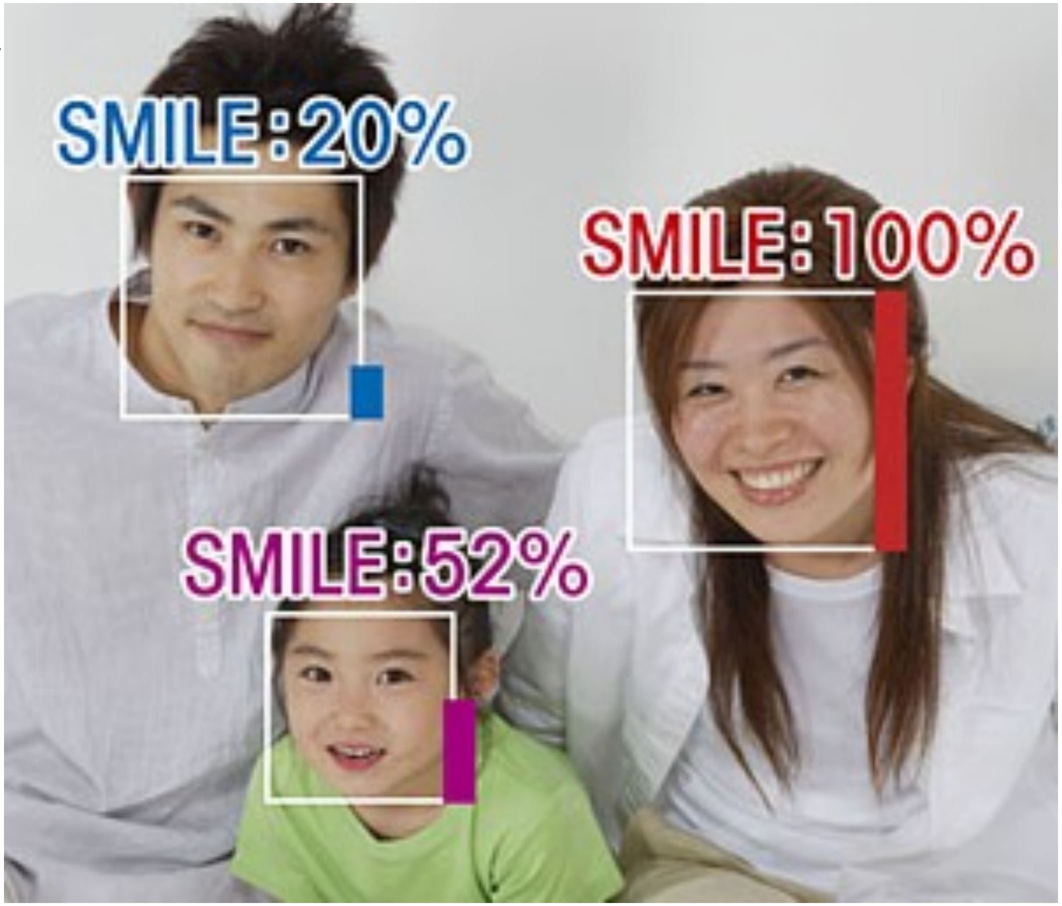
Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.



Face Recognition

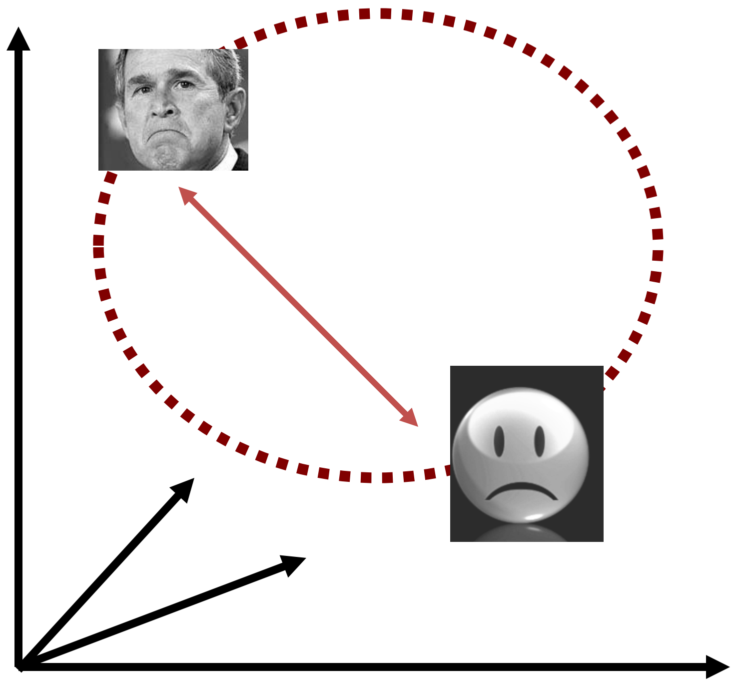
- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions



Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions
- Security/warfare
- Tele-conferencing
- Etc.

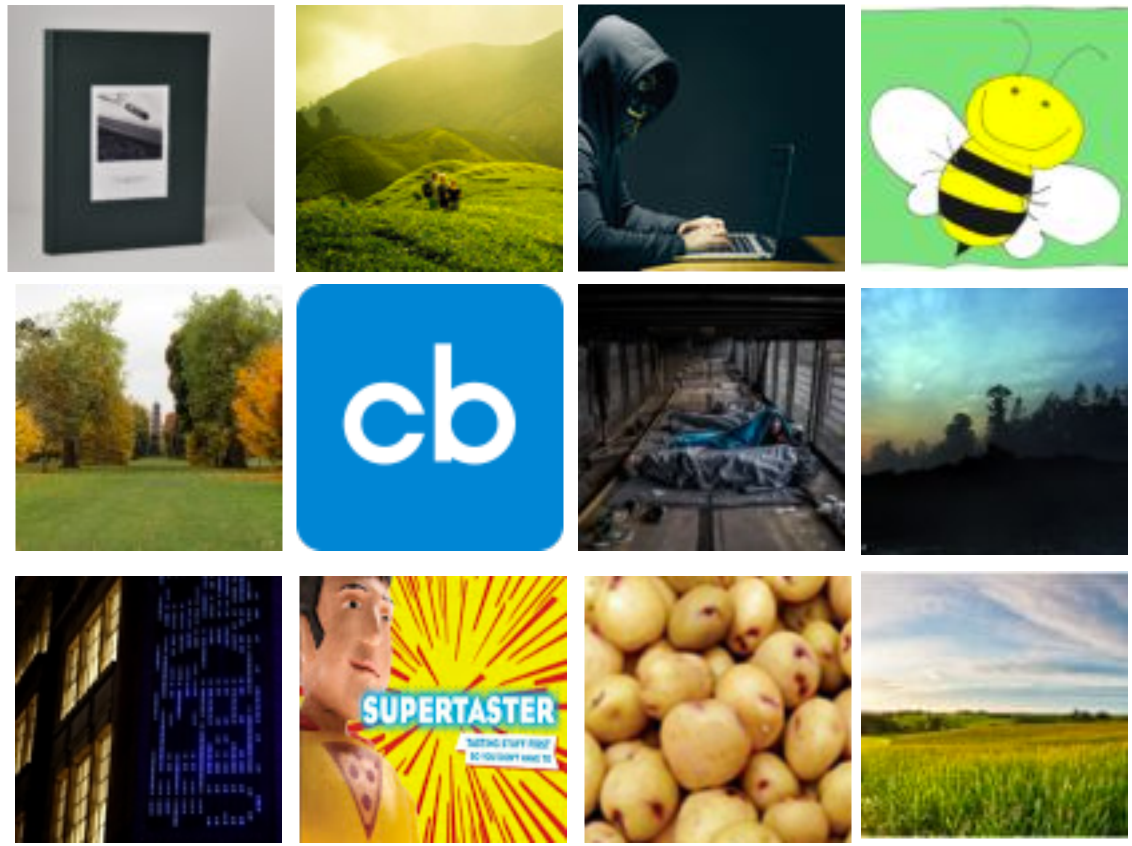
The Space of Faces



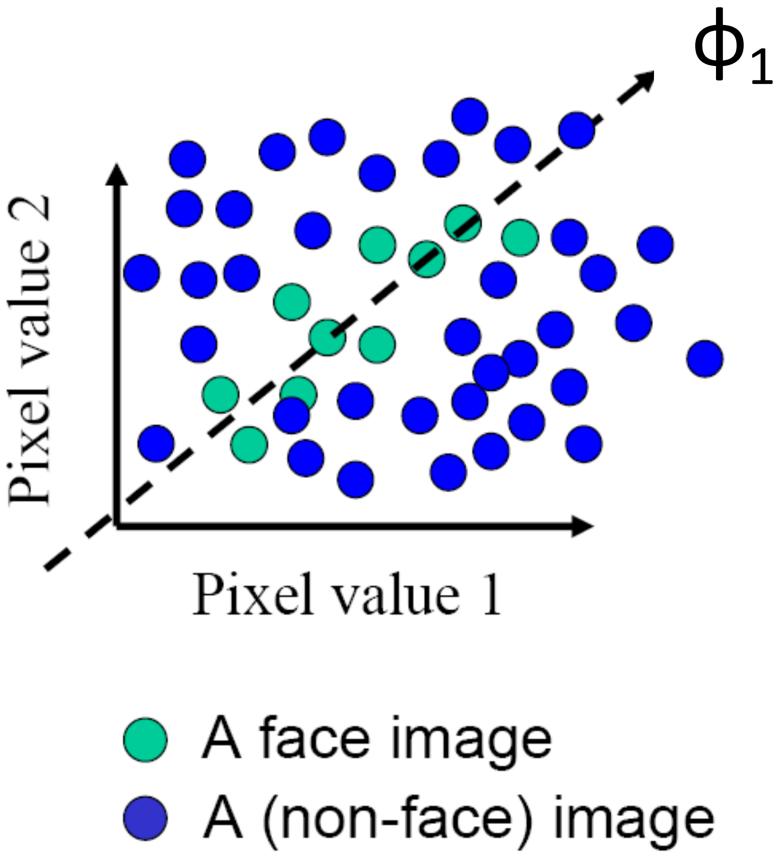
- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim

Slide credit: Chuck Dyer, Steve Seitz, Nishino

100x100 images can contain many things other than faces!



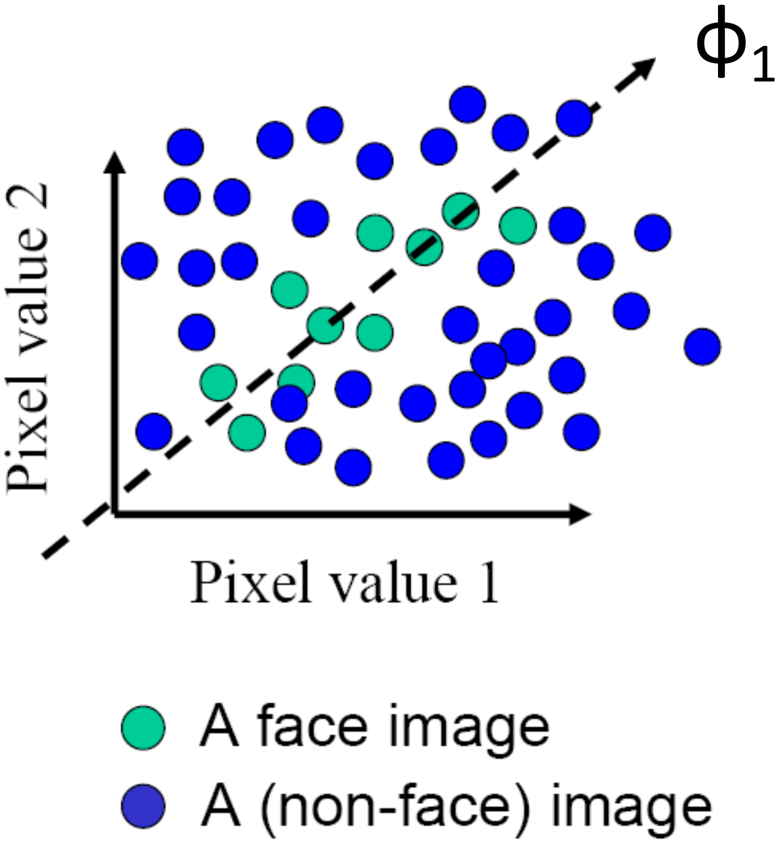
The Space of Faces

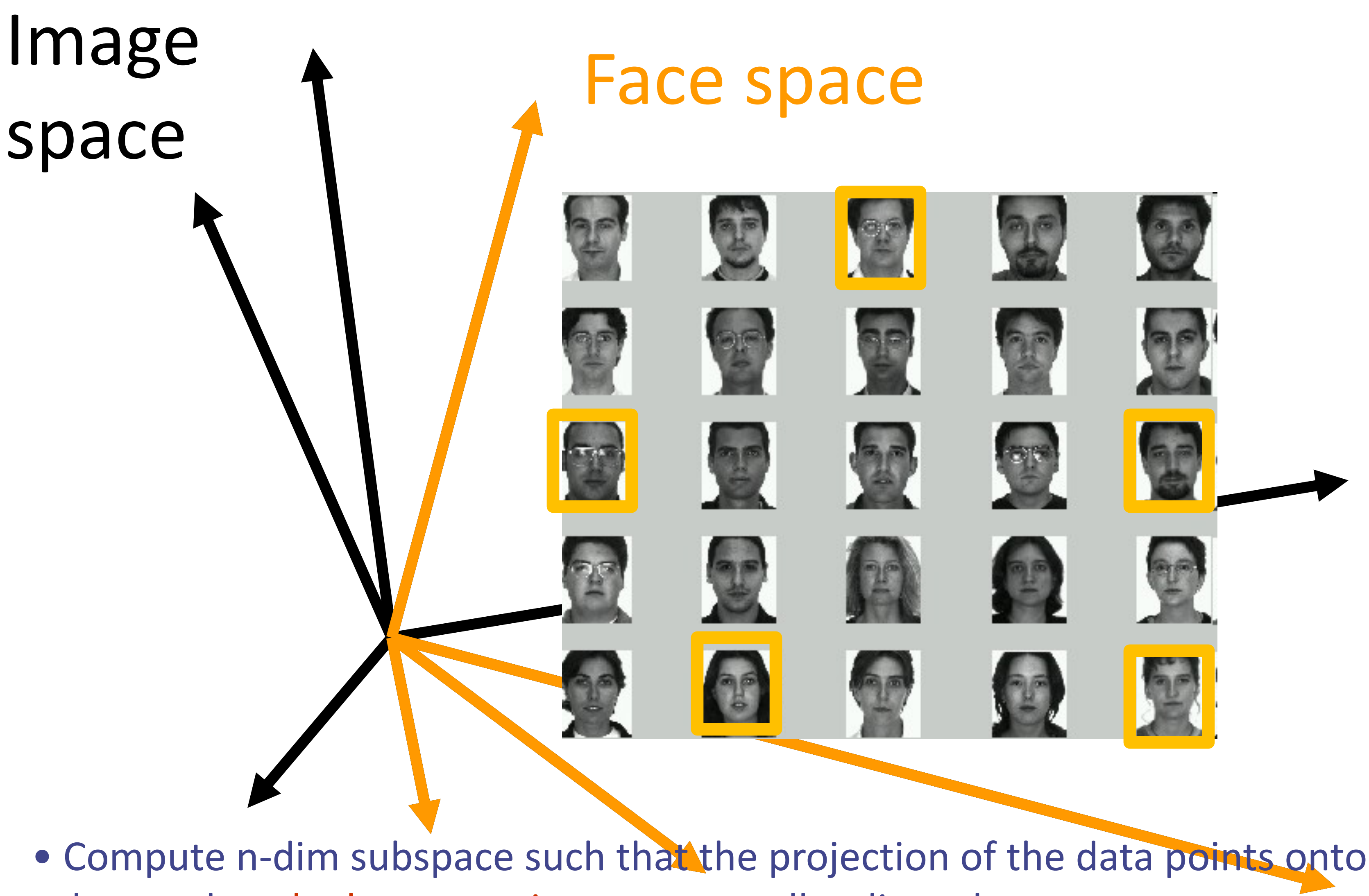


- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim
- However, relatively few high dimensional vectors correspond to valid face images
- We want to effectively model the subspace of face images

Slide credit: Chuck Dyer, Steve Seitz, Nishino

Where have we seen something like this before?





- Compute n-dim subspace such that the projection of the data points onto the subspace has the largest variance among all n-dim subspaces.
- Maximize the scatter of the training images in face space

Key Idea

- So, compress them to a low-dimensional subspace that captures key appearance characteristics of the visual DOFs.

- USE PCA for estimating the sub-space (dimensionality reduction)

- Compare two faces by projecting the images into the subspace and measuring the EUCLIDEAN distance between them.

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Eigenfaces: key idea

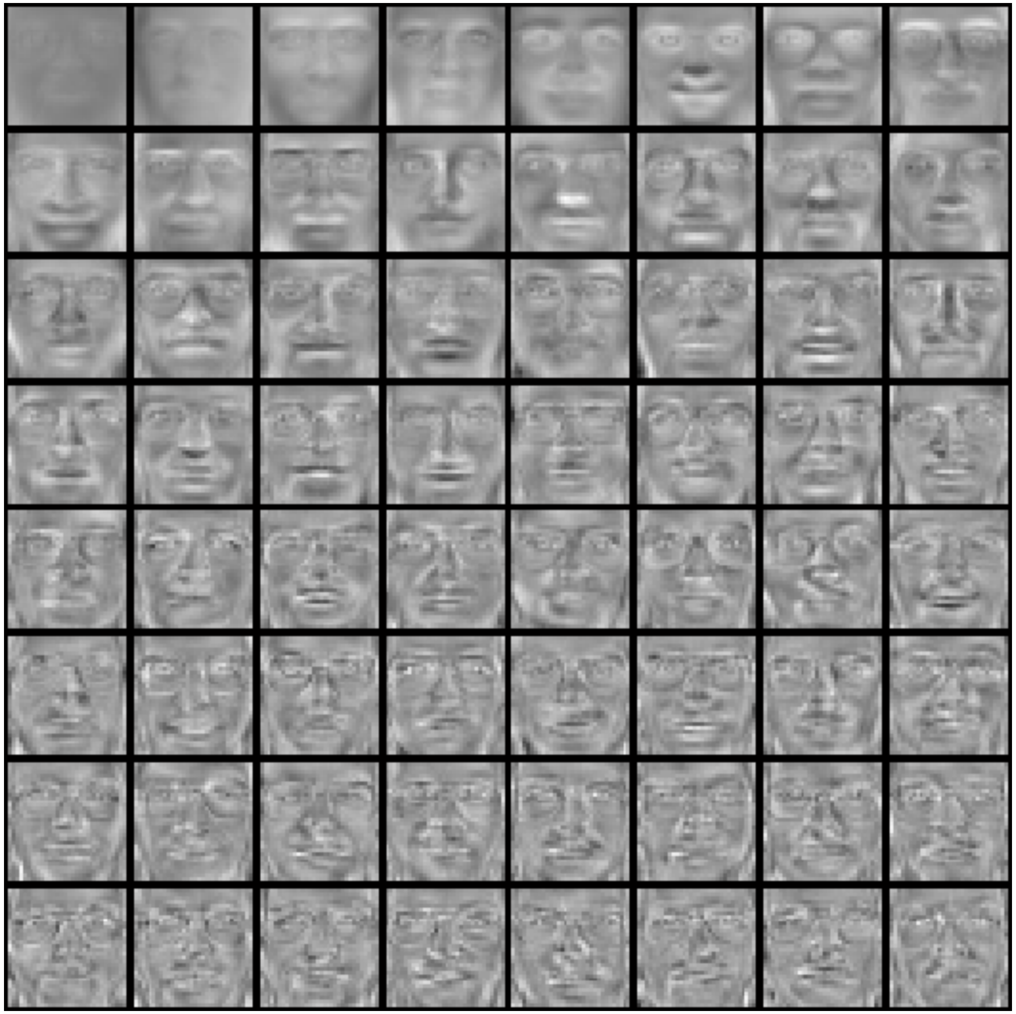
- Assume that most face images lie on a low-dimensional subspace determined by the first k (k<<d) directions of maximum variance
- Use PCA to determine the vectors or "eigenfaces" that span that subspace
- Represent all face images in the dataset as linear combinations of eigenfaces

M. Turk and A. Pentland, [Face Recognition using Eigenfaces](), CVPR 1991

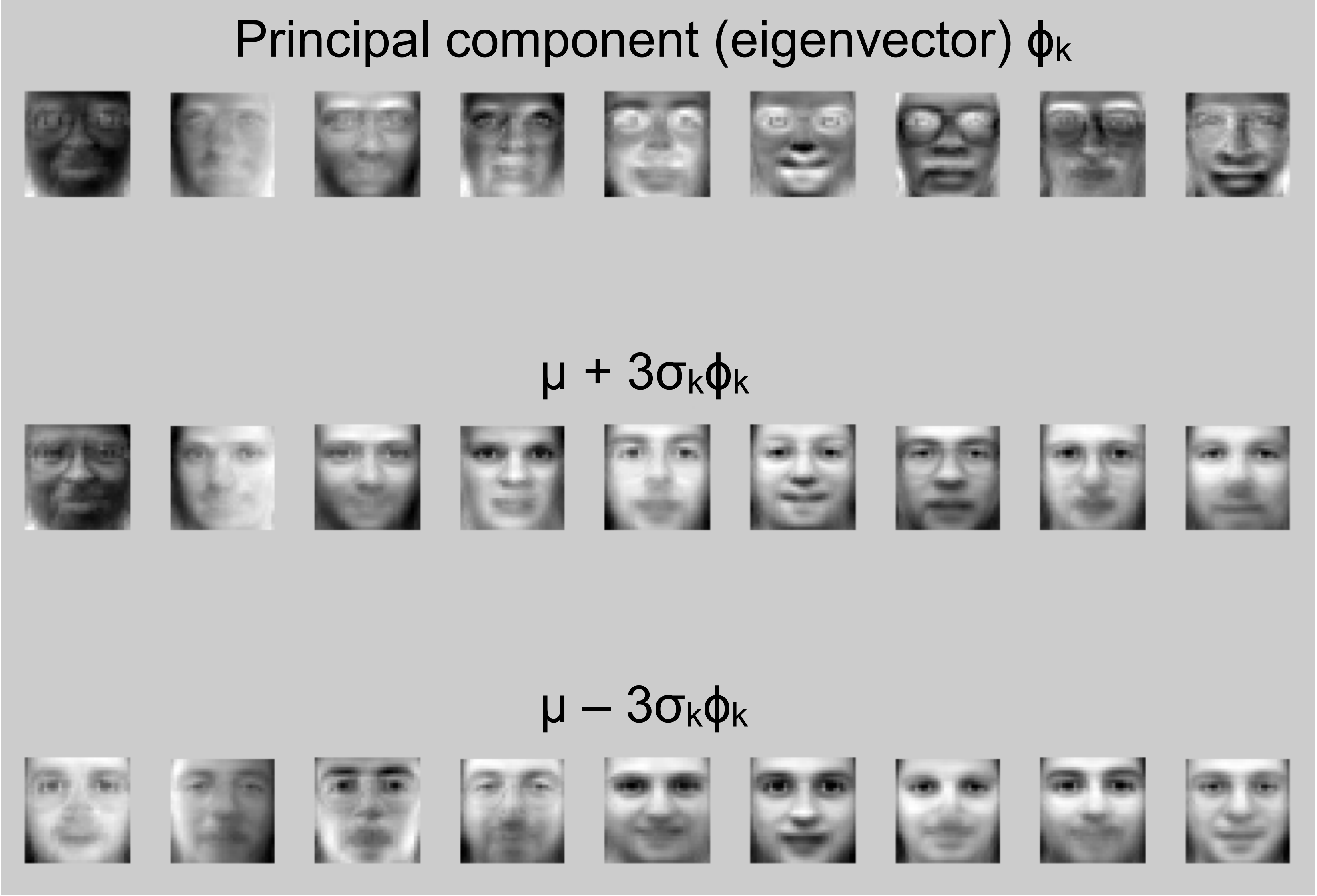
Training images: $\mathbf{x}_1,\ldots,\mathbf{x}_N$

Top eigenvectors: $\phi_1,\ldots,\phi_k$

Mean: μ



Visualization of eigenfaces



Eigenface algorithm

- Training
  1. Align training images $x_1, x_2, \ldots, x_N$

Note that each image is formulated into a long vector!

  2. Compute average face $\mu = \frac{1}{N}\sum x_i$

  3. Compute the difference image (the centered data matrix)

$$\boxed{\begin{aligned} X_c &= \begin{bmatrix} | & & | \\ x_1 & \ldots & x_n \\ | & & | \end{bmatrix} - \begin{bmatrix} | & & | \\ \mu & \ldots & \mu \\ | & & | \end{bmatrix} \\ &= X - \mu 1^T = X - \frac{1}{n} X 1 1^T = X\left(I - \frac{1}{n} 1 1^T\right) \end{aligned}}$$

Eigenface algorithm

4. Compute the covariance matrix

$$\boxed{\Sigma = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1^c & \cdots & x_n^c \\ | & & | \end{bmatrix}\begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix} = \frac{1}{n} X_c X_c^T}$$

5. Compute the eigenvectors of the covariance matrix Σ
6. Compute each training image x_i 's projections as

$$x_i \rightarrow \left(x_i^c \cdot \phi_1, x_i^c \cdot \phi_2, \ldots, x_i^c \cdot \phi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

7. Visualize the estimated training face x_i

$$x_i \approx \mu + a_1\phi_1 + a_2\phi_2 + \ldots + a_K\phi_K$$

Eigenface algorithm

x_i $\quad a_1\phi_1 \quad a_2\phi_2 \quad \cdots \quad a_K\phi_K$

Reconstructed training face

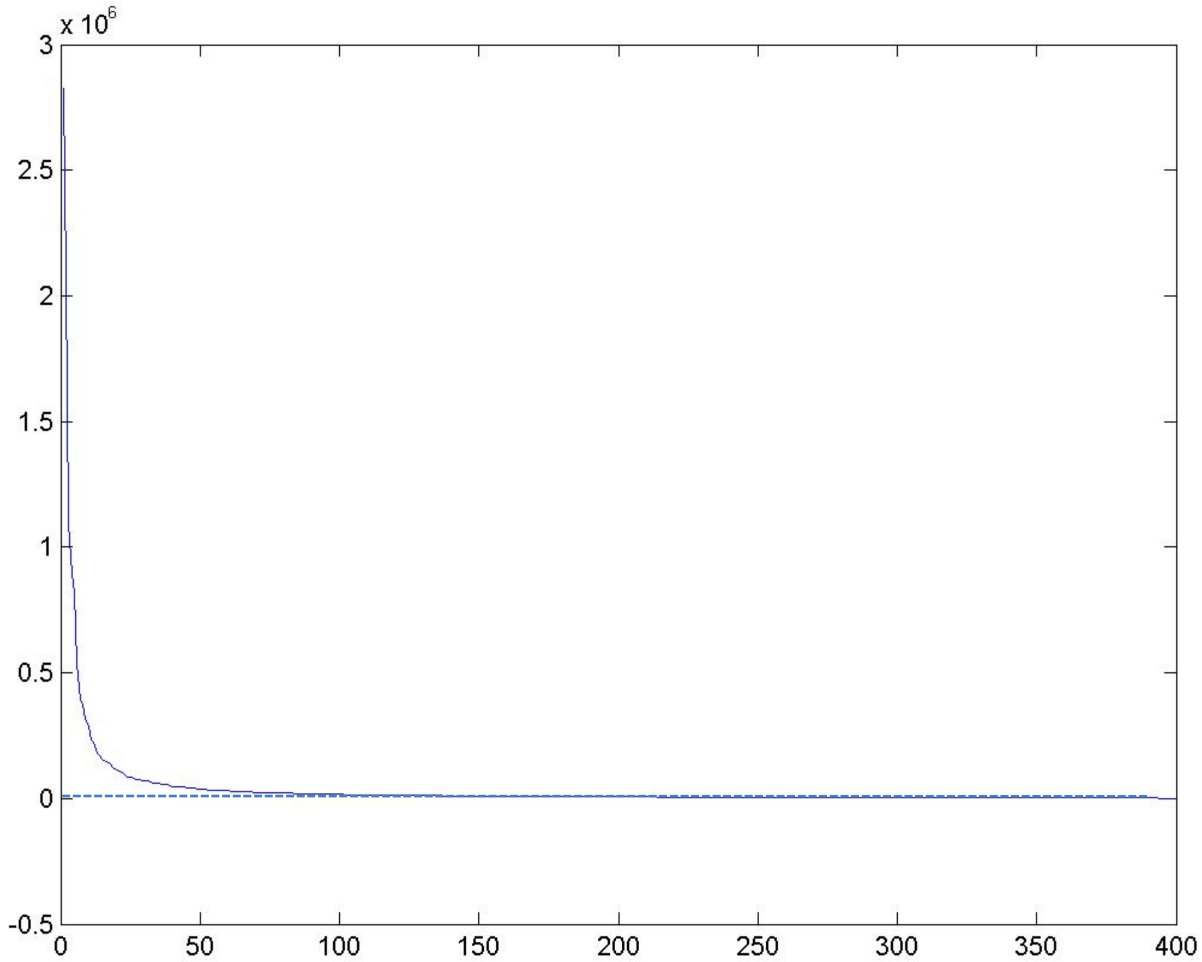
6. Compute each training image x_i 's projections as

$$x_i \rightarrow \left(x_i^c \cdot \phi_1, x_i^c \cdot \phi_2, \ldots, x_i^c \cdot \phi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

7. Visualize the reconstructed training face x_i

$$x_i \approx \mu + a_1\phi_1 + a_2\phi_2 + \ldots + a_K\phi_K$$

Eigenvalues (variance along eigenvectors)



Reconstruction and Errors

K = 4

K = 200

K = 400

- Only selecting the top K eigenfaces → reduces the dimensionality.
- Fewer eigenfaces result in more information loss, and hence less discrimination between faces.

Eigenface algorithm

- Testing
  1. Take query image t
  2. Project into eigenface space and compute projection

$$t \rightarrow \left((t-\mu)\cdot\phi_1, (t-\mu)\cdot\phi_2, \ldots, (t-\mu)\cdot\phi_K \right) \equiv \left(w_1, w_2, \ldots, w_K \right)$$

  3. Compare projection w with all N training projections
     - Simple comparison metric: Euclidean
     - Simple decision: K-Nearest Neighbor

       (note: this "K" refers to the k-NN algorithm, is different from the previous K's referring to the # of principal components)

Visualization of eigenfaces

Eigenfaces look somewhat like generic faces.

Shortcomings

- Requires carefully controlled data:
  - All faces centered in frame
  - Same size
  - Some sensitivity to angle
- Alternative:
  - "Learn" one set of PCA vectors for each angle
  - Use the one with lowest error
- Method is completely knowledge free
  - (sometimes this is good!)
  - Doesn't know that faces are wrapped around 3D objects (heads)
  - Makes no effort to preserve class distinctions

Summary for Eigenface

Pros

- Non-iterative, globally optimal solution

Limitations

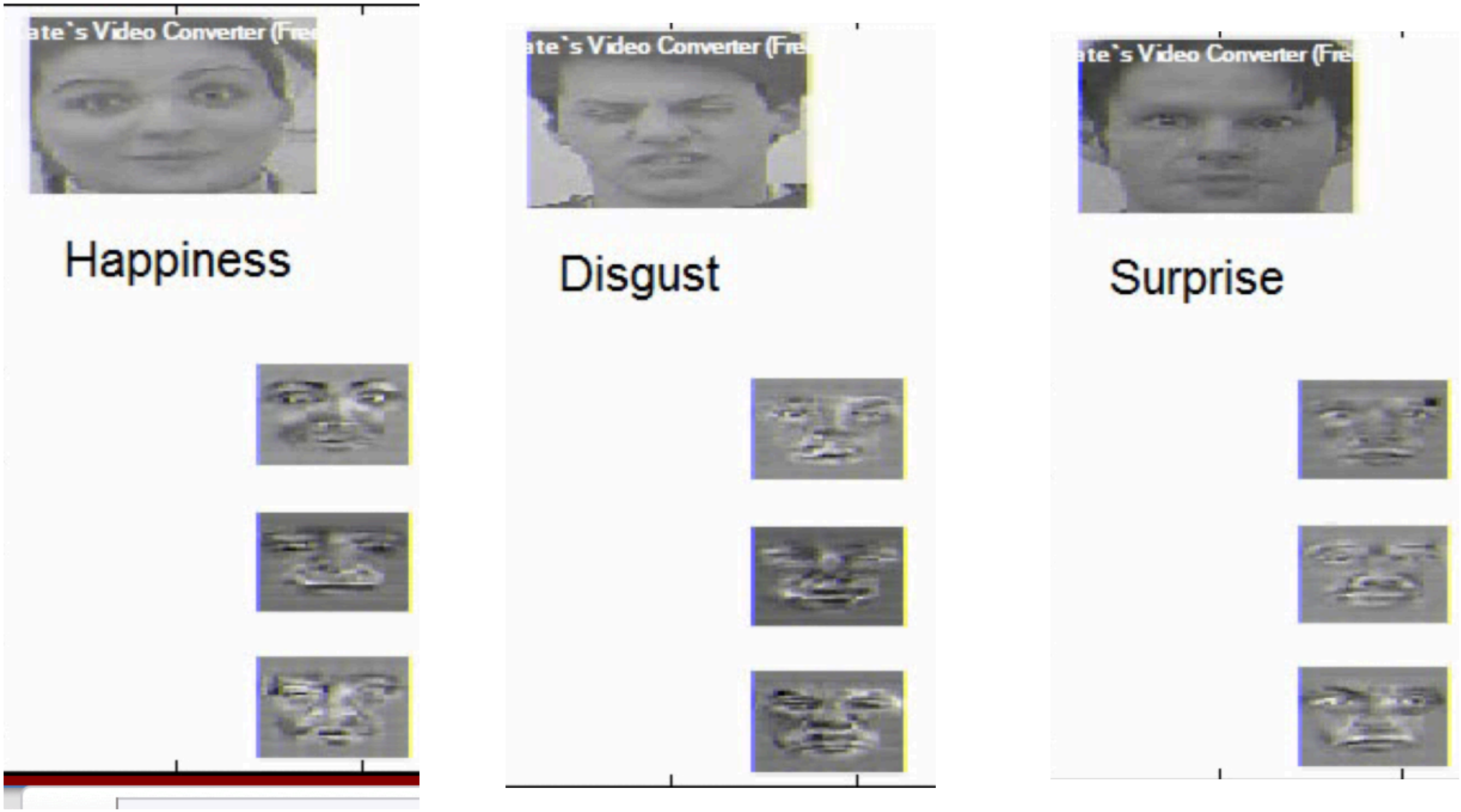
- PCA projection is **optimal for reconstruction** from a low dimensional basis, but **may NOT be optimal for discrimination…**

Besides face recognitions,
we can also do
Facial expression recognition

Happiness subspace (method A)

Disgust subspace (method A)

Facial Expression Recognition Movies (method A)



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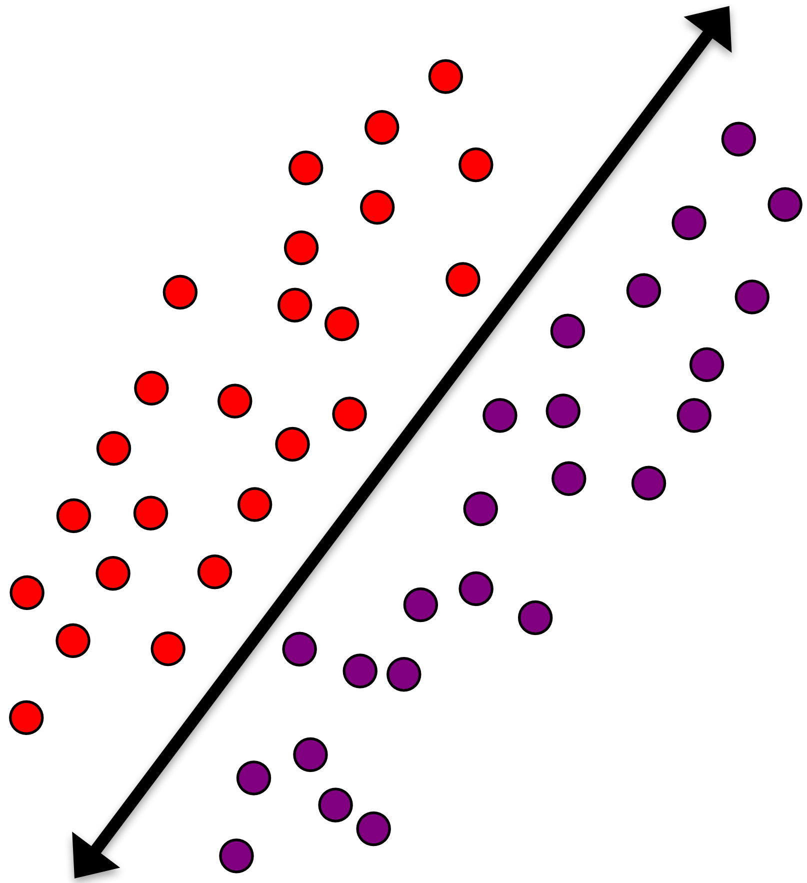
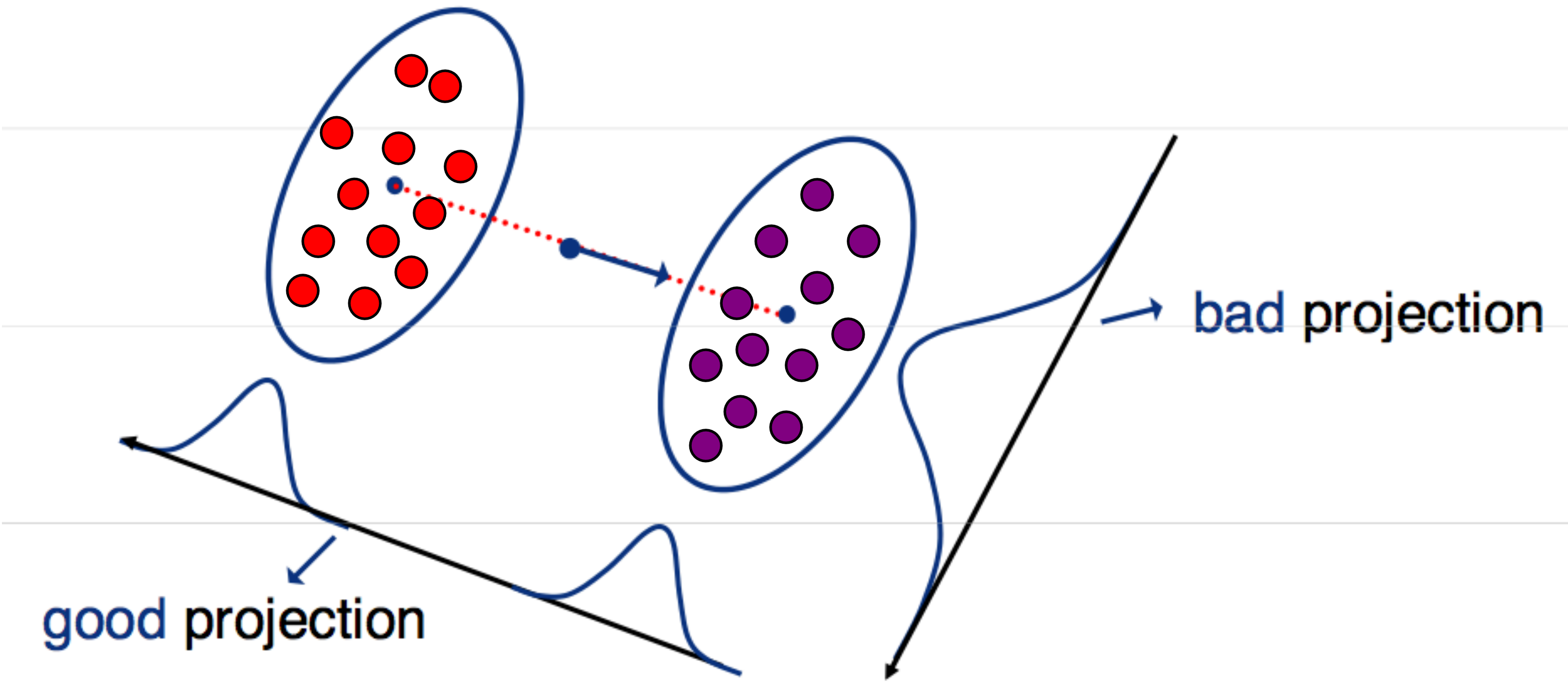
Which direction will is the first principle component?



Fischer’s Linear Discriminant Analysis

- Goal: find the best separation between two classes



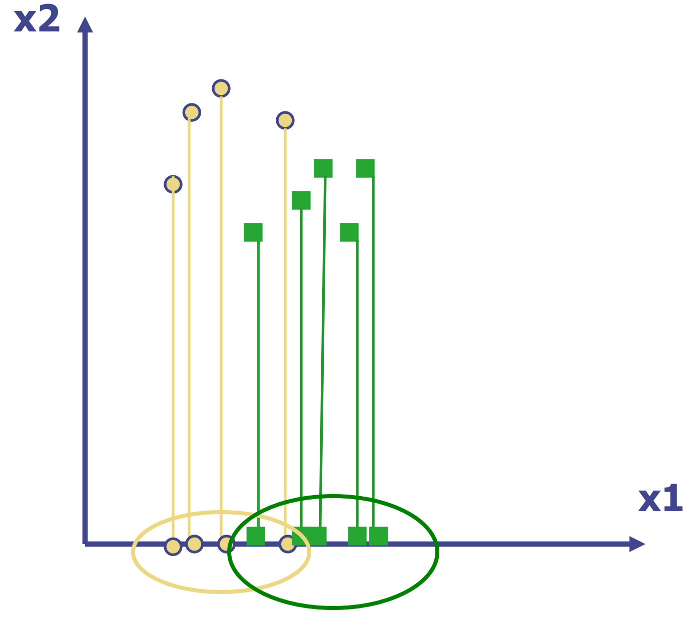
Slide inspired by N. Vasconcelos

Difference between PCA and LDA

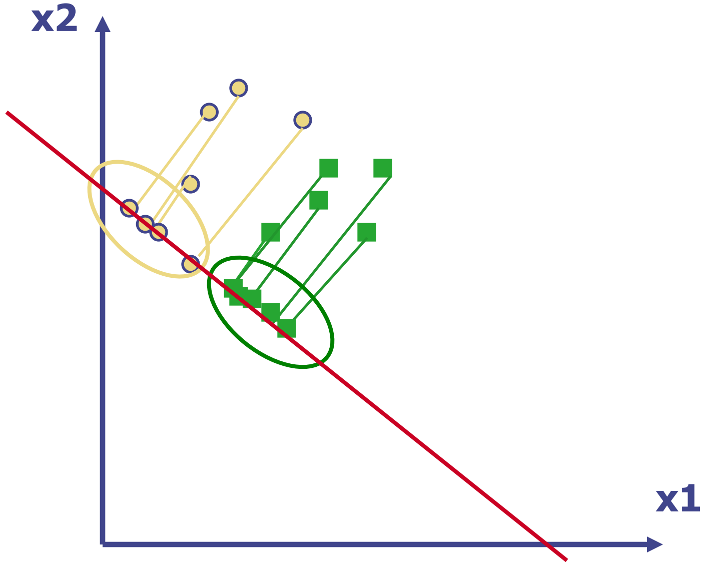
- PCA preserves maximum variance
- LDA preserves discrimination
  – Find projection that maximizes scatter between classes and minimizes scatter within classes

Illustration of the Projection

- Using two classes as example:

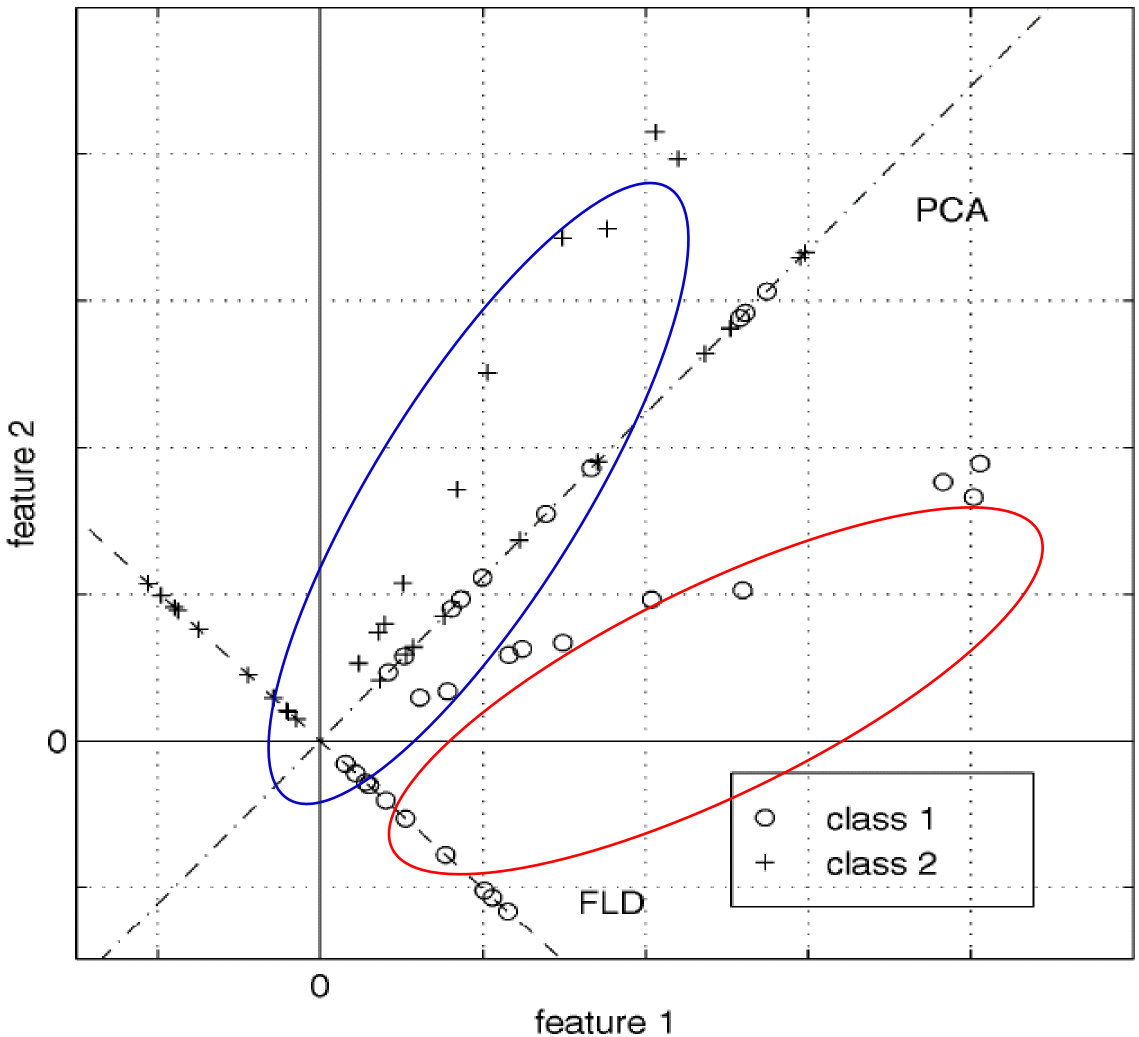


Poor Projection



Good

Basic intuition: PCA vs. LDA



LDA with 2 variables

- We want to learn a projection W such that the projection converts all the points from x to a new space (For this example, assume m == 1):

$$z = w^T x \qquad z \in \mathbf{R}^m \quad x \in \mathbf{R}^n$$

- Let the **per class** means be:

$$E_{X|Y}[X \mid Y = i] = \mu_i$$

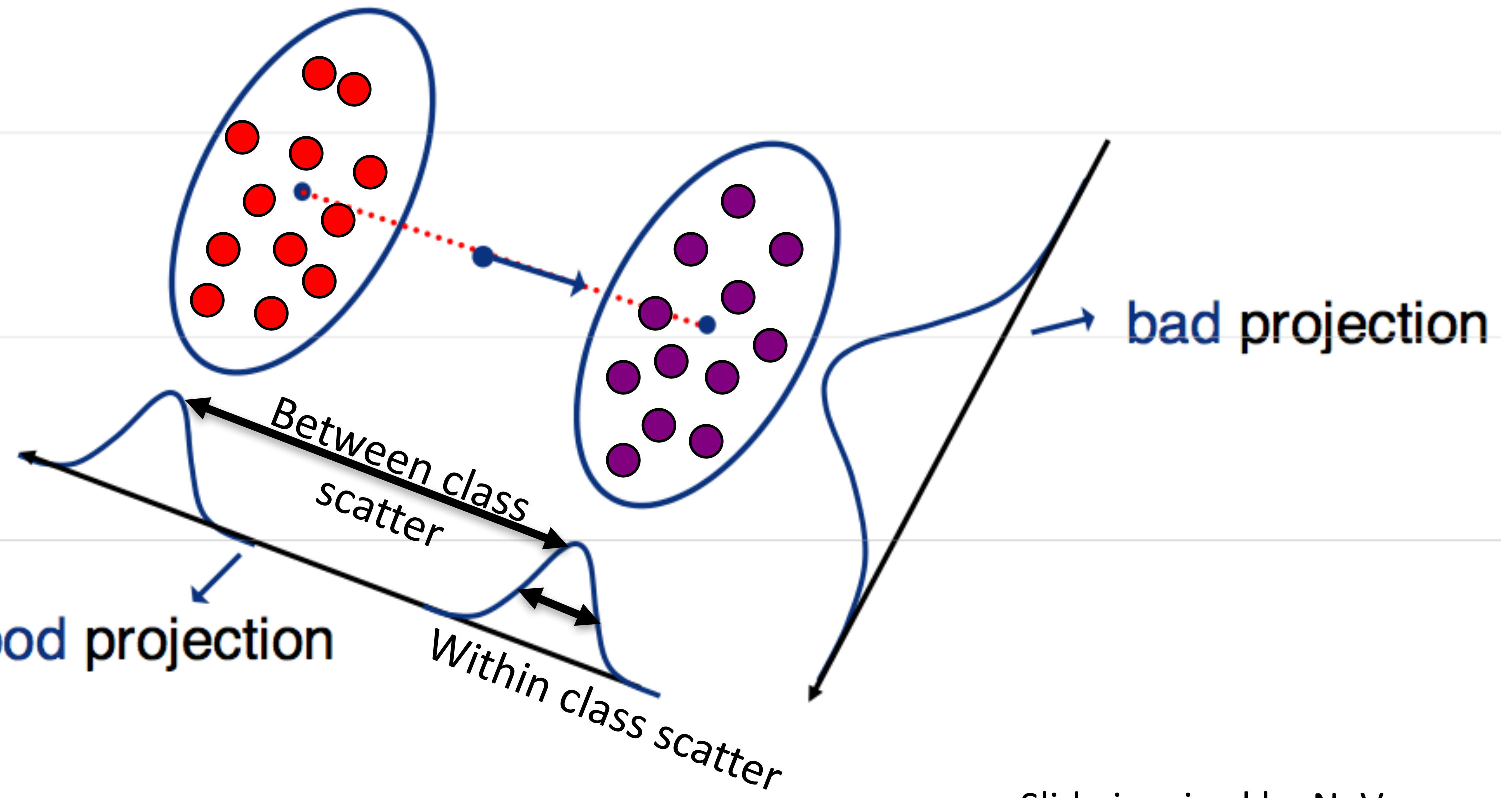
- And the **per class** covariance matrices be:

$$E_{X|Y}\left[(X - \mu_i)(X - \mu_i)^T \mid Y = i\right] = \Sigma_i$$

- We want a projection that maximizes:

$$J(w) = \max \frac{between\ \ class\ \ scatter}{within\ \ class\ \ scatter}$$

Fischer's Linear Discriminant Analysis



Slide inspired by N. Vasconcelos

LDA with 2 variables

The following objective function:

$$J(w) = \max \frac{between\ \ class\ \ scatter}{within\ \ class\ \ scatter}$$

Can be written as

$$J(w) = \frac{\left(E_{Z|Y}[Z \mid Y = 1] - E_{Z|Y}[Z \mid Y = 0]\right)^2}{\mathrm{var}[Z \mid Y = 1] + \mathrm{var}[Z \mid Y = 0]}$$

LDA with 2 variables

- We can write the between class scatter as:

$$\left(E_{Z|Y}[Z \mid Y=1]-E_{Z|Y}[Z \mid Y=0]\right)^2=\left(w^T[\mu_1-\mu_0]\right)^2$$
$$=w^T[\mu_1-\mu_0][\mu_1-\mu_0]^T w$$

- Also, the within class scatter becomes:

$$\operatorname{var}[Z \mid Y=i]=E_{Z|Y}\left\{\left(z-E_{Z|Y}[Z \mid Y=i]\right)^2 \mid Y=i\right\}$$
$$=E_{Z|Y}\left\{\left(w^T[x-\mu_i]\right)^2 \mid Y=i\right\}$$
$$=E_{Z|Y}\left\{w^T[x-\mu_i][x-\mu_i]^T w \mid Y=i\right\}$$
$$=w^T\Sigma_i w$$

Vasconcelos

LDA with 2 variables

- We can plug in these scatter values to our objective function:

$$\boxed{J(w) = \frac{w^T S_B w}{w^T S_W w}} \qquad \boxed{\begin{aligned} S_B &= (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T \\ S_W &= (\Sigma_1 + \Sigma_0) \end{aligned}}$$

between class scatter

within class scatter

- And our objective becomes:

$$J(w) = \frac{\left(E_{Z|Y}[Z \mid Y=1] - E_{Z|Y}[Z \mid Y=0]\right)^2}{\mathrm{var}[Z \mid Y=1] + \mathrm{var}[Z \mid Y=0]}$$

$$= \frac{w^T (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w}{w^T (\Sigma_1 + \Sigma_0) w}$$

Slide inspired by N. Vasconcelos

LDA with 2 variables

- The scatter variables

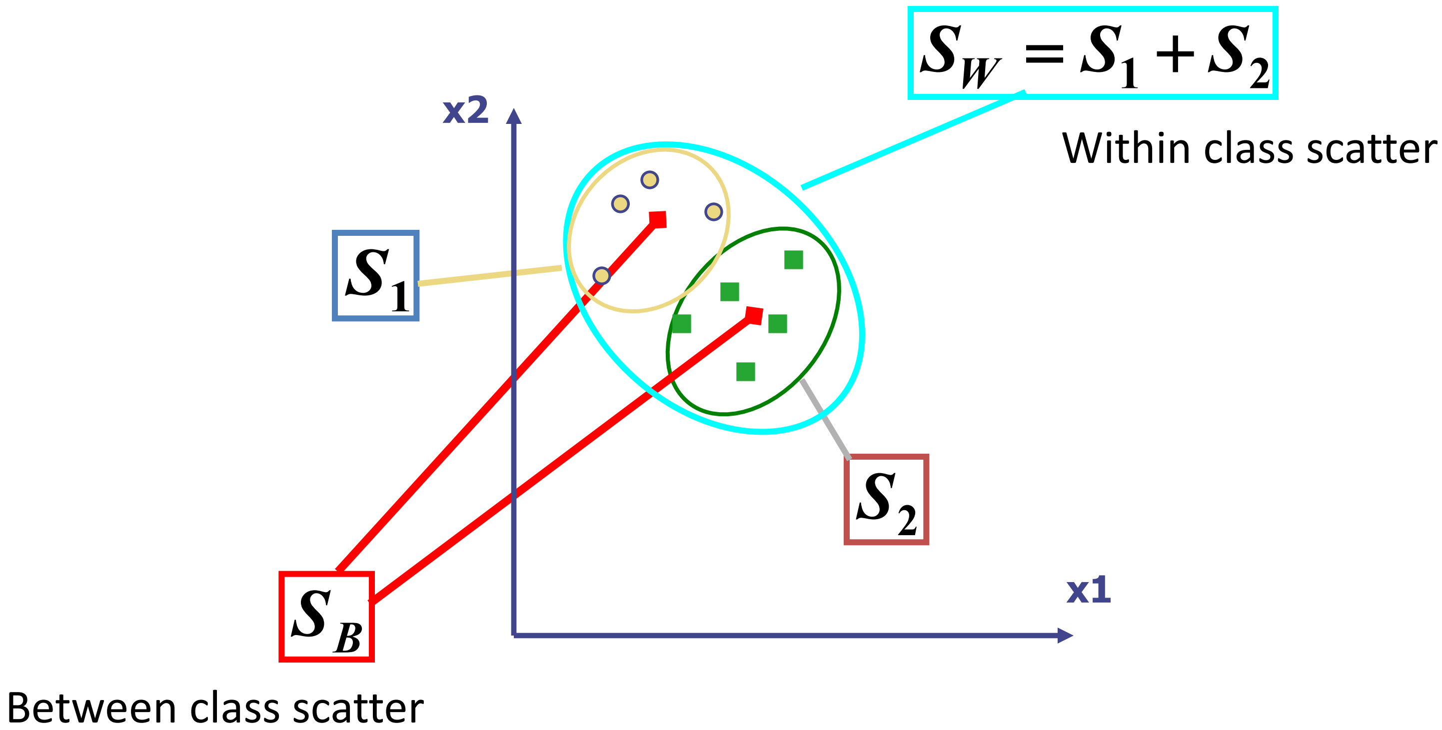
between class scatter

$$\boxed{\begin{aligned} S_B &= (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T \\ S_W &= (\Sigma_1 + \Sigma_0) \end{aligned}}$$

within class scatter

Slide inspired by N. Vasconcelos

Visualization



Linear Discriminant Analysis (LDA)

- Maximizing the ratio

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

- Is equivalent to maximizing the numerator while keeping the denominator constant, i.e.

$$\boxed{\max_w w^T S_B w \quad \text{subject to} \quad w^T S_W w = K}$$

- And can be accomplished using Lagrange multipliers, where we define the Lagrangian as

$$\boxed{L = w^T S_B w - \lambda\left(w^T S_W w - K\right)}$$

- And maximize with respect to both w and λ

Slide inspired by N. Vasconcelos

Linear Discriminant Analysis (LDA)

- Setting the gradient of

$$L = w^T\left(S_B - \lambda S_W\right)w + \lambda K$$

With respect to w to zeros we get

$$\nabla_w L = 2\left(S_B - \lambda S_W\right)w = 0$$

or

$$\boxed{S_B w = \lambda S_W w}$$

- This is a generalized eigenvalue problem
- The solution is easy when $\boxed{S_w^{-1} = \left(\Sigma_1 + \Sigma_0\right)^{-1}}$ exists

Slide inspired by N. Vasconcelos

Linear Discriminant Analysis (LDA)

- In this case

$$S_W^{-1} S_B w = \lambda w$$

- And using the definition of S_B

$$S_W^{-1}(\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w = \lambda w$$

- Noting that $(\mu_1-\mu_0)^T w=\alpha$ is a scalar this can be written as

$$S_W^{-1}(\mu_1 - \mu_0) = \frac{\lambda}{\alpha} w$$

- and since we don't care about the magnitude of w

$$\boxed{w^* = S_W^{-1}(\mu_1 - \mu_0) = (\Sigma_1 + \Sigma_0)^{-1}(\mu_1 - \mu_0)}$$

Slide inspired by N. Vasconcelos

LDA with N variables and C classes

Variables

- N Sample images: $\{x_1, \cdots, x_N\}$
- C classes: $\{Y_1, Y_2, \ldots, Y_c\}$
- Average of each class: $\mu_i = \frac{1}{N_i} \sum_{x_k \in Y_i} x_k$
- Average of all data: $\mu = \frac{1}{N} \sum_{k=1}^{N} x_k$

Scatter Matrices

- Scatter of class i: $$S_i = \sum_{x_k \in Y_i} (x_k - \mu_i)(x_k - \mu_i)^T$$
- Within class scatter: $$S_W = \sum_{i=1}^{c} S_i$$
- Between class scatter: $$S_B = \sum_{i=1}^{c} N_i (\mu_i - \mu)(\mu_i - \mu)^T$$

Mathematical Formulation

- Recall that we want to learn a projection W such that the projection converts all the points from x to a new space z:

$$z = w^T x \qquad z \in \mathbf{R}^m \quad x \in \mathbf{R}^n$$

- After projection:
  - Between class scatter $\tilde{S}_B = W^T S_B W$
  - Within class scatter $\tilde{S}_W = W^T S_W W$
- So, the objective becomes:

$$W_{opt} = \arg\max_{\mathbf{W}} \frac{\left|\tilde{S}_B\right|}{\left|\tilde{S}_W\right|} = \arg\max_{\mathbf{W}} \frac{\left|W^T S_B W\right|}{\left|W^T S_W W\right|}$$

Mathematical Formulation

$$W_{opt} = \arg\max_{\mathrm{W}} \frac{\left|W^T S_B W\right|}{\left|W^T S_W W\right|}$$

- Solve generalized eigenvector problem:

$$S_B w_i = \lambda_i S_W w_i \qquad\qquad i = 1, \ldots, m$$

Mathematical Formulation

- Solution: Generalized Eigenvectors

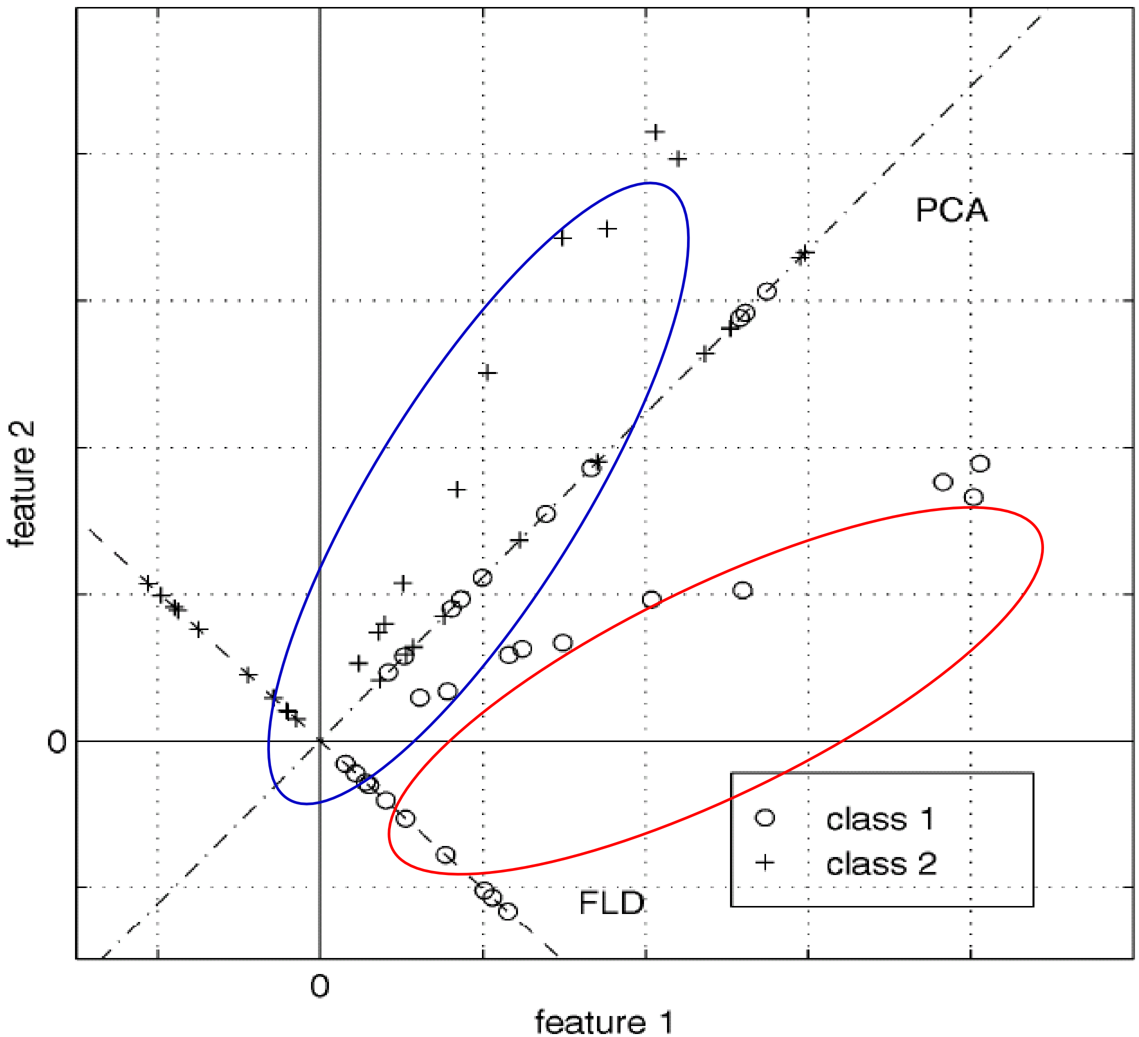
$$S_B w_i = \lambda_i S_W w_i \qquad i = 1, \ldots, m$$

- Rank of $\boldsymbol{W_{opt}}$ is limited
  - Rank(S_B) <= |C|-1
  - Rank(S_W) <= N-C

PCA vs. LDA

- Eigenfaces exploit the max scatter of the training images in face space
- Fisherfaces attempt to maximise the **between class scatter**, while minimising the **within class scatter**.

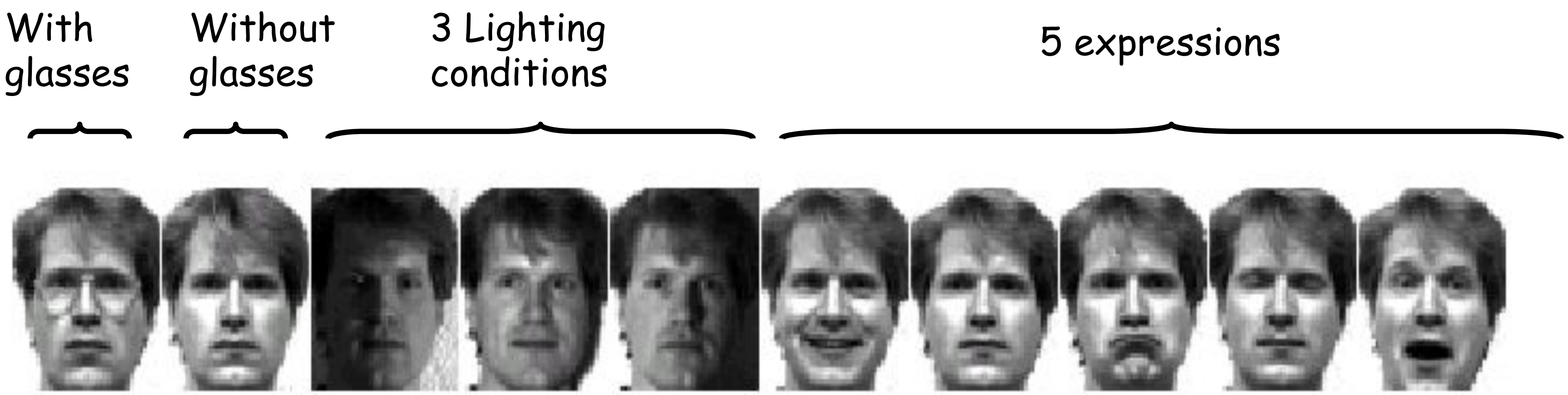
Basic intuition: PCA vs. LDA



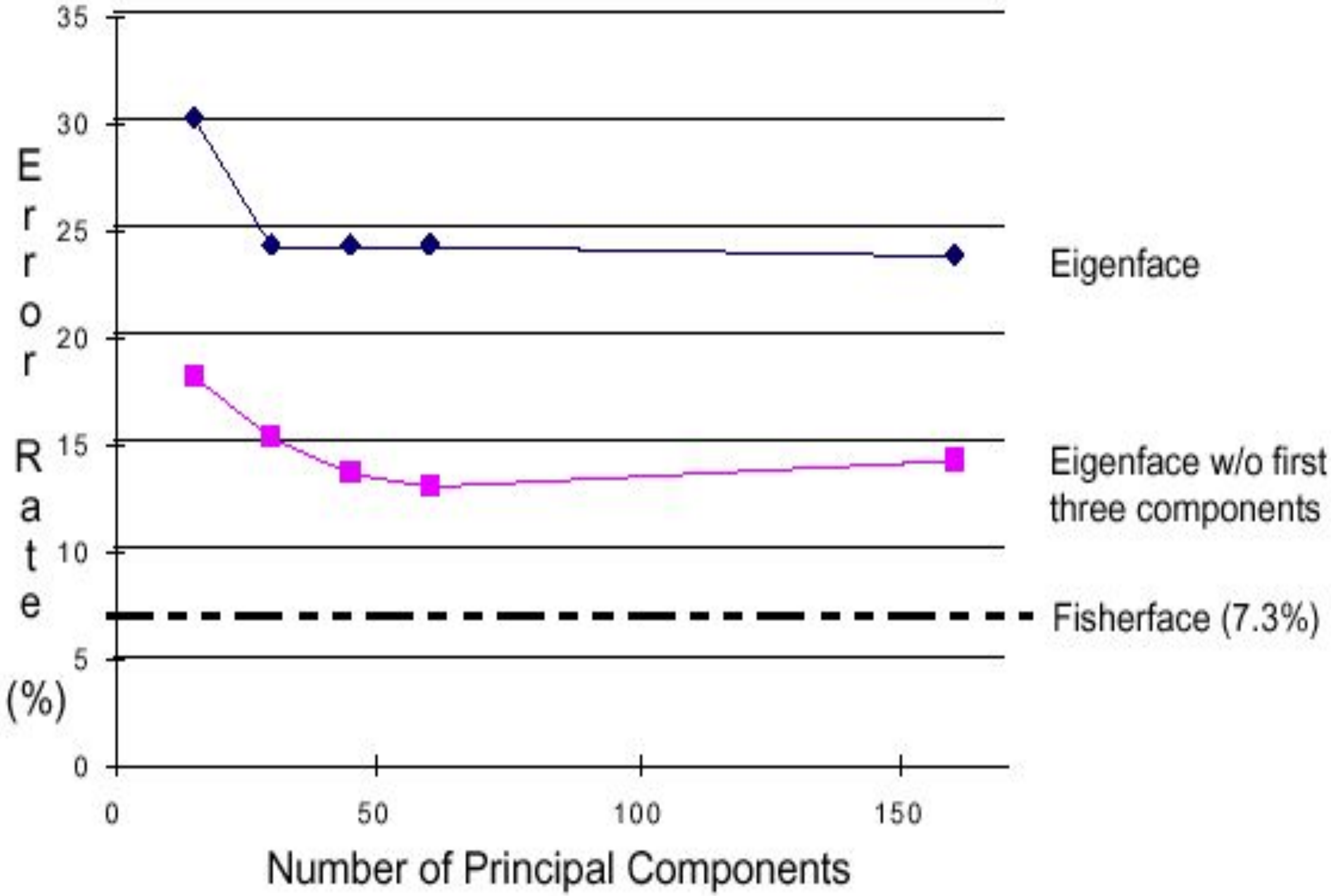
Results: Eigenface vs. Fisherface

- Input: 160 images of 16 people
- Train: 159 images
- Test: 1 image

- Variation in Facial Expression, Eyewear, and Lighting



Eigenface vs. Fisherface



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