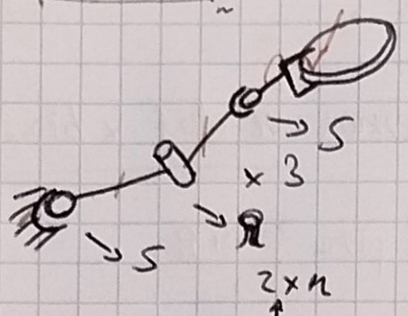
Ejercicio 1



$N = 1 + 1 + 6 = 8$

$J = 3 \times 3 = 9$

(3 brazos, 3 articulaciones × brazo)

$\Sigma f_i = 7 + 7 + 7 = 21$

Aplicando Grübler: $dof = 6(8 - 1 - 9) + 21 = 9$ ✓

(6(−2) + 21 = −12 + 21)

a) dof del sistema?

Base → 1 eslabón fijo

Disco → 1 eslabón

Brazo → 2 eslabones móviles, 3 articulaciones | 2×(S) → 3, 1×(R) → 1

Cada brazo aporta $f_i = 1 + 3 + 3 = 7$ dof

b) Con n brazos?

$dof = 6(\underbrace{1+1-1+2n}_{N} - \underbrace{3n}_{J}) + \underbrace{7n}_{\Sigma f_i} = 6 + n$ ✓

$N = 2$? $J = n$? ✓?

c) Remplazar S → U

(3 dof → 2 dof)

URU

$dof = 6(\overbrace{2+2n-1}^{N} - \overbrace{3n}^{J}) + \overbrace{5n}^{\Sigma f_i} = \underline{\underline{6 - n}}$

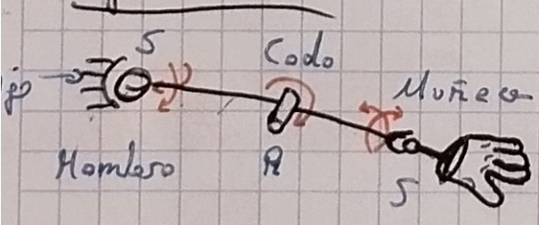
$f_i = URU = 2 + 1 + 2 = 5$

URS

$dof = 6(2+2n-1-3n) - 6n = \underline{\underline{6}}$

$f_i = URS = 2 + 1 + 3 = 6$

Ejercicio 2



$N_a = 4$; $N_b = 3$

$J = 3$

$f_a = 3 + 1 + 3$

$f_b = 3 + 1 + 3$ } Σf_i

a) Brazo libre: $dof = 3 + 1 + 3 = 7$ dof ✓

* $dof = 6(4 - 1 - 3) + 7 = 7$ dof

b) Brazo con la mano apoyada en la mesa:

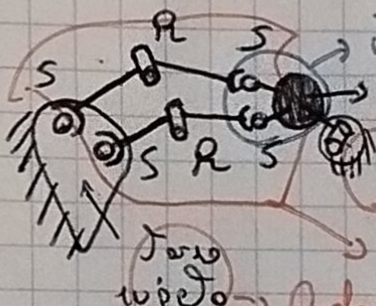
$dof = 7 - 6 = 1$ dof ✓

(7 = 3 + 1 + 3; 6 = 6(3 − 1 − 3) + 7)

* $dof = 6(3 - 1 - 3) + 7 = 1$ dof

Ejercicio 3

¿dof tiene el sistema brazos – volante?



→ Junta fija → $f = 0$

→ Volante

R → aporta 1 dof

Baja n dof x brazo

Torso sujeto → 0 dof

× Supunemos n dof x brazo

¿El volante gira? → + R

$N = 1$ (torso) $+ 2n$ (brazo) $+ 1$ (volante) $= 2 + 2n$

$J = 2n + 1 + 2 = 2n + 3$

$$dof = 6(N-1-J) + \sum_{i=1}^{J} f_i = 6(2+2n-1-2n-3)+2n+1$$
$$= 6(-2)+2n+1$$
$$= 2n-11 \checkmark$$

$\Sigma f_i = 2n + 1$ (brazos, volante)

Ejercicio 4

¡En su conjunto!

Brazo (6R) con una base móvil de una rueda.

↳ no derrape y la (rueda + base) gira en un eje ⊥ al suelo, manteniendose horizontal.

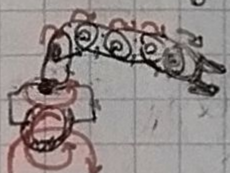
a) ¿Espacio de configuraciones?

La base móvil se mueve en el plano ($\mathbb{R}^2$).

↳ Componente traslacional

$dof = m(N-1-J) + \Sigma f_i$

(m → 6; N → 8; J → 1+6; Σf_i → 6+1+1)



$dof = 6 + 1 + 1 = 8$ ⇒ Espacio de configuraciones es $\mathbb{R}^2 \times T^8$ ✓

(1: giro base; 1: giro rueda; $\mathbb{R}^2$: mov traslacional; T^8: mov rotacional)

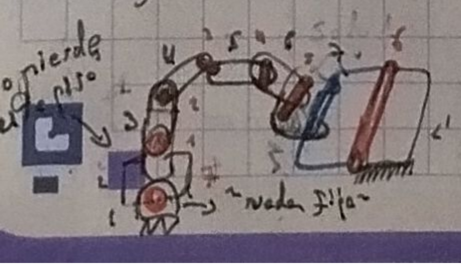
b) Fija la rueda y la base, además el robot sujeta el asa de la puerta. → Solo quedan móviles los elementos del brazo a la puerta.

Paso de cadena abierta con 6 uniones a cadena cerrada, conectando el brazo y la puerta como un conjunto.

$N = 7$; $J = 7$; $\Sigma f_i = 7$

$dof = 6(7-1-7)+7 = 1$ dof ✓

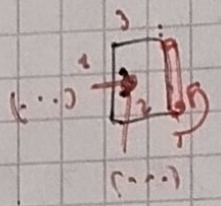
↳ Igual que la puerta por sí misma.



c) Añadimos otro róbot igual en la situación de b)

* Por lógica los dof deberían permanecer igual (dof = 1)

$\boxed{N = 7\,(\text{anteriores}) + \boxed{7\,(\text{nuevos}) - 1\,[\text{base}] - 1\,[\text{brazo conectado a la puerta}]} = 12}$



$J = 7\,[\text{brazo 1}] + 7\,[\text{brazo 2}] - 1\,[\text{unión entre brazo y puerta no se puede contar dos veces}]$

$\boxed{J = 13}$

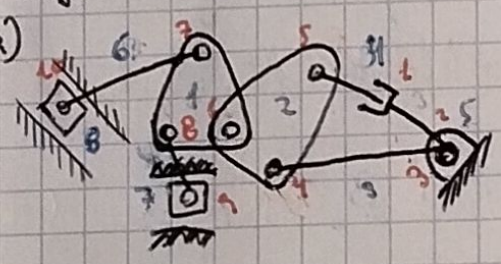
$\boxed{\Sigma f_i = 13}$

$dof = 6(12 - 1 - 13) + 13 = -12 + 13 = 1$

* La lógica anterior permanece

Ejercicio 5

a)



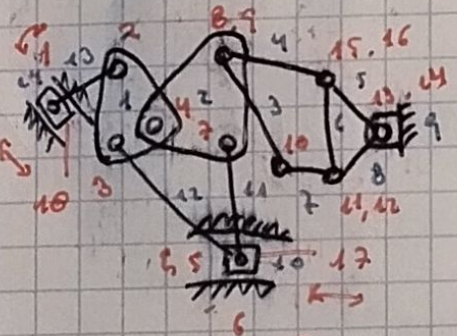
$N = 8$ ✓

$J = 10$ ✓

$\Sigma f_i = 1+1+1+1+1+1+1\times2+1+1\times2+1 = 12$

$dof = 3(8-1-10)+12 = 3$ ✓

b)



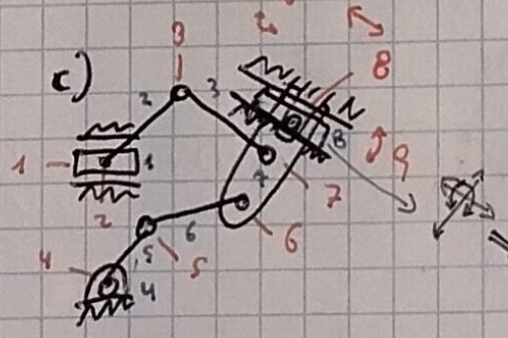
$N = 14$ ✓

$J = 18$ ✓

$\Sigma f_i = 18$ ✓

$dof = 3(14-1-18)+18 = 3$ ✓

c)



$N = 8$ ✓

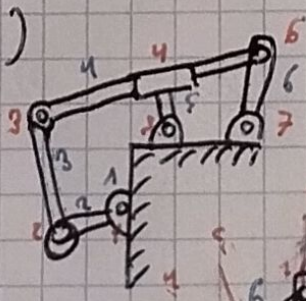
$J = 9$ ✓

$\Sigma f_i = 9+1 = 10$ ✓

2 articulaciones
3 dof!

$dof = 3(8-1-9)+6 = 4$ ✓

d)



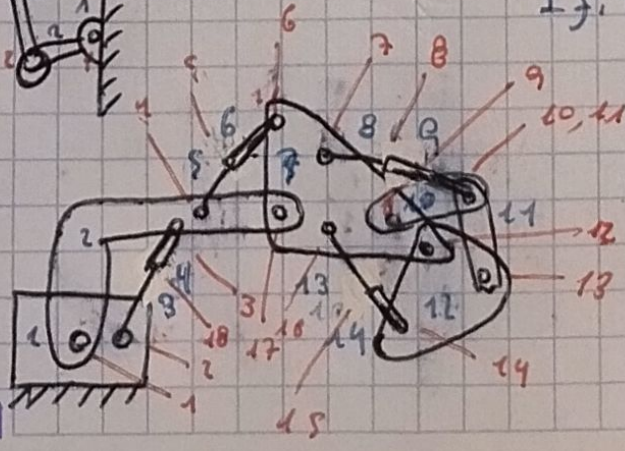
$N = 6$ ✓

$J = 7$ ✓

$\Sigma f_i = 7$ ✓

$dof\ 3(6-1-7)+7 = 1$ ✓

e)



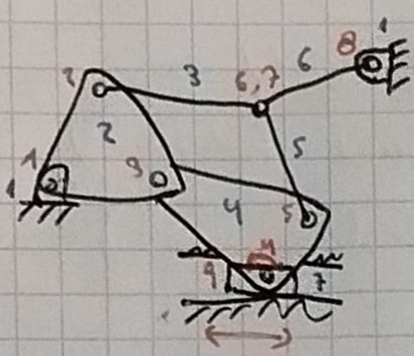
$N = 14$ ✓

$J = 18$ ✓

$\Sigma f_i = 18\times1 = 18$

$dof = 3(14-1-18)+18 = 3$ ✓

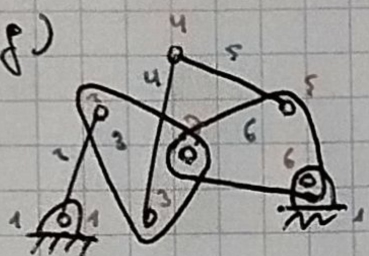
f)



$N = 7$ ✓
$J = 9$ ✓
$\Sigma f_i = 9$ ✓

$dof = 3(7-1-9)+9 = 0$ ✓
→ estructura fija

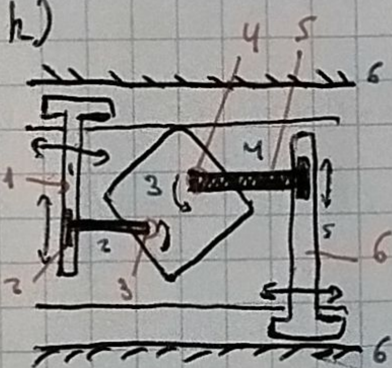
g)



$N = 6$ ✓
$J = 7$ ✓
$\Sigma f_i = 7$ ✓

$dof = 3(6-1-7)+7 = 1$ ✓

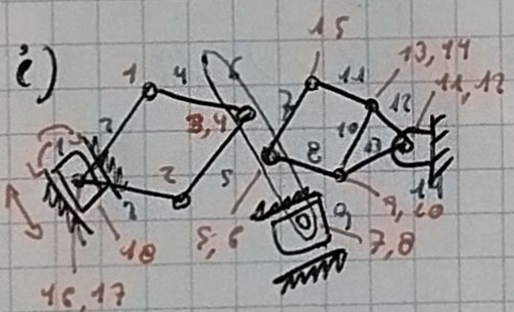
h)



$N = 5+1 = 6$ ✓
$J = 6$ ✓
$\Sigma f_i = 6$ ✓

$dof = 3(6-1-6)+6 = 3$ ✓

i)

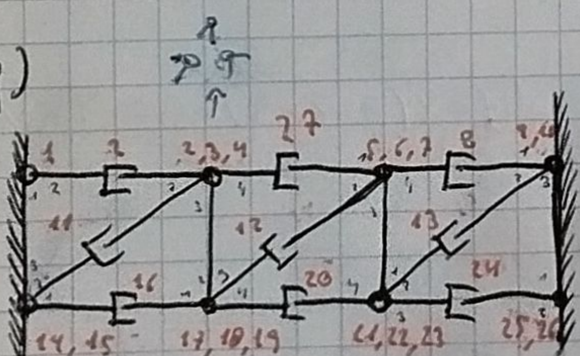


$N = 14$ ✓
$J = 18$ ✓
$\Sigma f_i = 18$ ✓

$dof = 3(14-1-18)+18 = 3$ ✓

x??!
✓
9P + 2Eslab + 1base

j)



$N = 9$ [prismaticas] + 2 [eslabones] + 1 [base] $= 9\times2+3 = 18+3 = 21$ ✓

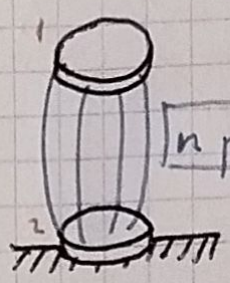
$J = 27$ ✓

$\Sigma f_i = 27$ ✓

Ejercicio 6

Aplicando Fórmula de Grübler: $dof = m(N-1-J) + \sum f_i$

En 3D → $\boxed{m=6}$



→ Disco móvil

n pares → Aporta p dof

→ Disco fijo

$dof = 6(N-1) - 6J + \sum f_i$

$dof = 6 - 6n + np = 6 - n(6-p)$

Para tener 6 dofs:

$dof \Rightarrow 6 - n(6-p) = 6$

$-n(6-p) = 0$

$p = 6$ → Se tienen 6 dofs siempre si los pares aportan 6 dof cada uno.

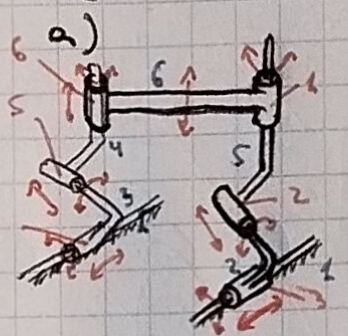
$N = 2$

$J = n$

$\sum f_i = n \cdot p$

Ejercicio 7 $m = 6$

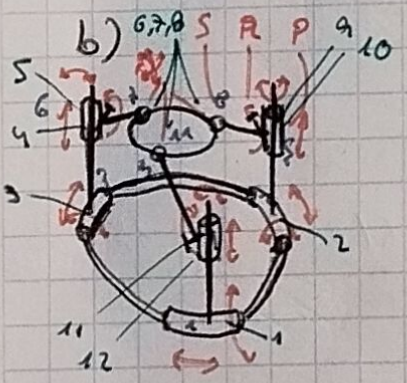
a)



$N = 6$
$J = 6$
$\sum f_i = 12$

$dof = 6(6-1-6) + 12 = 6$ ✓

b)

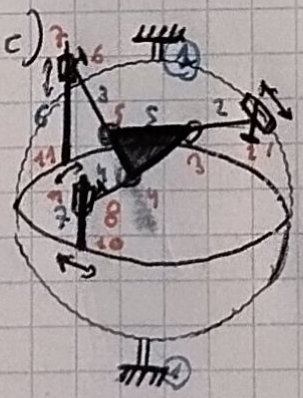


$N = 11$ ¿Se limitan pq?
$J = 12$
$\sum f_i = \underbrace{1 \times 3}_{P} + \underbrace{1 \times 3}_{P} + \underbrace{1 \times 3}_{R} + \underbrace{3 \times 3}_{S} = 18$

$3 + 3 + 3 + 9$

$dof = 6(11-1-12) + 18 = 6$ ✓

c)

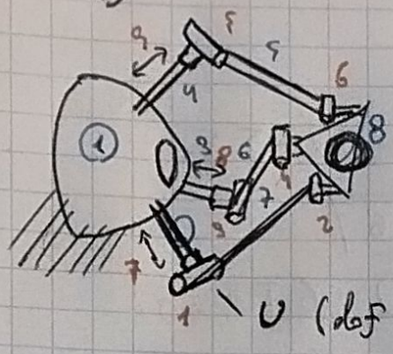


$N = 7$
$J = 8$
$\sum f_i = 2 \times 3 + 2 \times 2 + 1 \times 2 + 3 + 2 = 17$ (1×2 → 12, 3 → 45)

$dof = 6(7-1-8) + 17 = 5$ ✓

* 5, 4 y 8 ↗ → 8

d)

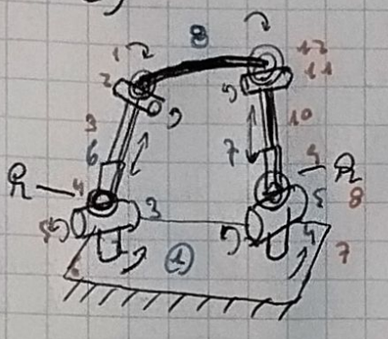


$N = 8$
$J = 9$
$\Sigma f_i = \underbrace{3\times1}_{3} + \underbrace{2\times6}_{12} = 15$

$dof = 6(8-1-9)+15 = 3$

U (dof 2)

e)

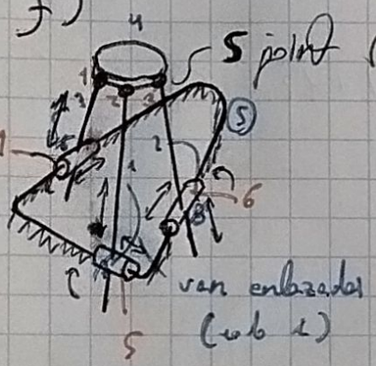


$N = 8$
$J = 12-4 = 8$
$\Sigma f_i = 12$

$dof = 6(8-1-8)+12 = 6$

~~1 → 12~~
~~6 → 7~~

f)



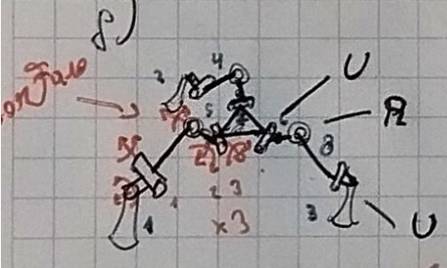
5 joint (3 dof)

van enbazada (wb 1)

$N = 5$
$J = 6$
$\Sigma f_i = \underbrace{3\times3}_{5} + \underbrace{3\times2}_{2} + \underbrace{1\times3}_{1} = 18$

$dof = 6(5-1-6)+18 = 6$

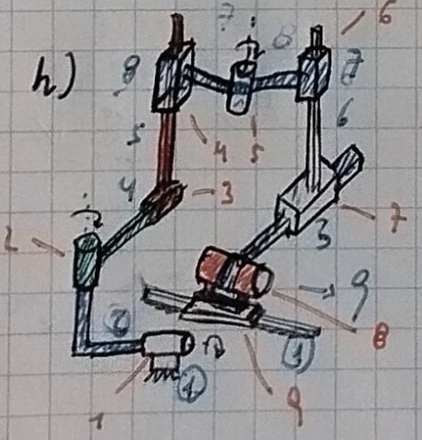
g)



$N = 8$
$J = 9$
$\Sigma f_i = 2\times6 + 1\times3 = 15$

$dof = 6(8-1-9)+15 = 3$

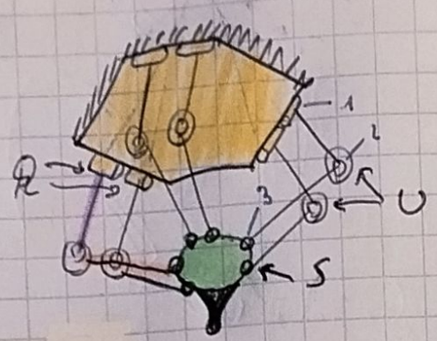
h)



$N = 9$
$J = 9$
$\Sigma f_i = 1\times9 = 9$

$dof = 6(9-1-9)+9 = 3$

i)



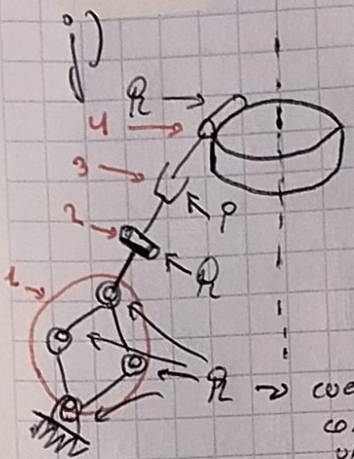
$N = 1 + 2\times6 + 1 = 14$

$U = 3\times6 = 18$

$\Sigma f_i = (1+2+3)\times6 = 36$

$dof = 6(14-1-18)+36 = 6$ ✓

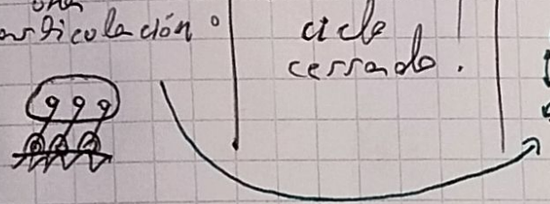
j)

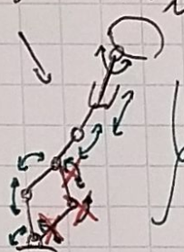


× 4 Brazos cerrados

$N = 1+1+(1+1+2+2)\times4 = 2$

$U = 4$ $\quad \Sigma f_i = 24$

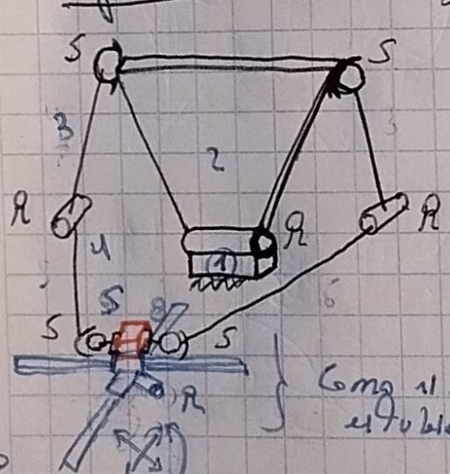




$3+3=6$ × 4 → 24

$dof = 6(2-1-4)+24 = 6$ ✓

Ejercicio 8 dof?

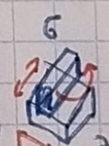


$N = 1[\text{Base}] + 1[R] + 1[S] + 1[R] + 1[S] + 2[\text{Unión}] =$

$3R + 4S + 1\ \text{Unión} \rightarrow J = 8$

$\Sigma f_i = \underbrace{1\times3}_{R} + \underbrace{3\times4}_{S} + \underbrace{3}_{\text{Unión}} = 18$

Como el tampón está bien apoyado



Unión plana 3 dof

2 dof de traslación
1 dof de rotación

$dof = 6(7-1-8)+18 = 6$ ✓