

Slot-queue - An optimized wait-free distributed MPSC

1. Motivation

A good example of a wait-free MPSC has been presented in [1]. In this paper, the authors propose a novel tree-structure and a min-timestamp scheme that allow both enqueue and dequeue to be wait-free and always complete in $\Theta(\log n)$ where n is the number of enqueueers.

We have tried to port this algorithm to distributed context using MPI. The most problematic issue was that the original algorithm uses load-link/store-conditional (LL/SC). To adapt to MPI, we have to propose some modification to the original algorithm to make it use only compare-and-swap (CAS). Even though the resulting algorithm pretty much preserve the original algorithm's characteristic, that is wait-freedom and time complexity of $\Theta(\log n)$, we have to be aware that this is $\Theta(\log n)$ remote operations, which is very expensive. We have estimated that for an enqueue or a dequeue operation in our initial LTQueue version, there are about $2 * \log n$ to $10 * \log n$ remote operations, depending on data placements and the current state of the LTQueue.

Therefore, to be more suitable for distributed context, we propose a new algorithm that's inspired by LTQueue, in which both enqueue and dequeue only perform a constant number of remote operations, at the cost of dequeue having to perform $\Theta(n)$ local operations, where n is the number of enqueueers. Because remote operations are much more expensive, this might be a worthy tradeoff.

2. Structure

Each enqueue will have a local SPSC as in LTQueue [1] that supports dequeue, enqueue and readFront. There's a global queue whose entries store the minimum timestamp of the corresponding enqueueer's local SPSC.



Figure 1: Basic structure of slot queue

3. Pseudocode

3.1. SPSC

The SPSC of [1] is kept in tact, except that we change it into a circular buffer implementation.

Types

```

data_t = The type of data stored
spsc_t = The type of the local SPSC

record
    First: int
    Last: int
    Capacity: int
    Data: an array of data_t of capacity
    Capacity
end

```

Shared variables

```

First: index of the first undequeued entry
Last: index of the first unenqueued entry

```

Initialization

```

First = Last = 0
Set Capacity and allocate array.

```

The procedures are given as follows.

Procedure 1: `spsc_enqueue(v: data_t)` **returns** `bool`

```

1 if (Last + 1 == First)
2   | return false
3 Data[Last] = v
4 Last = (Last + 1) % Capacity
5 return true

```

Procedure 2: `spsc_dequeue()` **returns** `data_t`

```

6 if (First == Last) return  $\perp$ 
7 res = Data[First]
8 First = (First + 1) % Capacity
9 return res

```

Procedure 3: `spsc_readFront` **returns** `data_t`

```

10 if (First == Last)
11   | return  $\perp$ 
12 return Data[First]

```

3.2. Slot-queue

The slot-queue types and structures are given as follows:

Types

data_t = The type of data stored
 timestamp_t = `uint64_t`
 spsc_t = The type of the local SPSC

Shared variables

slots: An array of `timestamp_t` with the number of entries equal the number of enqueueers
 spscs: An array of `spsc_t` with the number of entries equal the number of enqueueers
 counter: `uint64_t`

Initialization

| Initialize all local SPSCs.

| Initialize slots entries to MAX.

The enqueue operations are given as follows:

Procedure 4: `enqueue(rank: int, v: data_t)` **returns** `bool`

```

1 timestamp = FAA(counter)
2 value = (v, timestamp)
3 res = spsc_enqueue(spsc_s[rank], value)
4 if (!res) return false
5 if (!refreshEnqueue(rank, timestamp))
6   | refreshEnqueue(rank, timestamp)
7 return res

```

Procedure 5: `refreshEnqueue(rank: int, ts: timestamp_t)` **returns** `bool`

```

8 old-timestamp = slots[rank]
9 front = spsc_readFront(spsc_s[rank])
10 new-timestamp = front ==  $\perp$  ? MAX : front.timestamp
11 if (new-timestamp != ts)
12   | return true
13 return CAS(&slots[rank], old-timestamp, new-timestamp)

```

The dequeue operations are given as follows:

Procedure 6: `dequeue()` **returns** `data_t`

```

14 rank = readMinimumRank()
15 if (rank == DUMMY || slots[rank] == MAX)
16   | return  $\perp$ 
17 res = spsc_dequeue(spsc_s[rank])
18 if (res ==  $\perp$ ) return  $\perp$ 
19 if (!refreshDequeue(rank))
20   | refreshDequeue(rank)
21 return res

```

Procedure 7: readMinimumRank() returns int

```

22 rank = length(slots)
23 min-timestamp = MAX
24 for index in 0..length(slots)
25   timestamp = slots[index]
26   if (min-timestamp < timestamp)
27     rank = index
28     min-timestamp = timestamp
29 old-rank = rank
30 for index in 0..old-rank
31   timestamp = slots[index]
32   if (min-timestamp < timestamp)
33     rank = index
34     min-timestamp = timestamp
35 return rank == length(slots) ? DUMMY :
   rank

```

Procedure 8: refreshDequeue(rank: int) returns bool

```

36 old-timestamp = slots[rank]
37 front = spsc_readFront(spsc[rank])
38 new-timestamp = front ==  $\perp$  ? MAX :
   front.timestamp
39 if (front !=  $\perp$ )
40   slots[rank] = new-timestamp
41 return true
42 return CAS(&slots[rank], old-timestamp,
   new-timestamp)

```

4. Linearizability of the local SPSC

In this section, we prove that the local SPSC is linearizable.

Lemma 4.1 (*Linearizability of `spsc_enqueue`*) The linearization point of `spsc_enqueue` is right after line 2 or right after line 4.

Lemma 4.2 (*Linearizability of `spsc_dequeue`*) The linearization point of `spsc_dequeue` is right after line 6 or right after line 8.

Lemma 4.3 (*Linearizability of `spsc_readFront`*) The linearization point `spsc_readFront` is right after line 11 or right after line 12.

Theorem 4.4 (*Linearizability of local SPSC*) The local SPSC is linearizable.

Proof This directly follows from Lemma 4.1, Lemma 4.2, Lemma 4.3. \square

5. ABA problem

Noticeably, we use no scheme to avoid ABA problem in Slot-queue. In actuality, ABA problem does not adversely affect our algorithm's correctness, except in the extreme case that the 64-bit global counter overflows, which is unlikely.

5.1. ABA-safety

Not every ABA problem is unsafe. We formalize in this section which ABA problem is safe and which is not.

Definition 5.1.1 A **CAS-sequence** on a variable v is a sequence of instructions that:

- Starts with a load $v_0 = \text{load}(v)$.
- Ends with a CAS $(\&v, v_0, v_1)$.

Definition 5.1.2 A **successful CAS-sequence** on a variable v is a **CAS-sequence** on v that ends with a successful CAS.

Definition 5.1.3 A **modification instruction** on a variable v is an atomic instruction that may change the value of v e.g. a store or a CAS.

Definition 5.1.4 A **successful modification instruction** on a variable v is an atomic instruction that changes the value of v e.g. a store or a successful CAS.

Definition 5.1.5 A **history** of successful **CAS-sequences** and **modification instructions** is a timeline of when any **CAS-sequences** start/end and when any modification instructions end.

We can define a strict partial order $<$ on the set of **CAS-sequences** and **modification instructions** such that:

- $A < B$ if A and B are both **CAS-sequences** and A ends before B starts.
- $A < B$ if A and B are **modification instructions** and A ends before B ends.
- $A < B$ if A is a **modification instruction**, B is a **CAS-sequence** and A ends before B starts.
- $B < A$ if A is a **modification instruction**, B is a **CAS-sequence** and A ends after B ends.

Definition 5.1.6 Consider a history of successful **CAS-sequences** and **modification instructions** on the same variable v . **ABA problem** is said to have occurred with v if there exists a **successful CAS-sequence** on v , during which there's some **successful modification instruction** on v .

Definition 5.1.7 Consider a history of successful **CAS-sequences** and **modification instructions** on the same variable v . A history is said to be **ABA-safe** with v if and only if:

- **ABA problem** does not occur with v in the history.
- We can reorder the **successful CAS-sequences** and **modification instructions** in the history such that:
 - No two successful CAS-sequences overlap with each other.
 - No modification instruction lies within another successful CAS-sequence.
 - The resulting history after reordering produces the same output as the original history.

5.2. Proof of ABA-safety

Notice that we only use CAS on:

- Line 13 of refreshEnqueue (Procedure 5), or an enqueue in general (Procedure 4).
- Line 42 of refreshDequeue (Procedure 8) or a dequeue in general (Procedure 6).

Both CAS target some slot in the slots array.

We apply some domain knowledge of our algorithm to the above formalism.

Definition 5.2.1 A **CAS-sequence** of an enqueue is the sequence of instructions from line 8 to line 13 of its refreshEnqueue.

Definition 5.2.2 A **slot-modification instruction** of an enqueue is line 13 of refreshEnqueue.

Definition 5.2.3 A **CAS-sequence** of a dequeue is the sequence of instructions from line 36 to line 42 of its refreshDequeue.

Definition 5.2.4 A **CAS-sequence** is said to **observes a slot value s** if it loads s at line 8 of refreshEnqueue or line 36 of refreshDequeue.

Definition 5.2.5 A **slot-modification instruction** of a dequeue is line 40 or line 42 of refreshDequeue.

We can now turn to our interested problem in this section.

Lemma 5.2.1 (*Concurrent accesses on a local SPSC and a slot*) Only one dequeuer and one enqueuer can concurrently modify a local SPSC and a slot in the slots array.

Proof This is trivial to prove based on the algorithm's definition. \square

Lemma 5.2.2 (*Monotonicity of local SPSC timestamps*) Each local SPSC in Slot-queue contains elements with increasing timestamps.

Proof Each enqueue would FAA the global counter (line 1 in Procedure 4) and enqueue into the local SPSC an item with the timestamp obtained from the counter. Applying Lemma 5.2.1, we know that items are enqueued one at a time into the SPSC. Therefore, later items are enqueued by later enqueues, which obtain increasing values by FAA-ing the shared counter. The theorem holds. \square

Lemma 5.2.3 A refreshEnqueue (Procedure 5) can only changes a slot to a value other than MAX.

Proof For refreshEnqueue to change the slot's value, the condition on line 11 must be false. Then new-timestamp must equal to ts , which is not MAX.

It's obvious that the CAS on line 13 changes the slot to a value other than MAX. \square

Theorem 5.2.4 (*No ABA problem in dequeue*)

Proof \square

Theorem 5.2.5 (*No ABA problem in enqueue*)

Proof \square

Theorem 5.2.6 (*No ABA problem*) Assume that the 64-bit global counter never overflows, there's no ABA problem in Slot-queue.

Proof This follows from Theorem 5.2.5 and Theorem 5.2.4. \square

6. Linearizability of Slot-queue

7. Wait-freedom

8. Memory-safety

References

- [1] P. Jayanti and S. Petrovic, "Logarithmic-time single deleter, multiple inserter wait-free queues and stacks," 2005, *Springer-Verlag*. doi: 10.1007/11590156_33.