Slot-queue - An optimized wait-free distributed MPSC

1. Motivation

A good example of a wait-free MPSC has been presented in [1]. In this paper, the authors propose a novel tree-structure and a min-timestamp scheme that allow both enqueue and dequeue to be wait-free and always complete in $\Theta(\log n)$ where n is the number of enqueuers.

We have tried to port this algorithm to distributed context using MPI. The most problematic issue was that the original algorithm uses load-link/ store-conditional (LL/SC). To adapt to MPI, we have to propose some modification to the original algorithm to make it use only compare-and-swap (CAS). Even though the resulting algorithm pretty much preserve the original algorithm's characteristic, that is wait-freedom and time complexity of $\Theta(\log n)$, we have to be aware that this is $\Theta(\log n)$ remote operations, which is very expensive. We have estimated that for an enqueue or a dequeue operation in our initial LTQueue version, there are about $2 * \log n$ to $10 * \log n$ remote operations, depending on data placements and the current state of the LTQueue.

Therefore, to be more suitable for distributed context, we propose a new algorithm that's inspired by LTQueue, in which both enqueue and dequeue only perform a constant number of remote operations, at the cost of dequeue having to perform $\Theta(n)$ local operations, where n is the number of enqueuers. Because remote operations are much more expensive, this might be a worthy tradeoff.

2. Structure

Each enqueue will have a local SPSC as in LTQueue [1] that supports dequeue, enqueue and readFront. There's a global queue whose entries store the minimum timestamp of the corresponding enqueuer's local SPSC.



Figure 1: Basic structure of slot queue

3. Pseudocode

3.1. **SPSC**

The SPSC of [1] is kept in tact, except that we change it into a circular buffer implementation.

Types

```
data_t = The type of data stored
spsc_t = The type of the local SPSC
    record
    First: int
    Last: int
    Capacity: int
    Data: an array of data_t of capacity
    Capacity
    end
```

Shared variables

First: index of the first undequeued entry

Last: index of the first unenqueued entry

Initialization

```
First = Last = 0
Set Capacity and allocate array.
```

The procedures are given as follows.

Procedure 1: spsc_enqueue(v: data_t) returns bool

- 1 if (Last + 1 == First)
 2 | return false
- 3 Data[Last] = v
- 4 Last = (Last + 1) % Capacity
- 5 return true

Procedure 2: spsc_dequeue() returns data_t

- 6 **if** (First == Last) **return** \perp
- 7 res = Data[First]
- 8 First = (First + 1) % Capacity
- 9 return res

Procedure 3: spsc readFront returns data t

- 10 if (First == Last)
- 11 | return ⊥
- 12 return Data[First]

3.2. Slot-queue

The slot-queue types and structures are given as follows:

Types

data_t = The type of data stored
timestamp_t = uint64_t
spsc_t = The type of the local SPSC

Shared variables

slots: An array of timestamp_t with the number of entries equal the number of enqueuers spscs: An array of spsc_t with the number of entries equal the number of enqueuers counter: uint64_t

Initialization

| Initialize all local SPSCs.

Initialize slots entries to MAX.

The enqueue operations are given as follows:

Procedure 4: enqueue(rank: int, v: data_t)
returns bool

- 1 timestamp = FAA(counter)
- 2 value = (v, timestamp)
- 3 res = spsc_enqueue(spscs[rank], value)
- 4 if (!res) return false
- 5 if (!refreshEnqueue(rank, timestamp))
- 6 | refreshEnqueue(rank, timestamp)
- 7 return res

Procedure 5: refreshEnqueue(rank: int, ts:
timestamp_t) returns bool

- 8 old-timestamp = slots[rank]
- 9 front = spsc_readFront(spscs[rank])
- new-timestamp = front == \(\perp \) ? MAX :
 front.timestamp
- 11 if (new-timestamp != ts)
- 12 | return true
- return CAS(&slots[rank], old-timestamp,
 new-timestamp)

The dequeue operations are given as follows:

Procedure 6: dequeue() returns data_t

- 14 rank = readMinimumRank()
- 15 if (rank == DUMMY || slots[rank] == MAX)
- 16 | return ⊥
- 17 res = spsc_dequeue(spscs[rank])
- 18 **if** (res == \perp) **return** \perp
- 19 if (!refreshDequeue(rank))
- 20 | refreshDequeue(rank)
- 21 return res

Procedure 7: readMinimumRank() returns int

```
22 rank = length(slots)
23 min-timestamp = MAX
24 for index in 0..length(slots)
     timestamp = slots[index]
25
     if (min-timestamp < timestamp)</pre>
26
       rank = index
2.7
       min-timestamp = timestamp
28
29 \text{ old-rank} = \text{rank}
30 for index in 0..old-rank
     timestamp = slots[index]
31
     if (min-timestamp < timestamp)
32
       rank = index
33
      min-timestamp = timestamp
  return rank == length(slots) ? DUMMY :
35
   rank
```

Procedure 8: refreshDequeue(rank: int) returns bool

```
36 old-timestamp = slots[rank]
37 front = spsc_readFront(spscs[rank])
38 new-timestamp = front == \(\perp \)? MAX :
front.timestamp

39 return CAS(&slots[rank], old-timestamp,
new-timestamp)
```

4. Linearizability of the local SPSC

In this section, we prove that the local SPSC is linearizable.

Lemma 4.1 (*Linearizability of spsc_enqueue*) The linearization point of spsc_enqueue is right after line 2 or right after line 4.

Lemma 4.2 (*Linearizability of spsc_dequeue*) The linearization point of spsc_dequeue is right after line 6 or right after line 8.

Lemma 4.3 (*Linearizability of spsc_readFront*) The linearization point spsc_readFront is right after line 11 or right after line 12.

Theorem 4.4 (*Linearizability of local SPSC*) The local SPSC is linearizable.

Proof This directly follows from Lemma 4.1, Lemma 4.2, Lemma 4.3. □

5. ABA problem

Noticeably, we use no scheme to avoid ABA problem in Slot-queue. In actuality, ABA problem does not adversely affect our algorithm's correctness, except in the extreme case that the 64-bit global counter overflows, which is unlikely.

5.1. ABA-safety

Not every ABA problem is unsafe. We formalize in this section which ABA problem is safe and which is not.

Definition 5.1.1 A **modification instruction** on a variable v is an atomic instruction that may change the value of v e.g. a store or a CAS.

Definition 5.1.2 A successful modification instruction on a variable v is an atomic instruction that changes the value of v e.g. a store or a successful CAS.

Definition 5.1.3 A **CAS-sequence** on a variable v is a sequence of instructions of a method m such that:

- The first instruction is a load $v_0 = load(v)$.
- The last instruction is a CAS(&v, v_0 , v_1).
- There's no modification instruction on v between the first and the last instruction.

Definition 5.1.4 A **successful CAS-sequence** on a variable v is a **CAS-sequence** on v that ends with a successful CAS.

Definition 5.1.5 Consider a method m on a concurrent object S. m is said to be **ABA-safe** if and only if for any history of method calls produced from S, we can reorder any successful CAS-

sequences by an invocation of m in the following fashion:

- If a successful CAS-sequence is part of an invocation of m, after reordering, it must still be part of that invocation.
- If a successful CAS-sequence by an invocation of m precedes another in a method invocation, after reordering, this ordering is still respected.
- Any successful CAS-sequence by an invocation of m after reordering must not overlap with a successful modification instruction on the same variable.
- After reordering, all method calls' response events on the concurrent object *S* stay the same.

5.2. Proof of ABA-safety

Notice that we only use CAS on:

- Line 13 of refreshEnqueue (Procedure 5), or an enqueue in general (Procedure 4).
- Line 42 of refreshDequeue (Procedure 8) or a dequeue in general (Procedure 6).

Both CAS target some slot in the slots array.

We apply some domain knowledge of our algorithm to the above formalism.

Definition 5.2.1 A **CAS-sequence** on a slot s of an enqueue that corresponds to s is the sequence of instructions from line 8 to line 13 of its refreshEnqueue.

Definition 5.2.2 A **slot-modification instruction** on a slot s of an enqueue that corresponds to s is line 13 of refreshEnqueue.

Definition 5.2.3 A **CAS-sequence** on a slot s of a dequeue that corresponds to s is the sequence of instructions from line 36 to line 42 of its refreshDequeue.

Definition 5.2.4 A **slot-modification instruction** on a slot s of a dequeue that corresponds to s is line 40 or line 42 of refreshDequeue.

Definition 5.2.5 A CAS-sequence of a dequeue/ enqueue is said to observes a slot value of s_0 if it loads s_0 at line 8 of refreshEnqueue or line 36 of refreshDequeue.

We can now turn to our interested problem in this section.

Lemma 5.2.1 (Concurrent accesses on a local SPSC and a slot) Only one dequeuer and one enqueuer can concurrently modify a local SPSC and a slot in the slots array.

Proof This is trivial to prove based on the algorithm's definition. \Box

Lemma 5.2.2 (Monotonicity of local SPSC timestamps) Each local SPSC in Slot-queue contains elements with increasing timestamps.

Proof Each enqueue would FAA the global counter (line 1 in Procedure 4) and enqueue into the local SPSC an item with the timestamp obtained from the counter. Applying Lemma 5.2.1, we know that items are enqueued one at a time into the SPSC. Therefore, later items are enqueued by later enqueues, which obtain increasing values by FAA-ing the shared counter. The theorem holds.

Lemma 5.2.3 A refreshEnqueue (Procedure 5) can only changes a slot to a value other than MAX.

Proof For refreshEnqueue to change the slot's value, the condition on line 11 must be false. Then new-timestamp must equal to ts, which is not MAX. It's obvious that the CAS on line 13 changes the slot to a value other than MAX.

Theorem 5.2.4 (ABA safety of dequeue) Assume that the 64-bit global counter never overflows, dequeue (Procedure 6) is ABA-safe.

Proof Consider a **successful CAS-sequence** on slot s by a dequeue d.

Denote t_d as the value this CAS-sequence observes.

Due to Lemma 5.2.1, there can only be at most one enqueue at one point in time within d.

If there's no successful slot-modification instruction on slot s by an enqueue e within d's **successful CAS-sequence**, then this dequeue is ABA-safe.

Suppose the enqueue e executes the last successful slot-modification instruction on slot s within d's successful CAS-sequence. Denote t_e to be the value that e sets s.

If $t_e \neq t_d$, this CAS-sequence of d cannot be successful, which is a contradiction.

Therefore, $t_e = t_d$.

Note that e can only set s to the timestamp of the item it enqueues. That means, e must have enqueued a value with timestamp t_d . However, by definition, t_d is read before e executes the CAS. This means another process (dequeuer/enqueuer) has seen the value e enqueued and CAS s for e before t_d . By Lemma 5.2.1, this "another process" must be another dequeuer d' that precedes d because it overlaps with e.

Because d' and d cannot overlap, while e overlaps with both d' and d, e must be the first enqueue on s that overlaps with d. Combining with Lemma 5.2.1 and the fact that e executes the last successful slot-modification instruction on slot s within d's successful CAS-sequence, e must be the only enqueue that executes a successful slot-modification instruction on s within d's successful CAS-sequence.

During the start of d's successful CAS-sequence till the end of e, spsc_readFront on the local SPSC must return the same element, because:

- There's no other dequeues running during this time.
- There's no enqueue other than *e* running.
- The spsc_enqueue of e must have completed before the start of d's successful CAS sequence, because a previous dequeuer d' can see its effect.

Therefore, if we were to move the starting time of d's successful CAS-sequence right after e has ended, we still retain the output of the program because:

- The CAS sequence only reads two shared values: slots[rank] and spsc_readFront(), but we have proven that these two values remain the same if we were to move the starting time of *d*'s successful CAS-sequence this way.
- The CAS sequence does not modify any values except for the last CAS instruction, and the ending time of the CAS sequence is still the same.
- The CAS sequence modifies slots[rank] at the CAS but the target value is the same because inputs and shared values are the same in both cases.

We have proved that if we move *d*'s successful CAS-sequence to start after the *last* successful slot-modification instruction on slot s within *d*'s successful CAS-sequence, we still retain the program's output.

If we apply the reordering for every dequeue, the theorem directly follows. \Box

Theorem 5.2.5 (ABA safety of enqueue) Assume that the 64-bit global counter never overflows, enqueue (Procedure 4) is ABA-safe.

Proof Consider a **successful CAS-sequence** on slot s by an enqueue e.

Denote t_e as the value this CAS-sequence observes.

Due to Lemma 5.2.1, there can only be at most one enqueue at one point in time within e.

If there's no successful slot-modification instruction on slot s by an dequeue d within e's successful CAS-sequence, then this enqueue is ABA-safe.

Suppose the dequeue d executes the *last* successful slot-modification instruction on slot s within e's successful CAS-sequence. Denote t_d to be the value that d sets s.

If $t_d \neq t_e$, this CAS-sequence of e cannot be successful, which is a contradiction.

Therefore, $t_d = t_e$.

If $t_d=t_e=$ MAX, this means e observes a value of MAX before d even sets s to MAX. If this MAX value is the initialized value of s, it's a contradiction, as s must be non-MAX at some point for a dequeue such as d to run. If this MAX value is set by an enqueue, it's also a contradiction, as refreshEnqueue cannot set a slot to MAX. Therefore, this MAX value is set by a dequeue d'. If $d'\neq d$ then it's a contradiction, because between d' and d, s must be set to be a non-MAX value before d can be run. Therefore, d'=d. But, this means e observes a value set by d, which violates our assumption.

Therefore $t_d=t_e=t'\neq \text{MAX}$. e cannot observe the value t' set by d due to our assumption. Suppose e observes the value t' from s set by another enqueue/dequeue call other than d.

If this "another call" is a dequeue d' other than d, d' precedes d. By Lemma 5.2.2, after each dequeue, the front element's timestamp will be increasing, therefore, d' must have set s to a timestamp smaller than t_d . However, e observes $t_e = t_d$. This is a contradiction.

Therefore, this "another call" is an enqueue e' other than e and e' precedes e. We know that an enqueue only sets s to the timestamp it obtains.

Suppose e' does not overlap with d. e' can only set s to t' if e' sees that the local SPSC has the front element as the element it enqueues. Due to Lemma 5.2.1, this means e' must observe a local SPSC with only the element it enqueues. Then, when d executes readFront, the item e' enqueues must have been dequeued out already, thus, d cannot set s to t'. This is a contradiction.

Therefore, e' overlaps with d.

For e^\prime to set s to the same value as d, e^\prime 's spsc_readFront must serialize after d's spsc_dequeue.

Because e' and e cannot overlap, while d overlaps with both e' and e, d must be the *first* dequeue on s that overlaps with e. Combining with Lemma 5.2.1 and the fact that d executes the *last* **successful**

slot-modification instruction on slot s within e's **successful CAS-sequence**, d must be the only dequeue that executes a **successful slot-modification instruction** within e's **successful CAS-sequence**.

During the start of e's successful CAS-sequence till the end of d, spsc_readFront on the local SPSC must return the same element, because:

- There's no other enqueues running during this time
- There's no dequeue other than d running.
- The spsc_dequeue of d must have completed before the start of e's successful CAS sequence, because a previous enqueuer e' can see its effect.

Therefore, if we were to move the starting time of e's successful CAS-sequence right after d has ended, we still retain the output of the program because:

- The CAS sequence only reads two shared values: slots[rank] and spsc_readFront(), but we have proven that these two values remain the same if we were to move the starting time of *e*'s successful CAS-sequence this way.
- The CAS sequence does not modify any values except for the last CAS/store instruction, and the ending time of the CAS sequence is still the same.
- The CAS sequence modifies slots[rank] at the CAS but the target value is the same because inputs and shared values are the same in both cases.

We have proved that if we move *e*'s successful CAS-sequence to start after the *last* **successful slot-modification instruction** on slot s within *e*'s **successful CAS-sequence**, we still retain the program's output.

If we apply the reordering for every enqueue, the theorem directly follows. \Box

Theorem 5.2.6 (ABA safety) Assume that the 64-bit global counter never overflows, Slot-queue is ABA-safe.

Proof This follows from Theorem 5.2.5 and Theorem 5.2.4.

6. Linearizability of Slot-queue

Definition 6.1 For an enqueue or dequeue op, rank(op) is the rank of the enqueuer whose local SPSC is affected by op.

Definition 6.2 For an enqueuer whose rank is r, the value stored in its corresponding slot at time t is denoted as slot(r,t).

Definition 6.3 For an enqueuer with rank r, the minimum timestamp among the elements between First and Last in its local SPSC at time t is denoted as min-spsc-ts(r,t).

Definition 6.4 For an enqueue, **slot-refresh phase** refer to its execution of line 5-6 of Procedure 4.

Definition 6.5 For a dequeue, **slot-refresh phase** refer to its execution of line 19-20 of Procedure 6.

Definition 6.6 For a dequeue, **slot-scan phase** refer to its execution of line 24-34 of Procedure 7.

Definition 6.7 An enqueue operation e is said to **match** a dequeue operation d if d returns a timestamp that e enqueues. Similarly, d is said to **match** e. In this case, both e and d are said to be **matched**.

Definition 6.8 An enqueue operation e is said to be **unmatched** if no dequeue operation **matches** it.

Definition 6.9 A dequeue operation d is said to be **unmatched** if no enqueue operation **matches** it, in other word, d returns \bot .

We prove some algorithm-specific results first, which will form the basis for the more fundamental results.

Lemma 6.1 If an enqueue e begins its **slot-refresh phase** at time t_0 and finishes at time t_1 , there's always at least one successful

refreshEnqueue or refreshDequeue on rank(e) starting and ending its **CAS-sequence** between t_0 and t_1 .

Proof If one of the two refreshEnqueues succeeds, then the lemma obviously holds.

Consider the case where both fail.

The first refreshEnqueue fails because there's another refreshDequeue executing its **slot-modification instruction** successfully after t_0 but before the end of the first refreshEnqueue's **CAS-sequence**.

The second refreshEnqueue fails because there's another refreshDequeue executing its **slot-modification instruction** successfully after t_0 but before the end of the second refreshEnqueue's **CAS-sequence**. This another refreshDequeue must start its **CAS-sequence** after the end of the first successful refreshDequeue, due to Lemma 5.2.1. In other words, this another refreshDequeue starts and successfully ends its **CAS-sequence** between t_0 and t_1 .

We have proved the theorem. \Box

Lemma 6.2 If a dequeue d begins its **slot-refresh phase** at time t_0 and finishes at time t_1 , there's always at least one successful refreshEnqueue or refreshDequeue on rank(d) starting and ending its **CAS-sequence** between t_0 and t_1 .

Proof This is similar to the above lemma. \Box

Lemma 6.3 Given a rank r and a dequeue d that begins its **slot-scan phase** at time t_0 and finishes at time t_1 . If d finds that $slot(r,t')=s_0\neq$ MAX for some time t' such that $t_0\leq t'\leq t_1$, then $slot(r,t)=s_0\neq$ MAX for any t such that $t'\leq t\leq t_1$.

Proof Denote s_r as the slot of rank r.

 $slot(r,t')=s_0 \neq \text{MAX}$ because some processes have executed a successful slot-modification instruction on s_r to set it to s_0 .

Take op to be the enqueue/dequeue that executes the last successful slot-modification instruction on s_r before t'. By definition, op set s_r to s_0 .

Any dequeue before *d* would have finished before t_0 , and thus its **slot-fresh phase**. By Lemma 6.2, for each dequeue before d, there must be some successful refresh call whose spsc_readFront observes the state of the local SPSC after d's spsc dequeue. By definition, op's refresh call ended after all of these successful refresh call. In the process of proving Theorem 5.2.6, we have proved that the net effect is as if op starts after all of these successful refresh calls. Therefore, op can be treated as if it has seen the local SPSC after any of the previous dequeues' spsc dequeue calls. In other words, op has set s_r to the front element's timestamp after it has observed all previous spsc_dequeue before d. During t_0 to t_1 , there's no spsc_dequeue. Therefore, from after op's successful refresh call until t_1 , there is no new spsc_dequeue that can be observed. Any refresh calls after op until t_1 can only observe new spsc_enqueues, but because op set s_r to a non-MAX value, their corresponding refreshEnqueues cannot affect s_r . Therefore, the lemma holds.

Lemma 6.4 Given a rank r and a dequeue d that begins its **slot-scan phase** at time t_0 and finishes at time t_1 . If d finds that slot(r,t') = MAX for some time t' such that $t_0 \leq t' \leq t_1$, then $slot(r,t) \neq \text{MAX}$ for any t such that $t_0 \leq t \leq t'$.

Proof Because during d's **slot-scan phase**, no other dequeue can run and enqueues can only set a slot to non-MAX, if d finds that slot(r,t')= MAX for some time t' such that $t_0 \leq t' \leq t_1$, then $slot(r,t) \neq$ MAX for any t such that $t_0 \leq t \leq t'$.

The theorem holds.

Lemma 6.5 Given a rank r and a dequeue d that begins its **slot-scan phase** at time t_0 and finishes at time t_1 . If d finds that slot(r,t') = MAX for some time t' such that $t_0 \leq t' \leq t_1$, then $slot(r,t) \neq \text{MAX}$ for any t such that $t_0 \leq t \leq t'$.

Proof Because during d's **slot-scan phase**, no other dequeue can run and enqueues can only set a slot to non-MAX, if d finds that slot(r,t')= MAX for some time t' such that $t_0 \leq t' \leq t_1$, then $slot(r,t) \neq$ MAX for any t such that $t_0 \leq t \leq t'$.

The theorem holds. \Box

Lemma 6.6 Given a rank r, if at time t, there's no enqueue and no dequeue running on rank r, then slot(r,t) = min-spsc-ts(r,t).

Proof Take op to be the enqueue/dequeue that executes the last **slot-modification instruction** on the slot of rank r before t.

If op does not exist, that means either no enqueue/dequeue has started yet or they haven't performed the **slot-modification instruction**. If no enqueue/dequeue has started yet, it's trivial that slot(r,t)=min-spsc-ts(r,t)=MAX. If some has started but haven't performed the **slot-modification instruction** until t, this is a contradiction as at t, no enqueue and dequeue is running on rank r.

Consider op' to be any enqueue/dequeue that finishes before t. By definition, it must have finished its **slot-refresh phase** before t. By Lemma 6.1 and Lemma 6.2, there must be some successful refresh calls before t that observes the effect of $\mathsf{op}'s$'s $\mathsf{spsc_dequeue/spsc_enqueue}$. By our assumption, op 's $\mathsf{slot-modification}$ instruction happens after all of these refresh calls. In the process of proving Theorem 5.2.6, we have proved that op 's $\mathsf{CAS-sequence}$ can be treated as if it observe the state of the local SPSC after any of these $\mathsf{spsc_dequeue}$ / $\mathsf{spsc_enqueue}$. Therefore, it sets the slot of t to the minimum timestamp in the local SPSC after all $\mathsf{spsc_dequeue/spsc_dequeue}$

It follows that slot(r,t) = min-spsc-ts(r,t). \square

We now look at the more fundamental results.

Lemma 6.7 If d matches e, then either e precedes or overlaps with d.

Proof If d precedes e, none of the local SPSCs can contain an item with the timestamp of e. There-

fore, d cannot return an item with a timestamp of e. Thus d cannot match e. Therefore, e either precedes or overlaps with d. \square **Theorem 6.8** If an enqueue e precedes another dequeue d, then either: • *d* isn't matched. • d matches e. • e matches d' and d' precedes d. • d matches e' and e' precedes e. • d matches e' and e' overlaps with e. **Proof Lemma 6.9** If *d* matches *e*, then either *e* precedes or overlaps with d. **Proof Theorem 6.10** If a dequeue d precedes another enqueue e, then either: • d isn't matched. • d matches e' such that e' precedes or overlaps with e and $e' \neq e$. **Proof Theorem 6.11** If an enqueue e_0 precedes another enqueue e_1 , then either: • Both e_0 and e_1 aren't matched. • e_0 is matched but e_1 is not matched. • e_0 matches d_0 and e_1 matches d_1 such that d_0 precedes d_1 . **Proof Theorem 6.12** If a dequeue d_0 precedes another dequeue d_1 , then either: • d_0 isn't matched. • d_1 isn't matched.

• d_0 matches e_0 and d_1 matches e_1 such that e_0

Theorem 6.13 (*Linearizability of Slot-queue*) Slot-

precedes or overlaps with e_1 .

queue is linearizable.

Proof

7. Wait-freedom

The algorithm is trivially wait-free as there is no possibility of infinite loops.

8. Memory-safety

The algorithm is memory-safe: No memory deallocation happens and accesses are only made on allocated memory.

References

[1] P. Jayanti and S. Petrovic, "Logarithmic-time single deleter, multiple inserter wait-free queues and stacks," 2005, *Springer-Verlag*. doi: 10.1007/11590156_33.