ChE 597 Computational Optimization

Homework 3

February 4th 11:59pm

1. In this question we will model an LP Transshipment model for a heat exchanger network to determine the minimum utility consumption for the two hot and two cold streams given below:

   Data for the problem:

	Fep (MW/C)	Tin (C)	Tout (C)
H1	1	400	120
H2	2	340	120
C1	1.5	160	400
C2	1.3	100	250

   Steam : 500°C
   Cooling water: $20-30$°C
   Minimum recovery approach temperature (HRAT): 20°C

   The data for this problem are displayed below in Figure 1, where heat contents of the hot and cold processing streams are shown at each of the temperature intervals, which are based on the inlet and highest and lowest temperatures.

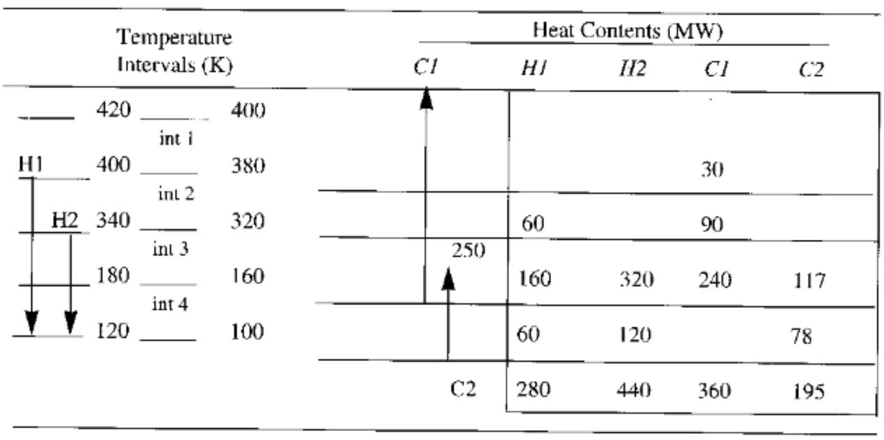

Figure 1: Data interpretation over temperature intervals

   The flows of the heat contents can be represented in the heat cascade diagram of Figure 2. Here the heat contents of the hot streams are introduced in the corresponding intervals, while the heat contents of the cold streams are extracted also from their corresponding intervals. If the cold stream cannot absorb all the heat from the heat streams at a given interval, the rest of the heat will be transported to the next heat interval as "heat residuals". The variables R_1, R_2, R_3, represent heat residuals, while variables $Q_s Q_w$ represent the heating and cooling loads respectively.

The values in Figure 2 are calculated as the $Fep \times \Delta T$. For example, the heat transported from stream H_1 to interval 2 is calculated as $1MW/C \times (400C - 340C) = 60MW$.

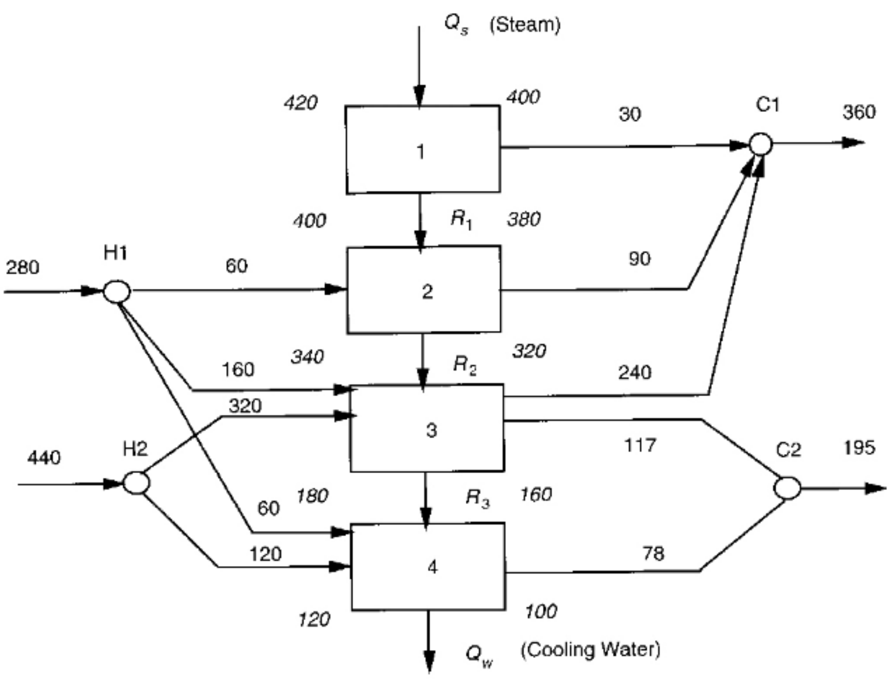

Figure 2: Heat Cascade Diagram

The usefulness of the heat cascade diagram in Figure 2 is that it can be regarded as a transshipment problem that we can formulate as a linear programming problem (Papoulias and Grossmann, 1983). In terms of the transshipment model, hot streams are treated as source nodes, and cold streams as destination nodes. Heat can then be regarded as a commodity that must be transferred from the sources to the destinations through some intermediate "warehouses" that correspond to the temperature intervals that guarantee feasible heat exchange. When not all of the heat can be allocated to the destinations (cold streams) at a given temperature interval, the excess is cascaded down to lower temperature intervals through the heat residuals.

Answer the following questions:

(a). Formulate the minimum utility ($\min Q_s + Q_w$) consumption problem as an LP transshipment problem, utilizing the cascade diagram provided in Figure 2. When formulating the constraints, you need to account for heat balances around each temperature level depicted in the cascade diagram in Figure 2.

(b). Solve the LP problem using Pyomo and determine the values of R_1, R_2, R_3, Q_s, Q_w at optimum

2. Consider the following linear programming problem:

   Maximize:

   $$Z = x_1 + 2x_2$$

   Subject to the constraints:

   1. $x_1 - x_2 \geq -2$
   2. $x_1 + x_2 \geq 1$
   3. $x_1 \geq 0$
   4. $x_2 \geq 0$

   (a) **Extreme Points:** Determine the vertices (extreme points) of the polyhedron.

   (b) **Extreme Rays:** Determine the extreme rays of the polyhedron.

   (c) **Optimal Solution:** Does this problem have a finite optimal solution? If yes, explain which of the extreme points is the optimal solution. If not, explain why the problem does not have a finite optimal solution.

3. Given is the following linear programming problem:

$$\begin{aligned}
\min \quad & Z = -2x_1 - 3x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 4 \\
& x_1 + 2x_2 \leq 6 \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned}$$

   (a) Plot the contours of the objective and the feasible region. Determine the optimum by inspection.

   (hint: do not add slack variables. Just plot it on a 2D plane)

   (b) Solve the above problem using the simplex algorithm.

   (hint: add slack variables first. You can either use the vanilla simplex algorithm or the full tableau method. Solving it by hand is enough. However, if you would like to try solving it by implementing a simplex algorithm-based solver in Python, it is encouraged as well.)

4. Let x be an element of the standard form polyhedron $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$. Prove that a vector $d \in \mathbb{R}^n$ is a feasible direction at x if and only if $Ad = 0$ and $d_i \geq 0$ for every i such that $x_i = 0$.

5. Let x be a basic feasible solution associated with some basis matrix B. Prove the following:

   (a) If the reduced cost of every nonbasic variable is positive, then x is the unique optimal solution.

   (b) If x is the unique optimal solution and is nondegenerate, then the reduced cost of every nonbasic variable is positive.