Hints for Problem 5

1. Argue that

$$\mathbb{P}(Z \in S) = \mathbb{P}(X_1 \in S) + \mathbb{P}(X_1 \notin A,\ X_2 \in S) + \mathbb{P}(X_1 \notin A,\ X_2 \notin A,\ X_3 \in S) + \cdots .$$

2. Use the fact that $X, X_1, X_2, \ldots$ are i.i.d.

3. Let $a = \mathbb{P}(X \notin A)$.

4. Use the fact that

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}.$$